Student Name

Mathematics Stage 4

EQUATIONS

Book 3 Solve quadratic equations

Version: 260207

Report mistakes & suggest improvements:
https://MrDingMaths.com

CONTENTS

OVERVIEW

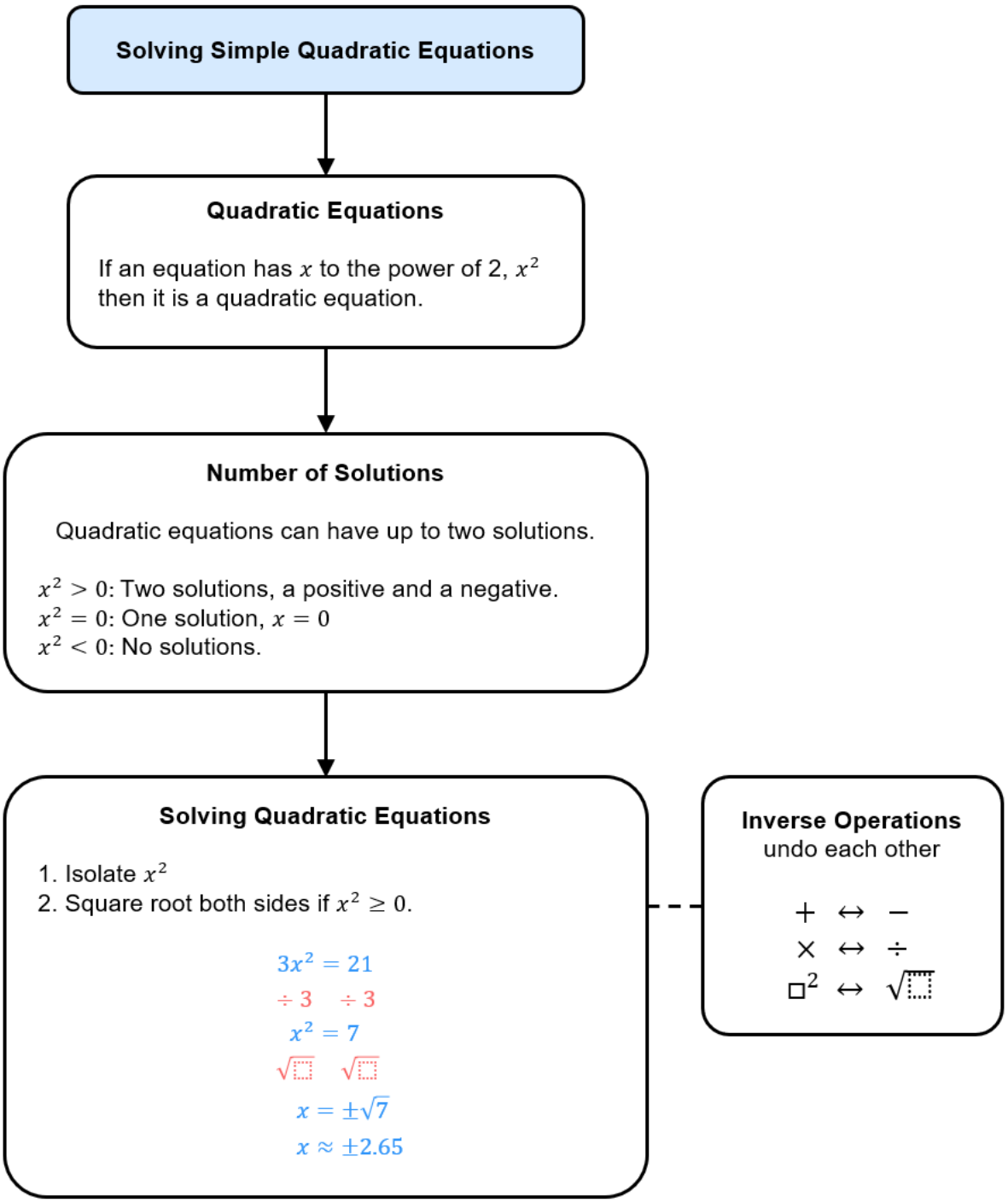
SYLLABUS CONTENT

MA4-EQU-C-01 solves linear equations of up to 2 steps and quadratic equations of the form $ax^2 = c$

Solve quadratic equations

- Reason why there are 2 values of x that satisfy a quadratic equation of the form $x^2 = c$ if $c = 0$
- Solve problems involving quadratic equations of the form $ax^2 = c$, giving answers in exact form and as decimal approximations
- Solve quadratic equations arising from substitution into a formula

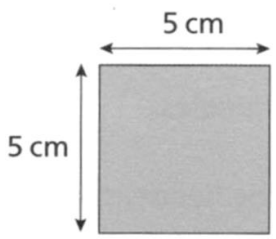
SUMMARY



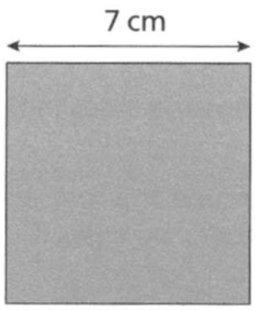
REVIEW OF PRIOR KNOWLEDGE

1 Find the area of each square.

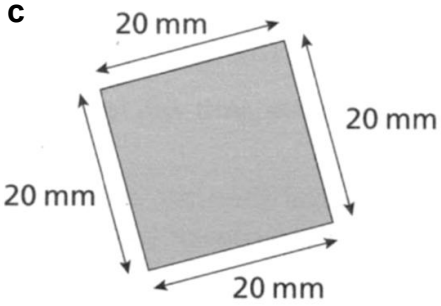
a



b

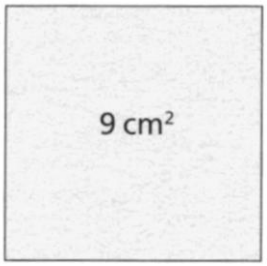


c

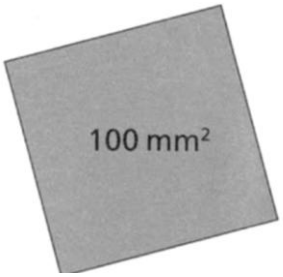


2 The area of each square is given. Find the side length.

a



b



c

3 Square these numbers.

a	1^2	**b**	$(-1)^2$	**c**	3^2	**d**	$(-3)^2$
e	5^2	**f**	$(-5)^2$	**g**	$(-12)^2$	**h**	13^2

4 Using a calculator, evaluate and round these square roots to two decimal places.

a	$\sqrt{12}$	**b**	$\sqrt{9}$	**c**	$\sqrt{0}$	**d**	$\sqrt{41}$
e	$\sqrt{-1}$	**f**	$\sqrt{180}$	**g**	$\sqrt{10.5}$	**h**	$\sqrt{66.25}$

5 A **surd** is a root that makes a non-terminating, non-recurring decimal.
Which of the following are surds?

a	$\sqrt{3} = 1.732050808\ldots$	**b**	$\sqrt{25} = 5$	**c**	$\sqrt{37} = 6.08276253\ldots$
d	$\sqrt{1.44} = 1.2$	**e**	$\sqrt{169} = 13$	**f**	$\sqrt{11} = 3.31662479\ldots$

ONE-STEP QUADRATIC EQUATIONS

- **Linear** equations have **variables with a power of 1**.
- **Quadratic** equations have **variables to the power of 2**: x^2 "x squared"

NUMBER OF SOLUTIONS

- Quadratic equations can have **up to two solutions**.
  - $x^2 > 0$: Two solutions, a positive and a negative.
  - $x^2 = 0$: One solution. $x = 0$
  - $x^2 < 0$: No solutions.

Example Identify number of solutions to simple quadratic equations.

Decide whether these equations have two, one, or no solution.

$x^2 = 9$	$x^2 = 0$	**a** $x^2 = -1$
$x^2 = 16$	$x^2 = -3$	**b** $x^2 = 0.25$
$x^2 = 1$	$x^2 = -25.7$	**c** $x^2 = 0$

Key ideas

1. A quadratic equation has a pronumeral raised to the …… of 2.
2. Quadratic equations can have ……, ……, or two solutions.
   a. There are two solutions when $x^2 > 0$, since x can be …… or ……
   b. There is …… solution when $x^2 = 0$, since x can only be ……
   c. There is …… solution when $x^2 < 0$, since it's not possible to have a …… number after squaring.

SOLVING QUADRATIC EQUATIONS

- The inverse operation of squaring is square rooting

- To solve quadratic equations $x^2 = c$:
  1. Square root both sides, as long as $x^2 \geq 0$.

- Square roots that don't make a nice terminating decimal should stay as roots.
  We call this leaving in **exact form** or **surd form**.
  - $\sqrt{225}$ should be simplified to 15.
  - Leave $\sqrt{146}$ as is.
  - Some questions will ask you to round to a specified decimal place.

Example Solve one-step quadratic equations.

$x^2 = 81$ $\sqrt{\ } = \sqrt{\ }$ $x = \pm\sqrt{81}$ $x = \pm 9$	What is the inverse operation of squaring? *The inverse operation of squaring is ....* What does $\pm$ mean? *$\pm$ means there are two solutions:* *a ..... and .....*
$x^2 = 23$ $\sqrt{\ } = \sqrt{\ }$ $x = \pm\sqrt{23}$ $x \approx \pm 4.8$	Why can there be up to two solutions for a quadratic equation? *In the first example $x^2 = 81$,* *$(\quad)^2 = 81$ and $(\quad)^2 = 81$* *So, there are two values that make the equation .....*

Solve these equations.

a $x^2 = 121$

$\sqrt{\ } \quad \sqrt{\ }$

$x = ...$

$x = \pm 11$

b $x^2 = 27$

$x = \pm\sqrt{27}$

c $87 = x^2$

$x = \pm\sqrt{87}$

d $x^2 = 169$

$x = \pm 13$

e $x^2 = 2.25$

$x = \pm 1.5$

f $x^2 = 0.25$

$x = \pm 0.5$

Key ideas

1. The inverse operation of squaring is ....

1 Complete these statements.

a If $x^2 = 16$, then $x =$ ..... or $x =$ .....

4 or -4

b If $x^2 = 0$, then $x =$ ..... only.

0 only

c If $x^2 = -16$, then .....

There is no solution

d Explain how you know whether a quadratic equation has two, one, or zero solutions.

If $x^2 > 0$, there are ..... solutions.

If $x^2 = 0$, there is ..... solution.

If $x^2 < 0$, there is ..... solution.

2, 1, no

2 Solve these equations. Evaluate answers to 2 decimal places, if required.

a $x^2 = 9$ $x = \pm 3$	**b** $x^2 = 121$ $x = \pm 11$	**c** $x^2 = 1$ $x = \pm 1$
d $x^2 = 18$ $x = \pm 4.24$	**e** $x^2 = 12.4$ $x = \pm 3.52$	**f** $x^2 = 0.16$ $x = \pm 0.4$

3 Solve these equations. Leave your answer in exact form.

a $x^2 = 42$ $x = \pm\sqrt{42}$	**b** $x^2 = -5$ No solution	**c** $x^2 = 72$ $x = \pm\sqrt{72}$
d $x^2 = 49$ $x = \pm 7$	**e** $x^2 = 0$ $x = 0$	**f** $x^2 = 56$ $x = \pm\sqrt{56}$

4 15 is subtracted from the square of a number and the result is 66.
What are the possible numbers?

$x^2 - 15 = 66$

$x = \pm 9$

5 Billy says to Maddy: 'When I add 10 to the square of a number, the result is 6.'
Maddy replies that this is impossible. Explain why Maddy is correct.

$x^2 + 10 = 6$

$x^2 = -4$

There is no solution because you cant square a number and get a negative.

TWO-STEP QUADRATIC EQUATIONS

- To solve quadratic equations in the form $ax^2 = c$
  1. Isolate x^2
  2. Square root both sides if $x^2 \geq 0$.

Example Solve two-step quadratic equations.

$2x^2 = 50$	$x^2 + 4 = 19$
1. Isolate x^2 $\div 2$ $\div 2$	1. Isolate x^2 -4 -4
$x^2 = 25$	$x^2 = 15$
2. Square root $\sqrt{\ }$ $\sqrt{\ }$	2. Square root $\sqrt{\ }$ $\sqrt{\ }$
$x = \pm\sqrt{25}$	$x = \pm\sqrt{15}$
$x = \pm 5$	$x \approx \pm 3.87$

Solve these equations. Evaluate all answers to 2 decimal places.

a $3x^2 = 147$

1. Isolate x^2 ....

$x^2 =$ .....

2. Square root $\sqrt{\ }$ $\sqrt{\ }$

$x =$ .....

$x = \pm 7$

b $x^2 - 11 = 75$

$x = \pm 9.27$

c $-4.5 = -2x^2$

$x = \pm 1.5$

d $28 + x^2 = 89$

$x = \pm 7.81$

e $\frac{x^2}{3} = 12$

$x = \pm 6$

f $12x^2 = -81$

No real solution

Key ideas

1. To solve quadratic equations, the first thing we do is ..... x^2,
   then .... both sides.

1 Solve these equations. Round your answers to 2 decimal places, if required.

a $x^2 = 81$ $x = \pm 9$	**b** $81x^2 = 0$ $x = 0$	**c** $x^2 + 81 = 0$ No solution
d $x^2 - 32 = 0$ $x = \pm\sqrt{32} = \pm 5.66$	**e** $2x^2 - 32 = 0$ $x = \pm 4$	**f** $2x^2 + 32 = 0$ No solution
g $5x^2 = 20$ $x = \pm 2$	**h** $6x^2 = 600$ $x = \pm 10$	**i** $8x^2 = 200$ $x = \pm 5$
j $4x^2 = 196$ $x = \pm 7$	**k** $\frac{1}{2}x^2 = 600$ $x = \pm\sqrt{1200} = \pm 34.64$	**l** $\frac{3}{2}x^2 = 200$ $x = \pm\sqrt{\frac{400}{3}} = \pm 11.55$
m $-3x^2 = -108$ $x = \pm 6$	**n** $-4x^2 = -64$ $x = \pm 4$	**o** $\frac{x^2}{2} = 32$ $x = \pm 8$
p $\frac{x^2}{3} = 27$ $x = \pm 9$	**q** $0.1x^2 = 1.6$ $x = \pm 4$	**r** $\frac{x^2}{0.01} = 0.36$ $x = \pm 0.06$

2 Solve these equations. Leave your answers in exact surd form.

a $3x^2 = 120$ $x = \pm\sqrt{40}$	**b** $x^2 + 4 = 0$ No solution	**c** $2x^2 - 44 = 0$ $x = \pm\sqrt{22}$
d $3x^2 - 75 = 0$ $x = \pm 5$	**e** $4 - x^2 = 0$ $x = \pm 2$	**f** $10 - x^2 = -12$ $x = \pm\sqrt{22}$
g $5x^2 - 32 = 43$ $x = \pm\sqrt{15}$	**h** $3x^2 = 2x^2 + 9$ $x = \pm 3$	**i** $x^2 + 3x = 3x + 4$ $x = \pm 2$
j $x^2 - 4x = 64 - 4x$ $x = \pm 8$	**k** $5x^2 + 3 = 3x^2 + 15$ $x = \pm\sqrt{6}$	**l** $7 - x^2 = 2x^2 + 4$ $x = \pm 1$

USING FORMULAS WITH QUADRATIC EQUATIONS

- We can use formulas to find any unknown value when we know all other quantities.

1. Write the formula.
2. Substitute given values.
3. Solve for the unknown value using inverse operations.

Example Substitute values into a formula to find the unknown.

$A = 3x^2$	$A = 3x^2$
Find A when $x = 6$.	Find x when $A = 15$
1. Write formula $A = 3x^2$	1. Write formula $A = 3x^2$
2. Substitute $A = 3(6)^2$	2. Substitute $15 = 3x^2$
3. Solve $A = 108$	3. Solve $5 = x^2$
	$\pm\sqrt{5} = x$

a $x = 25 - y^2$

Find x when $y = 4$

$x = 9$

Find y when $x = 16$

$y = \pm 3$

b $y = 3x^2 + 6$

Find y when $x = 12$

$y = 438$

Find x when $y = 27$

$x = \pm\sqrt{7}$

Key ideas

1. We can find any unknown value in a formula if know the value of the other ……
   a. Write the ……
   b. …… given values.
   c. …… the equation using inverse operations.

1 Use the formula $a^2 = c^2 - b^2$ to find the value of

a a when $c = 10$ and $b = 8$

$a = \pm 6$

b b when $a = 3$ and $c = 5$

$b = \pm 4$

2 Solve for the unknown variable.

a $v^2 = u^2 + 2as$,

Find s when $v = 22, u = 6, a = 12$

$s = \frac{56}{3}$

Find v when $u = 0, a = 10, s = 60$

$v = \pm\sqrt{1200} = \pm 34.64$

b $m = \sqrt{\frac{x}{y}}$

Find y when $m = 8, x = 4$

$y = \frac{1}{16}$

Find x when $y = 2, m = 10$

$x = 200$

3

a Write down the formula for the area of a triangle.

$A = \frac{bh}{2}$

b The area of a triangle is 8 cm^2.

i. Find the height of the triangle if the base is 10 cm

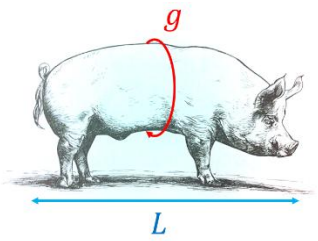
$h = 1.6$ cm

ii. Find the height of the triangle if it is equal to the base.

$h = 4$ cm

4 The weight of a pig can be estimated by measuring the girth g all the way around the pig and the length l from tail to head in centimetres. The weight, in kilograms, is:

$$w = \frac{Lg^2}{13781}$$



a A pig has a girth of 173 cm and length of 168 cm.
Estimate its weight, answer to 2 d.p.

364.86 kg

b Estimate the length of a pig that has a weight of 400 kg and a girth of 180 cm.
Answer to 2 d.p.

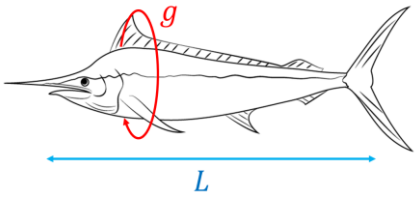
170.14 cm

c Estimate the girth of a pig that has a weight of 320 kg and a length of 150 cm.
Answer to 2 d.p.

171.46 cm

5 In the United States, the weight of a fish is estimated by measuring the girth g (all around the fish) at the thickest point in inches and the length L also in inches. The formula for the estimated weight, in pounds, is

$$w = \frac{Lg^2}{800}$$



a A blue marlin has a girth of 72 inches and a length of 156 inches. Could it weigh over 1000 pounds?

$w = 1010.88$ pounds

yes

b A swordfish is 120 inches in length and weighs 175 pounds. Estimate its girth to 2 d.p.

34.16 inches

2 **2004 HSC Standard 2 Band 4**

If $d = 6t^2$, what are two possible values of t when $d = 2400$?

$t = \pm 20$

3 **HSC Standard 2 Sample Question Band 4**

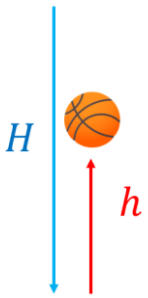
The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere.

If the volume is 220 cm^3, find the radius, to 1 decimal place.

$r \approx 3.7$ cm

6 The coefficient of restitution c measures how bouncy something is by comparing the height dropped H to the height bounced back h using the formula

$$c = \sqrt{\frac{h}{H}}$$



The regulations say that a basketball should have a c between 0.76 and 0.8.

a A basketball is dropped from a height of 100 cm and it bounces back up to 61 cm. Does this basketball fit the rules?

$c = 0.781$, yes

b A tennis ball has a coefficient of restitution of 0.82. If it is dropped from a height of 10 metres, how high would it bounce back up, to 2 d.p.?

$h = 6.72$ m

c A table tennis ball has a coefficient of restitution of 0.923. If it is dropped from a height of 10 metres, how high would it bounce back up, to 2 d.p.?

$h = 8.52$ m

d A cricket ball has a coefficient of restitution of 0.42. If it bounces back up to a height of 10 metres, how high was it dropped from, to 2 d.p.?

$H = 56.69$ m