Student Name

Mathematics Stage 5 Paths

PROBABILITY B

Book 2 Use the language, 'if … then', 'given', 'of' and 'knowing that', to examine conditional statements and identify common mistakes in interpreting the language

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-PRO-P-01 solves problems involving Venn diagrams, 2-way tables and conditional probability

Use the language, 'if … then', 'given', 'of' and 'knowing that', to examine conditional statements and identify common mistakes in interpreting the language

- Calculate the probabilities of events where a condition restricts the sample space
- Describe the effect of a given condition on the sample space
- Identify conditional statements used in descriptions of chance situations
- Explain the validity of conditional statements when describing chance situations, referring to dependent and independent events
- Identify and explain common misconceptions relating to chance experiments

CONDITIONAL PROBABILITY

- **Conditional probability** is how likely one event is to happen after we know another event has already happened.

Investigation Conditional Probability

Jenny will win a game if she rolls an odd number on a die.

The sample space is the possible numbers.: $S = \{1, 2, 3, 4, 5, 6\}$

The event space is the odd numbers: $A = \{1, 3, 5\}$

As she rolls the die, she closes her eyes. Someone shouts out "It's a prime number!"

From this information, we can write:

The **reduced sample space** is prime numbers: $B = \{$ ……, ……, …… $\}$

The **reduced event space** is prime odd numbers: $A \cap B = \{$ ……, …… $\}$

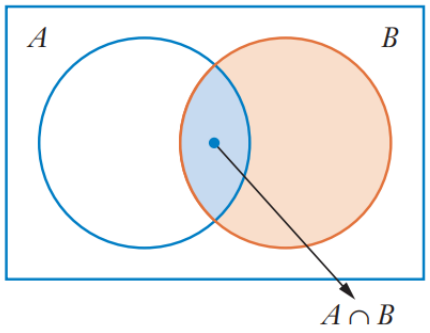
For her, what is the probability of winning?

$$P(\text{Jenny wins, given that the number is prime}) = \frac{\text{reduced event space}}{\text{reduced sample space}}$$

$$= \frac{\quad}{\quad}$$

The diagram shows conditional probability; $A|B$ "A given B:"

The sample space is reduced to B and the event space is reduced to $A \cap B$



- **Conditional probability** only applies if the two events are **dependent.**
- Language related to conditional probability include: 'if…then', 'given', 'knowing that'.

Conditional Probability Formula

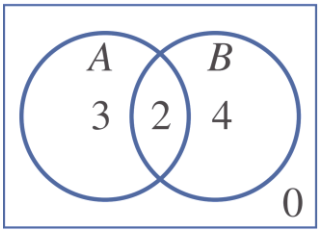
The probability of A, **given** B:

$$P(A|B) = \frac{A \cap B}{B} = \frac{P(A \cap B)}{P(B)}$$

Example Calculate conditional probability.

Consider this Venn diagram, displaying the number of elements in the events A and B.

Find the following probabilities.



$$P(A) = \frac{5}{9}$$

$$P(A \cap B) = \frac{2}{9}$$

$$P(A|B) = \frac{A \cap B}{B} = \frac{2}{6} = \frac{1}{3}$$

$$P(B|A) = \frac{A \cap B}{A} = \frac{2}{5}$$

A group of 15 hockey players all either played on the field or sat on the bench.
13 played on the field, 7 sat on the bench and 5 both played and sat on the bench.

Let A be the event 'the person played on the field'
Let B be the event 'the person sat on the bench'

Draw a Venn diagram of the situation.

Find the probability that a person only sat on the bench. $P(B \cap \bar{A})$

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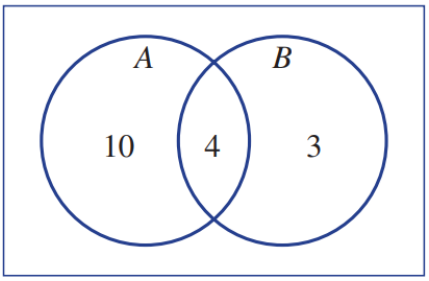
Find the probability that a person sat on the bench given that they played on the field $P(B|A)$.

5/13

Find the probability that the person played on the field given that they sat on the bench $P(A|B)$.

5/7

1 The following Venn diagram displays information about the number of elements associated with the events A and B. Find:



a P(A) ……

b P(A∩B) ……

c P(A| B) ……

d P(B| A) ……

2 Of a group of 20 English cricket fans at a match, 13 purchased a pie, 15 drank beer and 9 both purchased a pie and drank beer.
Let A be the event 'the fan purchased a pie'. Let B be the event 'the fan drank beer'.

a Represent the information in a Venn diagram.

b Find the probability that a fan in the group purchased only a pie (and did not drink beer).

……

c Find the probability that a fan in the group purchased a pie given that they drank beer.

……

d Find the probability that a fan in the group drank beer given that they purchased a pie.

……

3 In a particular class, 20 students study history, 15 study chemistry and 5 study both.
A student is chosen randomly.
What is the probability that they study chemistry, given that they study history?

4 Lauren is an athlete who enters a swimming and running race.
She has a 44% chance of winning the swimming race and a 37% chance of winning both races.
Find, to the nearest whole percentage, the probability that she wins the running race if she has won the swimming race.

5 Amy buys 4 tickets in a raffle in which 50 tickets are sold.
The tickets are drawn for first and second prizes. What is the probability that Amy wins:

a First prize?

b Second prize, given that she wins first prize?

c Second prize, given that she does not win first prize?

6 Two dice are rolled simultaneously. Find the probability that

a both die show a 5, given that one of the die showed a 5.

b the sum is a 6, given that one of the die showed a 5.

c a die showed a 5, given that the sum is 6.

7 In a certain population, 35% of people have blue eyes, 15% have blond hair, and 10% have blue eyes and blond hair. A person is chosen from this population at random.

a Find the probability that they have blond hair, given that they have blue eyes.

b Find the probability that they have blue eyes, given that they have blond hair.

8 A card is drawn from a standard deck of 52 playing cards. Find the probability that:

a the card is a king given that it is a heart.

b the card is a jack given that it is a red card.

c the card is a diamond given that it is a queen.

d the card is a black card given that it is an ace.

9 A number is chosen from the first 24 positive integers. Find the probability that:

a the number is divisible by 3 given that it is divisible by 4.

b the number is divisible by 6 given that it is divisible by 3.

10 Given that for two events A and B

a P(A) = 0.6, P(B) = 0.35, and P(B|A) = 0.5, find

i $P(A \cap B)$ **ii** $P(A|B)$

b P(A) = 0.7, P(B) = 0.4, and P(A∪B) = 0.8, find

i $P(A \cap B)$ **ii** $P(A|B)$

c P(A) = 0.6, P(B) = 0.4, and P(A|B) = 0.8, find

i $P(A \cap B)$ **ii** $P(B|A)$

11 In a sports club A, 60% of the members are male and 40% are female.
Of the male members, 20% prefer tennis, while 25% of the female members prefer tennis.
Let M represent 'males', F represent 'females', T represent preferring tennis and $\bar{T}$ represent not preferring tennis.

a Fill in the table.

	T	$\bar{T}$	Total
F			
M			
Total			

b A person is chosen at random.
What is the probability that they are a male and prefers tennis?

……

c A male is randomly selected. What is the probability that they prefer tennis?

……

d Consider parts **c** and **d,** which is asking about conditional probability?

……

12 2009 HSC Standard 2 Band 4

Some men and women were surveyed at a football game. They were asked which team they supported. The results are shown in the two-way table.

	Team A	*Team B*	*Totals*
Men	125	100	225
Women	75	90	165
Totals	200	190	390

What percentage of the women surveyed supported *Team B*, correct to the nearest percent?

55%

13 2005 HSC Standard 2 Band 4

On a television game show, viewers voted for their favourite contestant.
The results were recorded in the two-way table.

	Male viewers	**Female viewers**
Contestant 1	1372	3915
Contestant 2	2054	3269

One male viewer was selected at random from all of the male viewers.
What is the probability that he voted for Contestant 1?

$\frac{1372}{3426}$

14 HSC Sample Question Band 4

In a workplace of 25 employees, each employee speaks either French or German, or both.

If 36% of the employees speak German, and 20% speak both French and German.

Calculate the probability one person chosen could speak German if they could speak French. Give your answer to the nearest percent.

Calculate the probability one person chosen could not speak French if they could speak German. Give your answer to the nearest percent.

15 2022 HSC Advanced Band 5

In a bag there are 3 six-sided dice. Two of the dice have faces marked 1, 2, 3, 4, 5, 6. The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

What is the probability that the die shows a 5?

Given that the die shows a 5, what is the probability that it is the special die?

16 2017 VCE Band 5

For events A and B, $P(A|B) = \frac{1}{5}$ and $P(B|A) = \frac{1}{4}$. Let $P(A \cap B) = p$

Find $P(A)$ in terms of p.

Find $P(A' \cap B')$ in terms of p.

Given that $P(A \cup B) \leq \frac{1}{5}$, state the largest possible interval for p.