Student Name

Mathematics Stage 5 Paths

TRIGONOMETRY D

Book 2 Solve trigonometric equations using exact values and the relationships between supplementary and complementary angles

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-TRG-P-02 establishes and applies the properties of trigonometric functions and finds solutions to trigonometric equations

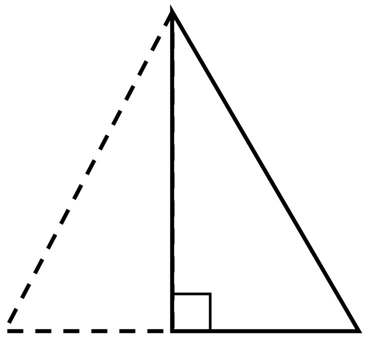
Solve trigonometric equations using exact values and the relationships between supplementary and complementary angles

- Derive and apply the exact sine, cosine and tangent ratios for angles of $30°$, $45°$ and $60°$
- Verify and use the relationships between the sine and cosine ratios of complementary angles in right-angled triangles: $\sin A = \cos(90° - A)$ and $\cos A = \sin(90° - A)$
- Find the possible acute and/or obtuse angles, given a trigonometric ratio
- Apply the sine rule and area rule to find angles involving the ambiguous case

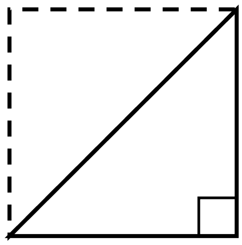
EXACT VALUES OF TRIGONOMETRIC RATIOS (ACUTE ANGLES)

Investigation Exact trigonometric values

Consider an equilateral triangle cut in half along its height.
Label the right-angled triangle.
Determine the exact values of $\sin 30°$, $\cos 30°$, $\tan 30°$, $\sin 60°$, $\cos 60°$, $\tan 60°$



Consider a unit square cut in half along the diagonal.
Label the right-angled triangle.
Determine the exact values of $\sin 45°$, $\cos 45°$, $\tan 45°$.



What type of right-angled triangle has a $45°$ internal angle? ....

What is the relationship between $\sin 45°$ and $\cos 45°$?

....

What is the relationship between $\sin 30°$ and $\cos 60°$?

....

What is the relationship between $\sin 60°$ and $\cos 30°$?

....

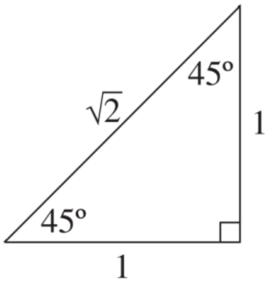
Using your calculator, check $\sin 10°$ and $\cos 80°$.

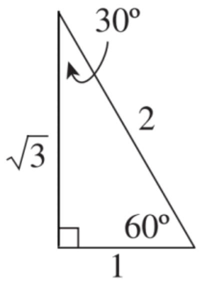
What can you say about complementary angles in sine and cosine functions?

....

....

EXACT VALUES OF TRIGONOMETRIC RATIOS





	0°	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

- The reason for the values for 0 degrees and 90 degrees will be covered when we redefine trigonometry using the unit circle.

1 Complete the table.

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

2 State the exact value of the following.

a $\cos 60°$ $\frac{1}{2}$	**b** $\tan 30°$ $\frac{1}{\sqrt{3}}$	**c** $\sin 60°$ $\frac{\sqrt{3}}{2}$	**d** $\cos 30°$ $\frac{\sqrt{3}}{2}$
e $\tan 60°$ $\sqrt{3}$	**f** $\sin 30°$ $\frac{1}{2}$	**g** $\cos 60°$ $\frac{1}{2}$	**h** $\tan 30°$ $\frac{1}{\sqrt{3}}$
i $\sin 30°$ $\frac{1}{2}$	**j** $\cos 30°$ $\frac{\sqrt{3}}{2}$	**k** $\tan 60°$ $\sqrt{3}$	**l** $\sin 60°$ $\frac{\sqrt{3}}{2}$
m $\cos 60°$ $\frac{1}{2}$	**n** $\tan 30°$ $\frac{1}{\sqrt{3}}$	**o** $\sin 30°$ $\frac{1}{2}$	**p** $\cos 30°$ $\frac{\sqrt{3}}{2}$

3 State the exact value of the following.

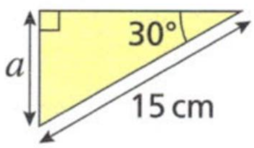
a $\cos 45°$ $\frac{1}{\sqrt{2}}$	**b** $\tan 0°$ 0	**c** $\sin 90°$ 1	**d** $\cos 0°$ 1
e $\tan 45°$ 1	**f** $\sin 45°$ $\frac{1}{\sqrt{2}}$	**g** $\cos 90°$ 0	**h** $\tan 90°$ undefined
i $\sin 0°$ 0	**j** $\tan 0°$ 0	**k** $\cos 45°$ $\frac{1}{\sqrt{2}}$	**l** $\sin 45°$ $\frac{1}{\sqrt{2}}$
m $\tan 45°$ 1	**n** $\sin 90°$ 1	**o** $\cos 0°$ 1	**p** $\cos 45°$ $\frac{1}{\sqrt{2}}$

4 State the exact value of the following.

a $\sin 0°$ 0	**b** $\cos 30°$ $\frac{\sqrt{3}}{2}$	**c** $\sin 45°$ $\frac{1}{\sqrt{2}}$	**d** $\cos 60°$ $\frac{1}{2}$
e $\sin 30°$ $\frac{1}{2}$	**f** $\tan 90°$ undefined	**g** $\sin 90°$ 1	**h** $\sin 30°$ $\frac{1}{2}$
i $\tan 60°$ $\sqrt{3}$	**j** $\cos 45°$ $\frac{1}{\sqrt{2}}$	**k** $\cos 60°$ $\frac{1}{2}$	**l** $\tan 0°$ 1
m $\tan 0°$ 0	**n** $\cos 0°$ 1	**o** $\cos 90°$ 0	**p** $\cos 90°$ 0
q $\tan 30°$ $\frac{1}{\sqrt{3}}$	**r** $\tan 45°$ 1	**s** $\tan 60°$ $\sqrt{3}$	**t** $\sin 90°$ 1
u $\tan 45°$ 1	**v** $\cos 45°$ $\frac{1}{\sqrt{2}}$	**w** $\sin 60°$ $\frac{\sqrt{3}}{2}$	**x** $\tan 90°$ undefined
q $\tan 60°$ $\sqrt{3}$	**r** $\sin 30°$ $\frac{1}{2}$	**s** $\cos 45°$ $\frac{1}{\sqrt{2}}$	**t** $\tan 60°$ $\sqrt{3}$

5 Find the lengths of the sides labelled with letters in exact form.

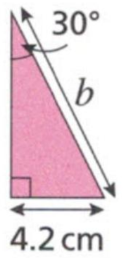
a



$\sin 30° = \frac{1}{2} = \frac{a}{15}$

$a = 7.5$ cm

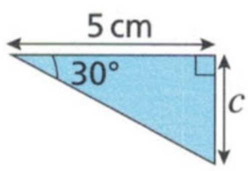
b



$\sin 30° = \frac{1}{2} = \frac{4.2}{b}$

$b = 8.4$ cm

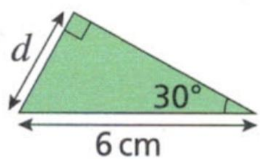
c



$\tan 30° = \frac{1}{\sqrt{3}} = \frac{c}{5}$

$c = \frac{5}{\sqrt{3}}$ cm

d

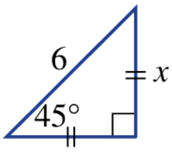


$\sin 30° = \frac{1}{2} = \frac{d}{6},$

$d = 3$ cm

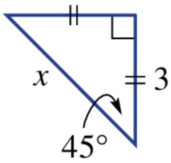
6 Find the exact value of x in each diagram. No calculators required.

a



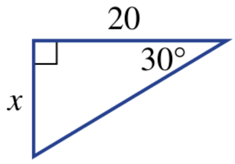
$3\sqrt{2}$

b



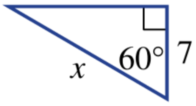
$3\sqrt{2}$

c



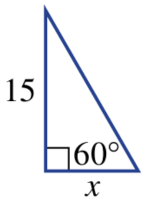
$\frac{20\sqrt{3}}{2}$

d



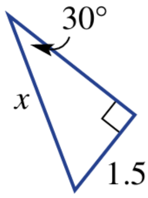
14

e



$5\sqrt{3}$

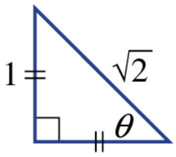
f



3

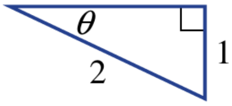
7 Find the exact value of θ in each diagram. No calculators required.

a



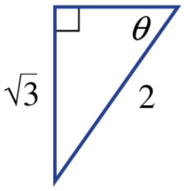
45°

b



30°

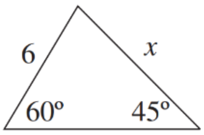
c



60°

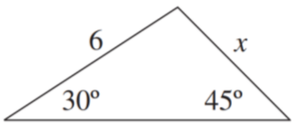
8 Find the exact value of x in each diagram. No calculators required.

a



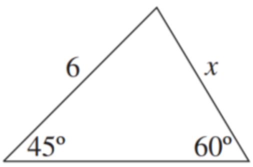
$3\sqrt{6}$

b



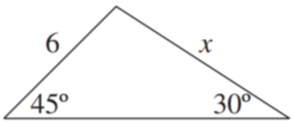
$3\sqrt{2}$

c



$2\sqrt{6}$

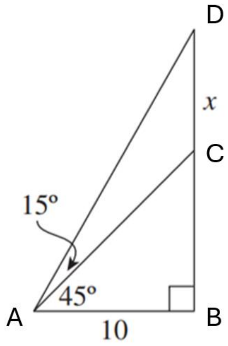
d



$6\sqrt{2}$

9

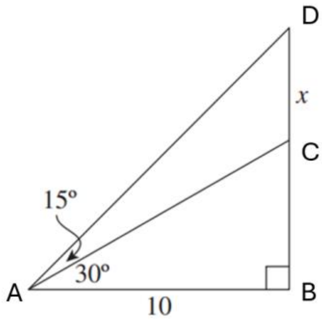
a



Show that $x = 10(\sqrt{3} - 1)$.

$$\begin{aligned}\tan 60° &= \frac{BD}{10}\\ BD &= 10\tan 60°\\ &= 10\sqrt{3}\\ \tan 45° &= \frac{BC}{10}\\ BC &= 10\tan 45°\\ &= 10\\ x &= BD - BC\\ &= 10\sqrt{3} - 10\\ &= 10(\sqrt{3} - 1)\end{aligned}$$

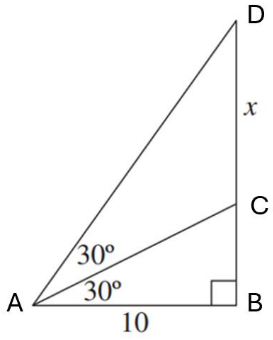
b



Show that $x = \frac{10}{3}(3 - \sqrt{3})$

$$\begin{aligned}\tan 45° &= \frac{BD}{10}\\ BD &= 10\tan 45°\\ &= 10\\ \tan 30° &= \frac{BC}{10}\\ BC &= 10\tan 30°\\ &= \frac{10}{\sqrt{3}}\\ x &= BD - BC\\ &= \frac{10}{3}(3 - \sqrt{3})\end{aligned}$$

c



Show that $x = \frac{20}{3}\sqrt{3}$

$$\begin{aligned}\tan 60° &= \frac{BD}{10}\\ BD &= 10\tan 60°\\ &= 10\sqrt{3}\\ \tan 30° &= \frac{BC}{10}\\ BC &= 10\tan 30°\\ &= \frac{10}{\sqrt{3}}\\ x &= BC - BC\\ &= 10\sqrt{3} - \frac{10}{\sqrt{3}}\\ &= 10\sqrt{3} - \frac{10\sqrt{3}}{3}\\ &= \frac{20\sqrt{3}}{3}\end{aligned}$$

10 Find the exact value of:

a $\sin 45° + \cos 30°$

$\frac{\sqrt{2} + \sqrt{3}}{2}$

b $\cos 45° + \sin 60°$

$\frac{\sqrt{2} + \sqrt{3}}{2}$

c $\sin 45° \cos 45° + \sin 30°$

1

d $\sin 60° \cos 30° - \cos 60° \sin 30°$

$\frac{1}{2}$

e $1 + \tan^2 60°$

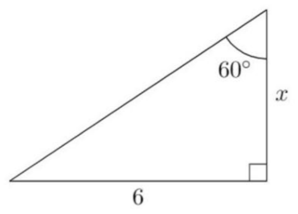
4

f $\frac{1}{\sin^2 30°} - \frac{1}{\tan^2 30°}$

1

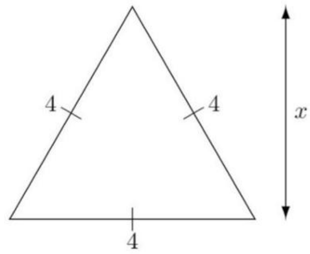
11 Find the exact value of x in these diagrams.

a



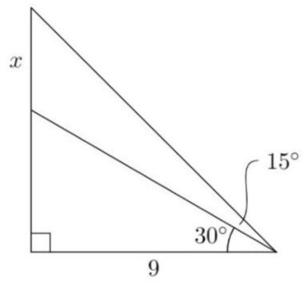
$2\sqrt{3}$

b



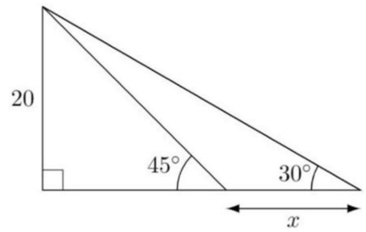
$2\sqrt{3}$

c



$9 - 3\sqrt{3}$

d



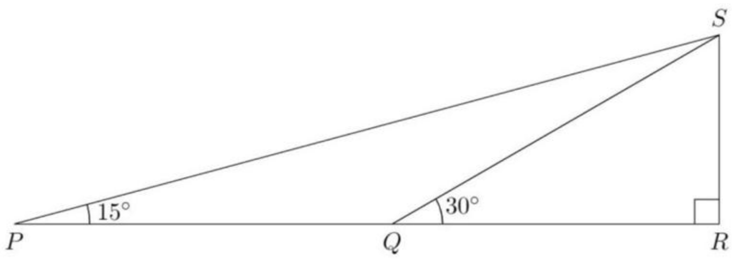
$20\sqrt{3} - 20$

12 Consider the diagram.

a Show that $PQ = QS$

b Given that the length of SR is 1 unit, write the lengths of QR, QS, and PQ

c Deduce that $\tan 15° = 2 - \sqrt{3}$



a) $\angle PQS = 150°$ (angles on a straight line)

$\angle PSQ = 180 - 150 - 15 = 15°$

$\therefore$ PQ = QS (sides opposite equal angles in isosceles triangles equal)

b) $\sin 30° = \frac{1}{2} = \frac{SR}{SQ} = \frac{1}{SQ}, \therefore SQ = 2, PQ = 2$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{1}{QR}, \therefore QR = \sqrt{3}$

c) $\tan 15° = \frac{SR}{PQ} = \frac{1}{2+\sqrt{3}} \frac{\times 2 - \sqrt{3}}{\times 2 - \sqrt{3}} = 2 - \sqrt{3}$

13 Show that:

a $2\sin 30°\cos 30° = \sin 60°$

$$LHS = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = RHS$$

b $\cos^2 60° - \cos^2 30° = -\frac{1}{2}$

$$LHS = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2} = RHS$$

c $\sin 60°\cos 45° - \cos 60°\sin 45° = \frac{\sqrt{6}-\sqrt{2}}{4}$

$$LHS = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \frac{\times \sqrt{2}}{\times \sqrt{2}} = \frac{6-\sqrt{2}}{4} = RHS$$

d $\frac{\tan 60° - \tan 45°}{1 + \tan 60°\tan 45°} = 2 - \sqrt{3}$

$$LHS = \frac{\sqrt{3}-1}{1+\sqrt{3}} \frac{\times 1-\sqrt{3}}{\times 1-\sqrt{3}} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3} = RHS$$

e $\frac{1}{\frac{1}{\cos 30°} - \tan 30°} = \sqrt{3}$

$$LHS = \frac{1}{\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3} = RHS$$

f $\frac{1+\tan 30°}{1-\tan 30°} = 2+\sqrt{3}$

$$LHS = \frac{\left(1+\frac{1}{\sqrt{3}}\right)}{\left(1-\frac{1}{\sqrt{3}}\right)} = \frac{\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)}{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \frac{\times(\sqrt{3}+1)}{\times(\sqrt{3}+1)} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3} = RHS$$

14 For this question, use the identity $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

a Let $t = \tan 15°$. Show that $\frac{2t}{1-t^2} = \frac{1}{\sqrt{3}}$

b Show that $t^2 + 2\sqrt{3}t - 1 = 0$

c Hence show that $\tan 15° = 2 - \sqrt{3}$

d Use a similar technique to find the exact value for $\tan 22.5°$.

a) Let $\theta = 15°$, $\quad \tan 30° = \frac{2\tan 15°}{1-\tan^2 15°} = \frac{1}{\sqrt{3}}$

$\therefore \frac{2t}{1-t^2} = \frac{1}{\sqrt{3}}$

b) cross multiply

$2\sqrt{3}t = 1 - t^2$

$\therefore t^2 + 2\sqrt{3}t - 1 = 0$

c) using quadratic for t

$t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$= -\sqrt{3} \pm 2$

$\therefore \tan 15° = -\sqrt{3} + 2$, (ignore negative as $\tan 15° > 0$)

d) $\sqrt{2} - 1$

15 For this question, use the identity $\cos 2\theta = 1 - 2\sin^2\theta$

a Let $s = \sin 15°$. Show that $s = \frac{\sqrt{2-\sqrt{3}}}{2}$

b Expand and simplify $\frac{(\sqrt{3}-1)^2}{2}$

c Hence, show that $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

a) Let $\theta = 15°$, $\quad \cos 30° = \frac{\sqrt{3}}{2} = 1 - 2\sin^2 15°$

$\frac{\sqrt{3}}{2} = 1 - 2s^2$

$2s^2 = 1 - \frac{\sqrt{3}}{2}$

$2s^2 = \frac{2-\sqrt{3}}{2}$

$s^2 = \frac{2-\sqrt{3}}{4}$

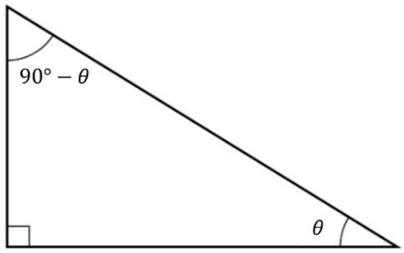
$s = \frac{\sqrt{2-\sqrt{3}}}{2}$ (ignore negative as $\sin 15° > 0$)

b) $\frac{(\sqrt{3}-1)^2}{2} = \frac{3-2\sqrt{3}+1}{2} = 2-\sqrt{3}$

c) $s^2 = \frac{2-\sqrt{3}}{4} = \frac{\left(\frac{(\sqrt{3}-1)^2}{2}\right)}{4} = \frac{(\sqrt{3}-1)^2}{8}$

$\therefore s = \frac{\sqrt{3}-1}{\sqrt{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

COMPLEMENTARY ANGLE IDENTITIES (SIN AND COS)

Trigonometric Ratios for Complementary Angles	
For an acute angle θ: $\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$	

- **Note**: Cosine is short for "complement of sine"

Example Apply the trigonometric ratios for complementary angles.

Find acute angle x such that:

$\sin 40° = \cos x$ $\sin 40° = \cos 50°$	$\cos 52° = \sin x$ $\cos 52° = \sin 38°$

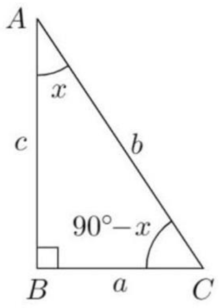
State the value of θ.

a $\sin 20° = \cos\theta$ 70°	**b** $\sin 85° = \cos\theta$ 5°	**c** $\cos 71° = \sin\theta$ 19°

1 θ is acute. Find its value.

a $\sin 10° = \cos\theta$ $80°$	**b** $\sin 28° = \cos\theta$ $62°$	**c** $\cos 54° = \sin\theta$ $36°$
d $\sin 59° = \cos\theta$ $31°$	**e** $\cos 73° = \sin\theta$ $17°$	**f** $\cos 83° = \sin\theta$ $7°$

2 Use the diagram to write a ratio for the following:



a $\sin x$ $\frac{a}{b}$	**b** $\cos x$ $\frac{c}{b}$	**c** $\tan x$ $\frac{a}{c}$
d $\sin(90-x)$ $\frac{c}{b}$	**e** $\cos(90-x)$ $\frac{a}{b}$	**f** $\tan(90-x)$ $\frac{c}{a}$

3 What is the relationship between $\tan x$ and $\tan(90-x)$?

They are reciprocals of each other.

4 Simplify without a calculator.

a $\cos 22° - \sin 68°$ 0	**b** $\frac{-\sin 57°}{\cos 53°}$ -1	**c** $\frac{\cos 28° + \sin 62°}{\sin 62°}$ 2
d $\frac{\sin 20° + \cos 70°}{\sin 20°}$ 2	**e** $\frac{\cos 42° + 2\sin 48°}{5\sin 48°}$ $\frac{3}{5}$	**f** $\sin 20° \cos 70°$ $\sin^2 20°$

5 For this question, simplify using the fact that $\tan(90-x) = \frac{1}{\tan x}$

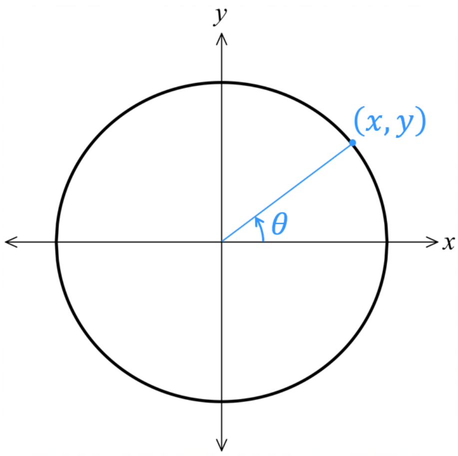
a $\tan 20° + \frac{1}{\tan 70°}$ $2\tan 20°$	**b** $\tan 30° \tan 60°$ 1	**c** $\tan 35° - \frac{1}{\tan 55°}$ 0

ANGLES OF ANY MAGNITUDE

- All angles of inclination θ can be written as an acute reference angle A.
- **We can express any trigonometric ratio using an acute angle.**

First Quadrant

$\theta = A$



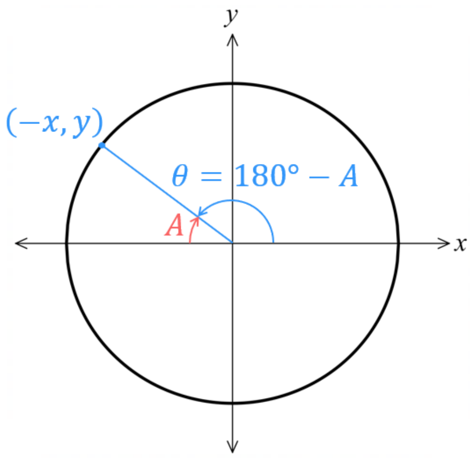
$\sin\theta = \sin A$

$\cos\theta = \cos A$

$\tan\theta = \tan A$

Second Quadrant

$\theta = 180 - A$



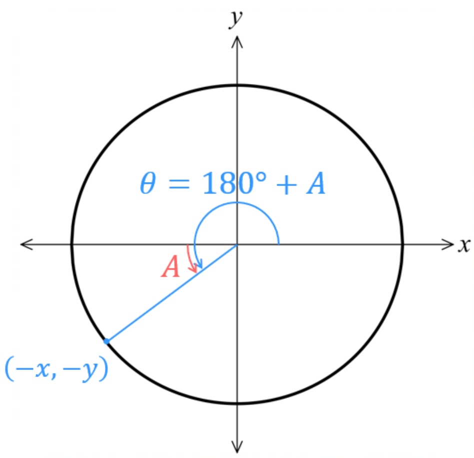
$\sin(180 - A) = \sin A$

$\cos(180 - A) = -\cos A$

$\tan(180 - A) = -\tan A$

Third Quadrant

$\theta \;=\; 180 + A$



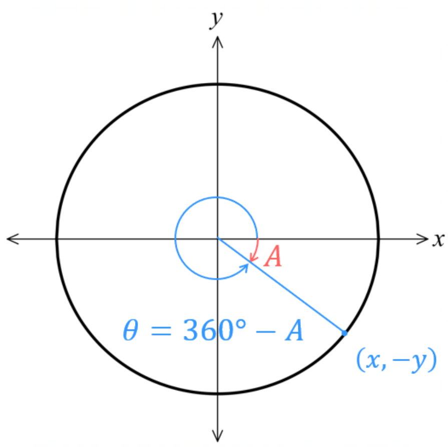
$\sin(180 + A) = -\sin A$

$\cos(180 + A) = -\cos A$

$\tan(180 + A) = \tan A$

Fourth Quadrant

$\theta \;=\; 360 - A$



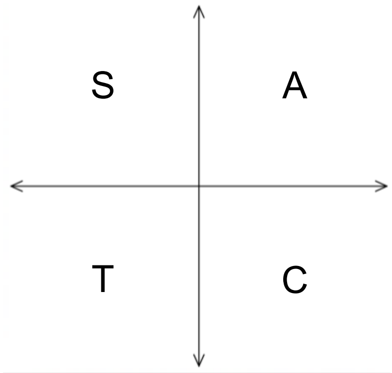
$\sin(360 - A) = -\sin A$

$\cos(360 - A) = \cos A$

$\tan(360 - A) = -\tan A$

- We can memorise the relationships on the previous page using the mnemonic:

ASTC
All positive in 1st quadrant.
Sin positive in 2nd quadrant; others are negative.
Tan positive in 3rd quadrant; others are negative.
Cos positive in 4th quadrant; others are negative.



Quadrant 1	**Quadrant 2**	**Quadrant 3**	**Quadrant 4**
$\theta = 180 - A$ $A = 180 - \theta$	$\theta = 180 + A$ $A = \theta - 180$	$\theta = 180 + A$ $A = \theta - 180$	$\theta = 360 - A$ $A = 360 - \theta$
All positive	**S**in positive, others negative	**T**an positive, others negative	**C**os positive, others negative

1 In which quadrants are these statements true?

a sin θ is positive — quadrants 1 and 2

b tan θ is negative — 2 and 4

c cos θ is negative — 2 and 3

d cos θ is positive — 1 and 4

e tan θ is positive — 1 and 3

f sin θ is negative — 3 and 4

2 Use ASTC to determine whether these trigonometric ratios are positive or negative.

a $\sin 20°$ — p

b $\cos 50°$ — p

c $\cos 100°$ — n

d $\tan 140°$ — n

e $\tan 250°$ — p

f $\sin 310°$ — n

g $\sin 200°$ — n

h $\cos 280°$ — p

i $\sin 340°$ — n

j $\cos 350°$ — p

k $\tan 290°$ — n

l $\cos 190°$ — n

m $\tan 170°$ — n

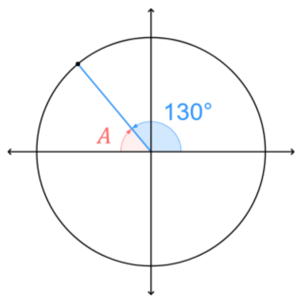
n $\sin 110°$ — p

o $\tan 80°$ — p

p $\cos 170°$ — n

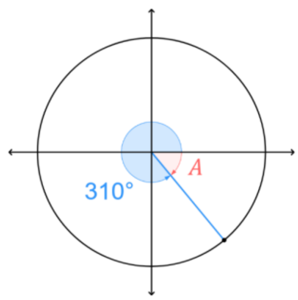
3 Rewrite these trigonometric ratios in terms of its acute reference angle.

a $\tan 130°$



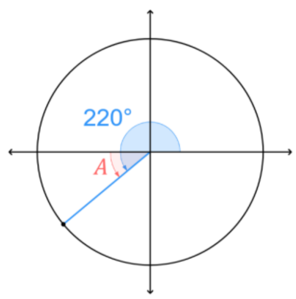
−tan 50°

b $\cos 310°$



cos 50°

c $\sin 220°$



−sin 40°

d $\tan 210°$ — tan 30°

e $\tan 315°$ — −tan 45°

f $\cos 240°$ — −cos 60°

g $\tan 260°$ — tan 80°

h $\cos 170°$ — −cos 10°

i $\sin 320°$ — −sin 40°

j $\cos 185°$ — −cos 5°

k $\sin 125°$ — sin 55°

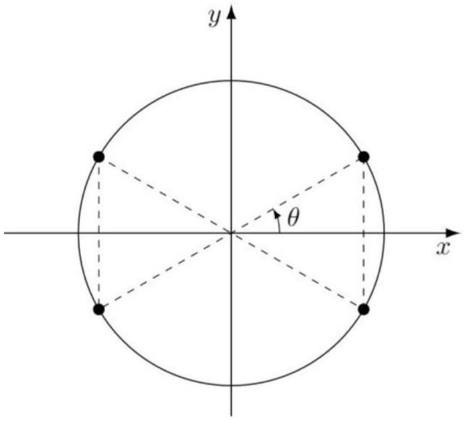
l $\tan 325°$ — −tan 35°

m $\sin 85°$ — sin 85°

n $\cos 95°$ — −cos 85°

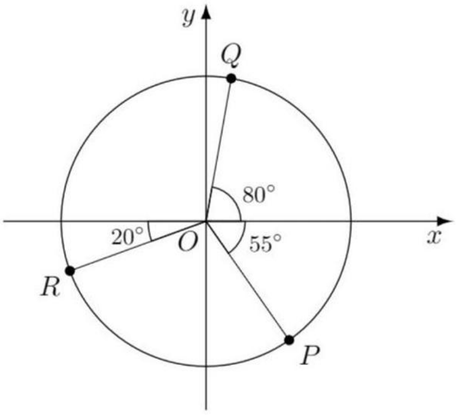
o $\tan 205°$ — tan 25°

4 Use the diagram to write these trigonometric ratios in terms of the acute angle, θ.



a $\sin(180-\theta)$ sin θ	**b** $\cos(180-\theta)$ -cos θ	**c** $\tan(180-\theta)$ -tan θ
d $\sin(180+\theta)$ -sin θ	**e** $\cos(180+\theta)$ -cos θ	**f** $\tan(180+\theta)$ tan θ
g $\sin(360-\theta)$ -sin θ	**h** $\cos(360-\theta)$ cos θ	**i** $\tan(360-\theta)$ -tan θ
j $\sin(-\theta)$ -sin θ	**k** $\cos(-\theta)$ cos θ	**l** $\tan(-\theta)$ -tan θ

5 The points P, Q, R, S are on the unit circle. $A(1,0)$ and O is the origin.
Find the coordinates of P, Q, R, S.
Leave your answer in terms of $\cos\theta$ and $\sin\theta$, where θ is the acute reference angle.



P (cos 55°, −sin 55°)
Q (cos 80°, sin 80°)
R (−cos 20°, −sin 20°)

6 Simplify:

a $\dfrac{\sin(180°+\beta)}{\cos(360°-\beta)}$ $-\tan\beta$	**b** $\dfrac{\cos(180°-\beta)}{\sin(90°-\beta)}$ -1	**c** $\sin(180°+\beta)\cos(90°-\beta)$ $-\sin^2\beta$

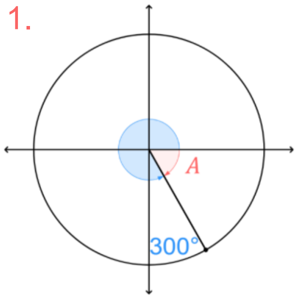
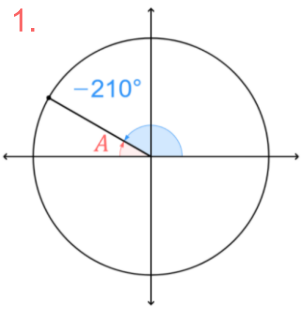
7 Given that sin 25° ≈ 0.42 and cos 25° ≈ 0.91, write down approximate values, without using a calculator, for:

a $\sin 155°$ $=\sin(180°-25°)$ $=\sin(\quad°)$ $=$ 0.42	**b** $\cos 205°$ $=\cos(180°+25°)$ $=-\cos(\quad°)$ $=$ −0.91	**c** $\cos 335°$ 0.91
d $\sin 335°$ −0.42	**e** $\sin 205° - \cos 155°$ 0.49	**f** $\cos 385° - \sin 515°$ 0.49

EXACT VALUES OF ANY TRIGONOMETRIC RATIO

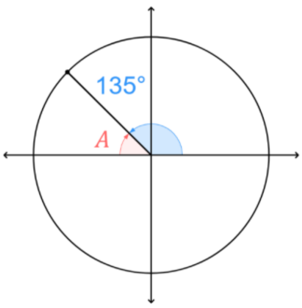
- To find the exact value of trigonometric ratios in any quadrant:
  1. Determine the quadrant and sketch the angle.
  2. Rewrite trig ratio with acute reference angle and the correct sign.
  3. Write with exact value.

Example Find exact value of a trigonometric function (all quadrants).

Find the exact value of $\tan 300\,°$ 1.  $\tan 300°$ 2. $= -\tan 60°$ 3. $= -\sqrt{3}$	How did we know the reference angle is 60? *Using angle sum of a …………………* How did we know the answer would be negative? *300° is in the ……th quadrant. Using ASTC, tan is negative.*
Find the exact value of $\sin(-210)\,°$ 1.  $\sin(-210°)$ $= \sin 150°$ 2. $= \sin 30°$ 3. $= \frac{1}{2}$	How did we know $\sin(-210°) = \sin 150°$? *By adding ………°* How did we know the reference angle is 30? *Angles on a ……………-………… sum to 180*

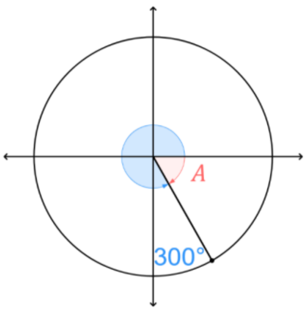
Find the exact value of these trigonometric functions.

a $\sin 135°$



$\frac{1}{\sqrt{2}}$

b $\cos 300°$



$\frac{1}{2}$

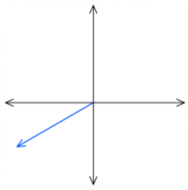
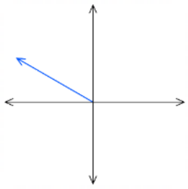
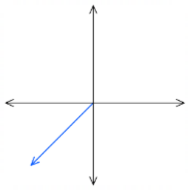
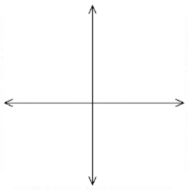
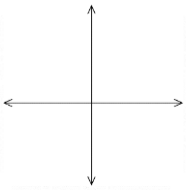
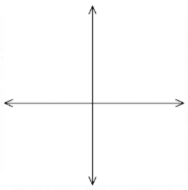
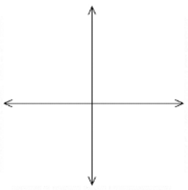
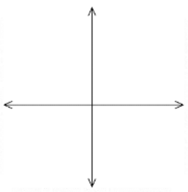
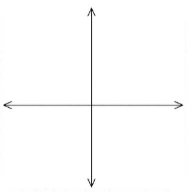
c $\tan 240°$

$\sqrt{3}$

d $\sin(-60)°$

$-\frac{\sqrt{3}}{2}$

1 Complete the table.

Expression	Diagram	Reference Angle	Positive or Negative	Equivalent Expression	Exact Value
a $\sin 210°$		$30°$	$-$	$-\sin 30°$	$-\frac{1}{2}$
b $\sin 150°$		$30°$	$+$	$\sin 30°$	$\frac{1}{2}$
c $\sin 225°$		$45°$	$-$	$-\sin 45°$	$-\frac{1}{\sqrt{2}}$
d $\cos 300°$		$60°$	$+$	$\cos 60°$	$\frac{1}{2}$
e $\cos 150°$		$30°$	$-$	$-\cos 30°$	$-\frac{\sqrt{3}}{2}$
f $\cos 300°$		$60°$	$+$	$\cos 60°$	$\frac{1}{2}$
g $\tan 300°$		$60°$	$-$	$-\tan 60°$	$-\sqrt{3}$
h $\tan 135°$		$45°$	$-$	$-\tan 45°$	-1
i $\tan 240°$		$60°$	$+$	$\tan 60°$	$\sqrt{3}$

2 Find the exact value of these trigonometric ratios.

a $\sin 60°$ $\frac{\sqrt{3}}{2}$	**b** $\sin 120°$ $\frac{\sqrt{3}}{2}$	**c** $\sin 240°$ $-\frac{\sqrt{3}}{2}$	**d** $\sin 300°$ $-\frac{\sqrt{3}}{2}$
e $\cos 45°$ $\frac{1}{\sqrt{2}}$	**f** $\cos 135°$ $-\frac{1}{\sqrt{2}}$	**g** $\cos 225°$ $-\frac{1}{\sqrt{2}}$	**h** $\cos 315°$ $\frac{1}{\sqrt{2}}$
i $\tan 30°$ $\frac{1}{\sqrt{3}}$	**j** $\tan 150°$ $-\frac{1}{\sqrt{3}}$	**k** $\tan 210°$ $\frac{1}{\sqrt{3}}$	**l** $\tan 330°$ $-\frac{1}{\sqrt{3}}$

3 Find the exact value of these trigonometric ratios.

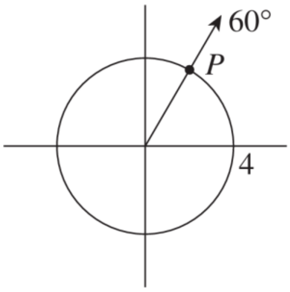
a $\cos 120°$ $-\frac{1}{2}$	**b** $\tan 225°$ 1	**c** $\sin 300°$ $-\frac{\sqrt{3}}{2}$	**d** $\sin 135°$ $\frac{1}{\sqrt{2}}$
e $\tan 240°$ $\sqrt{3}$	**f** $\cos 210°$ $-\frac{\sqrt{3}}{2}$	**g** $\tan 315°$ -1	**h** $\cos 300°$ $\frac{1}{2}$
i $\sin 225°$ $-\frac{1}{\sqrt{2}}$	**j** $\cos 150°$ $-\frac{\sqrt{3}}{2}$	**k** $\sin 210°$ $-\frac{1}{2}$	**l** $\tan 300°$ $-\sqrt{3}$

4 Find the exact value of these trigonometric ratios.

a $\cos(-60°)$ $\frac{1}{2}$	**b** $\sin(-120°)$ $-\frac{\sqrt{3}}{2}$	**c** $\tan(-120°)$ $\sqrt{3}$	**d** $\sin(-315°)$ $\frac{1}{\sqrt{2}}$
e $\tan(-210°)$ $-\frac{1}{\sqrt{3}}$	**f** $\cos(-225°)$ $-\frac{1}{\sqrt{2}}$	**g** $\tan 420°$ $\sqrt{3}$	**h** $\cos 510°$ $-\frac{\sqrt{3}}{2}$
i $\sin 495°$ $\frac{1}{\sqrt{2}}$	**j** $\sin 690°$ $-\frac{1}{2}$	**k** $\cos 600°$ $-\frac{1}{2}$	**l** $\tan 585°$ 1

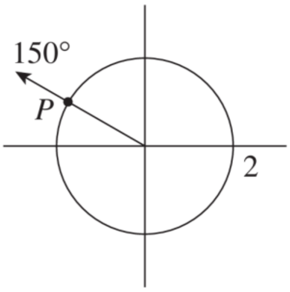
8 When the radius of the circle is not 1, the coordinates of a point on the circumference is given by $(r\cos\theta, r\sin\theta)$. Find the coordinates of the point P in each diagram in exact form.

a



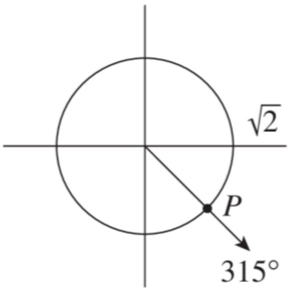
$(4\cos 60°, 4\sin 60°)$
$(2, 2\sqrt{3})$

b



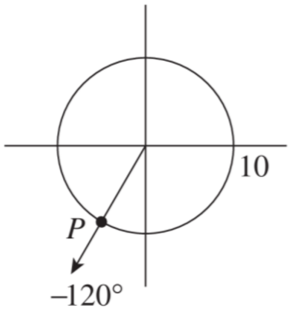
$(2\cos 150°, 2\sin 150°)$
$(-\sqrt{3}, 1)$

c



$(\sqrt{2}\cos 315°, \sqrt{2}\sin 315°)$
$(1, -1)$

d



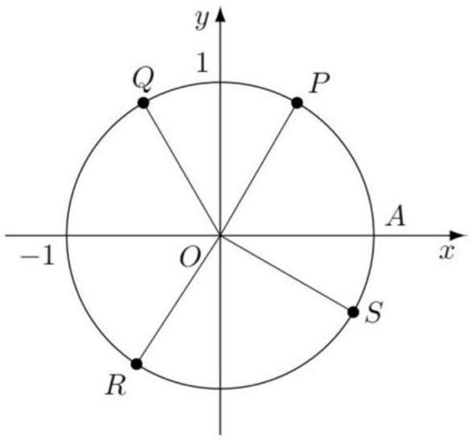
$(10\cos -120°, 10\sin -120°)$
$(-5, -5\sqrt{3})$

9 Find the exact value of:

a $\tan 225°$ 1	**b** $\tan 300°$ $-\sqrt{3}$	**c** $\cos 120°$ $-\frac{1}{2}$	**d** $\cos 330°$ $\frac{\sqrt{3}}{2}$
e $\cos(-300°)$ $\frac{1}{2}$	**f** $\cos(-45°)$ $\frac{1}{\sqrt{2}}$	**g** $\cos 570°$ $-\frac{\sqrt{3}}{2}$	**h** $\sin 480°$ $\frac{\sqrt{3}}{2}$
i $\sin 300°$ $-\frac{\sqrt{3}}{2}$	**j** $\cos(-225°)$ $-\frac{1}{\sqrt{2}}$	**k** $\tan(-300°)$ $\sqrt{3}$	**l** $\sin(-60°)$ $-\frac{\sqrt{3}}{2}$
m $\sin 690°$ $-\frac{1}{2}$	**n** $\sin 495°$ $\frac{1}{\sqrt{2}}$	**o** $\tan 675°$ -1	**p** $\cos 135°$ $-\frac{1}{\sqrt{2}}$

10 The points P, Q, R, S are on the unit circle. $A(1,0)$ and O is the origin.
Find the exact coordinates of P, Q, R, S if:

$\angle AOP = 60°$ $\quad \angle AOQ = 120°$ $\quad \angle AOR = 240°$ $\quad \angle AOS = 30°$



$P = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), Q = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right), R = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), S = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

11 Prove, using exact values, that:

a $\sin 330° \cos 150° - \cos 390° \sin 390° = 0$

$LHS = -\sin 30° \,(-\cos 30°) - \cos 30° \sin 30$

$= \sin 30° \cos 30° - \cos 30° \sin 30°$

$= 0 = \text{RHS}$

b $\sin 420° \cos 405° + \cos 420° \sin 405° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$LHS = \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} = RHS$

c $\frac{\sin 135° - \cos 120°}{\sin 135° + \cos 120°} = 3 + 2\sqrt{2}$

$LHS = \frac{\sin 45° - (-\cos 60°)}{\sin 45° + (-\cos 60°)}$

$= \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)}$

$= \frac{\left(\frac{2+\sqrt{2}}{2\sqrt{2}}\right)}{\left(\frac{2-\sqrt{2}}{2\sqrt{2}}\right)}$

$= \frac{2+\sqrt{2}}{2-\sqrt{2}} \frac{\times 2+\sqrt{2}}{\times 2+\sqrt{2}}$

$= \frac{4+4\sqrt{2}+2}{4-2}$

$= \frac{6+4\sqrt{2}}{2}$

$= 3 + 2\sqrt{2} = RHS$

d $(\sin 150° + \cos 270° + \tan 315°)^2 = \sin^2 135° \cos^2 225°$

$LHS = \left(\sin 30° + 0 + (-\tan 45°)\right)^2$

$= \left(\frac{1}{2} - 1\right)^2$

$= \frac{1}{4}$

$RHS = \sin^2 45° \,(- - \cos^2 45°)$

$= \left(\frac{1}{\sqrt{2}}\right)^2 \left(-\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{4}$

$\therefore LHS = RHS$

e $\dfrac{\sin 120°}{\tan 300°} - \dfrac{\cos 240°}{\left(\frac{1}{\tan 315°}\right)} = \tan^2 240° - \dfrac{1}{\cos^2 330°}$

$LHS = \dfrac{\sin 60°}{-\tan 60°} - \dfrac{-\cos 60°}{\left(\frac{1}{-\tan 45°}\right)}$

$= \dfrac{\left(\frac{\sqrt{3}}{2}\right)}{-\sqrt{3}} - \dfrac{\left(-\frac{1}{2}\right)}{-1}$

$= -\frac{1}{2} - \frac{1}{2}$

$= -1$

$RHS = \tan^2 60° - \dfrac{1}{\sin^2 30°}$

$= \left(\sqrt{3}\right)^2 - \dfrac{1}{\left(\frac{1}{2}\right)^2}$

$= 3 - \dfrac{1}{\left(\frac{1}{4}\right)}$

$= -3 - 4$

$\therefore LHS = RHS$

12 Prove that

$$\frac{\cos(90° - \theta)\ \sin(180° - \theta)}{\cos(180° + \theta)\ \cos(-\theta)} = -\tan^2 \theta$$

$LHS = \dfrac{\sin\theta \sin\theta}{-\cos\theta \cos\theta}$

$= \dfrac{\sin^2\theta}{-\cos^2\theta}$

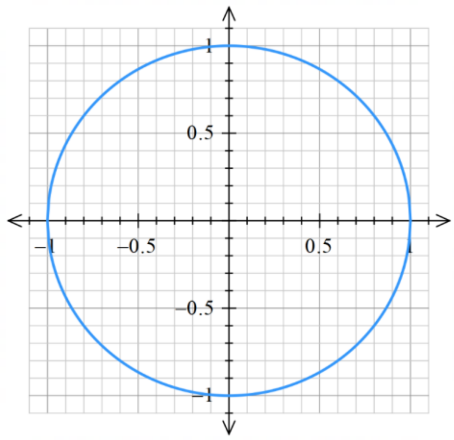
$= -\tan^2\theta = RHS$

TRIGONOMETRIC EQUATIONS IN THE DOMAIN 0° TO 360°

Investigation Solving Trigonometric Equations

Solve $\sin\theta = \frac{1}{2}$ for the domain $0 \leq \theta < 360°$

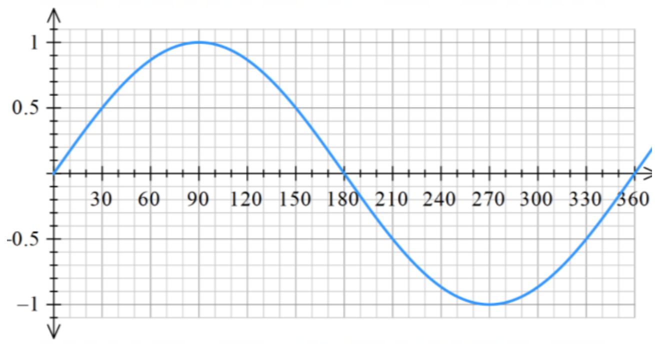
Method 1: Unit Circle



$\theta =$ ……….. or ………..

To solve $\sin\theta = \frac{1}{2}$ using the unit circle, look for ……………… on the unit circle where $y = \frac{1}{2}$, then find the ……………… of inclination formed by the radii to these points using right-angled-triangle trigonometry.

Method 2: Graph



$\theta =$ ……….. or ………..

To solve an equation graphically, we sketch $y =$ ……… and $y =$ ……… on the same axis, then find the x coordinates of the ……………… ……… ………………………

What are the disadvantages of these two methods?

Method 3: ASTC

Solve $\sin\theta = \frac{1}{2}$ for the domain $0 \leq \theta < 360°$

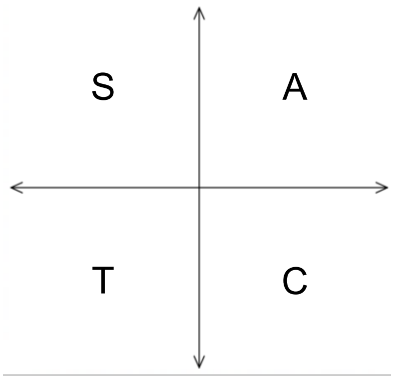
Step 1: Find the reference angle using a calculator.

$$\sin\theta = \frac{1}{2}$$

$$\theta = \ldots\ldots\ldots\ldots\ldots$$

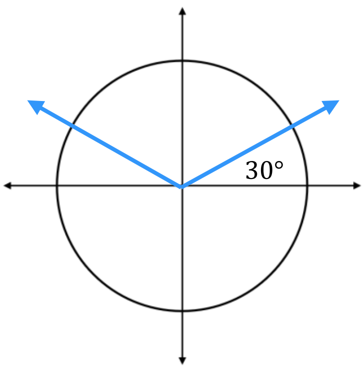
$$= \ldots\ldots\ldots\ldots\ldots$$

Step 2: Determine the quadrants of the solution using ASTC



$\sin\theta > 0$, so θ must be in quadrants ……… and ………

Step 3: Use the quadrants and the domain to find the solutions.



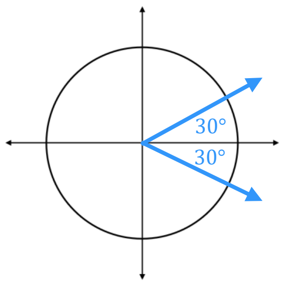
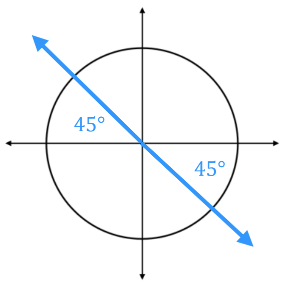
$\theta = \ldots\ldots\ldots\ldots$ or $\ldots\ldots\ldots\ldots$

The ASTC method involves finding the acute ……………… angle using a calculator, then finding the solutions by considering the …………………… in which the sign of the ratio matches. This method can solve any trigonometric function in any domain.

SOLVING TRIGONOMETRIC EQUATIONS USING THE ASTC METHOD

- To solve trigonometric equations and find all possible solutions:
  1. Find the acute reference angle using a calculator.
     Always use the positive ratio.
  2. Determine the quadrants of the solutions using ASTC.
  3. Use the quadrants to find the solutions.

Example Solve simple trigonometric equations where the domain is [0, 360°].

Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0 \le x < 360°$ 1. $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ 2. $\cos x > 0$, x is in Q1 or Q4. 3. $x = 30°$ or $330°$ 	How did we know the second solution is $330°$? *Since cos is also positive in the …th quadrant* *The reference angle of $30°$ in the …th quadrant represents an angle of inclination of ……$-30° = 330°$*
Solve $\tan\theta = -1$ for $0 \le \theta < 360°$ 1. $\theta = \tan^{-1}(1) = 45°$ 2. $\tan\theta < 0$, x is in Q2 or Q4. 3. $x = 135°$ or $315°$ 	Why is $45°$ not a solution? The question says $\tan\theta = -1$, and *tan is ……………… in the first quadrant.* Type in $\tan^{-1}(-1)$ into the calculator. Why do we always use the positive ratio? *By using the positive ratio, the calculator always gives us the ……………… angle.*

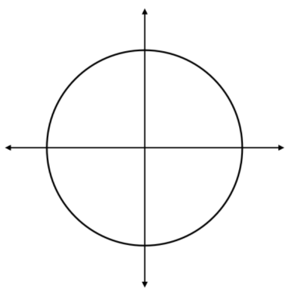
Solve these trigonometric equations for $0 \le x < 360°$

a $\cos\theta = \frac{1}{2}$

1. Reference angle

$\theta = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$

2. Quadrants



3. Solutions

$60°$ or $300°$

b $\sin\theta = -\frac{1}{8}$

1. Reference angle

2. Quadrants

3. Solutions

$187°11'$ or $352°49'$

c $\tan\theta = \frac{\sqrt{3}}{2}$

$40°54'$ or $220°54'$

1 For each equation, circle the quadrants that x would be in.

a $\sin x = \frac{1}{2}$ S A / T C — A S	**b** $\sin x = -\frac{1}{2}$ S A / T C — T C	**c** $\tan x = 3$ S A / T C — A T
d $\tan x = -3$ S A / T C — S C	**e** $\cos x = \frac{2}{5}$ S A / T C — A C	**f** $\cos x = -\frac{2}{5}$ S A / T C — S T

2 Solve each of these equations for $0 \le \theta < 360°$

a $\sin\theta = \frac{\sqrt{3}}{2}$

1. Reference angle

2. Quadrants

3. Solutions

θ =60° or 120°

b $\tan\theta = 1$

θ =45° or 225°

c $\tan\theta = \sqrt{3}$

θ =60° or 240°

d $\cos\theta = -\frac{1}{\sqrt{2}}$

θ =135° or 225°

e $\tan\theta = -\sqrt{3}$

θ =120° or 300°

f $\cos\theta = -\frac{\sqrt{3}}{2}$

θ =150° or 210°

3 These equations lie on boundary angles, i.e. they are on the edges of the quadrants.
We solve these using the graphs, a diagram of the unit circle, or knowledge of exact values.
Solve for $0 \le \theta < 360°$

a $\sin\theta = 1$

θ =90°

b $\cos\theta = 1$

θ = 0°

c $\tan\theta = 0$

θ = 0°

d $\sin\theta = -1$

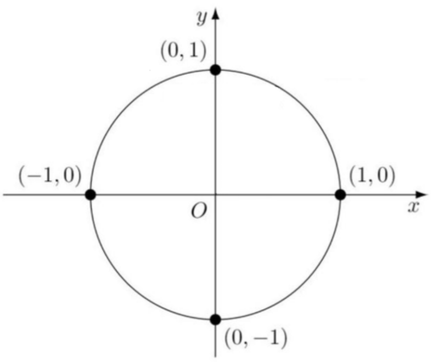
θ = 270°

e $\cos\theta = 0$

θ = 90° or 270°

f $\sin\theta = 0$

θ = 0°, 180°



4 Solve each of these equations for $0 \leq x < 360°$ to the nearest degree.

a $\cos x = \frac{3}{7}$

65° or 295°

b $\sin x = 0.1234$

7° or 173°

c $\tan x = 7$

82° or 262°

d $\sin x = -\frac{2}{3}$

222° or 318°

e $\tan x = -\frac{20}{9}$

114° or 294°

f $\cos x = -0.77$

140° or 220°

g $\tan x = -0.3$

163° or 343°

h $\cos x = 0$

90° or 270°

i $\sin x = -\frac{\sqrt{3}}{2}$

240° or 300°

j $\cos x = 0.191$

79° or 281°

k $\tan x = 1$

45° or 225°

l $\frac{1}{\cos x} = \sqrt{2}$

45° or 315°

5 Solve for $\theta \in [0°, 360°)$. Answer to the nearest degree.

a $2\sin\theta - 1 = 0$

30°, 150°

b $\sqrt{2}\cos\theta - 1 = 0$

45°, 315°

c $\sqrt{3}\tan\theta = -2 = -1$

30°, 210°

d $2\cos\theta + \sqrt{3} = 0$

150°, 210°

e $6\sin\theta + 3 = 0$

210°, 330°

f $1 + \tan\theta = 0$

135°, 315°

g $2\cos^2\theta - \cos\theta = 0$

60°, 90°, 270°, 300°

h $\sin^2\theta + \sin\theta = 0$

0°, 180°, 270°

i $\tan^2\theta = \tan\theta$

0°, 45°, 180°, 225°

j $\sin^2\theta = 1$

90°, 270°

k $\cos^2\theta = \frac{3}{4}$

30°, 150°, 210°, 330°

l $\tan^2\theta = 3$

60°, 120°, 240°, 300°

m $2\cos^2\theta - 1 = 0$

45°, 135°, 225°, 315°

n $3\tan^2 - 1 = 0$

30°, 150°, 210°, 330°

o $(2\sin\theta - 1)(\sin\theta + 1) = 0$

30°, 150°, 270°

p $2\sin^2\theta - 3\sin\theta + 1 = 0$

30°, 90°, 150°

q $2\cos^2\theta - \cos\theta - 3 = 0$

180°

r $\frac{1}{\cos^2\theta} + \frac{1}{\cos\theta} - 2 = 0$

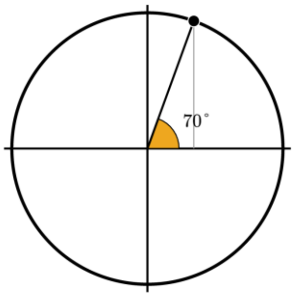
0°, 120°, 240°, 360°

THE AMBIGUOUS TRIANGLE

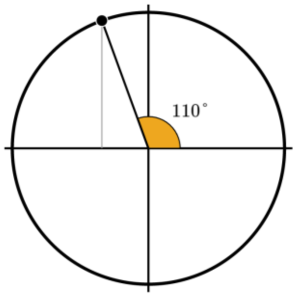
Investigation Ambiguous triangles

Due to symmetry in a unit circle, there are two possible angles for a given sine value, an acute angle and an obtuse angle.

$$\sin\theta = \sin(180 - \theta)$$



$\sin(\theta) \approx \quad 0.94$

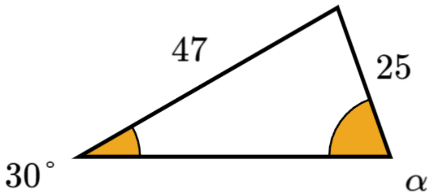


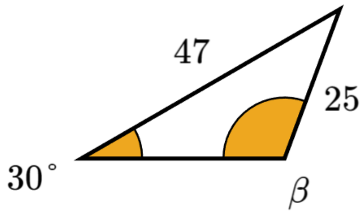
$\sin(\theta) \approx \quad 0.94$

We will investigate how this fact is important when applying the sine rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

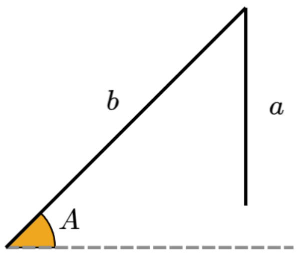
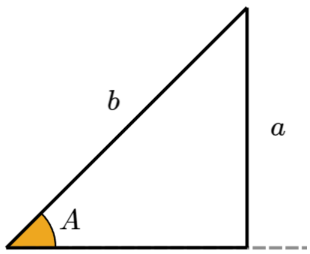
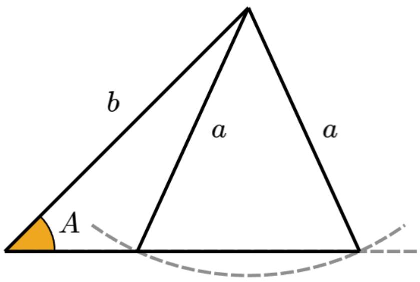
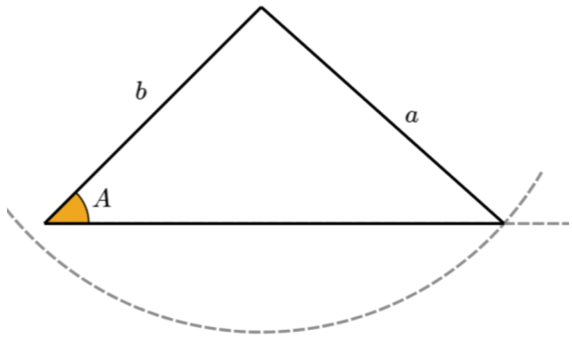
Using the sine rule, calculate the angles α and β in these triangles.





AMBIGUOUS CASE OF THE SINE RULE

- Suppose only side lengths a and b and angle A of a triangle are given.
- $b \sin A$ is the perpendicular height of the triangle.

$\boldsymbol{a < b \sin A}$	$\boldsymbol{a = b \sin A}$
The triangle is not possible.  	Exactly one triangle is possible 
$\boldsymbol{b \sin A < a < b}$	$\boldsymbol{a > b \sin \mathrm{A}}$
Two triangles are possible.  This is the ambiguous case.	Exactly one triangle is possible. 

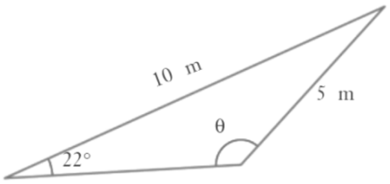
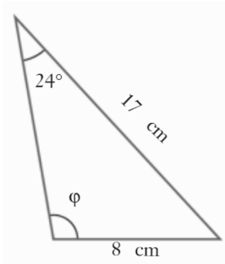
- When there are two possible triangles for the given information, we call this ambiguous.
- To remove this ambiguity, you need to be told whether the triangle is acute or obtuse, otherwise you must give two answers.

OBTUSE ANGLE USING SINE RULE

$$\sin\theta = \sin(180 - \theta)$$

- The calculator will always give the acute angle when using $\sin^{-1}$
- To get the obtuse angle, use $180° - \theta$

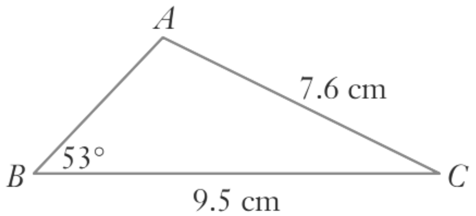
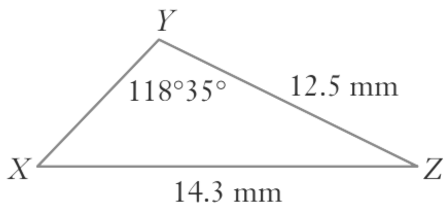
Example Calculate obtuse angles with sine rule.

Worked Example	Your Turn
Calculate θ to the nearest minute, given that it is obtuse.  $\frac{\sin\theta}{10} = \frac{\sin 22°}{5}$ $\sin\theta = \frac{10\sin 22°}{5}$ $\theta \approx 48°31'$ The obtuse angle $\approx 180 - 48°31' = 131°29'$	Calculate φ to the nearest minute, given that it is obtuse.  $120°12'$

AMBIGUOUS TRIANGLES

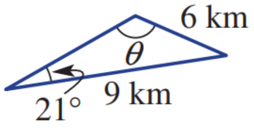
- If the question doesn't state whether the triangle is acute or obtuse, find both angles, then decide if the obtuse angle is possible using angle sum of a triangle.

Example Calculate angles when the triangle is ambiguous.

Worked Example	Your Turn
Find the value of $\angle A$ to the nearest degree.  $\frac{\sin A}{9.5} = \frac{\sin 53°}{7.6}$ $\sin A = \frac{9.5\sin 53°}{7.6}$ $A = \sin^{-1}\frac{9.5\sin 53°}{7.6}$ $A \approx 87°$ or $\approx 180° - 87°$ $= 93°$	Find the value of $\angle X$ to the nearest minute.  50°8′, can't be 129°52′ Explain why there is only one possible value of $\angle X$ in this triangle.

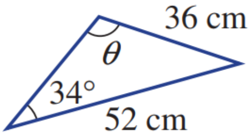
1 Calculate the unknown **obtuse** angle θ. Answer to 1 d.p.

a



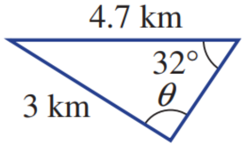
147.5°

b



126.1°

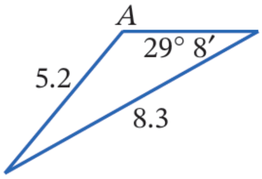
c



123.9°

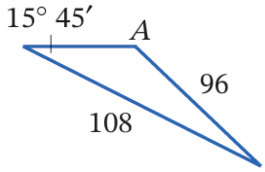
2 Calculate the unknown **obtuse** angle $\angle A$. Answer to nearest minute.

a



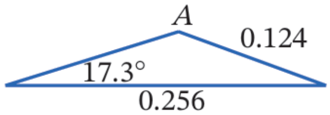
129° 0′

b



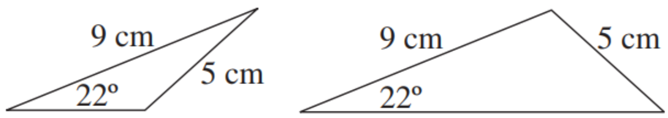
162° 13′

c



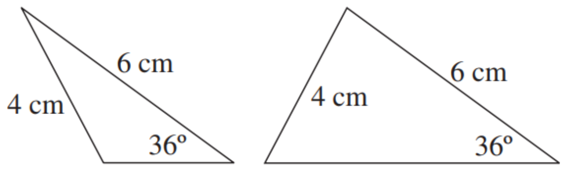
142° 8′

3 There are two triangles that have sides 9 cm and 5 cm. The angle opposite the 5cm side is 22°. Find, in each case, the size of angle opposite the 9 cm side, correct to the nearest degree.



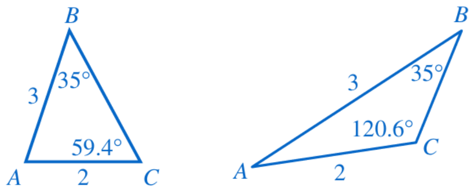
42°, 138°

4 Two triangles are shown, with sides 6 cm and 4 cm. the angle opposite the 4cm is 36°. Find, in each case, the angle opposite the 6cm side, correct to the nearest degree.



62°, 118°

5 A triangle ABC has AB = 3 cm, AC = 2 cm and ∠B = 35°
Find the two possible values of $\angle C$ correct to 1 d.p and sketch the triangles.



59.4° or 120.6°

6 Consider ΔPQR in which $p = 7\text{ cm}$, $q = 15\text{ cm}$ and $\angle P = 25°50'$.

a Find the two possible sizes of $\angle Q$ to the nearest minute then sketch the two triangles.

69°2′ or 110°58′

b For each possible size of $\angle Q$, find side length r in cm.

16.0 cm or 11.0 cm

7 A triangle ABC has AB = 6 m, AC = 10 m and ∠B = 120°.

a Find the two possible values of $\angle C$ correct to the nearest degree.

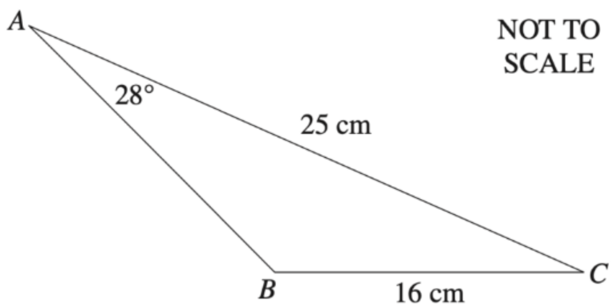
31.3° and 148.7°

b Explain why there is only one solution for $\angle C$.

A triangle can only have one obtuse angle as the angle sum is limited to 180°

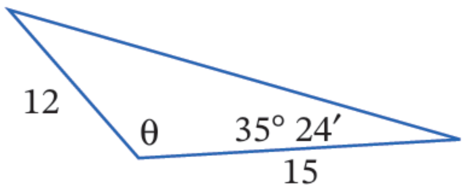
8 **2021 HSC Standard 2 Band 5**

Find obtuse $\angle ABC$ to the nearest degree.



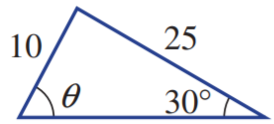
133°

9 Find θ in this diagram, correct to the nearest minute.



98° 12'

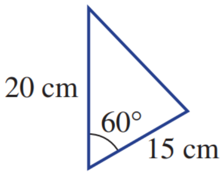
10 Try to find the angle θ in this triangle; explain why it is impossible.



$\theta = \sin^{-1}\left(\frac{25\sin 30^\circ}{10}\right)$, which is impossible bas $\frac{25\sin 30^\circ}{10} > 1$. Therefore, such a triangle is impossible to construct.

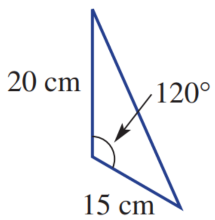
11 Find the area of these two triangles, correct to 2 decimal places.

Triangle 1



129.9 cm^2

Triangle 2

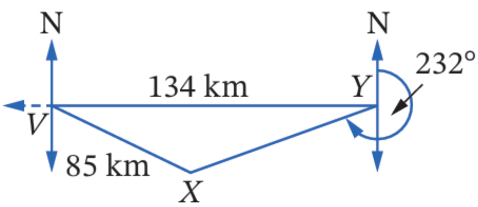


129.9 cm^2

What do you notice? How can you explain this?

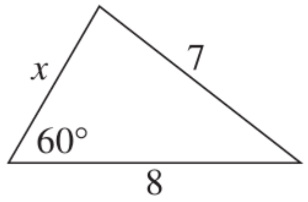
They are equal as sin 60° and sin 120° are equal.

12 Yorker (Y) is 134 km due east of Vendor (V).
Xena (X) is on a bearing of 232° from Yorker and is also 85 km from Vendor.
∠VXY is an obtuse angle. Find the bearing of Xena from Vendor.



128°

13 Use the cosine rule to find the two possible values of x in the diagram.



$7^2 = x^2 + 8^2 - 2(x)(8)\cos 60°$

$49 = x^2 + 64 - \dfrac{16x}{2}$

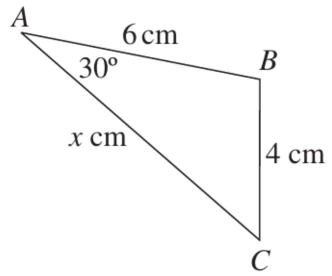
$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ or 5

14 Consider the diagram.

a Use cosine rule to show $x^2 - 6\sqrt{3}\,x + 20 = 0$



$4^2 = 6^2 + x^2 - 2(6)(x)\cos 30°$

$16 = 36 + x^2 - \dfrac{12x\sqrt{3}}{2}$

$0 = 20 + x^2 - 6x\sqrt{3}$ as required

b Use the quadratic formula to show AC has length $(3\sqrt{3} + \sqrt{7})$ cm or $(3\sqrt{3} - \sqrt{7})$ cm.

$$x = \frac{-(-6\sqrt{3}) \pm \sqrt{(-6\sqrt{3})^2 - 4(1)(20)}}{2}$$

$$= \frac{6\sqrt{3} \pm \sqrt{28}}{2}$$

$$= \frac{6\sqrt{3} \pm 2\sqrt{7}}{2}$$

$$= 3\sqrt{3} \pm \sqrt{7}$$

c Indicate on the diagram the other possible position of point C.