Student Name

Mathematics Stage 4

INDICES

Book 3 Use index notation to establish the index laws with positive-integer indices and zero index

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OVERVIEW

SYLLABUS CONTENT

MA4-INT-C-01 operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws

Use index notation to establish the index laws with positive-integer indices and the zero index

- Establish the multiplication, division and the power of a power index laws, by expressing each number in expanded form with numerical bases and positive-integer indices
- Verify through numerical examples that $(ab)^2 = a^2b^2$
- Establish the meaning of the zero index
- Apply index laws to simplify and evaluate expressions with numerical bases

SUMMARY

Index Laws

Law of Multiplication	Law of Division	Power of a Power	Power of Zero
$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$	$(a^m)^n = a^{m \times n}$	$a^0 = 1$

REVIEW OF PRIOR KNOWLEDGE

1 Write the following in index form.

a $5 \times 5 \times 5 \times 5$

b $2 \times 2 \times 2 \times 2 \times 2$

c $a \times a \times a \times a \times a$

2 Simplify these fractions.

a $\frac{24}{38}$

b $\frac{17}{34}$

c $\frac{44}{11}$

3 What operation are we doing to the numerator and denominator when we simplify fractions?

....

ESTABLISHING THE INDEX LAWS

Investigation Establish the index laws.

Index Law of Multiplication

$3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3)$

$= \ldots\ldots\ldots\ldots\ldots$

$2^3 \times 2^2 = (\ldots \times \ldots \times \ldots) \times (\ldots \times \ldots)$

$= \ldots\ldots\ldots\ldots\ldots$

English expression: *When multiplying two numbers with the ……… base, …… the powers.*

Algebraic expression: $a^m \times a^n = a^{m+n}$

Investigation Establish the index laws.

Index Law of Division

$2^5 \div 2^3 = \dfrac{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}}$

$= \ldots\ldots\ldots\ldots$

$3^6 \div 3^2 = \dfrac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$

$= \ldots\ldots\ldots\ldots$

English expression: *When dividing two numbers with the ……… base, …… the powers.*

Algebraic expression: ……………………………………………………………………………………

Investigation Establish the index laws.

Index Law of Powers

$(2^2)^3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

$= \dots\dots\dots\dots\dots\dots$

$(3^3)^4 = (\dots\dots\dots) \times (\dots\dots\dots) \times (\dots\dots\dots) \times (\dots\dots\dots)$

$= \dots\dots\dots\dots\dots\dots$

$(7^2)^5 = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots$

English expression: ....

Algebraic expression: ....

Investigation Establish the index laws.

Power of Zero

$2^3 =$	$3^3 =$	$(-2)^3 =$
$2^2 =$	$3^2 =$	$(-2)^2 =$
$2^1 =$	$3^1 =$	$(-2)^1 =$
$2^0 =$	$3^0 =$	$(-2)^0 =$

English expression: ....

Algebraic expression: ....

The index law of division can be used to show any base raised to the power of 0 is 1:

a $9^2 \div 9^2$	**b** $\dfrac{9^3}{9^3}$	**c** $\dfrac{(-5)^2}{(-5)^2}$

INDEX LAW OF MULTIPLICATION

Index Law of Multiplication
$a^m \times a^n = a^{m+n}$

Example Simplify expressions using the index law of multiplication.

$\mathbf{7^3 \times 7^5}$ $= 7^{3+5}$ $= 7^8$	When can we use the index law of multiplication? *When the ………………… are the same.*
$\mathbf{2^4 \times 2^3 \times 2}$ $= 2^{4+3+1}$ $= 2^8$	Where did the 1 in the index come from? *Any base number without an index has an index of ……*

Simplify these expressions. Do not evaluate the answer.

a $10^3 \times 10^4$

10^7

b $12^3 \times 12^6$

12^9

c 9×9^2

9^3

d $2^5 \times 2^4 \times 2^3$

2^{12}

Check your understanding

Determine whether you can use the index law of multiplication to simplify the expression.

a	$7^3 \times 7^2$	Yes | No	**e**	$6^3 \times 6^2$	Yes | No
b	$7^3 + 7^2$	Yes | No	**f**	$6^{-3} \times 6^{-2}$	Yes | No
c	$7^3 + 6^2$	Yes | No	**g**	$6^{-3} \times 6^2$	Yes | No
d	$7^3 \times 6^2$	Yes | No	**h**	$6^{0.5} \times 6^{2.5}$	Yes | No

Key ideas

1. Index law of multiplication: $8^6 \times 8^3 =$ ………
2. When multiplying two numbers with the same ……………, …………… the powers.

1 Write the following in expanded form:

a $5^2 = \ldots\ldots \times \ldots\ldots$
5×5

b $5^7 = \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots$
$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

c $5^2 \times 5^7 = \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots$
$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

d Write the answer to part **c** in index form: ………
5^9

e Does $5^2 \times 5^7 = 5^{2+7}$? …………………
yes

2 Simplify the following.

a $8^4 \times 8^{10} = 8^{\square + 10} = 8^{\square}$ 8^{14}	**b** $7^7 \times 7^2 = 7^{7 + \square} = 7^{\square}$ 7^9
c $9^7 \times 9^3 = 9^{\square + \square} = 9^{\square}$ 9^{10}	**d** $5^6 \times 5^{11} = 5^{\square + \square} = 5^{\square}$ 5^{17}

3 Simplify these expressions using the index law of multiplication.

a $3^5 \times 3^4$ 3^9	**b** $2^7 \times 2^5$ 2^{12}	**c** $7^2 \times 7^8$ 7^{10}	**d** $6^9 \times 6^4$ 6^{13}
e $5^4 \times 5$ 5^5	**f** $2^4 \times 2^4$ 2^8	**g** $8^9 \times 8^{12}$ 8^{21}	**h** 2×2^9 2^{10}
i $2^4 \times 2^6 \times 2^3$ 2^{13}	**j** $3^8 \times 3^3 \times 3^7$ 3^{18}	**k** $9^2 \times 9^5 \times 9^4$ 9^{11}	**l** $4^6 \times 4^3 \times 4$ 4^{10}

4 Simplify these expressions using the index law of multiplication.

a $5^5 \times 5^3 =$
5^8

b $5^5 \times 5^2 =$
5^7

c $5^2 \times 5^5 =$
5^7

d $5^2 \times 4^5 =$
Can't simplify

e $4^2 \times 4^5 =$
4^7

f $4 \times 4^5 =$
4^6

g $4^3 \times 4^5 =$
4^8

h $4^3 \times 4^{-5} =$
4^{-2}

i $4^{-3} \times 4^5 =$
4^2

j $4^{-3} \times 4^{-5} =$
4^{-8}

j $3^{-3} \times 3^{-5} =$
3^{-8}

k $3^3 \times 3^5 =$
3^8

l $(-3)^3 \times (-3)^5 =$
$(-3)^8$

m $3^{0.3} \times 3^{0.5} =$
$3^{0.8}$

n $3^{\frac{1}{3}} \times 3^{\frac{1}{5}} =$
$3^{\frac{8}{15}}$

o $3^a \times 3^a =$
3^{2a}

p $3^a \times 3^b =$
3^{a+b}

q $a^3 \times b^3 =$
$(ab)^3$

r $a^3 \times a^3 =$
a^6

k $\left(\frac{1}{a}\right)^3 \times \left(\frac{1}{a}\right)^3 =$
$\left(\frac{1}{a}\right)^6$

INDEX LAW OF DIVISION

Index Law of Division
$\frac{a^m}{a^n} = a^{m-n}$

Example Simplify expressions using the index law of division.

$7^8 \div 7^5$ $= 7^{8-5}$ $= 7^3$	When can we use the index law of division? *When the …………………are the same.*

Simplify these expressions. Do not evaluate the answer.

a $13^8 \div 13^5$ 13^3	**b** $\frac{7^{16}}{7^{12}}$ 7^4	**c** $3^{200} \div 3^{180}$ 3^{20}	**d** $\frac{89^{53}}{89^{52}}$ 89

Check your understanding

Determine whether you can use the index law of division to simplify the expression.

a	$11^3 \div 11^2$	Yes | No	**e**	$5^3 \div 5^2$	Yes | No
b	$11^3 - 11^2$	Yes | No	**f**	$5^{-3} \div 5^{-2}$	Yes | No
c	$11^3 \div 5^2$	Yes | No	**g**	$5^{-3} \div 5^2$	Yes | No
d	$11^3 \times 5^2$	Yes | No	**h**	$5^{0.5} \div 5^{2.5}$	Yes | No

Key ideas

1. Index law of division: $8^6 \div 8^3 = \ldots\ldots\ldots$
2. When dividing two numbers with the same ……………, …………… the powers.

1 Simplify each of the following by writing in expanded form.

a

$$4^7 \div 4^3 = \frac{4 \times 4 \times 4 \times \square \times \square \times \square \times \square}{4 \times 4 \times 4}$$
$$= 4^{\square}$$

4^4

b

$$9^6 \div 9^2 = \frac{9 \times 9 \times \square \times \square \times \square \times \square}{9 \times 9}$$
$$= 9^{\square}$$

9^4

c

$$5^8 \div 5^5 = \frac{5 \times 5 \times 5 \times 5 \times 5 \times \square \times \square \times \square}{5 \times 5 \times 5 \times 5 \times 5}$$
$$= \underline{\quad}^{\square}$$

5^3

2 Simplify the following.

a $3^7 \div 3^2 = 3^{\square}$ 3^5	**b** $7^5 \div 7^3 = 7^{\square}$ 7^2	**c** $2^8 \div 2^3 = 2^{\square}$ 2^5	**d** $9^8 \div 9^7 = 9^{\square}$ 9^1
e $3^5 \div 3^3$ 3^2	**f** $2^8 \div 2^5$ 2^3	**g** $5^{10} \div 5^4$ 5^6	**h** $4^9 \div 4^5$ 4^4
i $3^5 \div 3^1$ 3^4	**j** $5^6 \div 5^1$ 5^5	**k** $2^7 \div 2$ 2^6	**l** $10^4 \div 10$ 10^3

3 Simplify the following.

a $6^6 \div 6^3 =$

6^3

b $6^6 \div 6^2 =$

6^4

c $6^6 \div 5^2 =$

Can't simplify

d $5^6 \div 5^2 =$

5^4

e $5^2 \div 5^6 =$

5^{-4}

f $5^{-2} \div 5^6 =$

5^{-8}

g $5^2 \div 5^{-6} =$

5^8

h $5^{-2} \div 5^{-6} =$

5^4

i $5^{-6} \div 5^{-2} =$

5^{-4}

j $(-5)^6 \div (-5)^2 =$

$(-5)^4$

k $(-4)^6 \div (-5)^2 =$

Can't simplify

h $\dfrac{4^6}{4^2} =$

4^4

i $\dfrac{4^{0.6}}{4^{0.2}} =$

$4^{0.4}$

j $\dfrac{4^{\frac{1}{2}}}{4^{\frac{1}{6}}} =$

$4^{\frac{1}{3}}$

h $\dfrac{4^6}{4^6} =$

1

i $\dfrac{4^m}{4^m} =$

1

j $\dfrac{4^m}{4^p} =$

4^{m-p}

k $\dfrac{m^4}{p^4} =$

$\left(\dfrac{m}{p}\right)^4$

l $\dfrac{m^4}{m^6} =$

m^{-2}

m $\dfrac{4^{0.6}}{4^{0.2}} =$

$4^{0.4}$

n $\dfrac{4^6}{4^6} =$

$4^{\frac{1}{3}}$

INDEX LAWS OF POWERS

Index Law of Powers
$(a^m)^n = a^{m\times n}$

Example Simplify expressions using index laws of power of a power and zero.

$(\mathbf{7^3})^5$ $= 7^{3\times 5}$ $= 7^{15}$	When do we use the index law of a power of a power? *When raising a term with a ……………… to another power.*

Simplify these expressions. Do not evaluate the answer.

a $(7^5)^8$ 7^{40}	**b** $(4^5)^3$ 4^{15}	**c** $(4^7)^2$ 4^{14}	**d** $(125^3)^{25}$ 125^{75}

Key ideas

1. Index law of power of a power: $(8^6)^3 =$ ………

1 Simplify each of the following by writing in expanded form.

a $(4^4)^3 = (4 \times \_\_\_ \times \_\_\_ \times 4)^3$
$= (4 \times \_\_\_ \times \_\_\_ \times 4) \times (4 \times \_\_\_ \times \_\_\_ \times 4) \times (4 \times \_\_\_ \times \_\_\_ \times 4)$
$= 4^{\square}$

4^{12}

b $(8^3)^2 = (8 \times \_\_\_ \times \_\_\_)^2$
$= (8 \times \_\_\_ \times \_\_\_) \times (8 \times \_\_\_ \times \_\_\_)$
$= 8^{\square}$

8^6

c $(2^5)^3 = (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_)^3$
$= (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_) \times (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_) \times (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_)$
$= 2^{\square}$

2^{15}

d $(7^3)^5 = (7 \times 7 \times \_\_\_)^5$
$= (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_)$
$= 7^{\square}$

7^{15}

2 Simplify the following.

a $(4^4)^3 = 4^{\square}$

12

b $(8^3)^2 = 8^{\square}$

6

c $(2^5)^3 = 2^{\square}$

15

d $(7^3)^5 = 7^{\square}$

15

3 Simplify these expressions using the index law of power of a power.

a $(6^2)^5 =$

6^{10}

b $(3^2)^{\frac{1}{2}} =$

3

c $(6^3)^5 =$

6^{15}

d $\left(3^{\frac{1}{2}}\right)^2 =$

3

e $(6^4)^5 =$

6^{20}

f $\left(3^{\frac{1}{2}}\right)^{\frac{1}{2}} =$

$3^{\frac{1}{4}}$

g $(2^4)^5 =$

2^{20}

h $(x^2)^2 =$

x^4

i $(2^5)^4 =$

2^{20}

j $(x^2)^7 =$

x^{14}

k $(2^5)^1 =$

2^5

l $(2^7)^x =$

2^{7x}

m $(2^5)^{-1} =$

2^{-5}

n $(7^x)^2 =$

7^{2x}

o $(2^{-5})^{-1} =$

2^5

p $(x^2)^5 =$

x^{10}

q $(3^{-5})^{-1} =$

3^5

r $(x^2)^a =$

x^{2a}

4

a By evaluating both sides of the equation, show that $6^2 = 2^2 \times 3^2$.

$36 = 4 \times 9$

b Therefore, write 6^3 as a product of powers of 2 and 3.

$6^3 = 2^3 \times 3^3$

c Write 10^2 as a product of powers of 2 and 5.

$2^2 \times 5^2$

5 In general, $(ab)^2 = a^2 \times b^2$.

Use this rule to find the prime factorisation of these numbers.

a 6^4 $2^4 \times 3^4$	**b** 10^3 $2^3 \times 5^3$	**c** 14^2 $2^2 \times 7^2$
d 22^3 $2^3 \times 11^3$	**e** 20^3 $2^6 \times 5^3$	**f** 12^4 $2^8 \times 3^4$

INDEX LAW OF ZERO

Index Law of Zero
$a^0 = 1$

Example Simplify expressions using index laws of power of a power and zero.

14^0	$(7^3)^0$	6×6^0	$(-6)^0$	-6^0
$= 1$	$= 7^{3\times0}$ $= 7^0 = 1$	$= 6 \times 1$ $= 6$	$= 1$	$= -1$

Evaluate these expressions.

a 4^0 1	**b** 12^0 1	**c** $(91^{5412})^0$ 1	**d** $(11^0)^5$ 1
e $72 + 72^0$ 73	**f** 3×72^0 3	**g** $(-3)^0$ 1	**h** $-(3)^0$ -1

Key ideas

1. Index law of zero power: $8^0 =$ ………
2. ……………… tell you what part of an expression is raised to zero.

1 Simplify these expressions using the power of zero.

a $2^0 =$ 1

b $6^0 =$ 1

c $3^0 =$ 1

d $(-6)^0 =$ 1

e $4^0 =$ 1

f $-(6)^0 =$ -1

g $5^0 =$ 1

h $\left(\frac{1}{6}\right)^0 =$ 1

i $0^5 =$ 0

j $(0.6)^0 =$ 1

k $0^4 =$ 0

l $(6p)^0 =$ 1

m $0^0 =$ Undefined :0

n $6p^0 =$ 6

2 Simplify these expressions.

a $5^0 \times 13^0$ 1	**b** $4^0 + 7^0$ 2	**c** $11^0 - 6^0$ 0	**d** 18×7^0 18

3

a Use the index law for division to write $\frac{5^2}{5^2}$ as a power of 5.

$5^{2-2} = 5^0$

b If a number is divided by itself, what is the answer?

1, so $5^0 = 1$

4 Simplify these expressions.

a $(-5)^0$ 1	**b** -5^0 -1	**c** $6^0 - 5^0$ 0	**d** $6^0 - (-5)^0$ 0
e $6^0 + (-5)^0$ 2	**f** $(-6)^0 + (-5)^0$ 2	**g** $-6^0 + (-5)^0$ 0	**h** $-6^0 - 5^0$ -2

MIXED PRACTICE

Index Law of Multiplication	Index Law of Division
$a^m \times a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$

Index Law of Powers	Index Law of Zero
$(a^m)^n = a^{m \times n}$	$a^0 = 1$

Check your understanding

Determine whether these statements are true or false.

a	$x^2 \times x^3 = x^6$	True | False	**a**	$x^8 \div x^2 = x^4$	True | False
b	$x^4 \times x^5 = x^{20}$	True | False	**b**	$x^6 \div x^3 = x^3$	True | False
c	$x^3 \times x^6 = x^9$	True | False	**c**	$x^{27} \div x^3 = x^{24}$	True | False
d	$x \times x^7 = x^7$	True | False	**d**	$x^5 \div x^5 = x$	True | False
e	$x^5 \times x = x^6$	True | False	**e**	$x^4 \div x^4 = x^0 = 1$	True | False

Review of prior knowledge

1 Simplify the following expressions using index laws. Leave your answer in index form.

a $3^5 \times 3^7$	**b** 5^0	**c** $(-6)^0$
d $7^4 \div 7^2$	**e** $8^9 \times 8$	**f** $2^4 \times 2^3$
g $-(4^0)$	**h** 4^0	**i** -4^0
j $(4^3)^5$	**k** $(7^5)^2$	**l** $(3^2)^3$
m $12^5 \div 12^3$	**n** $3^3 \times 2^3$	**o** $7^{11} \div 7^9$

1 Simplify these expressions.

a $5^8 \times 5^2$ 5^{10}	**b** $6^7 \times 6^9$ 6^{16}	**c** $10^{11} \times 10^4$ 10^{15}	**d** $12^6 \div 12^2$ 12^4
e $8^7 \div 8^2$ 8^5	**f** $3^6 \div 3^1$ 3^5	**g** $(4^2)^3$ 4^6	**h** $(5^3)^6$ 5^{18}
i $(9^2)^{11}$ 9^{22}	**j** 7^0 1	**k** 12^0 1	**l** 5.6^0 1
m $2^{18} \div 2^{15}$ 2^3	**n** $(8^4)^3$ 8^{12}	**o** $(5^2)^{14}$ 5^{28}	**p** $\left(\frac{1}{4}\right)^0$ 1
q $4^9 \times 4^3$ 4^{12}	**r** $(59)^0$ 1	**s** $14^{10} \div 14^4$ 14^6	**t** $(136)^0$ 1
u $17^8 \div 17^5$ 17^3	**v** $12^7 \times 12^2$ 12^9	**w** $(6^8)^3$ 6^{24}	**x** $9^{15} \times 9^2$ 9^{17}

2 Determine whether you can use the index law of division to simplify these expressions.
If so, write the simplified form.

a $9^5 \div 9^3$ 9^2	**b** $9^4 \div 4^5$ no	**c** $6^4 \div 6$ 6^3	**d** $10^3 \div 10^{-1}$ 10^4
e $8^2 \div 8^2$ 1	**f** $5^4 - 5^3$ no	**g** $6^{-3} \div 6^2$ 6^{-5}	**h** $4^{1.5} \div 4^{0.5}$ 4

3 Determine whether you can use the index law of multiplication to simplify these expressions.
If so, write the simplified form.

a $5^4 \times 5^3$ 5^7	**b** $5^4 + 5^3$ no	**c** $5^4 \times 6^3$ no	**d** $2^7 \times 2^{-2}$ no
e $3^5 \times 3$ 3^6	**f** $8^2 + 8^4$ no	**g** $6^{-3} \times 6^2$ 6^{-5}	**h** $4^{1.5} \times 4^{0.5}$ 4^2

4 Write true or false for these statements.

a $(5^2)^{12} = 5^{24}$ T	**b** $(3^6)^2 = 3^8$ F	**c** $1^0 = 1$ T
d $(3^2)^{10} = 9^{20}$ F	**e** $3(4)^0 = 1$ F	**f** $0^3 = 0$ T

5 Fill in the missing indices so each of these expressions is true.

a $4^2 \times 4^{\square} = 4^7$ 5	**b** $7^{\square} \times 7 = 7^5$ 4	**c** $(-3)^4 \times (-3)^{\square} = (-3)^6$ 2
d $\frac{7^{\square}}{7^5} = 7^4$ 9	**e** $\frac{3^5}{3^{\square}} = 3^3$ 2	**f** $\frac{(-2)^{10}}{(-2)^{\square}} = (-2)^5$ 5

6 Complete these equations:

a $5^2 \times \ldots\ldots\ldots = 5^6$ 5^4	**b** $\ldots\ldots\ldots \div 3^4 = 3^3$ 3^7	**c** $2^7 \div \ldots\ldots\ldots = 2^{-1}$ 2^8

7 Find the missing value to make the equation true.

a $(5^3)^{\square} = 5^{15}$ 5	**b** $(2^{\square})^4 = 2^{12}$ 3
c $(3^2)^3 \times 3^{\square} = 3^{11}$ 5	**d** $(7^4)^{\square} \times (7^3)^2 = 7^{14}$ 2

8 Determine whether these equations are true or false.

a $3^7 \times 3^5 = 3^{12}$ T	**b** $3^7 \times 2^5 = 6^{12}$ F	**c** $3^7 \times 3^5 = 9^{12}$ F
d $5^8 \div 5^5 = 5^3$ T	**e** $6^7 \div 2^4 = 3^3$ F	**f** $5^9 \div 5^3 = 1^6$ F
g $2^5 \times 2^4 = 2^9$ T	**h** $2^5 \times 5^4 = 10^9$ F	**i** $2^5 \times 2^4 = 4^9$ F
j $6^5 \div 3^2 = 2^3$ F	**k** $2^6 \div 2^2 = 1^4$ F	**l** $3^9 \div 3^3 = 1^6$ F
m $4^6 \times 4^7 = 4^{13}$ T	**n** $4^6 \times 3^7 = 12^{13}$ F	**o** $4^6 \times 4^7 = 16^{13}$ F
p $8^{10} \div 2^4 = 4^6$ F	**q** $7^{12} \div 7^4 = 7^8$ T	**r** $3^9 \div 3^4 = 1^5$ F
s $6^{15} \div 6^8 = 6^7$ T	**t** $2^{10} \div 2^6 = 1^4$ F	**u** $4^7 \div 4^6 = 4$ T
v $5^4 \times 3^6 = 15^{10}$ F	**w** $7^5 \times 7^6 = 49^{11}$ F	**x** $10^7 \times 10^8 = 10^{15}$ T

9 Simplify the following using index laws.

a $(8^4)^2 \times 8^3$ $= 8^8 \times 8^{\square}$ $= 8^{\square}$ 8^{11}	**b** $10^9 \times (10^2)^6$ $= 10^9 \times 10^{\square}$ $= 10^{\square}$ 1)^21	**c** $(3^6)^5 \times 3^3$ 3^{33}
d $(7^4)^3 \times 7^2$ 7^{14}	**e** $(5^5)^2 \times 5^4$ 5^{14}	**f** $(9^2)^4 \times 9^3$ 9^{11}
g $8^5 \times (8^3)^2$ 8^{11}	**h** $6^2 \times (6^4)^4$ 6^{18}	**i** $10^3 \times (10^2)^5$ 10^{13}
j $7^6 \times (7^2)^5$ 7^{16}	**k** $(2^3)^2 \times (2^4)^3$ 2^{18}	**l** $(5^2)^4 \times (5^3)^2$ 5^{14}
m $(3^4)^5 \times (3^2)^4$ 3^{28}	**n** $(9^2)^5 \times (9^3)^4$ 9^{22}	**o** $(4^6)^2 \times (4^7)^3$ 4^{33}

10 Simplify the following using index laws.

a $3^6 \times 3^4 \div 3^5$ 3^5	**b** $2^5 \times 2^8 \div 2^6$ 2^7	**c** $7^{10} \times 7^8 \div 7^{16}$ 7^2	**d** $(5^6)^3 \div 5^{10}$ 5^8
e $(4^5)^2 \div 4^7$ 4^3	**f** $(3^3)^5 \div 3^7$ 3^8	**g** $5^8 \div 5^3 \times 5^4$ 5^9	**h** $3^6 \div 3^3 \times 3^4$ 3^7
i $2^{10} \div 2^4 \times 2^3$ 2^9	**j** $5^{12} \times 5^2 \div 5^4$ 5^{10}	**k** $(2^4)^3 \div 2^9$ 2^3	**l** $3^{10} \div 3^8 \times 3^5$ 3^7
m $2^6 \times 2^4 \div 2^7$ 2^3	**n** $(5^2)^3 \times (5^4)^2 \div 5^{11}$ 5^3	**o** $3^{20} \div 3^8 \div 3^7$ 3^5	**p** $7^{25} \div (7^3)^5$ 7^{10}

11 Simplify each of the following by combining index laws.

a $4 \times (4^3)^2$ 4^7

b $(3^4)^2 \times 3$ 3^9

c $7^8 \div (7^3)^2$ 7^2

d $(4^2)^3 \div 4^5$ 4

e $(3^6)^3 \div (3^5)^2$ 3^8

f $2^2 \times (2^2)^8 \div (2^3)^3$ 2^9

12 Simplify these expressions.

a $(2^4)^3 \times 2^2$ 2^{10}

b $(6^5)^2 \times 6^3$ 6^{13}

c $(8^2)^2 \times 8^4$ 8^8

d $(9^5)^0$ 1

e $(7^0)^2 \times 3$ 3

f $(8^0)^3 \times 4$ 4

g $(3^2)^0 \times (5^4)^0$ 1

h $(4^5)^2 \times (4^3)^3 \div (4^6)^2$ 4^7

i $(5^3)^2 \times (5^2)^0 \div (5^2)^3$ 1

j $(9^2)^3 \times (9^4)^2 \times 9^0$ 9^{14}

k $(7^6)^2 \div (7^3)^4 \times 7$ 7

l $(6^5)^4 \div (6^3)^3 \times (6^8)^0$ 6^{11}

13 Fill in the empty boxes to make the expressions equal.

a

$2^{\square} \times 3^{\square} \div 32$ $\quad$ $2^{\square} \times 3^{\square} \times 12$

$2^{\square} \times 3^{\square} \times 3/8$ $\quad$ $2^{\square} \times 3^{\square} \times 18$

$2^5 \times 3^4$

$2^{\square} \times 3^{\square} \div 24$ $\quad$ $2^{\square} \times 3^{\square} \times 72$

$2^{\square} \times 3^{\square} \div 54$ $\quad$ $2^{\square} \times 3^{\square} \times 108$

Clockwise from top left
$2^{10} \times 3^4 \div 32$
$2^3 \times 3^3 \times 12$
$2^4 \times 3^2 \times 18$
$2^2 \times 3^2 \times 72$
$2^3 \times 3^1 \times 108$
$2^6 \times 3^7 \div 54$
$2^8 \times 3^5 \div 24$
$2^8 \times 3^3 \times 3/8$

b

$2^{\square} \times 5^{\square} \div 50$ $\quad$ $2^{\square} \times 5^{\square} \times 100$

$2^{\square} \times 5^{\square} \times 5/8$ $\quad$ $2^{\square} \times 5^{\square} \times 160$

$2^7 \times 5^5$

$2^{\square} \times 5^{\square} \div 10{,}000$ $\quad$ $2^{\square} \times 5^{\square} \times 1000$

$2^{\square} \times 5^{\square} \div 1250$ $\quad$ $2^{\square} \times 5^{\square} \times 1280$

Clockwise from top left
$2^8 \times 5^7 \div 50$
$2^5 \times 5^3 \times 100$
$2^2 \times 5^4 \times 160$
$2^4 \times 5^2 \times 1000$
$2^{-1} \times 5^4 \times 1280$
$2^8 \times 5^9 \div 1250$
$2^{11} \times 5^9 \div 10000$
$2^{10} \times 5^4 \times 5/8$

14 Given that $2^{12} = 4096$ and $2^8 = 256$, find the number that makes this equation true:

$$256 \times ? = 4096$$

2^4, or 16

15 If $\frac{2^a}{2^b} = 8$, then what is $\frac{5^a}{5^b}$?

$2^{a-b} = 2^3$

$a - b = 3$,

so $5^{a-b} = 5^3 = 125$

16

a Use the fact that $(2^2)^3 = 2^6$ to simplify $((2^2)^3)^4$

2^{24}

b Simplify $((2^3)^4)^5$

2^{60}

c Arrange these numbers in ascending order.

$$2^{100}, (2^7)^{10}, ((2^5)^6)^7, ((2^3)^4)^5$$

$((2^3)^4)^5, (2^7)^{10}, 2^{100}, ((2^5)^6)^7$

17 Find the prime factorisation of the following without a factor tree by using index laws.

a 100 $= 10^2$ $= (2 \times 5)^2$ $= 2 \times 5 \times 2 \times 5$ $= 2^2 \times 5^2$	**a** 1000 $2^3 \times 5^3$	**b** 25^7 5^{14}	**c** 18^{30} $2^{30} \times 3^{60}$
b $4^6 \times 8^6$ 2^{30}	**d** $9^3 \times 18$ 2×3^8	**e** 12×2^{100} $2^{102} \times 3$	**f** $1\,000\,000^{100}$ $2^{600} \times 5^{600}$

18 Which two whole numbers, neither ending in zero, equal 1 000 000 000 when multiplied?
(hint: start by looking for numbers that multiply to 10, 100, 1000, etc.)

$2^9 \times 5^9$

EVALUATING EXPRESSIONS BY APPLYING INDEX LAWS

- To **evaluate** means to work out the final number by following the **order of operations.**
- Apply index laws according to the order of operations.

Example Evaluate expressions using index laws

$\mathbf{4 + 4^5 \div 4^3}$ $= 4 + 4^2$ $= 4 + 16$ $= 20$	Which index law did we use to get to the first line? *Index law of …………………* Which law did we use to get to the second line? *Index law of …………………*
$\mathbf{\frac{4^5 \times 4^3}{4^7}}$ $= \frac{4^8}{4^7}$ $= 4^1$ $= 4$	Which operation did we do first, and why? We did $4^5 \times 4^3$ first, as there are invisible ……………… enclosing the numerator. Explain why we subtract the powers when there is a fraction. *Fractions mean to ………………, so we use the index law of ……………… if the bases are the same.*

Evaluate by applying index laws.

a $5^2 \times (2^6 \div 2^3)$

$= 5^2 \times (\ldots\ldots\ldots\ldots)$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

200

b $\frac{5^7 \times 5^4}{5^{11}}$

1

c $(4^0 \times 3^4 \div 3^3)^2$

9

d $(2^5 \div 2^4 \times 3)^2$

36

1 Evaluate the following expressions, give your answer as a basic numeral.

a $7^7 \div 7^5$ 49	**b** $10^6 \div 10^5$ 10	**c** $101^5 \div 101^4$ 101
d $2^3 \times 2^2$ 32	**e** $3^2 \times 3$ 27	**f** $4^4 \div 4^4$ 1
g $200^{30} \div 200^{28}$ 40000	**h** $7 \times (31^{16} \div 31^{15})$ 217	**i** $3 \times (50^{200} \div 50^{198})$ 7500

2 Evaluate these using your knowledge of the zero index.

a 4^0 1	**b** $5^2 \div 4^0$ 25	**c** $10^3 \div 15^0$ 1000	**d** $4^6 \times 4^0$ 4096
e $(5^6)^0$ 1	**f** $(2^4)^0$ 1	**g** $(3^6)^0$ 1	**h** $(3^0)^6$ 1
i $-(x^0)^6$ -1	**j** $(a^0)^3$ 1	**k** $(q^5)^0$ 1	**l** $(5^6)^0$ 1

3 Evaluate these. An example is provided.

$$5m^0 + 7$$
$$= 5 \times 1 + 7 = 12$$

a $2v^0$ $= 2 \times 1 =$ 2	**b** $8h^0$ 8	**c** $h^0 + 7$ 8	**d** $7h^0 - 4$ 3
e x^0 1	**f** $2x^0$ 2	**g** $2x^0 + 2$ 4	**h** $x^0 + 2x^0$ 3
i n^0 1	**j** 4^0 1	**k** $4n^0$ 4	**l** $(4n)^0$ 1
m $4n^0 + 4$ 8	**n** $(n^0 + 4)^2$ 25	**o** $4n^0 + (4n)^0$ 5	**p** $4^0 n$ n

4 Study the example and simplify these.

$$(3x^0)^3 = (3\times1)^3 = (3)^3 = 27$$

a $(5n^0)^4 = (\square \times \square)^4 = \square^4 = \square$

625

b $(8p^0)^3 = (\square \times \square)^3 = \square^3 = \square$

512

c $(2h^0)^9 = (\square \times \square)^9 = \square^9 = \square$

512

d $(2h^0)^3$

8

e $(8k^0)^2$

64

f $(3x^0y^0)^4$

81

g $(2h^3)^0$

1

h $(8k^2)^0$

1

i $(3x^2y^4)^0$

1

5 Evaluate each of the following by combining index laws.

a $(2^4)^8 \div 2^{30}$

4

b $(10^3)^7 \div 10^{18}$

1000

c $((-1)^{11})^2 \times ((-1)^2)^{11}$

1

6

a According to the index laws, what would $(9^{0.5})^2$ be?

9

b Using what you got above, what is $9^{0.5}$?

3

c What would $16^{0.5}$ be?

4

d What would $25^{0.5}$ be?

5

e What would $36^{0.5}$ be?

6

f What would $x^{0.5}$ be?

$\sqrt{x}$

7 **a** Darius says that it is not possible to write 8×4 as a single power.

Beth says that you can think of each number of a power of 2.

Complete Beth's method.

$4 = 2^{\square}$ 2

$8 = 2^{\square}$ 3

$4 \times 8 = 2^{\square} \times 2^{\square} = 2^{\square}$ 2, 3, 5

b Write each of these as single power.

i 9×27 3^5	**ii** $16 \times 8 \times 8$ 2^{10}	**iii** $5 \times 25 \times 125$ 5^6
iv $100 \times 1000 \times 10\,000 \times 100$ 10^{11}	**v** $64 \times 128 \times 32 \times 8$ 2^{21}	

8 Look at Abdullah's method for writing 8^6 as a power of 2

$8^6 = (2^3)^6 = 2^{18}$

a Explain Abdullah's method.

He changed 8 to 2^3, then used index law of power of a power.

b Find the missing value in each of these expressions.

i $9^4 = 3^{\square}$ 8	**ii** $100^5 = 10^{\square}$ 10	**iii** $1000^3 = 10^{\square}$ 9	**iv** $125^4 = 5^{\square}$ 12

c Solve this equation.

$16^x = 2^{60}$

$2^{4x} = 2^{60}$

$x = 15$

9 Fill in the blanks.

a $4^8 \div 2^3 = (2^{\square})^8 \div 2^3$ $= 2^{\square} \div 2^3$ $= 2^{\square}$ 2^{13}	**b** $27^5 \div 9^4 = 3^{\square} \div 3^{\square}$ $= \square^{\square}$ 3^7

10 Given that $4 = 2^2$, simplify these expressions to a single smallest base number.

a 4×2

2^3

b $8 \times 4^2 \times 2^3$

2^{19}

c $3 \times 9 \times 27^2$

3^9

d $25^5 \div 5^3$

5^7

e $128^9 \div 256^2$

2^{47}

f $100000^{10} \div 10000^2$

10^{42}

11 Evaluate these

a $\dfrac{2^{1001} \times 2^{2002}}{(2^{150})^{20}}$

8

b $\dfrac{5^{1000} \times 3^{1001}}{15^{999}}$

45

c $\dfrac{8^{50} \times 4^{100} \times 2^{200}}{(2^{250})^2 \times 2^{48}}$

4

12 Evaluate the following expression.

$$-\left(\frac{2}{3}\right)^0 \times (-2)^2 \times ((2)^3)^3 \div (4)^2$$

-128

13 Using a calculator, evaluate $(2^3)^4$ and $(2^6)^2$. ....

4096

a Explain why they are the same

The bases are the same, and $3 \times 4 = 6 \times 2$

b Which of the following are also equal to $(2^3)^4$?

A $(2^4)^3$ **B** $(2^2)^6$ **C** $(4^2)^3$ **D** $(4^3)^2 \times (6^2)^2$

A, B, C

14 Fill in the blanks in as many ways as possible.

$$(2^5)^6 = (4^{\square})^{\square}$$

Many possible answers, as long as the two numbers multiply to make 30

15 Determine x

$$(5^7)^x = 5^x \times 5^4$$

$x = \frac{2}{3}$

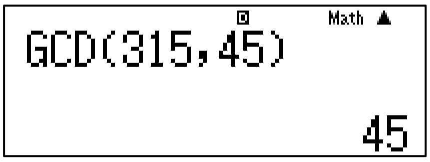
FINDING HCF AND LCM USING PRIME FACTORISATION

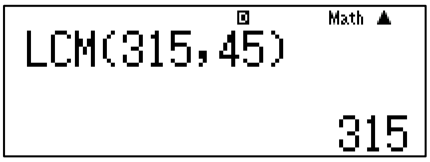
- Before we begin, we will introduce new notation to make writing multiplication neater.
- The dot product operator · will show multiplication from now on.

$$5 \times 7 \rightarrow 5 \cdot 7$$

- A calculator can find the HCF and LCM of two numbers. Use this to check answers.

FX-82 AU





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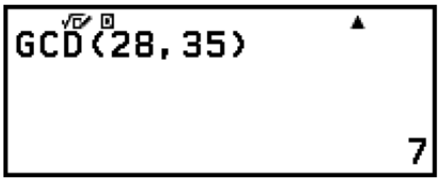
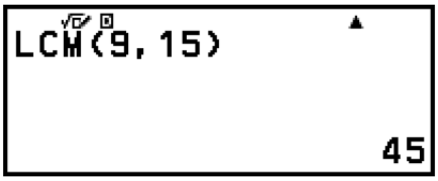
– [Numeric Calc] > [GCD]
28 ⬆) (,)35) EXE

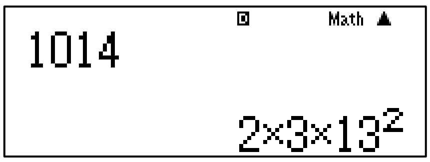


– [Numeric Calc] > [LCM]
9 ⬆) (,)15) EXE



A calculator can also find the prime factorisation of a number.

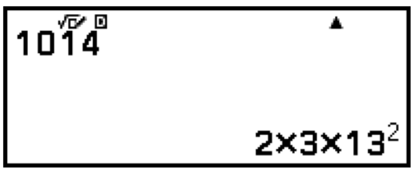
FX-82 AU



FX-8200 AU

1014 EXE

FORMAT – [Prime Factor]



Finding HCF (GCD)
GCD (or HCF) of two numbers is the **minimum** of the indices of their prime factorisations.

- To find the HCF of any two numbers:
  1. Express each number as a product of its prime factors
  2. List all unique prime factors - use a zero power for any prime not shared between numbers.
  3. Choose the smallest power for each prime factor and multiply them to find the HCF

Example Find HCF using prime factorisation.

Find the HCF of 294 and 364.	Rewriting with unique prime factors is not necessary, but useful. Why?
1. Prime factorisations $294 = 2^1 \cdot 3^1 \cdot 7^2$ $364 = 2^2 \cdot 7^2 \cdot 13^1$ 2. List unique prime factors $294 = 2^1 \cdot 3^1 \cdot 7^2 \cdot 13^0$ $364 = 2^2 \cdot 3^0 \cdot 7^2 \cdot 13^1$ 3. Choose smallest powers for each factor $294 = 2^1 \cdot 3^1 \cdot 7^2 \cdot 13^0$ $364 = 2^2 \cdot 3^0 \cdot 7^2 \cdot 13^1$ $\text{HCF}(294{,}364) = 2^1 \cdot 3^0 \cdot 7^2 \cdot 13^0$ $= 14$	*By lining up the same bases, we can easily identify the …………… of the indices.*

Determine the HCF, based on these prime factorisations.

a $735 = 3 \cdot 5 \cdot 7^2$

$98 = 2 \cdot 7^2$

49

b $360 = 2^3 \cdot 3^2 \cdot 5$

$700 = 2^2 \cdot 5^2 \cdot 7$

200

c $330 = 2 \cdot 3 \cdot 5 \cdot 11$

$27370 = 2 \cdot 5 \cdot 7 \cdot 17 \cdot 23$

10

d $720 = 2^4 \cdot 3^2 \cdot 5$

$1080 = 2^3 \cdot 3^3 \cdot 5$

360

e $3150 = 2 \cdot 3^2 \cdot 5^2 \cdot 7$

$4725 = 3^3 \cdot 5^2 \cdot 7$

1575

f $15470 = 2 \cdot 5 \cdot 7 \cdot 13 \cdot 17$

$17640 = 3 \cdot 5^2 \cdot 7 \cdot 17$

70

Finding LCM
LCM of two numbers is the **maximum** of the indices of their prime factorisations.

- To find the LCM of any two numbers:
  1. Express each number as a product of its prime factors
  2. List all unique prime factors - use a zero power for any prime not shared between numbers.
  3. Choose the largest power for each prime factor and multiply them to find the LCM

Example Find LCM using prime factorisation.

Find the LCM of 294 and 364.

$294 = 2^1 \cdot 3^1 \cdot 7^2 \cdot 13^0$

$364 = 2^2 \cdot 3^0 \cdot 7^2 \cdot 13^1$

$\text{LCM}(294{,}364) = 2^2 \cdot 3^1 \cdot 7^2 \cdot 13^1$

$= 7644$

Determine the LCM, based on these prime factorisations.

a $30 = 2 \cdot 3 \cdot 5$

$70 = 2 \cdot 5 \cdot 7$

210

b $39 = 3 \cdot 13$

$2366 = 2 \cdot 7 \cdot 13^2$

7098

c $735 = 3 \cdot 5 \cdot 7^2$

$98 = 2 \cdot 7^2$

1470

d $210 = 2 \cdot 3 \cdot 5 \cdot 7$

$294 = 2 \cdot 3 \cdot 7^2$

1470

e $1274 = 2 \cdot 7^2 \cdot 13$

$273 = 3 \cdot 7 \cdot 13$

3822

f $1547 = 7 \cdot 13 \cdot 17$

$4420 = 2^2 \cdot 5 \cdot 13 \cdot 17$

30940

1 Write each pair of numbers as their prime factors and hence determine the HCF and LCM. Check your answer using a calculator.

a 315 and 90 HCF: 45, LCM: 630	**b** 180 and 252 HCF: 36, LCM: 1260	**c** 900 and 1620 HCF: 180, LCM: 8100
d 756 and 1008 HCF: 252, LCM: 3024	**e** 1152 and 1728 HCF: 576, LCM: 3456	**f** 2450 and 3675 HCF: 1225, LCM: 7350

2 For the following questions, use a calculator to find the prime factorisation.

a Find the HCF and LCM of 123 552 and 234 360.

HCF: 216, LCM: 134 053 920

b Find the HCF and LCM of 2760, 4140, and 4830.

HCF: 690, LCM: 57 960

c Find the HCF and LCM of 1260, 1680, and 2520.

HCF: 420, LCM: 5040

3 Notice that

$2^3 \cdot 3 \cdot 5^4 = 15000$	with 3 trailing zeros.
$2^5 \cdot 5^2 \cdot 7 = 5600$	with 2 trailing zeros.
$2^7 \cdot 5^5 \cdot 11^2 = 48\,400\,000$	with 5 trailing zeros.

a How can you tell the number of trailing zeros from the prime factorisation? Explain why this works.

It is the number of factors of 10 (how many lots of 2×5 there are)
as multiplying a number by 10 has the effect of "adding" a trailing zero to a number.

b How many zeroes are on the end of $2^{100} \cdot 3^{40} \cdot 5^{80}$?

80

c Without using your calculator, what is the last *non-zero* digit of $2^7 \cdot 3^2 \cdot 5^5$?

Remove all factors of 10

$2^2 \times 3^2 = 36$

So, 6 is the last non-zero digit.

4

a Write 315 and 90 in index form.

$315 = 3^2 \cdot 5 \cdot 7$

$90 = 2 \cdot 3^2 \cdot 5$

b Find the LCM and HCF of 315 and 90. Answer in index form.

HCF: $3^2 \cdot 5$, LCM: $2 \cdot 3^2 \cdot \mathbf{5^2} \cdot 7$

c Find the product of the LCM and HCF. Answer in index form.

$2 \cdot 3^2 \cdot 5^2 \cdot 7$

d Find the product of 315 and 90. Answer in index form.

$2 \cdot 3^2 \cdot 5^2 \cdot 7$

e Explain, using your knowledge of index laws, why the $HCF \times LCM$ of two numbers is **always** equivalent to the product of the given numbers.

The product of two numbers is the sum of the indices of their prime factors.

The HCF is the minimum of the indices and LCM is the maximum of the indices.

If you multiply the HCF and LCM, then you are adding the maximum and the minimum indices of the original numbers.

Since there are only two numbers, the sum of the minimum and maximum is the same as the sum of the two numbers.

5 Explain why we can find the LCM of two numbers by multiplying them then dividing by the HCF.

For two numbers a and b, $HCF \times LCM = a \times b$

So, $LCM = a \times b \div HCF$

6 What is the smallest non-zero multiple of 2, 4, 7 and 8 which is a square?

LCM of these numbers is $2^3 \times 7$

So we need to add a factor of 2 and 7 to make it a perfect square.

$2^4 \times 7^2 = 784$

7 The HCF of two numbers, a and b, is 12. The LCM is 180.

a Explain why the two numbers can be written as $a = 12x$ and $b = 12y$, where x and y are coprime (co-prime means two whole numbers have no common factors)

Since the HCF is 12, both numbers must be multiples of 12.

x and y are coprime, because if they shared a factor, the HCF would not be 12.

b Explain why the product of the two numbers is 12×180.

As established earlier, the product of any two numbers is the product of the HCF and LCM

c Write an equation using parts **a** and **b**.

$ab = 2160$

$ab = 144xy$

d Solve your equation in part **c** for possible whole number values of x and y.

$144xy = 2160$

$xy = 15$

x and y could be: (1, 15) or (3, 5)

e Hence find the unknown numbers a and b (there are two possible pairs)

$a = 12, b = 180$ or

$a = 36, b = 60$

8 The HCF of two numbers is 15, and the LCM is 420. Find the two numbers. There are two possible pairs.

15, 420

60, 105

9 The HCF of two numbers is 21, and the LCM is 2310. Find the possible two numbers.

21, 2310

42, 1155

105, 462

210, 231

10 The HCF of two numbers is 504
The LCM of two numbers is 66 528
What could the two numbers be?

504, 66528

1512, 22176

2016, 16632

3024, 11088

5544, 6048

11 The GCD of three numbers is 30 and their LCM is 900.
Two of the numbers are 60 and 150.

a By finding the prime factorisations of the GCD, LCM, 60, and 150, explain why the third number must have a factor of 3^2.

$60 = 2^2 \cdot 3 \cdot 5$

$150 = 2 \cdot 3 \cdot 5^2$

GCD: $2 \cdot 3 \cdot 5$

LCM: $2^2 \cdot 3^2 \cdot 5^2$

Since the LCM has 3^2, the third number must have 3^2

b By considering the GCD and your answer to part **a**, find the possibilities for the third number (there are four).

$2 \cdot 3^2 \cdot 5 = 90$

$2^2 \cdot 3^2 \cdot 5 = 180$

$2 \cdot 3^2 \cdot 5^2 = 450$

$2^2 \cdot 3^2 \cdot 5^2 = 900$

c [alternate method]

i What is the LCM of 60 and 150?

$2^2 \cdot 3 \cdot 5^2 = 300$

ii What factor is missing to get an LCM of 900?

3

iii Hence explain why the third number must be a multiple of 90 and a factor of 900.

The third number must be a multiple of 90 to have the 3^2 factor

It must be a factor of 900 to ensure the LCM doesn't exceed 900

iv Therefore, find the possibilities for the third number.

$90, 180, 450, 900$

12 The GCD of three numbers is 84 and their LCM is 205 800.
Two of the numbers are 1176 and 2100. What could the third number be?

Third number must have factors of 7^3 and 3^1

It can have either $2^{2 \text{ or } 3}$ or $5^{0\ 1 \text{ or } 2}$

4116

8232

20 580

41 160

102 900

205 800

13 Mr Ding asked ChatGPT for a good exam question. It said:
"The GCD of three numbers is 42 and their LCM is 16 800.
Two of the numbers are 294 and 210. What could the third number be?"
Explain why this question is impossible.

HCF: $42 = 2 \cdot 3 \cdot 7$

LCM: $16\,800 = 2^5 \cdot 3 \cdot 5^2 \cdot 7$

$294 = 2 \cdot 3 \cdot 7^2$

$210 = 2 \cdot 3 \cdot 5 \cdot 7$

By looking at the prime factorisations, 16 800 is not divisible by 294,

so, it is impossible for the LCM of a set of numbers including 294 to be 16 800

CHECK YOUR UNDERSTANDING

Index law of multiplication and division

1	$4^5 \times 4^3$							
	a	16^8	**b**	16^{15}	**c**	4^{15}	**d**	4^8
2	$3^4 \times 3^3$							
	a	3^7	**b**	3^{12}	**c**	9^7	**d**	6^7
3	3×3^2							
	a	3^3	**b**	9^3	**c**	9^2	**d**	27
4	$5^{10} \div 5^5$							
	a	2	**b**	5^2	**c**	5^5	**d**	1^5
5	$\frac{2^{10}}{2^5}$							
	a	2^2	**b**	2^5	**c**	1^2	**d**	0^5
6	$\frac{2^7}{2^3}$							
	a	$2^{\frac{7}{3}}$	**b**	2^4	**c**	4	**d**	2^{10}
7	$\frac{2^4 \times 2^6}{2^2}$							
	a	2^{12}	**b**	2^5	**c**	2^8	**d**	None of these

Index law of power of a power

1	$(5^4)^3$							
	a	5^{12}	**b**	5^7	**c**	125^{12}	**d**	None of these
2	$(2^3)^4$							
	a	2^{12}	**b**	2^7	**c**	6^4	**d**	None of these
3	$(6^2)^3$							
	a	6^5	**b**	6^6	**c**	12^3	**d**	36^3
4	7^0							
	a	70	**b**	1	**c**	0	**d**	7
5	$(4 \times 4)^0$							
	a	16	**b**	0	**c**	1	**d**	4
6	$4^0 \times 4$							
	a	$\frac{1}{4}$	**b**	0	**c**	1	**d**	4
7	5×3^0							
	a	15^0	**b**	1	**c**	3	**d**	5