Student Name

Mathematics Stage 5 Paths

TRIGONOMETRY D

Book 1 Use the unit circle to define trigonometric functions and represent them graphically

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

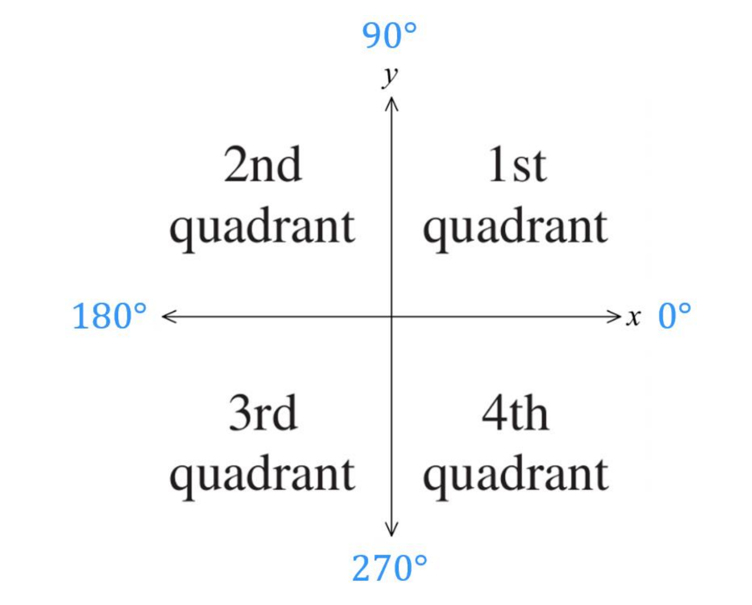
MA5-TRG-P-02 establishes and applies the properties of trigonometric functions and finds solutions to trigonometric equations

Use the unit circle to define trigonometric functions and represent them graphically

- Redefine the sine and cosine ratios in terms of the unit circle
- Verify that the tangent ratio can be expressed as a ratio of the sine and cosine ratios
- Use graphing applications to examine the sine, cosine and tangent ratios for (at least) $0^{\circ} \leq x \leq 360^{\circ}$, and graph the results
- Use graphing applications to examine graphs of the sine, cosine and tangent functions for angles of any magnitude, including negative angles
- Use the unit circle or graphs of trigonometric functions to establish and apply the relationships $\sin A = \sin(180° - A)$, $\cos A = -\cos(180° - A)$, and $\tan A = -\tan(180° - A)$ for obtuse angles when $0° \leq A \leq 90°$
- Establish and apply the relationship $m = \tan\theta$, where m is the gradient of the line and θ is the angle of inclination of a line with the x -axis on the Cartesian plane

REVIEW OF PRIOR KNOWLEDGE

Quadrants of the Cartesian Plane

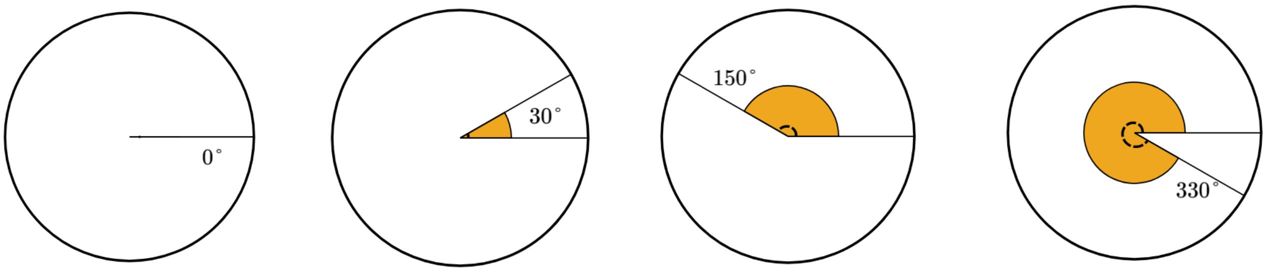


1 Which quadrant corresponds to these values of θ?

a $0° < \theta < 90°$	**b** $180° < \theta < 270°$	**c** $270° < \theta < 360°$	**d** $90° < \theta < 180°$

ANGLE OF INCLINATION

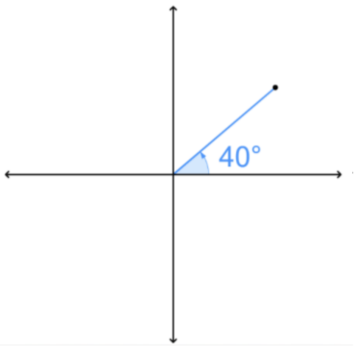
Angle of Inclination
The **angle of inclination θ** measures **anticlockwise turn from the positive x axis**.



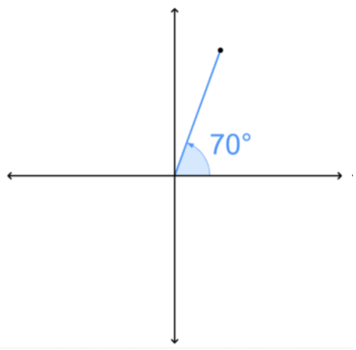
FOUNDATION

1 Sketch these angles of inclination:

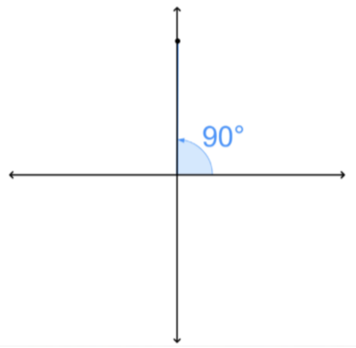
a 40°



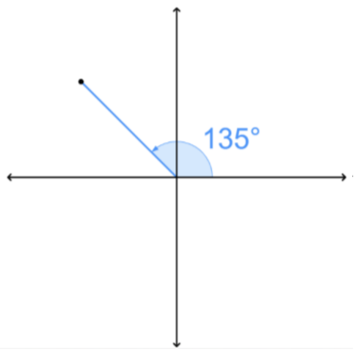
b 70°



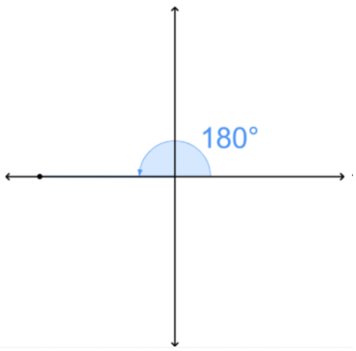
c 90°



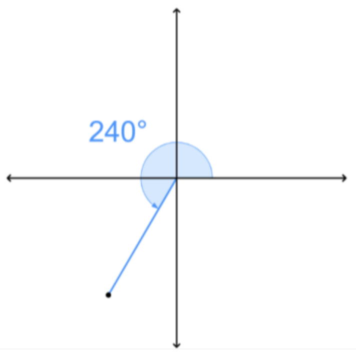
d 135°



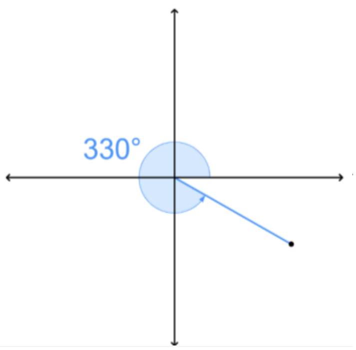
e 180°



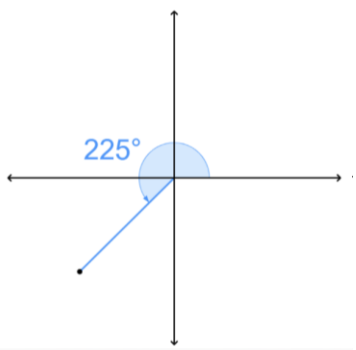
f 240°



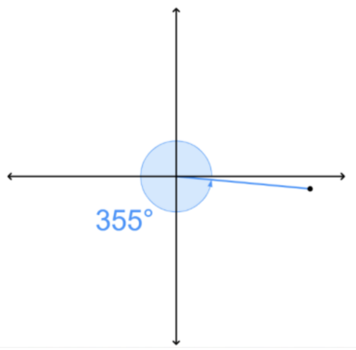
g 330°



h 225°

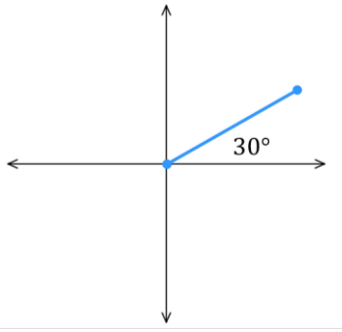


i 355°



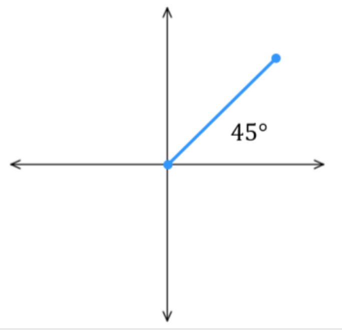
2 What is the angle of inclination between the positive x-axis and each line?

a



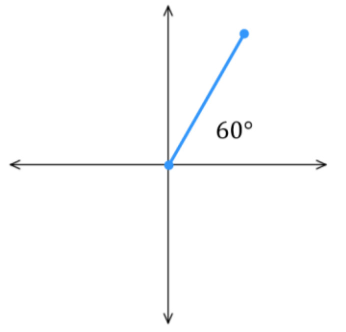
30°

b



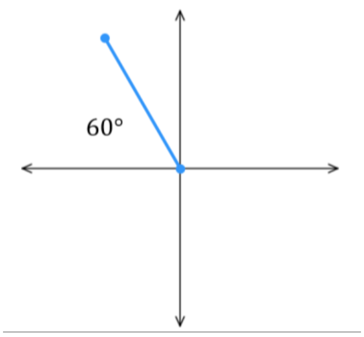
45°

c



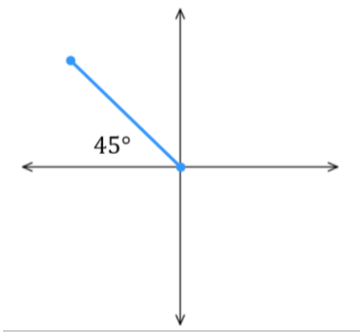
60°

d



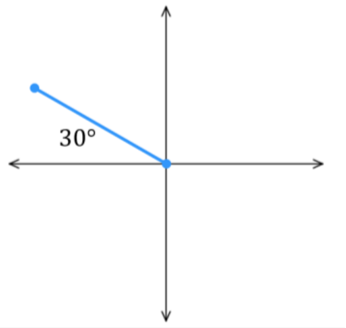
120°

e



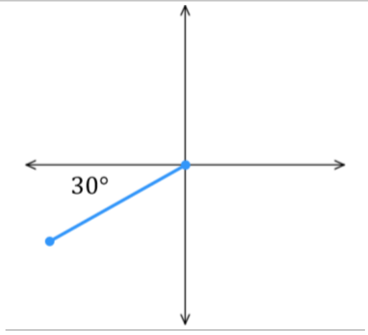
135°

f



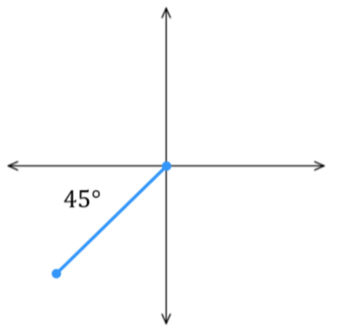
150°

g



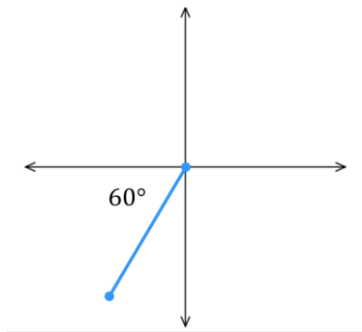
210°

h



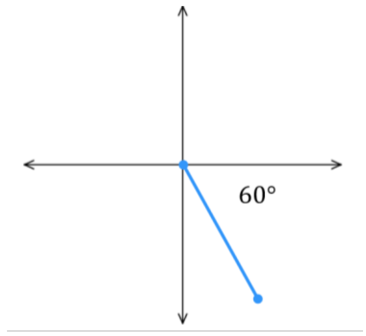
225°

i



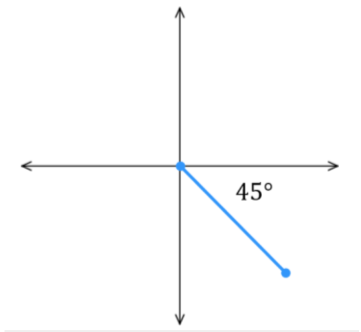
240°

j



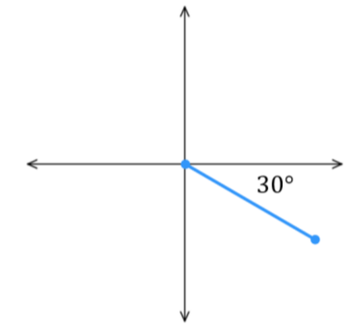
300°

k



315°

l



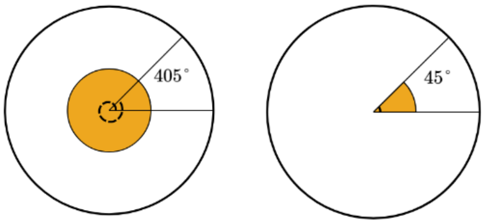
330°

PERIODICITY

Periodicity

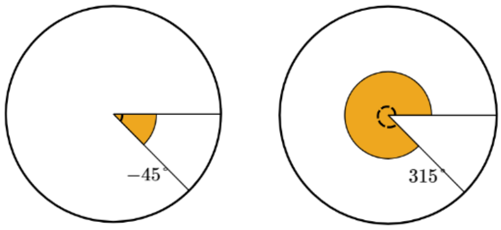
All angles can be reduced to an equivalent angle between 0° and 360°.

To turn more than $360°$; continue rotating around the circle.



Large angles: subtract 360° repeatedly until the angle is between 0° and 360°.

When an angle is negative, rotate clockwise.

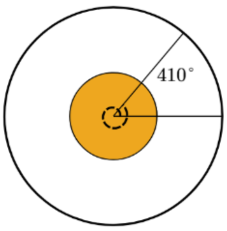


Negative angles: add 360° repeatedly until the angle is between 0° and 360°.

Example Rewrite angles to be between 0 and 360 degrees.

Rewrite these angles as an identical angle θ where $0 \leq \theta < 360°$

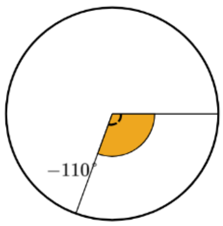
$410°$



$\theta = 410° - 360°$

$= 50°$

$-110°$



$\theta = -110° + 360°$

$= 250°$

How do we know whether to add or subtract 360?

If the angle is greater than ………, subtract multiples of $360°$.

If the angle is less than ………, add multiples of $360°$

Rewrite these angles as an identical angle θ where $0 \leq \theta < 360°$

a $500°$

$140°$

b $-50°$

$310°$

c $800°$

$80°$

d $420°$

$60°$

e $-10°$

$350°$

f $-330°$

$30°$

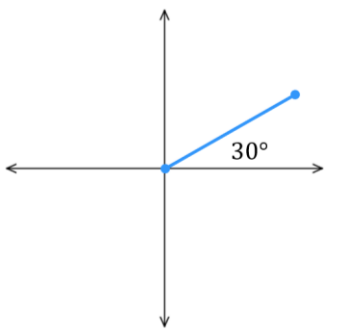
FOUNDATION

1 Rewrite these angles of inclination as an angle $0° \leq \theta < 360°$.

a $361°$ — 1°	**b** $-10°$ — 350°	**c** $720°$ — 0°
d $-500°$ — 220°	**e** $895°$ — 175°	**f** $-90°$ — 270°
g $1000°$ — 280°	**h** $-1000°$ — 80°	**i** $-45°$ — 315°
j $540°$ — 180°	**k** $-135°$ — 225°	**l** $-315°$ — 45°
m $660°$ — 300°	**n** $-45°$ — 315°	**o** $540°$ — 180°
p $-720°$ — 0°	**q** $-300°$ — 60°	**r** $1080°$ — 0°

2

a List 6 possible angles of inclination that this ray could represent.



Many answers possible, $\theta = 30 \pm 360n$, where n is an integer.

b Is there a limit to how many angles this ray could represent?

No.

REDEFINING TRIGONOMETRY USING THE UNIT CIRCLE

- So far, we have considered trigonometric ratios in terms of triangles.
- We will now redefine trigonometric ratios in terms of the **unit circle**.

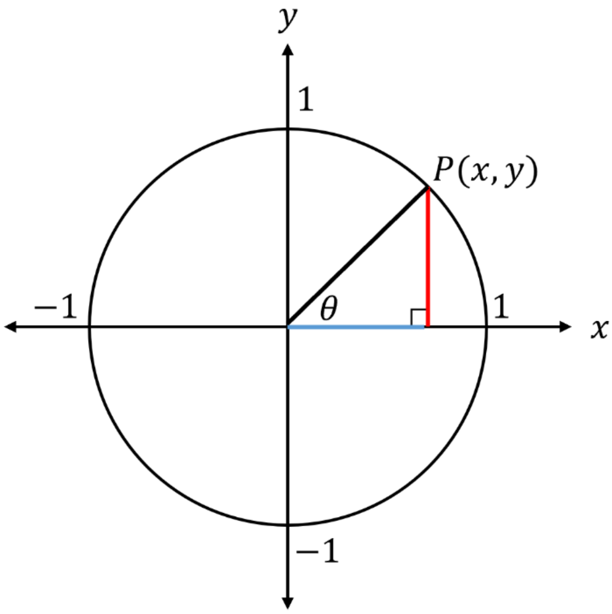
Investigation Trigonometry in terms of the unit circle.

Consider a circle centred at the origin $(0,0)$ with radius 1.

A radius extends out from the centre of the circle to a point (x, y) on the circumference.

A right-angled triangle is formed by the radius and the x-axis and y-axis.

Label the right-angled triangle and calculate:



a $\sin\theta = \frac{\text{Opp}}{\text{Hyp}} =$

$\therefore \sin\theta =$

b $\cos\theta = \frac{\text{Adj}}{\text{Hyp}} =$

$\therefore \cos\theta =$

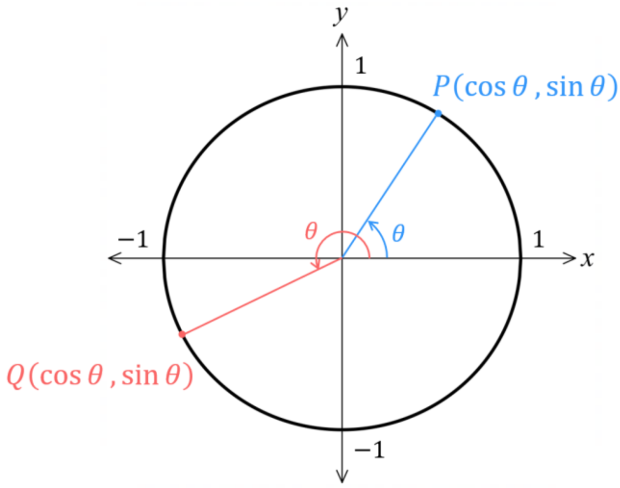
c $\tan\theta = \frac{\text{Opp}}{\text{Adj}} =$

$\therefore \tan\theta =$

Unit Circle Definition of Trigonometry

$P(\cos\theta, \sin\theta)$ is a point on a unit circle.

- The radius extends from the centre to P.
- θ is the angle of inclination between the positive x-axis and the radius.
- $\cos\theta$ is the x-coordinate, $\sin\theta$ is the y-coordinate.



$x = \cos\theta$	$y = \sin\theta$	$\tan\theta = \dfrac{y}{x} = \dfrac{\sin\theta}{\cos\theta}$

Example Determine the coordinates of a point on a unit circle.

Show the approximate location of these points on the unit circle.

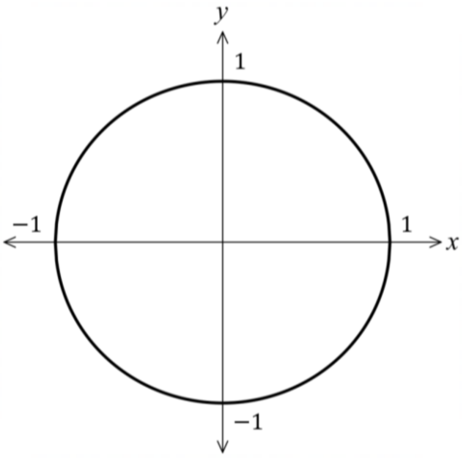
a $A(\cos 60°, \sin 60°)$

b $B(\cos 90°, \sin 90°)$

c $C(\cos 150°, \sin 150°)$

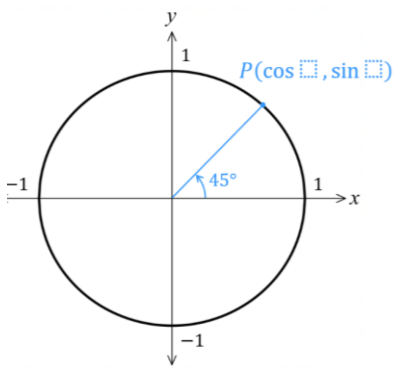
d $D(\cos 250°, \sin 250°)$

e $E(\cos 300°, \sin 300°)$



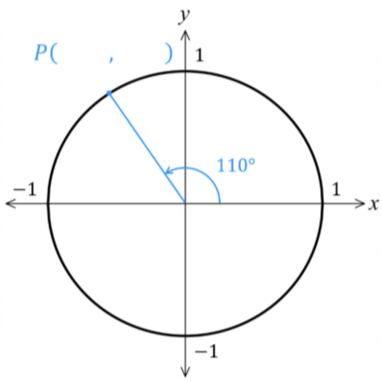
Write the coordinates of these points.

a



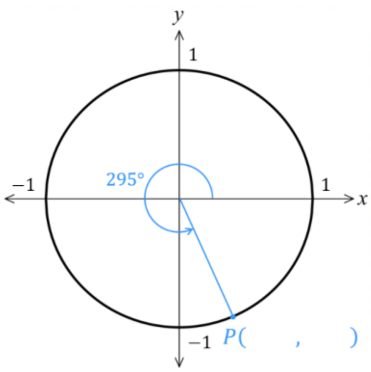
$(\cos 45°, \sin 45°)$

b



$(\cos 110°, \sin 110°)$

c



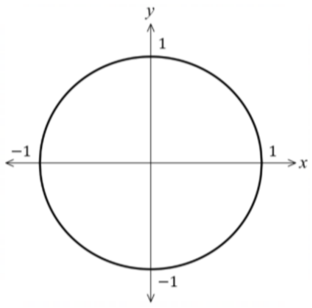
$(\cos 295°, \sin 295°)$

Example Determine the coordinates of a point on a unit circle.

$P(\cos\theta, \sin\theta)$ is a point on the unit circle.

Using the given angle of inclination, plot the point, then find the value of x and y.

a $\theta = 0°$



$x = \cos\theta$ $y = \sin\theta$

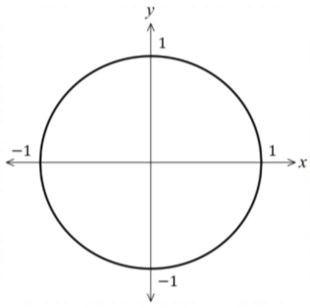
$= \cos ...$ $= \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(1,0)$

b $\theta = 90°$



$x = \cos\theta$ $y = \sin\theta$

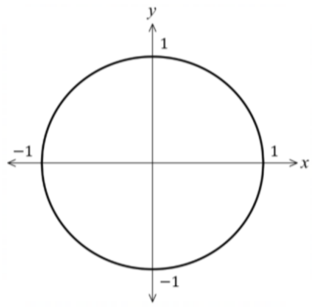
$= \cos ...$ $= \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(0,1)$

c $\theta = 180°$



$x = \cos\theta$ $y = \sin\theta$

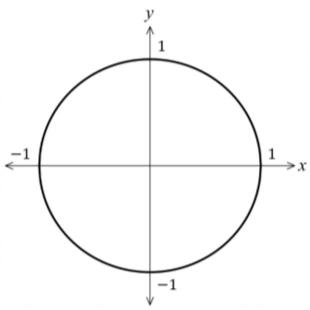
$= \cos ...$ $= \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(-1,0)$

a $\theta = 270°$



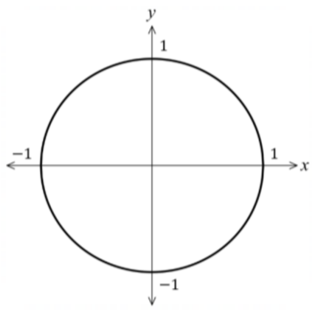
$x = \cos ...$ $y = \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(0,-1)$

b $\theta = 30°$



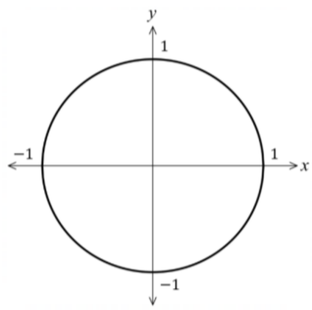
$x = \cos ...$ $y = \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(0.866, 0.500)$

c $\theta = 225°$



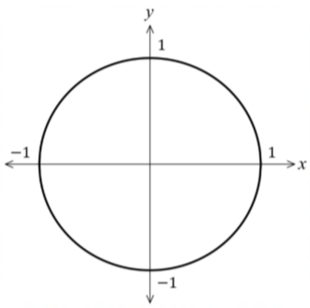
$x = \cos ...$ $y = \sin ...$

$= ......$ $= ......$

$\therefore P = (......, ...)$

$(-0.707, -0.707)$

d $\theta = 420°$

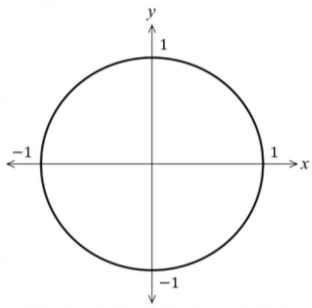


$x =$ $y =$

$\therefore P = (......, ...)$

$(0.5, 0.866)$

e $\theta = -30°$

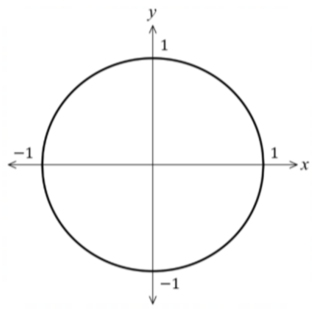


$x =$ $y =$

$\therefore P = (......, ...)$

$(0.866,, -0.500)$

f $\theta = -150°$

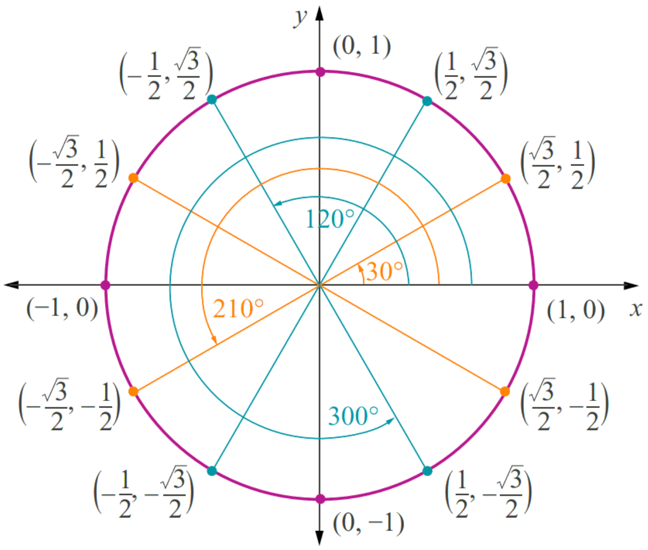


$x =$ $y =$

$\therefore P = (......, ...)$

$(-0.866, -0.500)$

1 The unit circle shown has exact values.
Use the unit circle to find the exact value of these trigonometric functions.



a	$\sin 0°$ 0	**b**	$\cos 0°$ 1	**c**	$\tan 0°$ 0
d	$\sin 120°$ $\frac{\sqrt{3}}{2}$	**e**	$\cos 120°$ $-\frac{1}{2}$	**f**	$\tan 120°$ $-\sqrt{3}$
g	$\sin 300°$ $-\frac{\sqrt{3}}{2}$	**h**	$\cos 300°$ $\frac{1}{2}$	**i**	$\tan 300°$ $-\sqrt{3}$
j	$\cos 210°$ $-\frac{\sqrt{3}}{2}$	**k**	$\sin 240°$ $-\frac{\sqrt{3}}{2}$	**l**	$\tan 150°$ $-\frac{1}{\sqrt{3}}$
m	$\sin 60°$ $\frac{\sqrt{3}}{2}$	**n**	$\cos 30°$ $\frac{\sqrt{3}}{2}$	**o**	$\tan 240°$ $\sqrt{3}$

p	$\cos 150°$ $-\frac{\sqrt{3}}{2}$	**q**	$\cos 90°$ 1	**r**	$\sin 180°$ 0	**s**	$\sin 210°$ $-\frac{1}{2}$	**t**	$\tan 330°$ $-\frac{1}{\sqrt{3}}$	**u**	$\tan 270°$ undefined

2 Use the unit circle to find all the angles $0° \leq \theta < 360°$ that have these values:

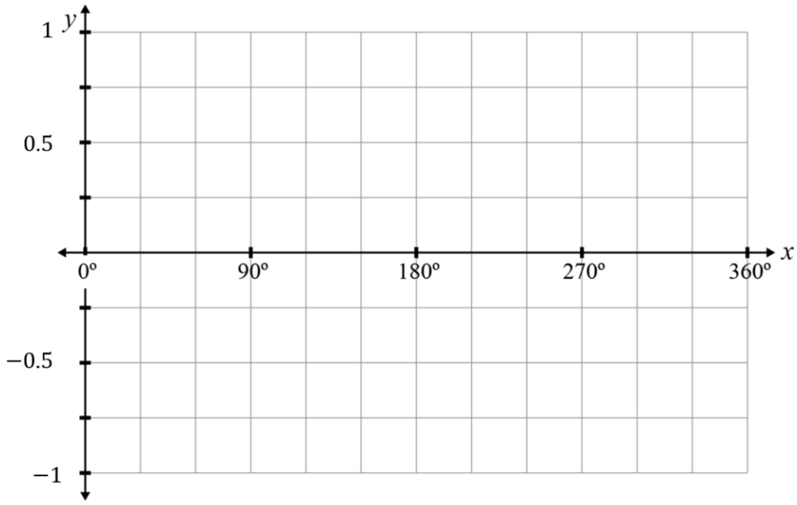
a	$\cos\theta = \frac{\sqrt{3}}{2}$ $\theta = 30°$ and ……° 30° and 330°	**b**	$\sin\theta = \frac{1}{2}$ 30° and 150°	**c**	$\sin\theta = -\frac{1}{2}$ 210° and 330°
d	$\sin\theta = \frac{\sqrt{3}}{2}$ 60° and 120°	**e**	$\sin\theta = -\frac{\sqrt{3}}{2}$ 240° and 300°	**f**	$\cos\theta = \frac{1}{2}$ 60° and 300°
g	$\cos\theta = 0$ 90° and 270°	**h**	$\sin\theta = -1$ 270°	**i**	$\cos\theta = 1$ 0°

GRAPHS OF TRIGONOMETRIC FUNCTIONS

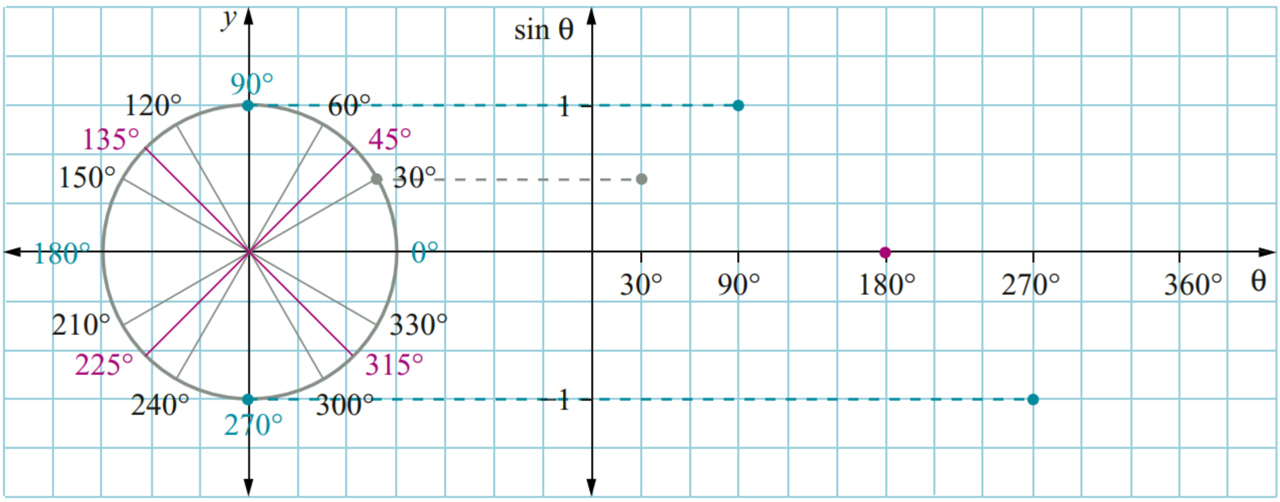
Investigation Graphs of trigonometric functions.

The trigonometric ratios should be thought of as a function.
$\sin(\theta)$ takes an angle input and outputs the y-coordinate on a unit circle.
The table has been completed for $y = \sin\theta$ (you can use a calculator to find these values)
Graph the function.

$\boldsymbol{\theta}$	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
$\boldsymbol{\sin\theta}$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0



We can also graph the sine function by considering the y coordinates on a unit circle.



a Examine the graph for θ between 180° and 270°. What do you notice about all the values of sin θ? How does this compare with the expected sign of sin θ in quadrant 3?

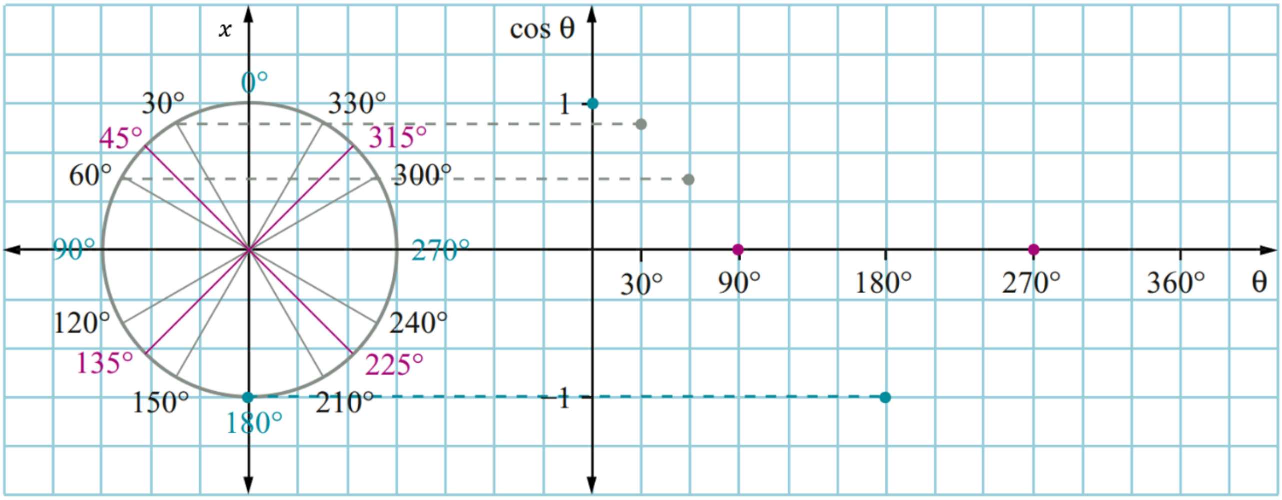
....

b Examine the graph for θ between 270° and 360°. What do you notice about all the values of sin θ? How does this compare with the expected sign of sin θ in quadrant 4?

....

c What would happen beyond $360°$? ....

Graph $\cos\theta$ by considering the x **coordinates** on a unit circle.



a Examine the graph for θ between 90° and 180°. What do you notice about all the values of cos θ? How does this compare with the expected sign of sin θ in quadrant 2?

....

b Look at the circle diagram for $\cos\theta$ and $\sin\theta$. What is different, and why?

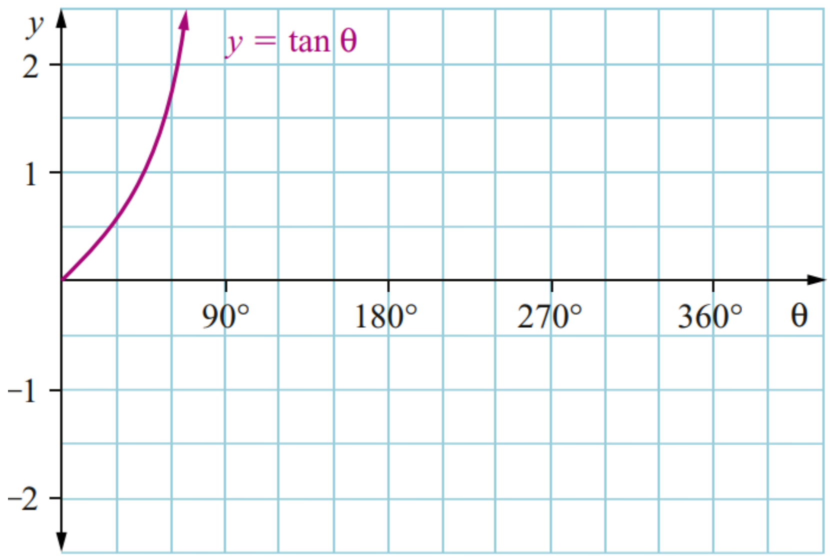
....

c What are the similarities and differences between the graphs of $\cos\theta$ and $\sin\theta$?

....

Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we will use a table of values once again.

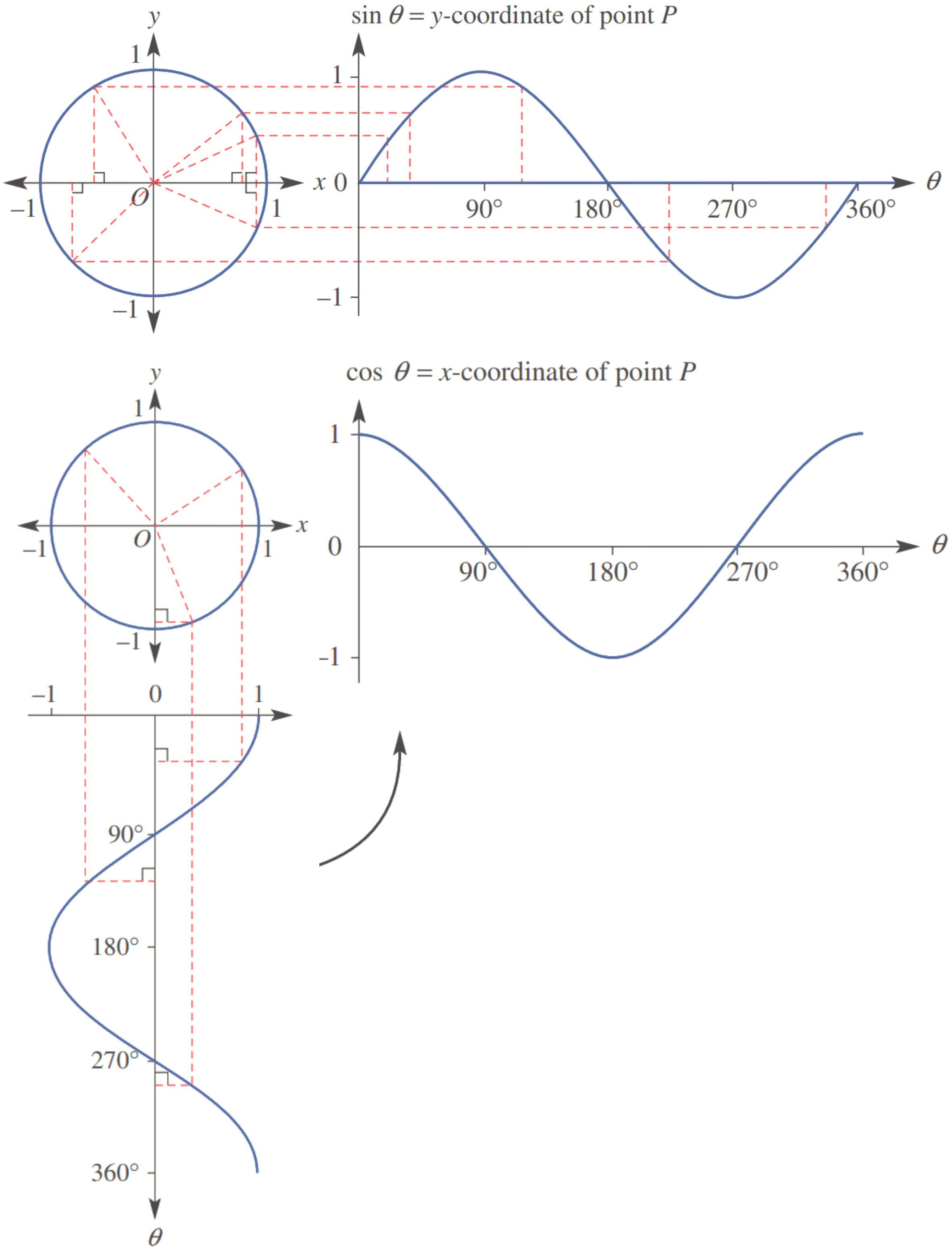
$\boldsymbol{\theta}$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\tan\boldsymbol{\theta}$	0	0.58	1.73	und	−1.73	−0.58	und	0.58	1.7	und	−1.7	−0.58	und



a Why are there asymptotes at $\theta = 90°$ and $\theta = 270°$?

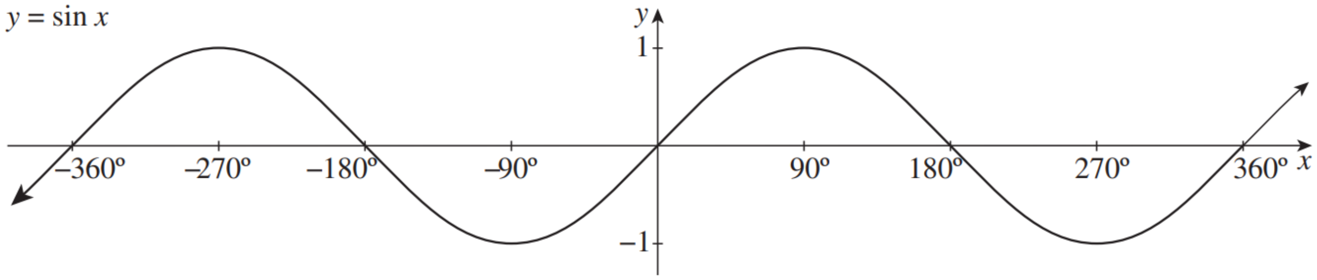
....

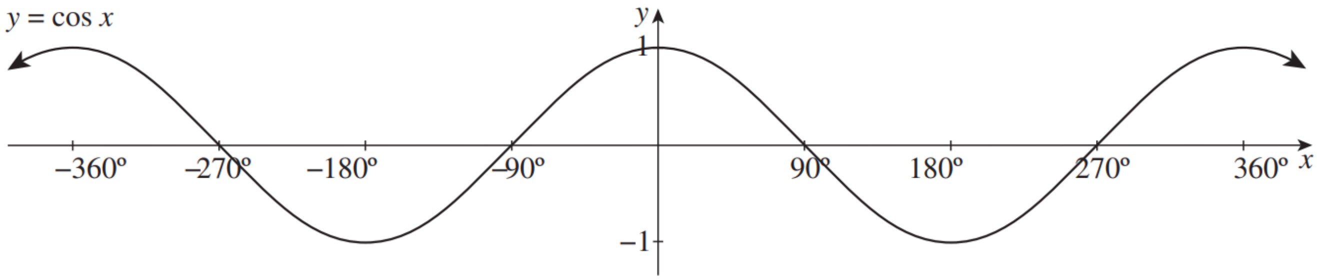
Sin and cos on the unit circle: https://www.geogebra.org/m/mynd2psx

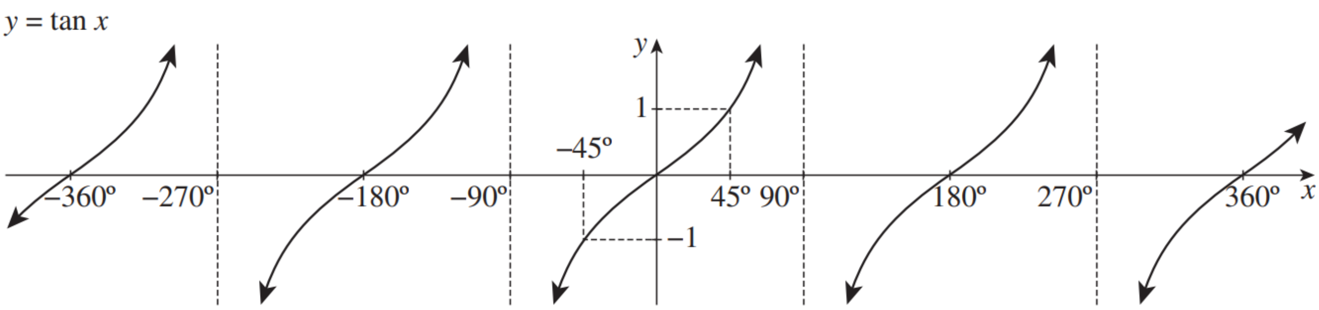


GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

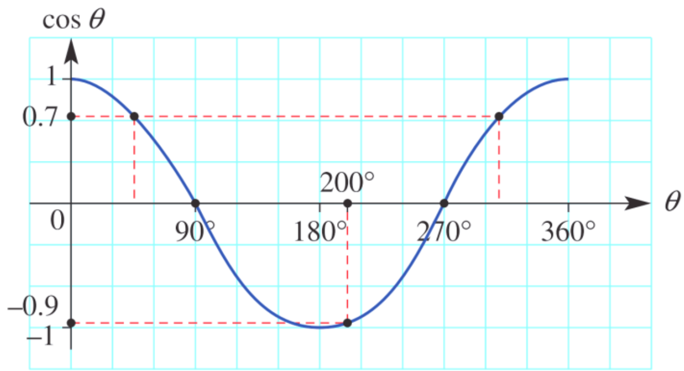
- The graphs trigonometric functions are **periodic**, they keep repeating.
- The **period** is how long it takes for the graph to complete a full cycle.
- $\sin x$ and $\cos x$ are identical, except $\cos x$ is shifted by $90°$: $\cos x = \sin(90 - x)$
- $\sin x$ and $\cos x$ are called **sinusoidal;** they occur whenever you analyse waves.







Example Interpret graphs of trigonometric functions.



Use the graph to estimate $\cos 200°$

$\cos 200° \approx -0.9$

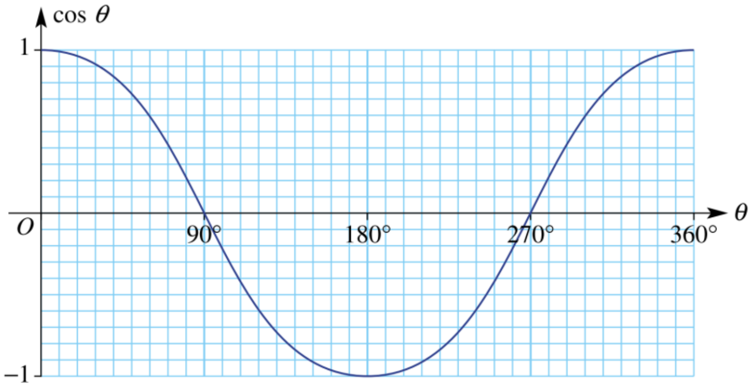
Find two values of θ for which $\cos\theta = 0.7$

$\theta \approx 46°$ or $314°$

What is the advantage of using graphs vs the calculator? (hint: type in $\cos^{-1}(0.7)$).

....

1 This graph shows $\cos\theta$. Use it to evaluate the following to 2 d.p.

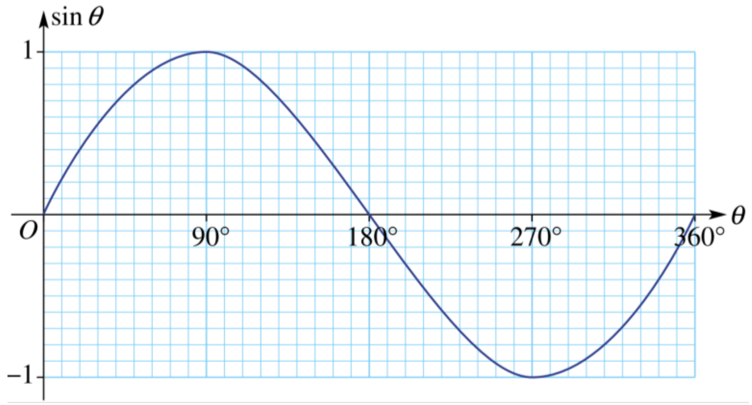


a $\cos 35°$ 0.82	**b** $\cos 190°$ − 0.98	**c** $\cos 330°$ 0.87	**d** $\cos 140°$ − 0.77

2 Use the same graph as in Q1 to estimate the **values** of θ between 0° and 360° for:

a $\cos\theta = 0.8$ 37°, 323°	**b** $\cos\theta = 0.6$ 53°, 307°	**c** $\cos\theta = 0$ 90°, 270°	**d** $\cos\theta = -0.4$ 114°, 246°

3 Use the graph of $\sin\theta$ to evaluate/solve:

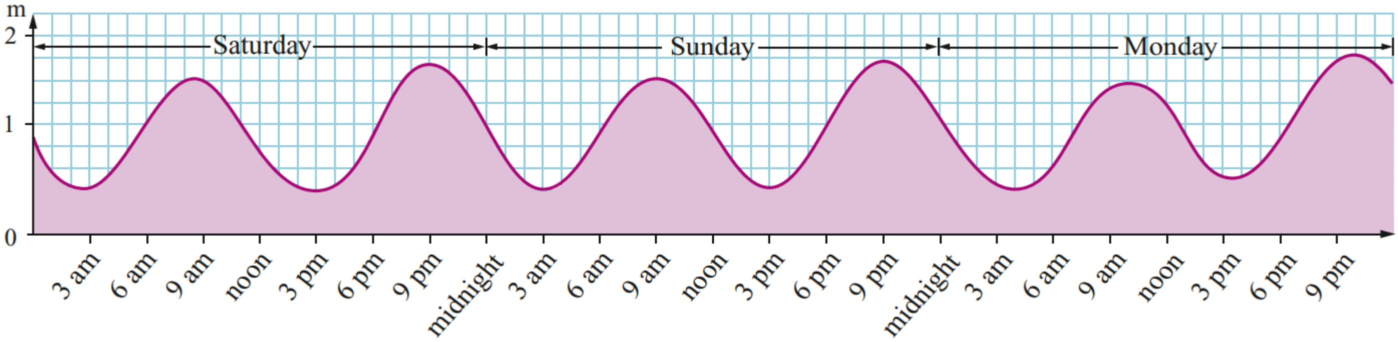


a $\sin 25°$ 0.42	**b** $\sin 115°$ 0.91	**c** $\sin 220°$ − 0.64	**d** $\sin 310°$ − 0.77
e $\sin\theta = 0.6$ 37°, 143°	**f** $\sin\theta = 0.2$ 12°, 168°	**g** $\sin\theta = -0.4$ 204°, 336°	**h** $\sin\theta = -0.7$ 233°, 307°

4 Use the trigonometric graphs to find the values (if they exist) of these trigonometric ratios of boundary angles.

a $\sin 0°$ 0	**b** $\cos 180°$ −1	**c** $\cos 90°$ 0	**d** $\tan 0°$ 0
e $\sin 90°$ 1	**f** $\cos 0°$ 1	**g** $\sin 270°$ −1	**h** $\cos 270°$ 0
i $\sin 180°$ 0	**j** $\tan 270°$ undefined	**k** $\tan 90°$ undefined	**l** $\tan 180°$ 0

5 A tidal chart is shown below.



a How many low tides occur in a day?

2

b How many high tides occur in a day?

2

c What word is used to describe graphs of this shape?

sinusoidal

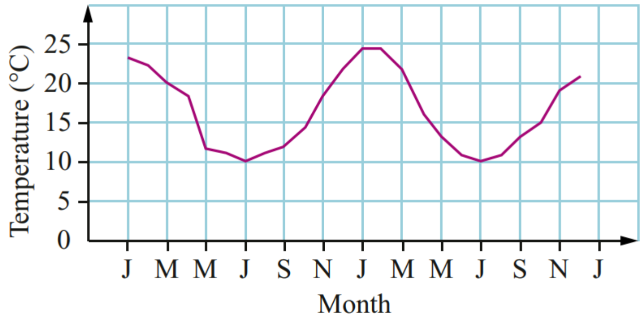
d Estimate the low tide height and high tide height for Saturday between noon and midnight.

low tide: 0.25 m, high tide: 1.7 m

e Estimate the period for the tide graph.

Around 12 hours

6 The graph shows the average monthly temperature of a city over 2 years, starting in Jan.



a Would you expect this city to be in the Northern or Southern Hemisphere?

Southern, temperatures are high in January.

b Which trigonometric graph does this resemble? A cosine or sine graph?

Cosine

c If temperatures of a city in the Northern hemisphere were graphed, what graph would you expect, and why?

Sine; temperatures would be cold in January.

d What is the difference between the highest and lowest values?

Around 14°C

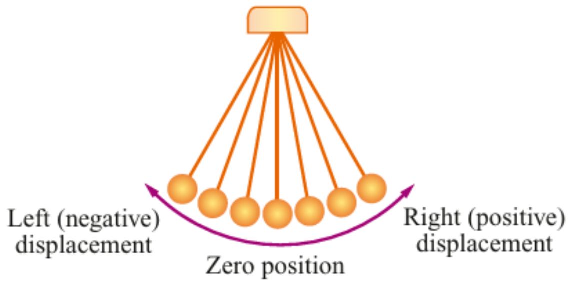
e What is the period of this graph?

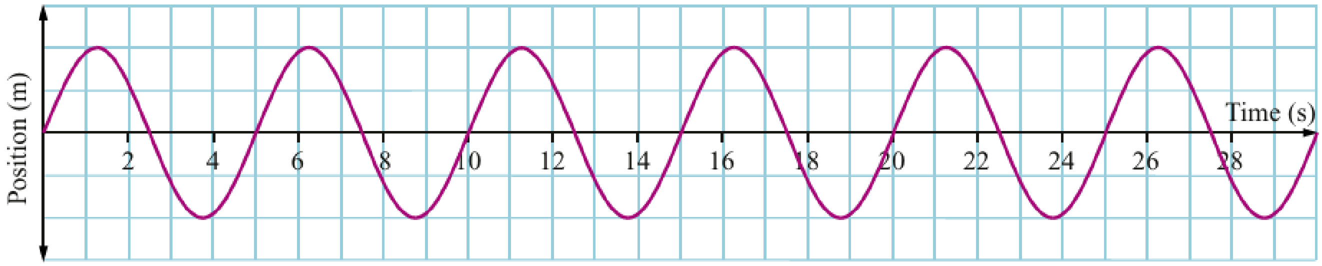
1 year

7 The diagram shows the motion of a swinging pendulum and its graph.

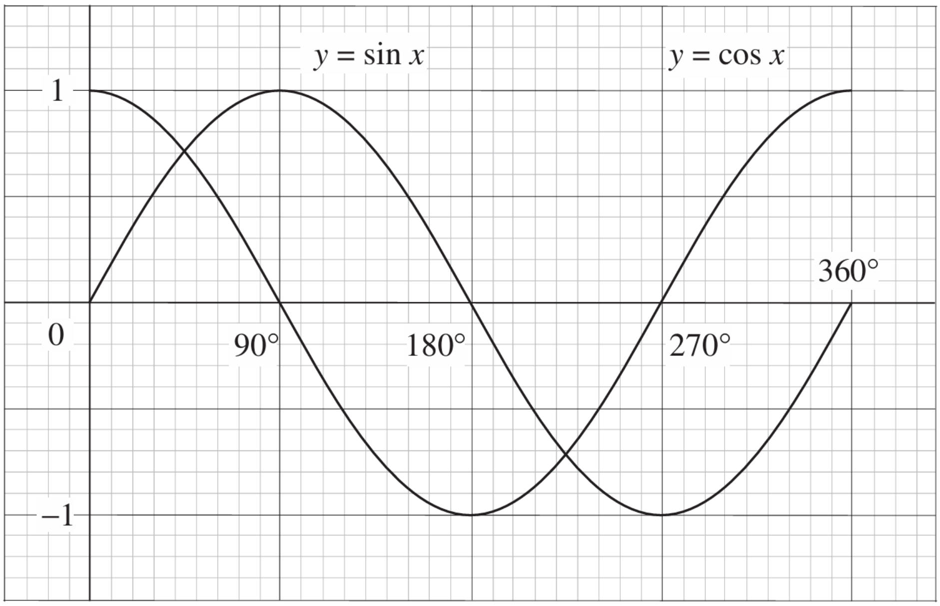
What is the period if this graph?

5 seconds





8 Consider the graphs of $y = \cos x$ and $y = \sin x$ on the same axis:



Decide whether the following statements are true or false.

a sin 60° > sin 200° T	**b** sin 100° < sin 300° F	**c** sin 135° < sin 10° F
d sin 200° = sin 340° T	**e** cos 70° < cos 125° F	**f** cos 315° > cos 135° T
g cos 310° = cos 50° T	**h** cos 95° > cos 260° T	**i** sin 90° = cos 360° T
j cos 180° = sin 180° F	**k** sin 210° > sin 285° T	**l** cos 15° >cos 115° T

9 Using the graph in the previous question, for what angles are the values of $\sin x$ and $\cos x$ equal in the domain $0° \leq x < 360°$?

45° and 135°

10 What is the domain and range of $y = \sin\theta$?

D: all real θ

R: $[-1,1]$

11 What is the domain and range of $y = \cos\theta$?

D: all real θ

R: $[-1,1]$

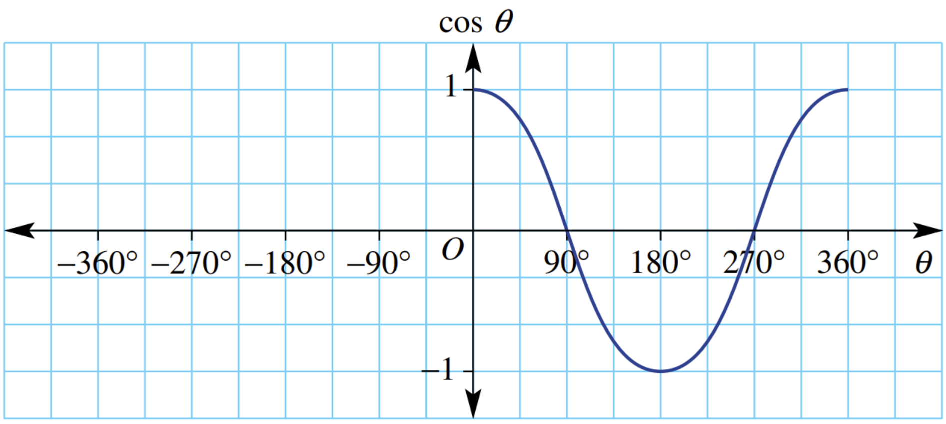
12 Explain why no values of θ satisfy sin θ = 2.

No solution as 2 is not in the range of $y = \sin\theta$.

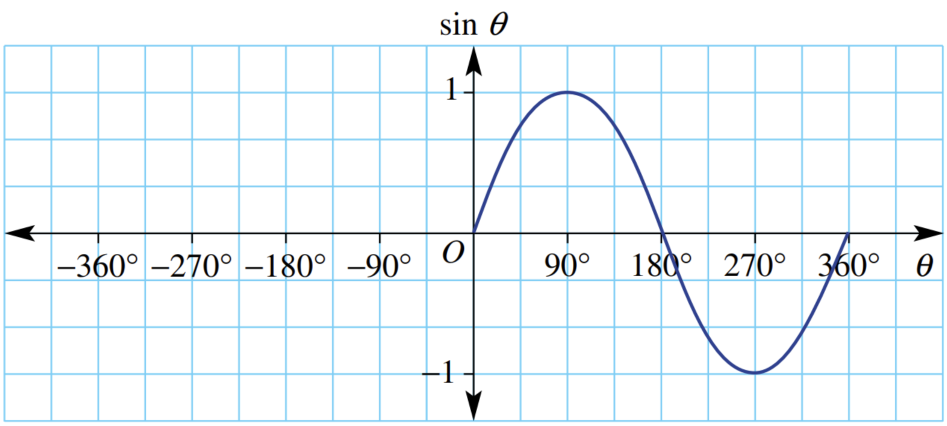
13 How many values of θ satisfy cos θ = −4? Give a reason.

None. -4 is not in the range of $y = \cos\theta$.

14 $y = \cos\theta$ is an even function as $\cos(-\theta) = \cos\theta$.
Complete the graph for the domain $-360° \le \theta \le 360°$



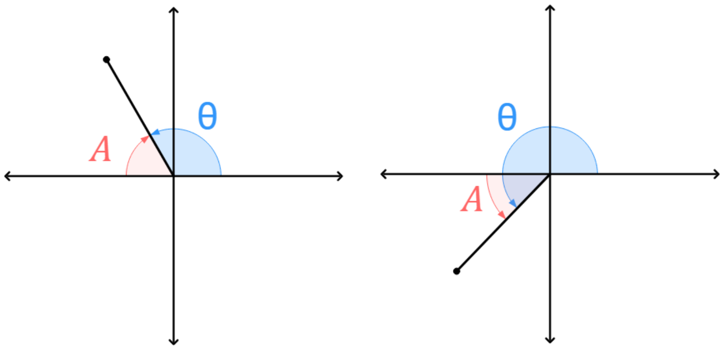
15 $y = \sin\theta$ is an odd function as $\sin(-\theta) = -\sin\theta$.
Complete the graph for the domain $-360° \le \theta \le 360°$



REFERENCE ANGLES

Reference Angle

The **reference angle,** or **related angle** A is the acute angle between the x-axis and the radius.

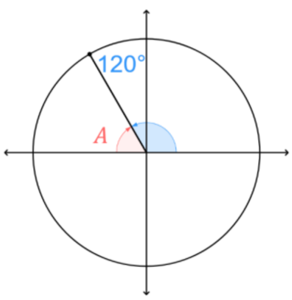


- In this booklet, the angle of inclination will be referred to as θ and the reference angle as A.

Example Determine the reference angle.

Determine the reference angle for these angles of inclination.

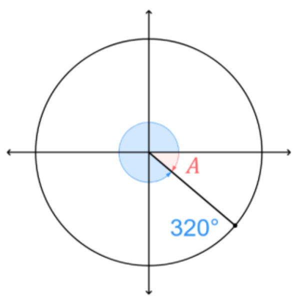
$\theta = 120°$



$A = 180° - 120°$

$= 60°$

$\theta = 320°$



$A = 360° - 320°$

$= 40°$

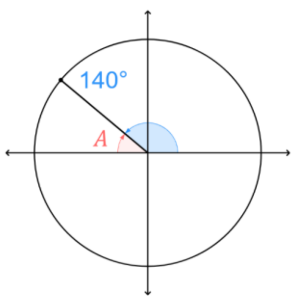
What is the difference between the angle of inclination θ and the reference angle A?

The angle of inclination is how much to turn ……………-wise from the …………… …-axis.

The reference angle is the …………… angle between the ray and the ……-axis.

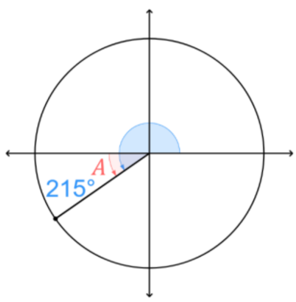
Determine the reference angle for these angles of inclination.

a $\theta = 140°$



40°

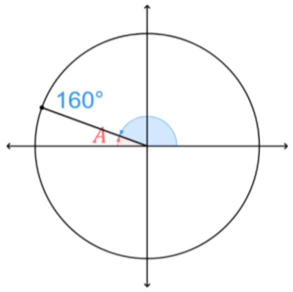
b $\theta = 215°$



35°

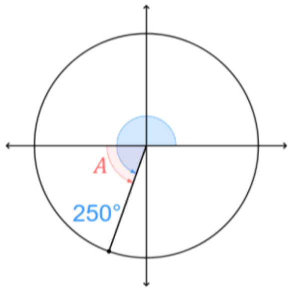
1 Find the reference angle, or related angle, of these angles of inclination. A sketch will help.

a $\theta = 160°$



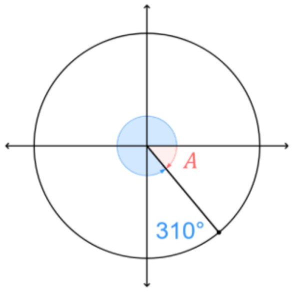
20°

b $\theta = 250°$



70°

c $\theta = 310°$



50°

d $\theta = 200°$

20°

e $\theta = 290°$

70°

f $\theta = 250°$

70°

g $\theta = 10°$

10°

h $\theta = 215°$

35°

i $\theta = 240°$

60°

j $\theta = 300°$

60°

k $\theta = 135°$

45°

l $\theta = 120°$

30°

m $\theta = 315°$

45°

n $\theta = 135°$

45°

o $\theta = 150°$

30°

p $\theta = 100°$

80°

q $\theta = 350°$

10°

r $\theta = 160°$

20°

s $\theta = 225°$

45°

t $\theta = 210°$

30°

u $\theta = 300°$

60°

v $\theta = 420°$

60°

w $\theta = -300°$

60°

x $\theta = -135°$

45°

y $\theta = 495°$

45°

z $\theta = -30°$

30°

aa $\theta = 675°$

45°

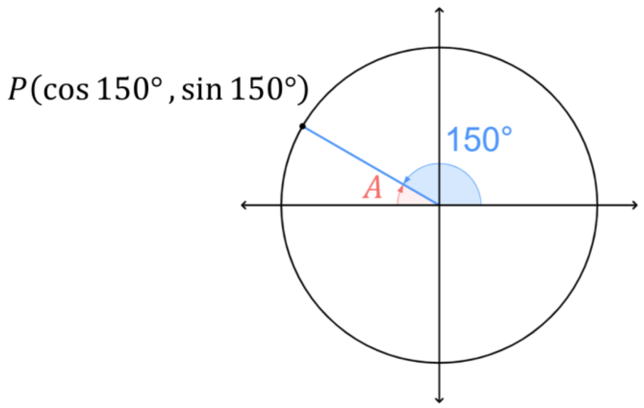
TRIGONOMETRIC RATIOS FOR OBTUSE ANGLES

Investigation Trigonometric ratios for obtuse angles.

How might we find the value of $\sin 150°$, $\cos 150°$, and $\tan 150°$?

Consider a point P with **angle of inclination $150°$** on a unit circle.

What is the acute reference angle? …………



We will reflect the point across the y-axis so that it is in the first quadrant.

Sketch this on the diagram above and label the coordinates of the new point P'.

What do you notice about the value of $\sin 30°$ and $\sin 150°$?

$\sin 150°$ *is ………… to the value of* $\sin 30°$.

What do you notice about the value of $\cos 30°$ and $\cos 150°$?

$\cos 150°$ *is ………… to the value of* $\cos 30°$, *except it is ……………………*

Therefore, we can write:

$$\sin 150° = \sin 30°$$

$$\cos 150° = -\cos 30°$$

$$\tan 150° = \frac{\sin 150°}{\cos 150°} =$$

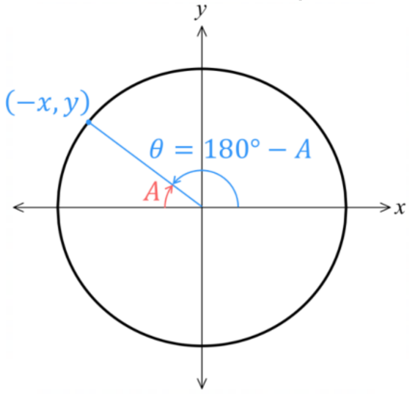
How are $150°$ and $30°$ related?

For a $150°$ *angle of inclination, the ………………… angle is* $30°$

Trigonometric Ratios for Obtuse Angles

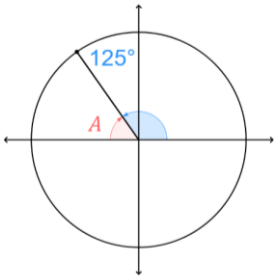
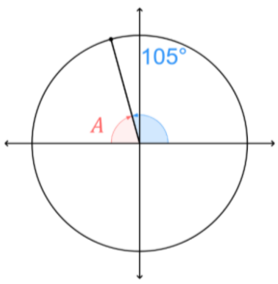
We can express trigonometric ratios for an **obtuse angle $\boldsymbol{\theta}$** according to its acute reference angle $\boldsymbol{A}$:

$$\sin\theta = \sin A$$
$$\cos\theta = -\cos A$$
$$\tan\theta = -\tan A$$



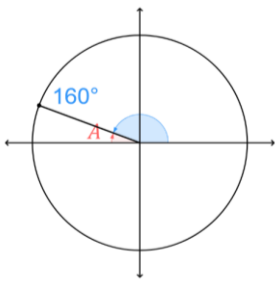
- To write a trigonometric ratio with an acute angle:
  1. Determine whether the trig ratio is positive or negative.
  2. Rewrite trig ratio with acute reference angle and the correct sign.

Example Rewrite trigonometric ratios with the acute reference angle (2nd quadrant).

Write $\sin 125°$ using its acute reference angle.  1. sin is positive in Q2 2. $\sin 125° = \sin 55°$	What is A? *The ………………… angle.* How did we know $A = 55°$? *Angles on a ……………-line sum to ………* Why is $\sin 125°$ positive? *The …-coordinate in the …………… quadrant of a unit circle is positive.*
Write $\cos 105°$ using its acute reference angle.  1. cos is negative in Q2 2. $\cos 105° = -\cos 75°$	How did we know $\theta = 75°$? *Angles on a ……………-line sum to ………* Why is $\cos 105°$ negative? *The …-coordinate in the …………… quadrant of a unit circle is negative.*

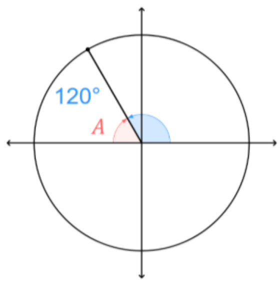
Rewrite these trigonometric ratios in terms of its acute reference angle.

a $\sin 160°$



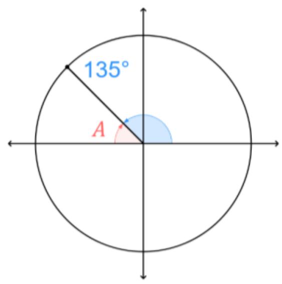
$\sin 20°$

b $\cos 120°$



$-\cos 60°$

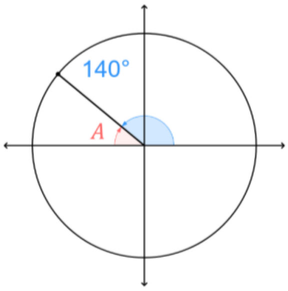
c $\tan 135°$



$-\tan 45°$

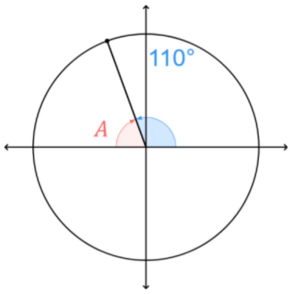
1 Rewrite these trigonometric ratios in terms of its acute reference angle.

a $\cos 140°$



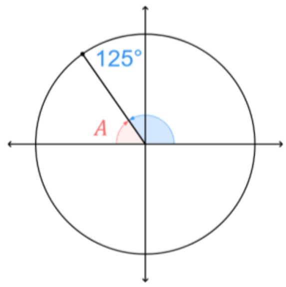
–cos40°

b $\sin 110°$



sin70°

c $\cos 125°$



–cos55°

d $\cos 175°$

–cos5°

e $\tan 170°$

–tan10°

f $\tan 95°$

–tan85°

g $\sin 175°$

sin5°

h $\sin 180°$

sin0°

i $\cos 155°$

–cos25°

j $\tan 120°$

–tan60°

k $\cos 150°$

–cos30°

l $\sin 160°$

sin20°

m $\sin 140°$

sin40°

n $\cos 125°$

–cos55°

o $\tan 123°$

tan57°

p $\tan 105°$

–tan75°

q $\cos 155°$

–cos25°

r $\tan 100°$

–tan80°

s $\cos 148°$

–cos32°

t $\sin 167°$

sin13°

u $\tan 129°$

–tan51°

2 Find the exact values of these trigonometric ratios by first rewriting them with the acute reference angle.

a $\sin 150°$

$= \sin 30°$

$= \ldots\ldots\ldots$

$\frac{1}{2}$

b $\cos 150°$

$= -\cos 30°$

$= \ldots\ldots\ldots$

$-\frac{\sqrt{3}}{2}$

c $\tan 120°$

$-\sqrt{3}$

d $\cos 135°$

$-\frac{1}{\sqrt{2}}$

e $\sin 120°$

$\frac{\sqrt{3}}{2}$

f $\cos 120°$

$-\frac{1}{2}$

g $\tan 150°$

$-\frac{1}{\sqrt{3}}$

h $\sin 135°$

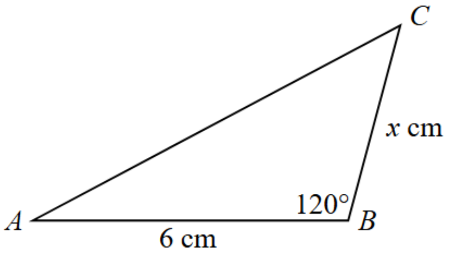
$\frac{1}{\sqrt{2}}$

i $\tan 135°$

-1

3 HSC Advanced Sample Question Band 4

The triangle shown below has an exact area of $\left(12 - \sqrt{3}\right)$ cm^2.



The exact value of x is:

A. $\dfrac{24 - 2\sqrt{3}}{3}$

B. $\dfrac{8\sqrt{3} - 2}{3}$

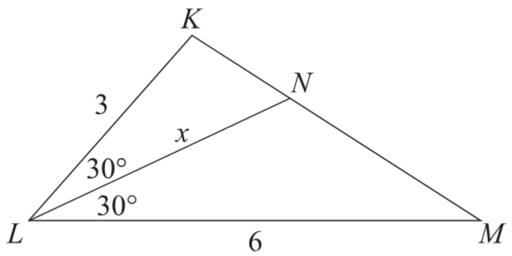
C. $8\sqrt{3} - 1$

D. $12\sqrt{3} + 1$

B

4 2018 HSC Advanced Band 5

Find the exact value of the area of ΔKLM and hence find the exact value of x.

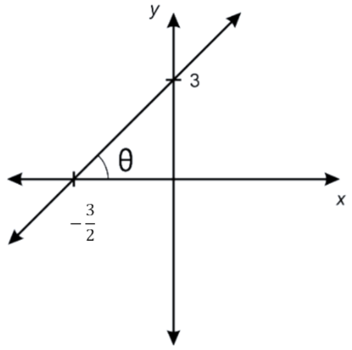


$$\text{Area } \Delta KLM = \frac{1}{2}(3)(6)(\sin 60°)$$
$$= \frac{9\sqrt{3}}{2}$$

$$\text{Area } \Delta KLM + \text{Area } \Delta NLM = \text{Area } \Delta KLM$$
$$\frac{3}{4}x + \frac{3}{2}x = \frac{9\sqrt{3}}{2}$$
$$\frac{9x}{4} = \frac{9\sqrt{3}}{2}$$
$$x = x\sqrt{3}$$

TAN AND GRADIENT

Investigation Gradient and Tangent



Find the gradient m

$m = \dfrac{\text{rise}}{\text{run}} =$

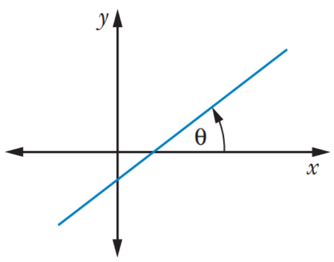
Find $\tan\theta$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} =$

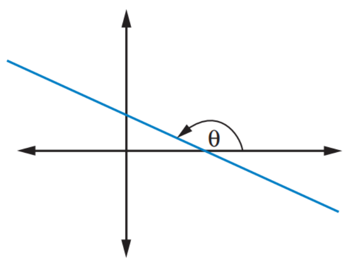
Gradient and Angle of Inclination

$$m = \tan\theta$$

For acute angles, the gradient is positive.

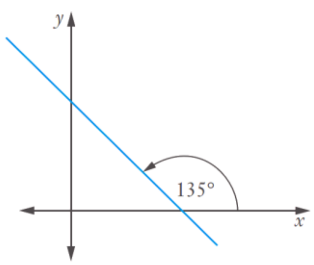


For obtuse angles, the gradient is negative.



Example Determine gradient of a line from the angle of inclination.

Find the gradient of the line that makes an angle of inclination of $135°$



$m = \tan\theta$

$m = \tan 135°$

$= -1$

How do we find the value of $\tan 135°$?

We put it into the ....

Does it make sense that the gradient is negative?

Yes, the line is sloping .... from left to right.

Find the gradient of the line with these angles of inclination, answer rounded to 2 d.p.

a $\theta = 45°$

1

b $\theta = 80°$

5.67

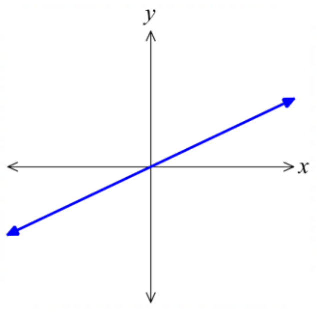
c $\theta = 155°$

-0.47

Example Determine angle of inclination from the gradient.

Find, correct to the nearest minute, the angle of inclination of a straight line with gradient:

$m = 0.5$



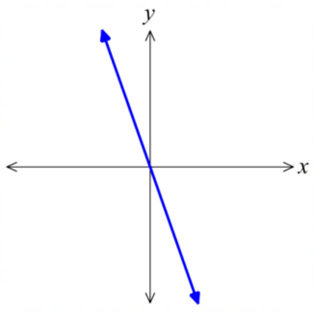
$\tan\theta = m$

$\tan\theta = 0.5$

$\theta = \tan^{-1}(0.5)$

$\theta \approx 26°34'$

$m = -3$



$\tan\theta = m$

$\tan\theta = -3$

$\theta = \tan^{-1}(-3)$

$\theta \approx -71°35'$

$\approx 108°26'$

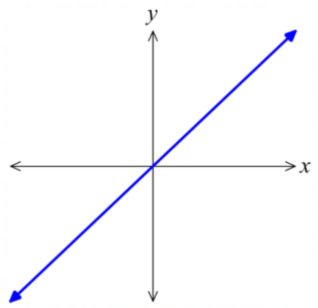
How did we change $-71°35'$ to $108°26'$?

An angle of inclination of $-71°35'$ means to turn ………-wise from the positive x axis.

The angle to turn anticlockwise is the s……………… angle, so, we subtract from $180°$.

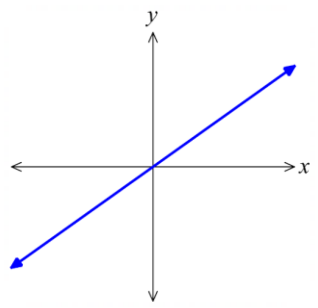
Find, correct to the nearest minute, the angle of inclination of a straight line with gradient:

a $m = 1$



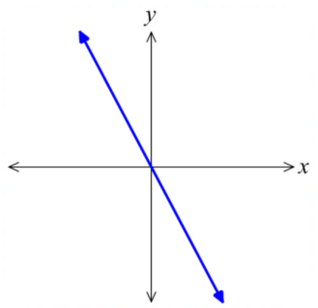
$45°$

b $m = \frac{3}{4}$



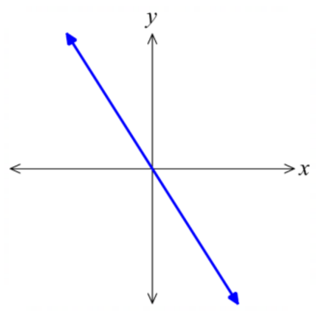
$36°52'$

c $m = -2$



$116°34'$

d $m = -\frac{5}{3}$



$120°58'$

1 Use the formula $m = \tan\theta$ to find the gradient of a line with these angles of inclination. Answer to 2 decimal places.

a $\theta = 15°$

0.27

b $\theta = 85°$

11.43

c $\theta = 135°$

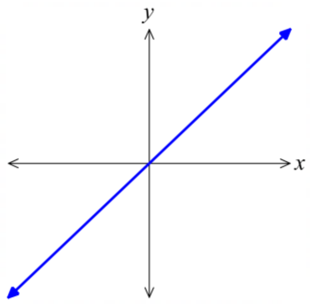
–1

d $\theta = 180°$

0

2 Use the formula $\tan\theta = m$ to find the angle of inclination of a line with these gradients. Answer to the nearest degree. A sketch will help you visualise the problem.

a $m = 1$



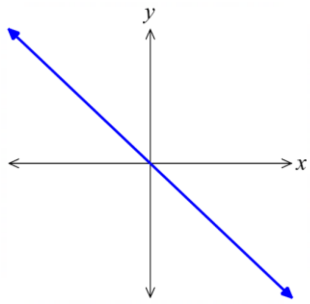
$\tan\theta = 1$

$\theta = \tan^{-1}(\qquad)$

$=$

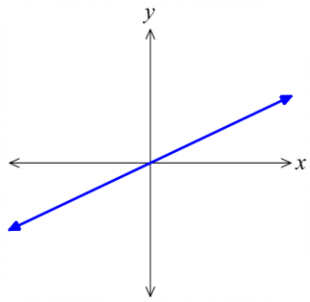
45°

b $m = -1$



135°

c $m = 0.5$



27°

m $m = -0.5$

153°

n $m = 1.73$

60°

o $m = \frac{5}{2}$

68°

p $m = 4$

76°

q $m = -0.58$

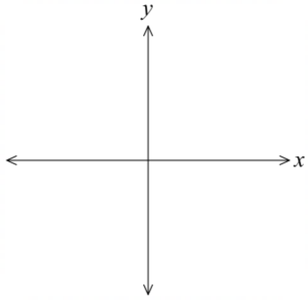
150°

r $m = -12$

95°

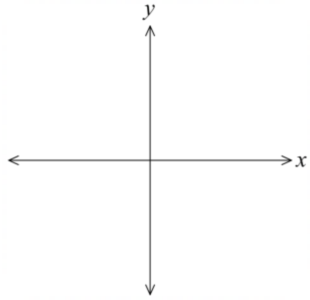
3 Sketch these lines then find the angle of inclination to the nearest degree.

a $y = 3x + 6$



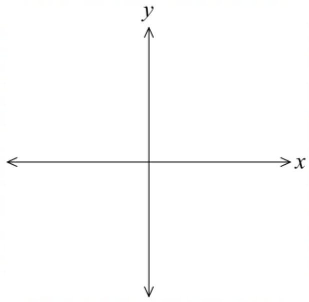
72°

b $y = -\frac{1}{2}x + 1$



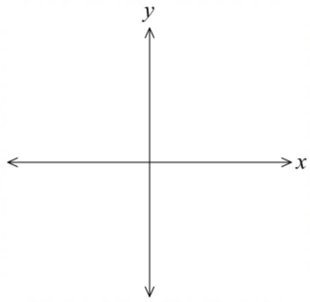
153°

c $3x + 4y = -12$



143°

d $\frac{x}{3} - \frac{y}{2} = 1$



34°

4 What is the angle of inclination of the line $x = -2$?

90°

5 The angle of inclination of a line is 135° and passes through the point $(4,2)$ and $(x, -3)$. Find the value of x.

$$m = \tan 135° = -1$$

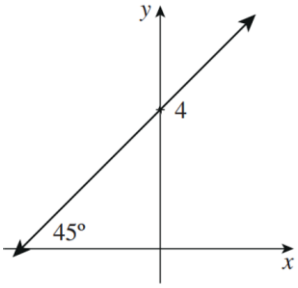
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{x - 4} = -1$$

$$\frac{-5}{x - 4} = -1$$

$$x = 9$$

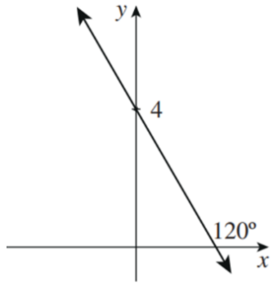
6 Determine the equation of each straight line sketched below in exact general form.

a



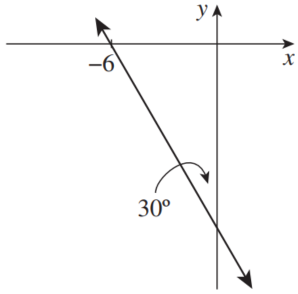
$x - y + 4 = 0$

b



$\sqrt{3}x + y - 4 = 0$

c



$x\sqrt{3} + y + 6\sqrt{3} = 0$

7 In each part, the angle of inclination α and the y-intercept c of a line are given,
Use $m = \tan\alpha$ to find the gradient of each line, then find its equation in exact general form.

a $\alpha = 45°$, $c = (0,3)$

$x - y + 3 = 0$

b $\alpha = 60°$, $c = (0,-1)$

$-\sqrt{3}x + y + 1 = 0$

c $\alpha = 30°$, $c = (0,-2)$

$x - \sqrt{3}y - 2\sqrt{3} = 0$

d $\alpha = 135°$, $c = (0,1)$

$x + y - 1 = 0$

8 In each part, the angle of inclination α and a point A on the line is given,
Use $m = \tan\alpha$ to find the gradient of each line, then find its equation in exact general form.

a $\alpha = 45°$, $A = (1,0)$

$x - y - 1 = 0$

b $\alpha = 120°$, $A = (-1,0)$

$\sqrt{3}x + y + \sqrt{3} = 0$

c $\alpha = 30°$, $A = (4,-3)$

$x - y\sqrt{3} - 4 - 3\sqrt{3} = 0$

d $\alpha = 150°$, $c = (-2,-5)$

$x + \sqrt{3}y + 2 + 5\sqrt{3} = 0$

9 A triangle is formed by the x-axis and the lines $5y = 9x$ and $5y + 9x = 45$.

a Find, correct to the nearest degree, the angles of inclination of the two lines.

b What sort of triangle has been formed?

a) They are about 61° and 119°

b) Since the two interior angles with the x-axis are equal, it is isosceles (a sketch really helps!)

10 Consider the diagram.

a Find the gradient of line AC and hence determine θ, correct to the nearest degree.

b Derive the equation of AC.

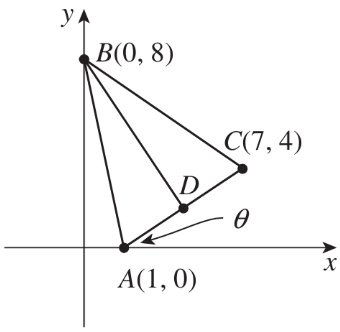
c Find the coordinates of the midpoint D of AC.

d Show that AC is perpendicular to BD.

e What type of triangle is ABC?

f Find the area of this triangle.

g Write the coordinates of a point E such that $ABCE$ is a rhombus.



a) $\frac{2}{3}$, $34°$

b) $2x - 3y - 2 = 0$

c) $D(4,2)$

d) $m_{AB} \times m_{BD} = \frac{2}{3} \times -\frac{3}{2} = -1$

e) isosceles

f) $A = \frac{1}{2} \times AC \times BD = \frac{1}{2}(\sqrt{52})(\sqrt{52}) = 26$

g) $E(8, -4)$