Student Name

Mathematics Stage 5 Paths

LINEAR RELATIONSHIPS C

Book 3 Use various forms of the equation of a straight line

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-LIN-P-01 describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems

Use various forms of the equation of a straight line

- Rearrange linear equations from gradient–intercept form $(y = mx + c)$ to general form $(ax + by + c = 0)$ and vice versa
- Find the x- and y-intercepts of a straight line in any form
- Graph the equation of a straight line in any form
- Use the point–gradient form $(y - y_1 = m(x - x_1))$ or the gradient–intercept form $(y = mx + c)$ to find the equation of a line passing through a point (x_1, y_1), with a given gradient m
- Use the gradient and the point–gradient form to find the equation of a line passing through 2 points
- Find the equation of a line that is parallel or perpendicular to a given line in any form
- Determine and justify whether 2 given lines are parallel or perpendicular

REARRANGING THE EQUATION OF A LINE

- **Gradient-intercept form:** $y = mx + c$
- **General form:** $ax + by + c = 0$
- When graphing lines, convert all equations to gradient-intercept form.

Gradient-Intercept Equation of a Straight Line

$$y = \boldsymbol{m}x + \boldsymbol{c}$$

Coefficient of x — Gradient

Constant term — y-intercept

Example Identify gradient and y-intercept after rearranging the equation.

$y - 7 = 2x$ $y = 2x + 7$ Gradient: 2 y-intercept: 7	$3y + 15 - 2x = 0$ $3y = 2x - 15$ $y = \frac{2x}{3} - \frac{15}{3}$ Gradient: $\frac{2}{3}$ y-intercept: -5	$4x + 2y = 1$ $2y = 1 - 4x$ $y = \frac{1}{2} - \frac{4x}{2}$ $y = 0.5 - 2x$ Gradient: -2 y-intercept: 0.5

Identify the gradient and y-intercept of these lines.

a $y = 2x + 5$

$m = 2, c = 5$

b $y + 2x = 5$

$m = -2, c = 5$

c $x - y = 7$

$m = 1, c = -7$

d $-2x - 5y = 3$

$m = -\frac{2}{5}, c = 3$

1 Write each of these equations in the form $y = mx + c$

a $y + 9 = 2x$	**b** $y - 8x = 11$	**c** $y + x = 1$	**d** $y - 11 = 3x$
e $12 + y = 4x$	**f** $y - \frac{1}{2} = 10x$	**g** $y - 5x - 1 = 0$	**h** $0 = 11x - 7 - y$

2 Identify the gradient of each of these lines.

a $7y = 21x - 35$	**b** $y - 10 = 6x$	**c** $11x + y = 15$	**d** $20 + 4x = 2y$
e $y + 20 = x$	**f** $20y = 5 - 10x$	**g** $x + y = 16$	**h** $0.5y = 2x + 1$

3 Identify the y-intercept in each of these lines.

a $x + y = 0$	**b** $2y = 8 - 4x$	**c** $100y = 20x + 300$	**d** $7 = 15x + y$
e $60y = x + 60$	**f** $-15x + y = 17$	**g** $3(2x + 5) = 6y$	**h** $2y = 5(11 - 6x)$

4 Write each of these equations in the form $y = mx + c$.

a $2y = 18x + 20$	**b** $3y = 6x + 21$	**c** $10y = 40x + 50$
d $6y = 24x + 6$	**e** $5y = 90 + 10x$	**f** $12x + 24 = 3y$
g $35 + 77x = 7y$	**h** $4y = 10 + 8x$	**i** $2y = 7x + 8$
j $10y = 15 + 60x$	**k** $15x + 45 = 30y$	**l** $21y = 14x + 7$

5 Determine the gradient and y-intercept of these equations.
You will need to rearrange any equations not in the form $y = mx + c$.

a $y = x + 7$ $m =$ $c =$

$m = 1, c = 7$

b $y - x = 7$ $m =$ $c =$

$m = 1, c = 7$

c $y - \frac{x}{2} = 7$ $m =$ $c =$

$m = \frac{1}{2}, c = 7$

d $2y - \frac{x}{2} = 7$ $m =$ $c =$

$m = \frac{1}{4}, c = \frac{7}{2}$

e $2y + \frac{x}{2} = 7$ $m =$ $c =$

$m = -\frac{1}{4}, c = \frac{7}{2}$

f $3y + \frac{x}{2} = 7$ $m =$ $c =$

$m = -\frac{1}{6}, c = \frac{7}{3}$

g $3y - 2x = 7$ $m =$ $c =$

$m = \frac{2}{3}, c = \frac{7}{3}$

h $3x + 2y = -4$ $m =$ $c =$

$m = -\frac{3}{2}, c = -2$

i $3x - 2y = -4$ $m =$ $c =$

$m = \frac{3}{2}, c = 2$

6 **HSC Standard 2 Sample Question Band 3**

What is the gradient of the line $2x + 3y + 4 = 0$?

7 Match the equations that represent the same straight lines.

a	$y - 3x = 11$	$20y = 220 - 20x$
b	$x + y = 11$	$y - \frac{x}{3} = \frac{11}{3}$
c	$3y - x = 11$	$\frac{y}{8} = 1.375 - \frac{3}{8}x$
d	$11 - 3x = y$	$121 + 33x = 11y$

8 A student has made the following mistakes when identifying the gradient and y-intercept. Determine the correct gradient and y-intercept.

a $y = \frac{2x+1}{2}$ $m = 2$ ✗

b $y = 0.5(x + 3)$ $c = 3$ ✗

c $3x + y = 7$ $m = 3$ ✗

d $x - 2y = 4$ $m = 1$ ✗

SKETCHING BY FINDING INTERCEPTS OF A LINE

- y intercept: substitute $x = 0$ and solve.
- x intercept: substitute $y = 0$ and solve.

Example Identify x and y intercept from the equation.

Find the x and y intercepts of

$$y = 4x + 2$$

y-int: $x = 0$

$y = 4(0) + 2$

$y = 2$

y-int is at $(0,2)$

x-int: $y = 0$

$0 = 4x + 2$

$-2 = 4x$

$-\frac{2}{4} = x$

$-0.5 = x$

x-int is at $(-0.5, 0)$

Find the x and y intercepts of

$$x + 2y - 6 = 0$$

y-int: $x = 0$

$0 + 2y - 6 = 0$

$2y = 6$

$y = 3$

y-int is at $(0,3)$

x-int: $y = 0$

$x + 2(0) - 6 = 0$

$x - 6 = 0$

$x = 6$

x-int is at $(6, 0)$

What is a fast way to find the y-intercept when in gradient intercept form $y = mx + c$?

If an equation is in gradient intercept form, the y-intercept is the .....

Find the x and y intercepts of these lines.

a $y = 2x - 8$

y int: $x = 0$

$y = 2(0) - 8$

x int: $y = 0$

$0 = 2x - 8$

x-int: $(4,0)$, y-int: $(0,-8)$

b $x - y = 7$

y int: $x = 0$

x int: $y = 0$

x-int: $(7,0)$, y-int: $(0,-7)$

c $-3x - 2y = 6$

x-int: $(-2,0)$, y-int: $(0,-3)$

d $4x + 4y = 7$

x-int: $\left(\frac{7}{4}, 0\right)$, y-int: $\left(0, \frac{7}{4}\right)$

GRAPHING USING THE INTERCEPTS

1. Find the y intercept ($x = 0$)
2. Find the x intercept ($y = 0$)
3. Connect the x and y intercept and extend the straight line.

Example Sketch graphs from the equation by finding the intercepts.

$y = 2x - 4$

1. y-int: $x = 0$

$y = 2(0) - 4$

$= -4$

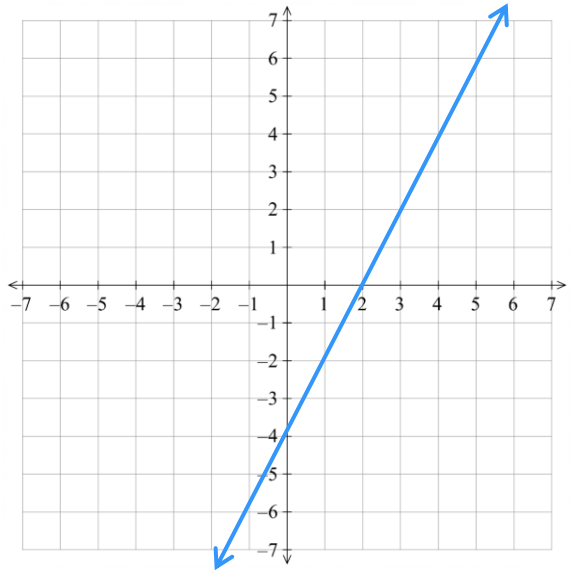
2. x-int: $y = 0$

$0 = 2x - 4$

$4 = 2x$

$2 = x$

3.



$x + 2y + 6 = 0$

1. y-int: $x = 0$

$2y + 6 = 0$

$2y = -6$

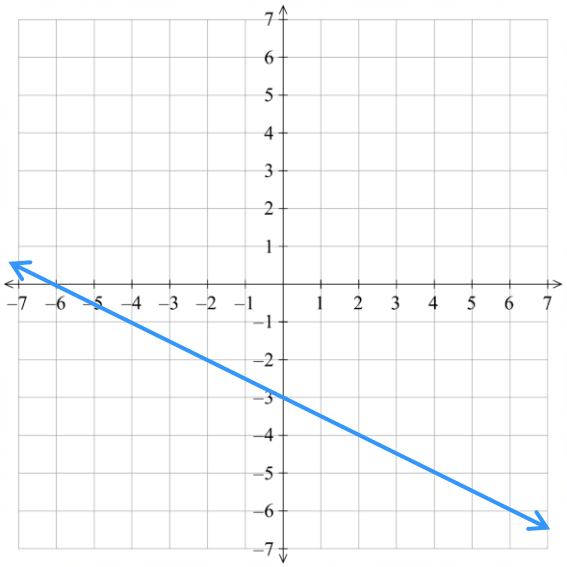
$y = -3$

2. x-int: $y = 0$

$x + 6 = 0$

$x = -6$

3.



Sketch these graphs by finding the intercepts first.

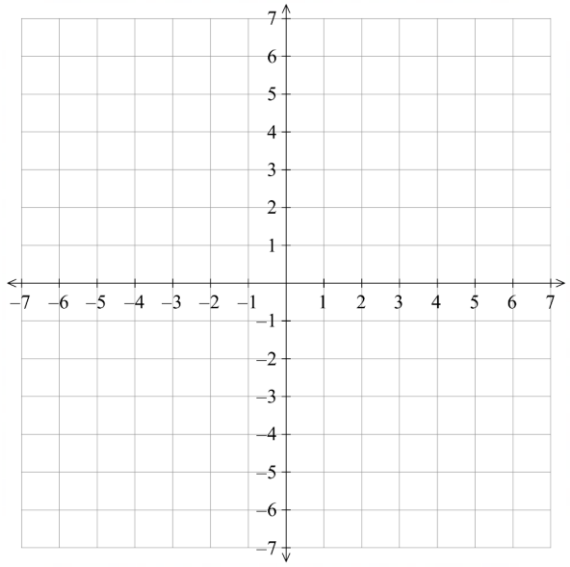
a $y = 3x - 5$

y-int: $x = 0$

$y = 3(0) - 5$

x-int: $y = 0$

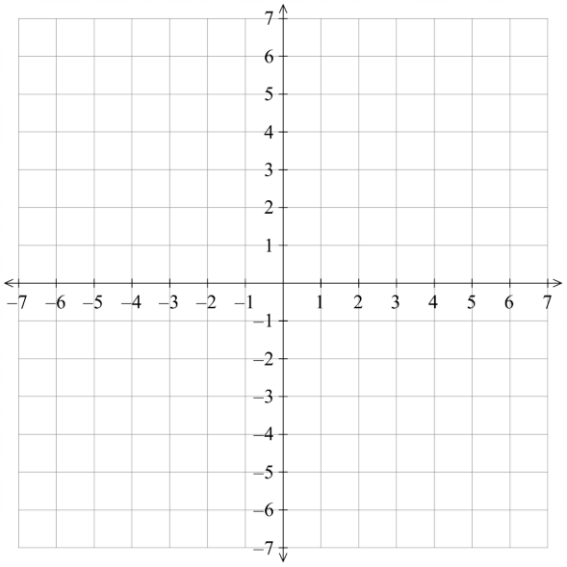
$0 = 3x - 5$



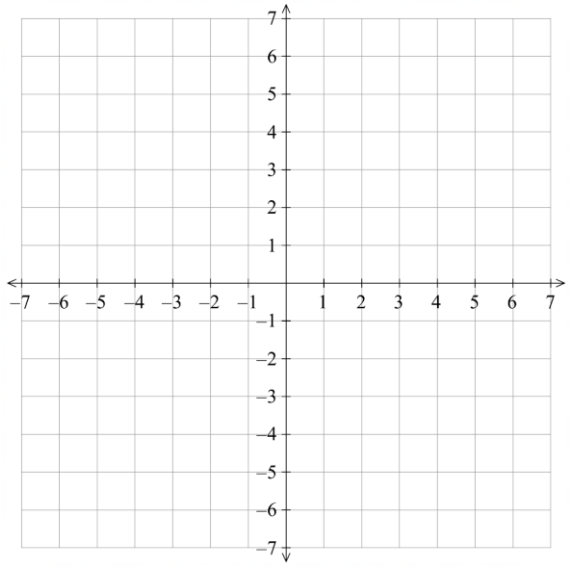
b $y = 2x + 3$

y-int: $x = 0$

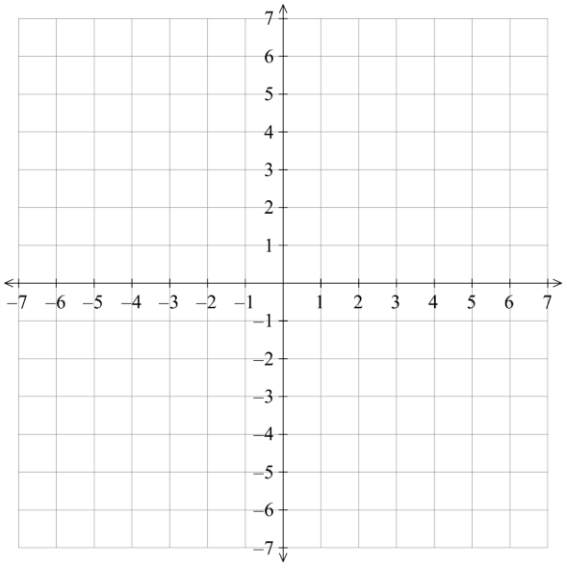
x-int: $y = 0$



c $3y = 4x - 6$



d $4 - 2y = 16$



Key ideas

1. The x-intercept has a ……-coordinate of 0.
2. The y-intercept has a ……-coordinate of 0.

1 Determine the x and y intercepts for these lines.

a $y = x + 1$

y int: $x = 0$

x int: $y = 0$

b $y = \frac{1}{2}x - 3$

c $y = x$

d $y - 3x = 0$

e $x + y - 1 = 0$

f $x - 3y - 3 = 0$

g $2x - 3y - 12 = 0$

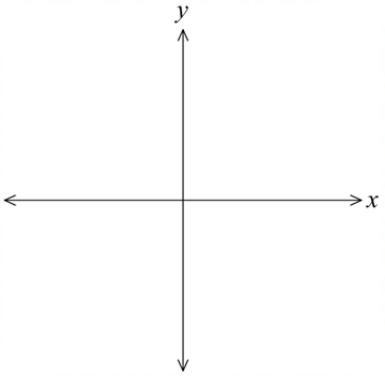
h $5x + 2y - 10 = 0$

2 Follow these steps for the equation $y = 2x - 2$

a Find the y-intercept by substituting $x = 0$.

b Find the x-intercept by substituting $y = 0$ then solving.

c Sketch the graph on the axis.

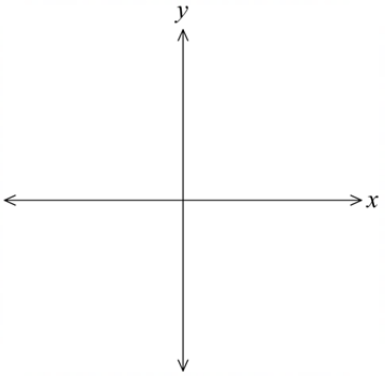


3 Follow these steps for the equation $x + 2y - 4 = 0$

a Find the y-intercept by substituting $x = 0$.

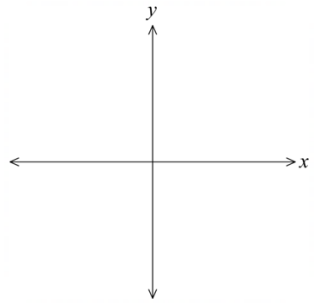
b Find the x-intercept by substituting $y = 0$ then solving.

c Sketch the graph on the axis.

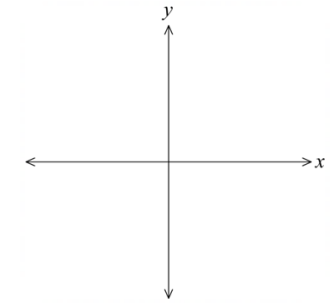


4 Sketch these lines. Show the y-intercept and x-intercept.

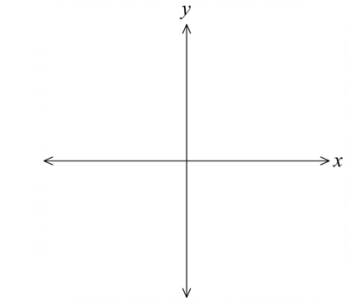
a $y = 2x - 4$



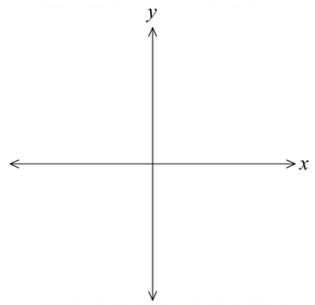
b $y = -2x + 1$



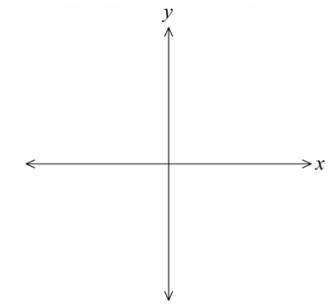
c $y = 5x - 3$



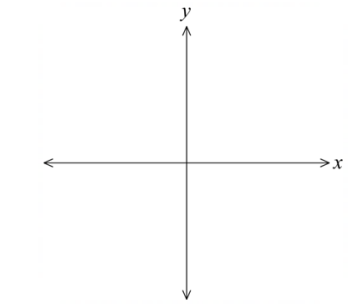
d $y = -\frac{3}{2}x + 1$



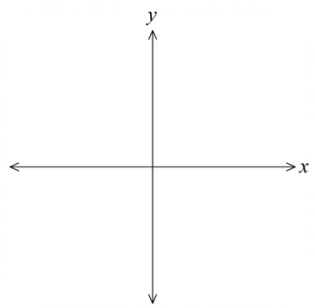
e $y = \frac{7}{2}x + 6$



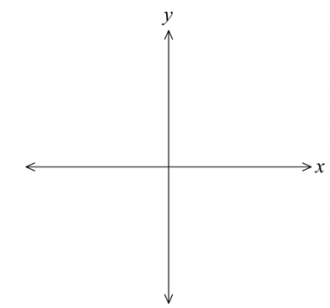
f $y = 3 + \frac{2}{3}x$



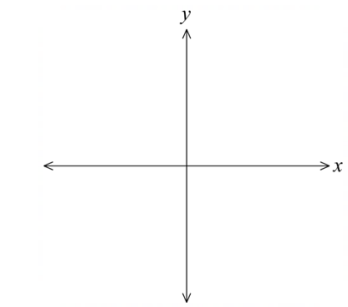
g $y = \frac{1}{2}x - 2.5$



h $y = \frac{4}{3}x - 2$

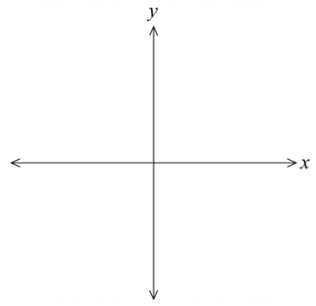


i $y = 4 - 2x$

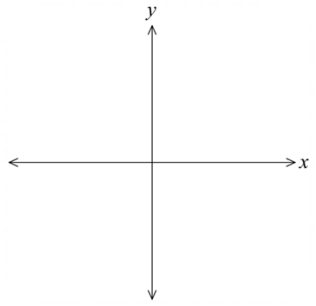


5 Sketch these graphs by finding the x- and y-intercepts.

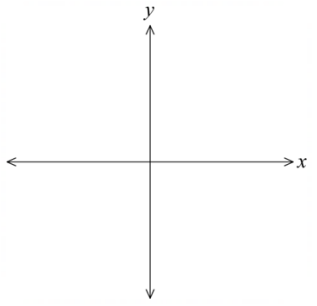
a $3x + y = 12$



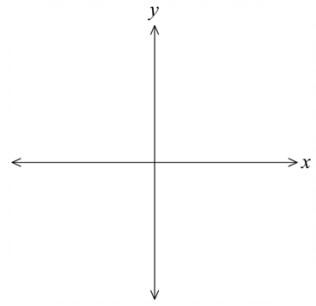
b $x - y = 7$



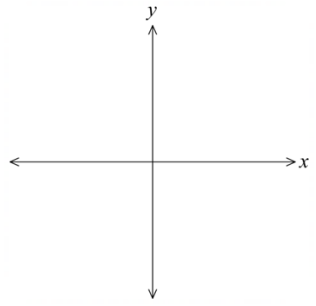
c $4x - 3y = 9$



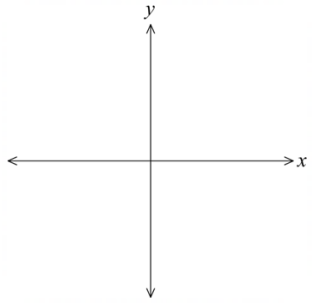
d $10x + 2y = 5$



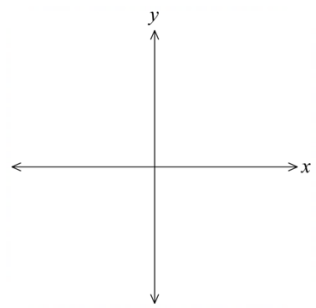
e $2x - y + 2 = 0$



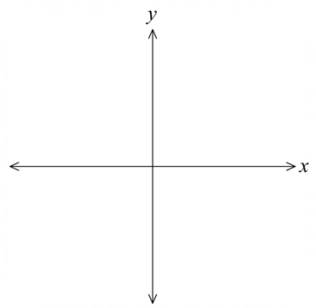
f $3x + 2y = 12$



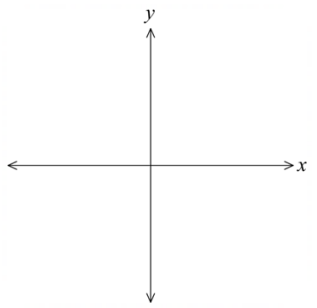
g $5y - 2x = 12$



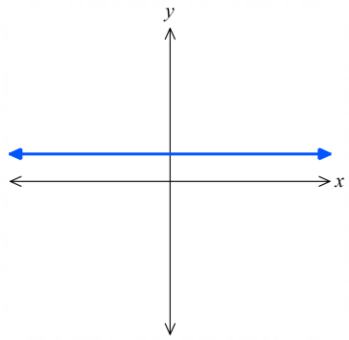
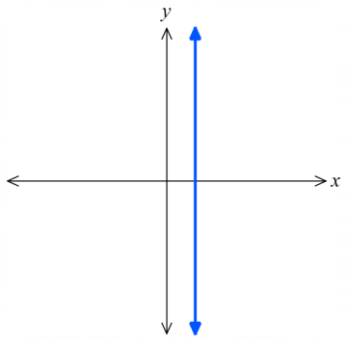
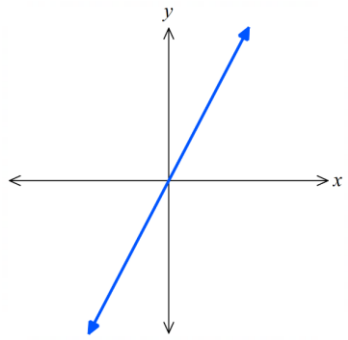
h $4y - 3x = 24$



i $-5x + 2y + 15 = 0$



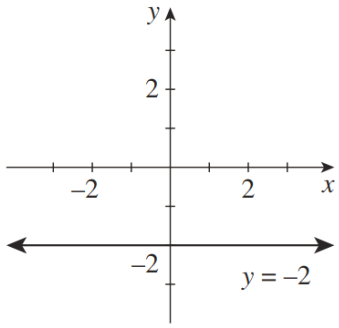
SPECIAL LINES

$y = c$	$x = k$	$y = mx$
Horizontal line through $y = c$	Vertical line though $x = k$	Line through origin and $(1, m)$
		

Example Sketch special lines.

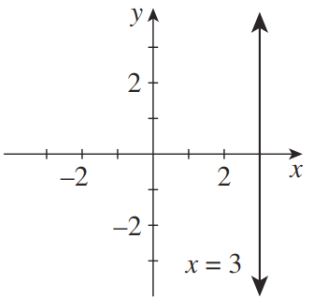
a $y + 2 = 0$

$y = -2$



b $x - 3 = 0$

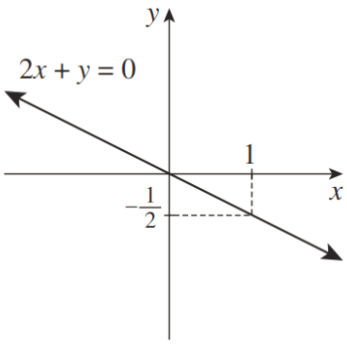
$x = 3$



c $x + 2y = 0$

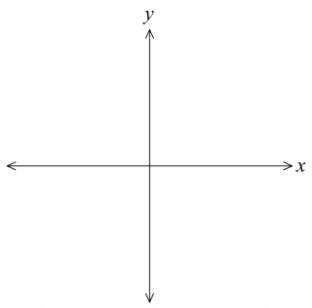
$2y = -x$

$y = -\frac{x}{2}$

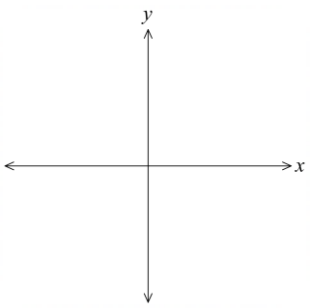


Sketch these lines.

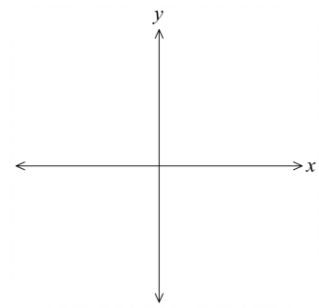
a $x = -1$



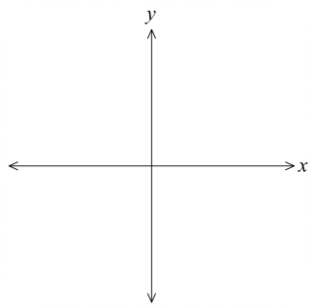
b $y - 2 = 0$



c $2y = -3$



d $3x + 5y = 0$

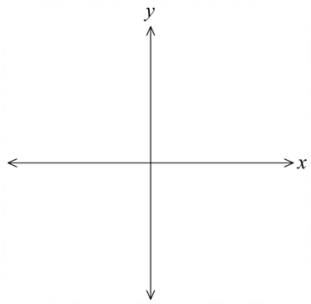


Key ideas

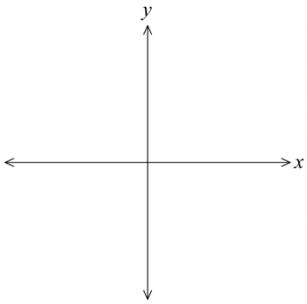
1. $y = mx$ is a straight line passing through the ..... and $(1, ...)$
2. $y = c$ is a ..... line passing through $y = c$.
3. $x = k$ is a ..... line passing through $x = k$.

1 Sketch these special lines.

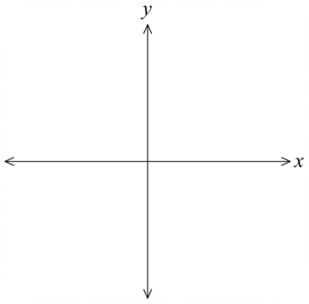
a $y = -8$



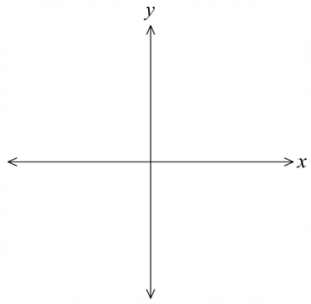
b $x = 2$



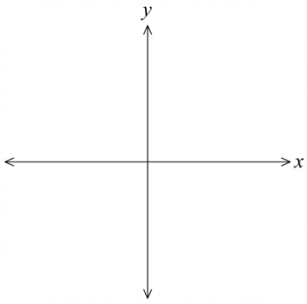
c $y = 0$



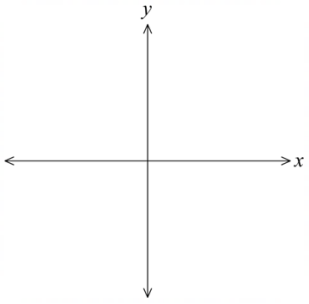
d $y = -3x$



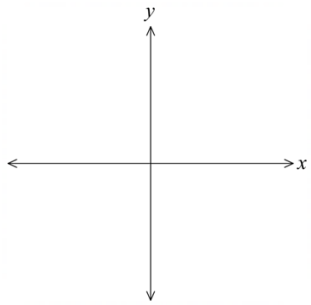
e $y = \frac{5x}{2}$



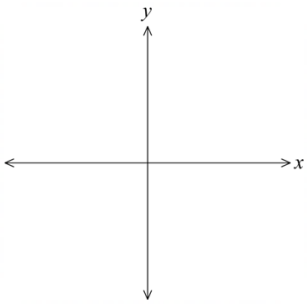
f $y = \frac{5}{2}x$



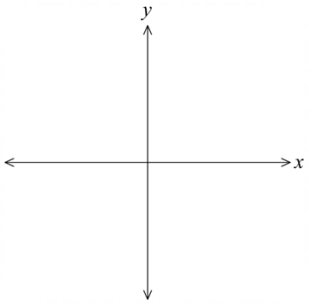
g $4 - y = 0$



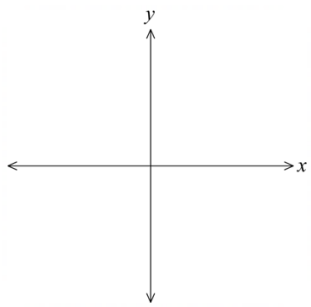
h $x = -3y$



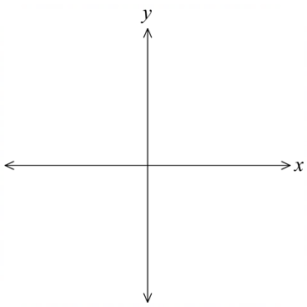
i $y = 0$



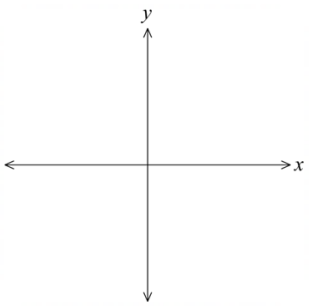
j $2y = -3$



k $3x = 5$



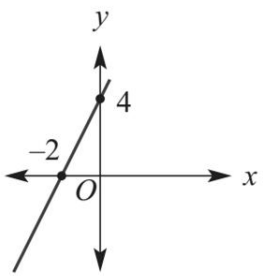
l $x = \frac{2}{3}$



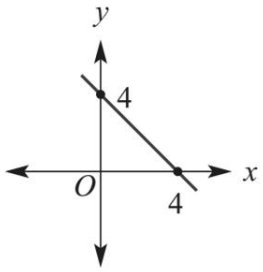
6 Match the equation to the graph.

a $y=-\frac{2}{3}x+2$	**b** $y=-x+4$	**c** $y=x+3$
d $y=2x+4$	**e** $y=4$	**f** $y=7x$
g $y=-3x+6$	**h** $x=2$	**i** $y=-3x$

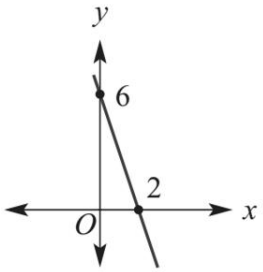
A



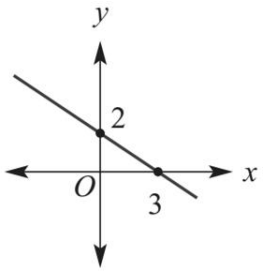
B



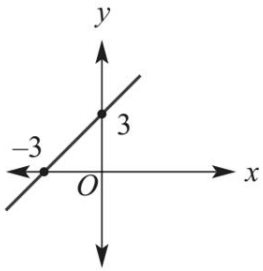
C



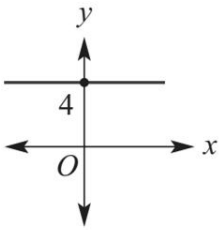
D



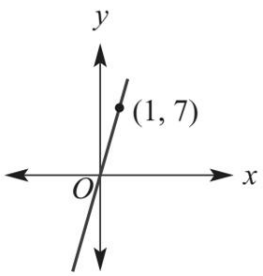
E



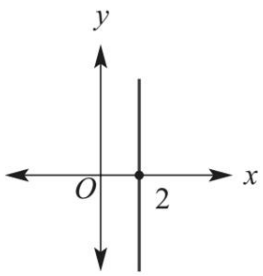
F



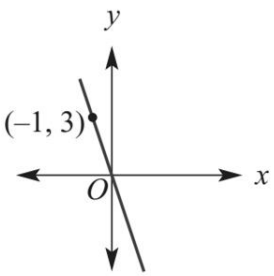
G



H

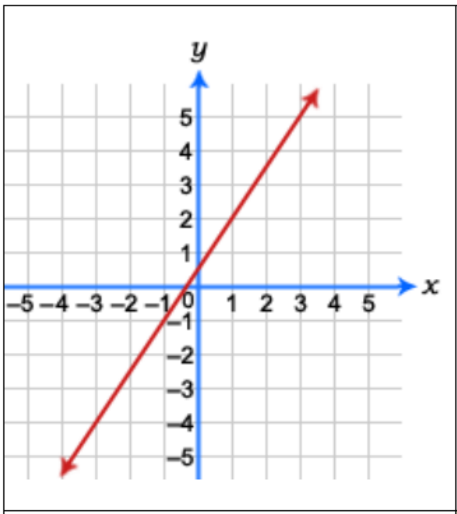


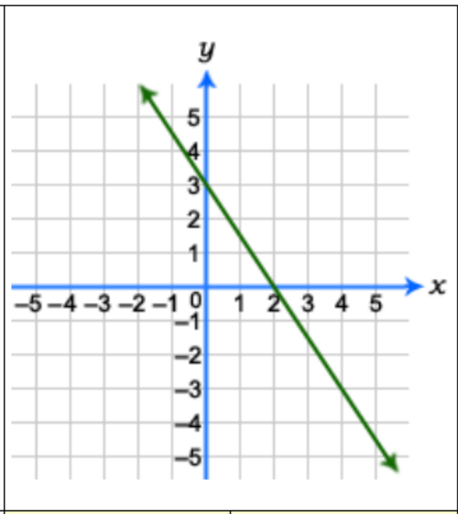
I

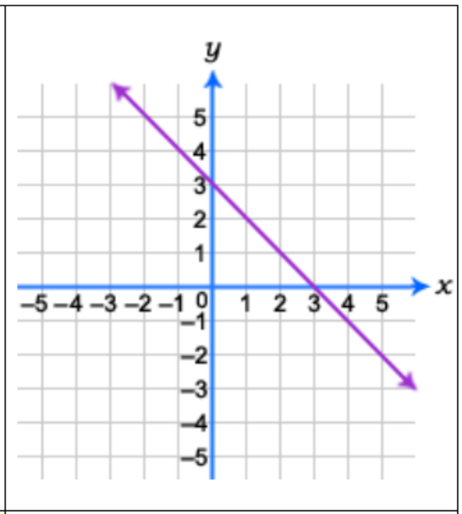


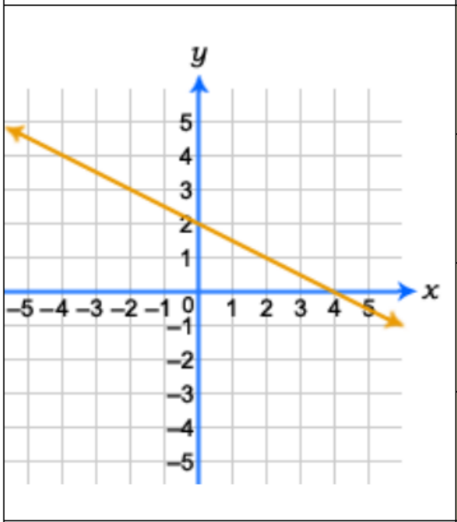
a-D, b-B, c-E
d-A, e-F, f-G
g-C, h-H, i-I

7 Match the equation to the graph.

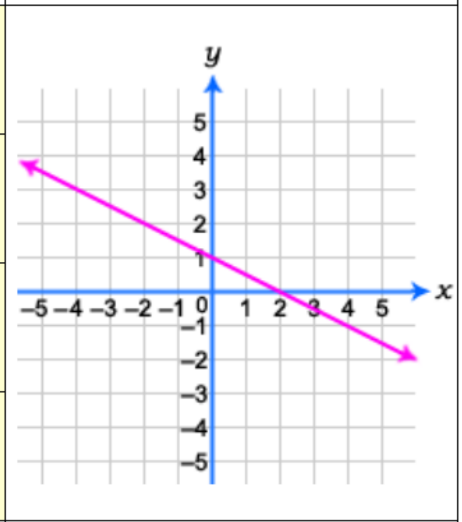


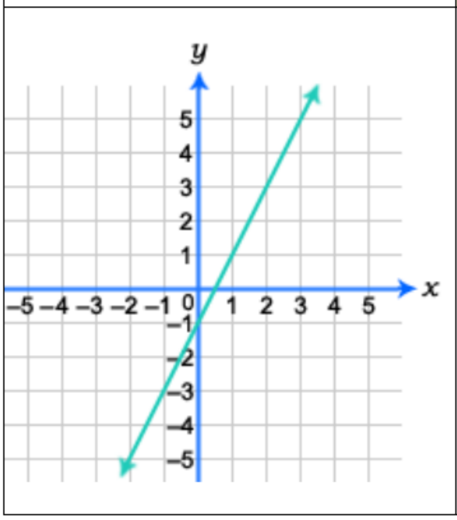


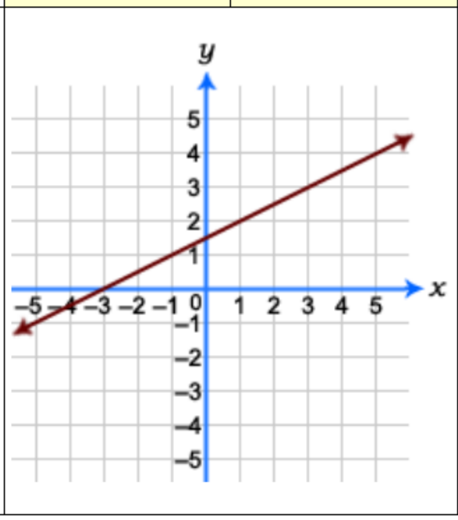


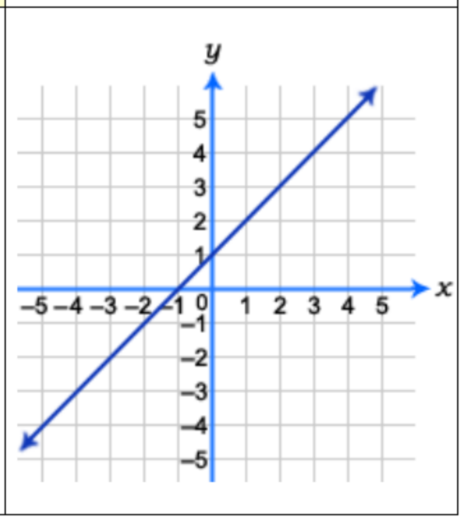


a $x + y = 3$	b $x + 2y = 4$
c $x - y = -1$	d $y = \dfrac{3x+1}{2}$
e $y = 2x - 1$	f $3x + 2y = 6$
g $y = 1 - \dfrac{1}{2}x$	h $x = 2y - 3$









FINDING THE EQUATION GIVEN GRADIENT AND A POINT

- If you are asked to find the equation of a line, given the gradient and a point, use the **point-gradient form**.

Point-Gradient Formula

Given a gradient m and a point on the line (x_1, y_1):

Gradient

$$y - y_1 = m(x - x_1)$$

y coordinate of a point

x coordinate of a point

Example Find equation of a line given gradient and a point.

Find the equation of the line with gradient 3 passing through $(6,4)$.

m $x_1\ y_1$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 3(x - 6)$$
$$y - 4 = 3x - 18$$
$$y = 3x - 14$$

Find the equation of the line with gradient -4 and x intercept 1.

m $(x_1\ y_1) = (1,0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -4(x - 1)$$
$$y = -4x + 4$$

Find the equation of the line with:

a Gradient 4 and passes through $(-1,2)$

$y = 4x + 6$

b Gradient 3 and passes through $(-3,5)$

$y = 3x + 14$

c Gradient $\frac{1}{3}$ and passes through $(9,2)$

$y = \frac{x}{3} - 1$

d Gradient -3 passes through $(3,-3)$

$y = -3x + 6$

1 The gradient of each line is given. Use $y - y_1 = m(x - x_1)$ to find the equation of these lines.

a $(3,2)$ and $m = 4$

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 3)$

b $(4, -1)$ and $m = -1$

c $(-7,2)$ and $m = \frac{1}{2}$

d $(2,5)$ and $m = 2$

e $(-6,4)$ and $m = -1$

f $(3, -8)$ and $m = 4$

2 Find the equation of the line:

a through (1,1) with gradient 2

b with gradient −1 through (3, 1)

c through (0,0) with gradient −5

d through (−1,3) with gradient $-\frac{1}{3}$

e with gradient $-\frac{4}{5}$ through (3, −4).

f through $(-5,3)$ with gradient $-\frac{3}{5}$

3 A line passes through the y-axis at (0, 6) and has a gradient of $\frac{5}{7}$. What is its equation?

$y = \frac{5x}{7} + 6$

4 Find the equation of a line with a gradient of -4 and an x-intercept of 5.

$y = -4x + 20$

FINDING THE EQUATION GIVEN GRADIENT AND TWO POINTS

- If you are asked to find the equation of a line, given two points:

1. Calculate gradient using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Substitute into point-gradient form. $y - y_1 = m(x - x_1)$
3. Rearrange to $y = mx + c$.

Example Find equation of a line given two points.

Find the equation of the line passing through (6, 0) and (0, 9)

1. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9-0}{0-6}$

$= -\frac{3}{2}$

2. Sub $m = -\frac{3}{2}$ and (6,0)

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{3}{2}(x - 6)$

3. $y = -\frac{3x}{2} + 9$

Find the equation of the line passing through (1, 5) and (4, −1)

1. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-5}{4-1}$

$= -2$

2. Sub $m = -2$ and (1,5)

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x - 1)$

3. $y - 5 = -2x + 2$

$y = -2x + 7$

Determine the equation of the line passing through:

a (2,6) and (4,10)

1. $m = \frac{y_2 - y_1}{x_2 - x_1}$

2. $y - y_1 = m(x - x_1)$

3. $y =$

$y = 2x + 2$

b (1,7) and (3, −1)

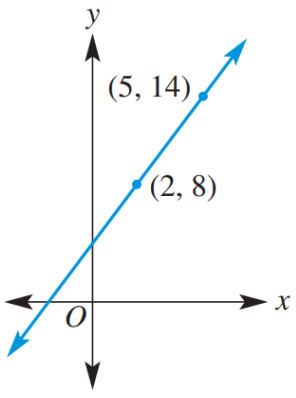
1. $m = \frac{y_2 - y_1}{x_2 - x_1}$

2. $y - y_1 = m(x - x_1)$

3. $y =$

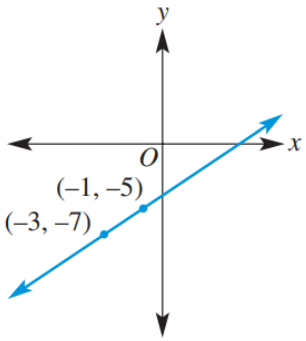
$y = -4x + 11$

c



$y = 2x + 4$

d



$y = x - 4$

1 Determine the equation of the line that pass through these two points.

a $(4,2)$ and $(12,4)$

b $(1,4)$ and $(3,8)$

c $(-1,3)$ and $(2,0)$

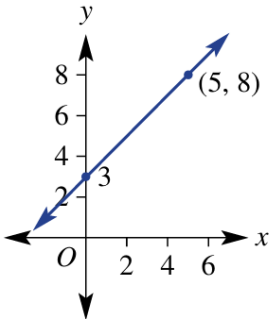
d $(7,3)$ and $(10,6)$

e $(2,-2)$ and $(-1,6)$

f $(-10,2)$ and $(1,-4)$

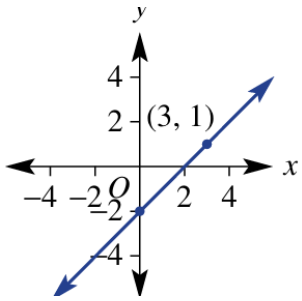
2 Find the equation of these lines using an appropriate method.
(Hint: some of these graphs already give you the y-intercept, so you only need the gradient)

a



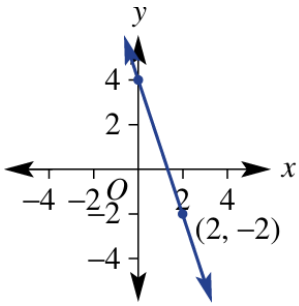
$y = x + 3$

b



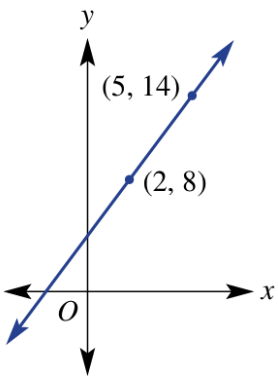
$y = x - 2$

c



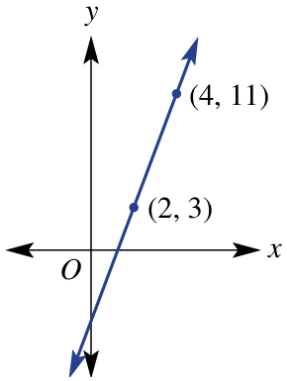
$y = -3x + 4$

d



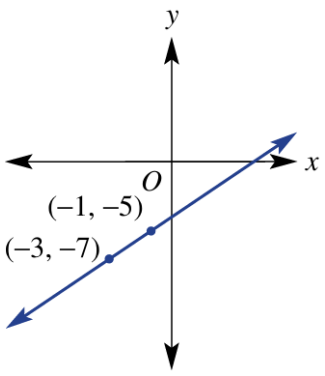
$y = 2x + 4$

e



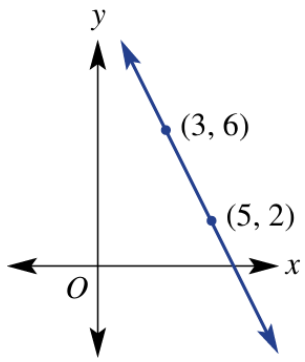
$y = 4x - 5$

f



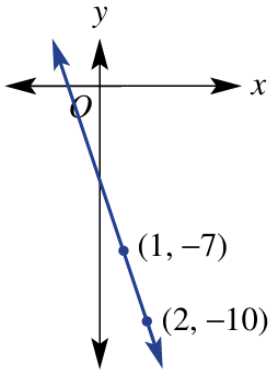
$y = x - 4$

g



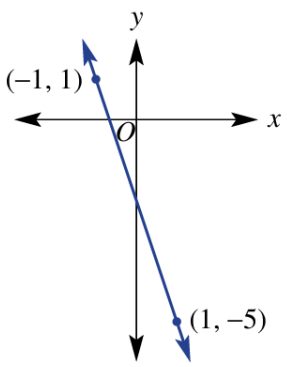
$y = -2x + 12$

h



$y = -3x - 4$

i



$y = -3x - 2$

3 Consider the points P(4, 7) and Q(6, −1).

a Calculate the length of PQ.

b Find the midpoint of the PQ.

c What is the gradient of PQ?

d Find the equation of the line passing through P and Q.

4 A line with gradient $\frac{2}{3}$ passes through the midpoint of $(5,6)$ and $(1,10)$.

Find the equation of the line.

b

5 A line with gradient $-\frac{3}{5}$ passes through the midpoint of $(-8,2)$ and $(-2,20)$.

Find the equation of the line.

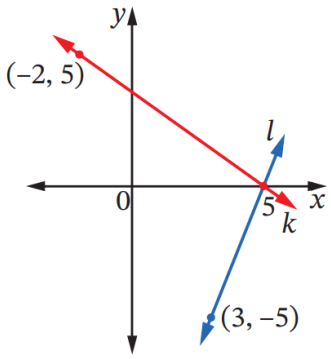
6 Determine the equation of a line with gradient -4 and passes through $(3, -10)$.

Method 1: Substitute into $y - y_1 = m(x - x_1)$

Method 2: Substitute into $y = mx + c$ and solve for c

7 The diagram shows line k and line l. Find:

a The equation of line l in general form.



$5x - 2y - 25 = 0$

b The equation of line k in general form.

$5x + 7y - 25 = 0$

c w if the point $(7, w)$ lies on l.

$w = 5$

d t if the point $(t, 11)$ lies on k.

$t = -\frac{52}{5}$

8

a A line with gradient 4 passes through $(1, 5)$ and $(-2, k)$. Find the value of k.

b Find the value of a if $2x + 5y - 6 = 0$ passes through $(a, -2)$.

c Find the value of a if $ax - 8y + 3 = 0$ passes through $(-7, 3)$.

d Find the value(s) of a if $ax + y = 6$ passes through $(a, 5a)$

9 This is the **intercept form of a straight line**.

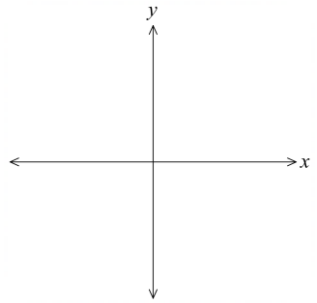
$$\frac{x}{a}+\frac{y}{b}=1$$

a Find the x and y-intercepts of this line and hence explain why it's called the intercept from of a straight line.

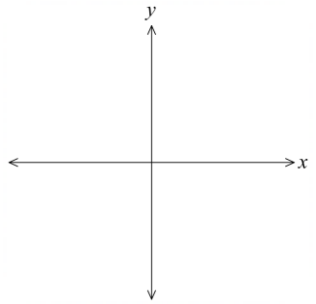
b Use the intercept form to find the equation of a line with x-intercept at -2 and y-intercept at 3.

c Use this knowledge to quickly sketch these lines without any calculations.

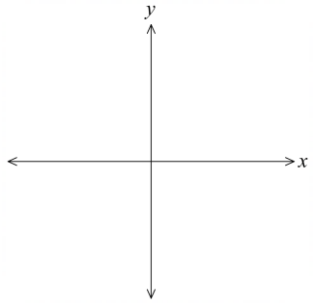
i $\frac{x}{6}-\frac{y}{3}=1$



ii $-x+\frac{y}{2}=1$



iii $\frac{x}{2}+\frac{y}{5}=1$



10 Another form of a line is the **two-point form** that states the equation of a line that passes through any two points $(x_1 y_1)$ and (x_2, y_2). We will derive it in this section.

a What is the expression for the gradient between the two points $(x_1 y_1)$ and (x_2, y_2)?

b Use the formula $y - y_1 = m(x - x_1)$ and the expression in part **a** to show that the two-point form of a line is $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

c Use the two-point form to find the equation of a line passing through $(7,3)$ and $(10,6)$. Answer in gradient-intercept form. Check your answer using DESMOS.

d Find the equation of a line passing through $(-1,10)$ and $(10, -17.5)$. Answer in gradient-intercept form. Check your answer using DESMOS.

PARALLEL AND PERPENDICULAR LINES

Gradient of Parallel Line	Gradient of Perpendicular Line
$m_{\text{par}} = m_{\text{original}}$	$m_{\text{perp}} = -\dfrac{1}{m_{\text{original}}}$

1. Identify gradient of original line.
2. Determine gradient m_{new} of the parallel/perpendicular line.
3. Write the equation $y - y_1 = m_{\text{new}}(x - x_1)$
4. Substitute new gradient and given point into the formula.
5. Rearrange to $y = mx + c$

Example Find the equation of a parallel/perpendicular line.

Find the equation of a line that is parallel to $y = -2x - 7$ and passes through $(1,9)$

1. $m_{\text{orig}} = -2$
2. $m_{\text{new}} = -2$
3. $y - y_1 = m_{\text{new}}(x - x_1)$
4. $y - 9 = -2(x - 1)$
5. $y - 9 = -2x + 2$

   $y = -2x + 11$

Find the equation of a line perpendicular to $y = \frac{3}{4}x - 1$ and passes through $(3, -2)$

1. $m_{\text{orig}} = \frac{3}{4}$
2. $m_{\text{new}} = -\frac{4}{3}$
3. $y - y_1 = m_{\text{new}}(x - x_1)$
4. $y - (-2) = -\frac{4}{3}(x - 3)$
5. $y + 2 = -\frac{4x}{3} + 4$

   $y = -\frac{4x}{3} + 2$

Find the equation of the line that is:

a Parallel to $y = x + 3$ and passes through $(1, 5)$

1. $m_{\text{orig}} =$ .....
2. $m_{\text{new}} =$ .....
3. $y - y_1 = m_{\text{new}}(x - x_1)$
4.
5.

$y = x + 4$

b Parallel to $y = \frac{2}{3}x + 1$ and passes through $(3, -4)$

1. $m_{\text{orig}} =$ .....
2. $m_{\text{new}} =$ .....
3. $y - y_1 = m_{\text{new}}(x - x_1)$
4.
5.

$y = \frac{2x}{3} - 6$

c Perpendicular to $y = 2x + 3$ and passes through $(2, 5)$

$y = -\frac{x}{2} + 6$

d Perpendicular to $y = -4x + 1$ and passes through $(-4, -3)$

$y = \frac{x}{4} - 2$

1 Find the gradient of a line that is parallel to each line.

a $y = 4x + 6$

4

b $y = -7x - 1$

-7

c $y = -\frac{3}{4}x + 2$

$-\frac{3}{4}$

d $y = \frac{8x}{7} - \frac{1}{2}$

$\frac{8}{7}$

2 Find the gradient of a line that is perpendicular to each line.

a $y = 3x - 1$

$-\frac{1}{3}$

b $y = -2x + 6$

2

c $y = \frac{7}{8}x - \frac{2}{3}$

$-\frac{8}{7}$

d $y = \frac{-4x}{9} - \frac{4}{7}$

$\frac{9}{4}$

3 Decide if these pairs of lines are parallel, perpendicular, or neither.

a $y = 3x - 1$
$y = 3x + 7$

parallel

b $y = \frac{1}{2}x - 6$
$y = \frac{x}{2} - 4$

parallel

c $y = -\frac{2}{3}x + 1$
$y = \frac{2}{3}x - 3$

neither

d $y = -8x + 4$
$y = \frac{1}{8}x - 2$

perpendicular

e $x - y = 4$
$y = -x + \frac{1}{2}$

perpendicular

f $3x - y = 2$
$x + 3y = 5$

perpendicular

4 Find the equation of:

a the line through (2, 5) and parallel to $y = 2x + 5$

$y = 2x + 1$

b the line through (2, 5) and perpendicular to $y = 2x + 5$

$y = -\frac{x}{2} + 6$

c the line through (−7, 6) and parallel to $y = \frac{3x}{7} - 8$

$y - \frac{3x}{7} + 9$

d the line through (−4, 0) and perpendicular to y = $-\frac{2}{5}x$

$y = \frac{5x}{2} + 10$

5 Write down the line that is parallel to the given lines and passes through the given point.

Parallel to	Goes through	Answer
$y = 3x + 2$	$(0,7)$	$y = 3x + 7$
$y = 3x + 2$	$(3,0)$	$y = 3x - 9$
$y = 3x + 2$	$(3,7)$	$y = 3x - 2$
$y = 3x + 10$	$(3,7)$	$y = 3x - 2$
$y = 2x - 2$	$(3,7)$	$y = 2x + 1$
$y = 2x - 2$	$(-3,7)$	$y = 2x + 13$
$y = 2x - 2$	$(0,0)$	$y = 2x$
$y = -3$	$(8,11)$	$y = 11$

6 Write down the line that is perpendicular to the given lines and passes through the given point.

Perpendicular to	Goes through	Answer
$y = -x + 2$	$(0,7)$	$y = x + 7$
$y = 2x + 2$	$(0,7)$	$y = -\frac{x}{2} + 7$
$y = 2x + 2$	$(12,7)$	$y = -\frac{x}{2} + 13$
$y = 3x + 2$	$(12,7)$	$y = -\frac{x}{3} + 11$
$y = 3x + 2$	$(10,7)$	$y = -\frac{x}{3} + \frac{31}{3}$
$y = 3x + 2$	$(-12,7)$	$y = -\frac{x}{3} + 3$
$y = 2x + 2$	$(0,0)$	$y = -\frac{x}{2}$
$x = 7$	$(0,3)$	$y = 3$

7 Find the equation of the line that is:

a Perpendicular to $y = \frac{2x-1}{3}$ passing through $(0,5)$

$y = -\frac{3x}{2} + 5$

b Parallel to $y = \frac{3-5x}{7}$ passing through $(1,7)$

$y = -\frac{5x}{7} + \frac{54}{7}$

c Parallel to $3y - 2x = 3$ passing through $(-2,4)$

$y = \frac{2x}{3} + \frac{16}{3}$

d Perpendicular to $7x - y = -1$ passing through $(-3,-1)$

$y = -\frac{x}{7} - \frac{10}{7}$

e Parallel to $y = -\frac{3}{7}$ passing through $\left(\frac{2}{3}, \frac{1}{2}\right)$

$y = \frac{1}{2}$

f Perpendicular to $3x + 4y - 3 = 0$ passing through $(-2, 1)$.

$y = \frac{4x}{3} + \frac{11}{3}$

g parallel to the x-axis passing through $(-2,6)$.

$y = 6$

h parallel to the y-axis passing through $(3, 5)$.

$x = 3$

8 Find the value of a if:

a $y = \frac{a}{7}x + b$ is parallel to $y = 2x - 4$

$a = 14$

b $y = \frac{2a+1}{3}x + b$ is parallel to $y = -x - 3$

$a = -\frac{3}{2}$

c $y = \left(\frac{1-a}{2}\right)x + b$ is perpendicular to $y = \frac{1}{2}x - \frac{3}{5}$

$a = 5$

d $ay = 3x + b$ is perpendicular to $y = -\frac{3}{7}x - 1$

$a = \frac{9}{7}$

Explain why your answers above don't depend on the value of b

Parallel and perpendicular depends only on the gradient.

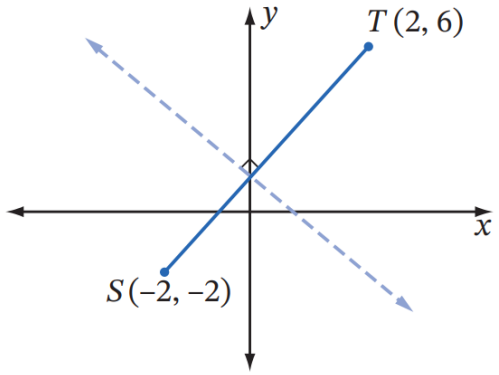
9 The line $3x - 8y + 10 = 0$ and another line intersect at right angles at the point $(10, 5)$.
Find the equation of the other line.

$y = \frac{-8x + 95}{3}$

10 In this question, you will find the *perpendicular bisector* of an interval.

a Find the gradient of interval ST

$m = 2$



b Find the midpoint of ST

$M(0,2)$

c The dotted line is perpendicular to ST and passes through its midpoint. What is its gradient?

$-\frac{1}{2}$

d Find the equation of the dotted line.
Answer in gradient-intercept form $y = mx + c$.

$y = -\frac{1}{2}x + 2$

11

a Find the equation of the perpendicular bisector of the line segment joining $(1,1)$ with $(3,5)$ in general form.

$x + 2y - 8 = 0$

b Find where this bisector cuts the x-axis.

$(8,0)$

12 The line $ax + by + 1 = 0$ is perpendicular to $4x + 3y = 7$ and passes through $(9,7)$.
Find the values of a and b.

$a = 3, b = -4$

13 The interval PQ has gradient −3. A second line passes through $A(-2,4)$ and $B(1,k)$.

a Find k if AB is parallel to PQ.

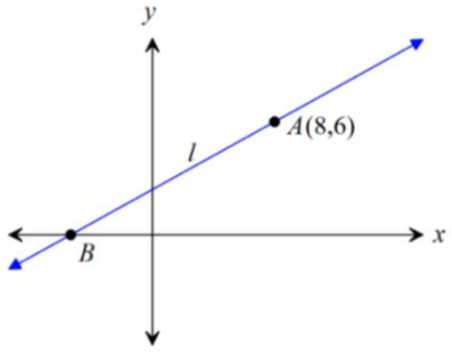
$k = -5$

b Find k if AB is perpendicular to PQ

$k = 5$

14 The line l with the equation $x - 2y + 4 = 0$ passes through the point $A(8,6)$ and intersects the x-axis at B as shown on the diagram below.
The line through A and perpendicular to l intersects the x-axis at C.

a Find the equation of AC.

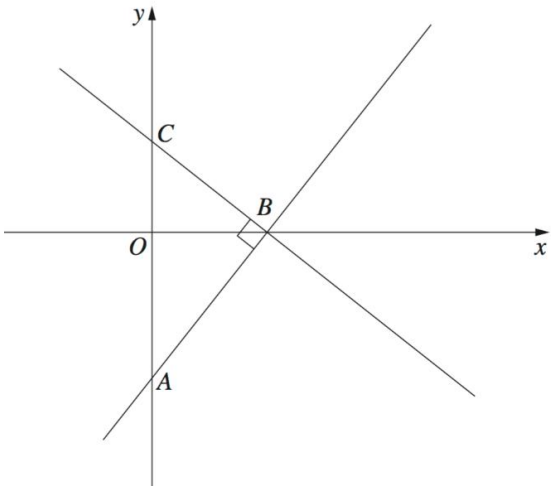


$2x + y - 22 = 0$

b Find the coordinates of C

$(11,0)$

15 The points A and C lie on the y-axis and the point B lies on the x-axis.
The line AB has equation $5x - 3y = 15$.
The line BC is perpendicular to AB.



a Find the coordinates of B.

(3,0)

b Find the equation of the line BC.

$3x + 5y - 9 = 0$

16 Line A has equation $2x + ky = 5$.
Line B passes through the points $(1,5)$ and $(4 + k, 6)$.
Find the value of k if the line A and B do not intersect.

$k = -2$

17 Given the points $X(-1, 0), Y(1, a)$ and $Z(a, 2)$, find a if $\angle YXZ = 90°$.

$a = -\frac{1}{2}$

18 For the four points $P(k, 1), Q(-2, -3), R(2, 3)$ and $S(1, k)$, it is known that PQ is parallel to RS.
Find the possible values of k.

$k = 2$ or $k = -1$

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Which equation represents the line perpendicular to $2x - 3y = 8$ passing through $(2,0)$?

A. $3x + 2y = 4$

B. $3x + 2y = 6$

C. $3x - 2y = -4$

D. $3x - 2y = 6$

B

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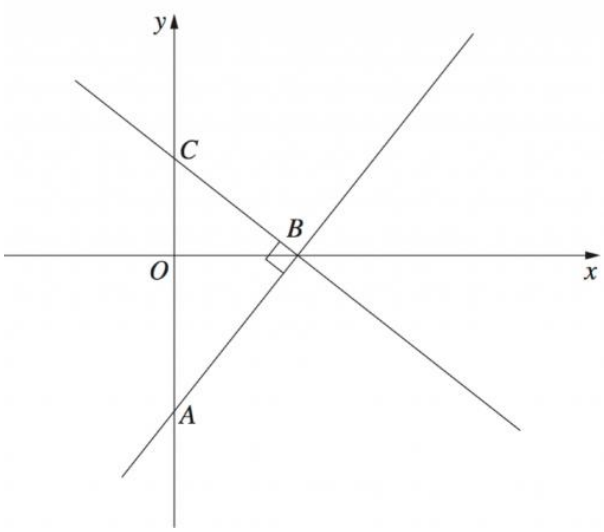
In the diagram, A and C lie on the y-axis and the point B lies on the x-axis. The line AB has equation $y = \sqrt{3}x - 3$. The line BC is perpendicular to AB.

Find the equation of line BC



$y = -\frac{1}{\sqrt{3}}x + 1$

Find the area of triangle ABC.

$2\sqrt{3}$ units^2