Student Name

Mathematics Stage 5 Paths

EQUATIONS C

Book 3 Solve quadratic equations using a variety of methods

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-EQU-P-02 solves linear equations of more than 3 steps, monic and non-monic quadratic equations, and linear simultaneous equations

Solve quadratic equations using a variety of methods

- Solve equations of the form $ax^2 + bx + c = 0$ by factorisation and by completing the square
- Apply the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve quadratic equations
- Apply the most appropriate method to solve a variety of quadratic equations
- Use substitution to verify solutions to quadratic equations
- Identify whether a given quadratic equation has real solutions, and if there are real solutions, whether or not they are equal
- Solve quadratic equations resulting from substitution into formulas in various contexts
- Model and solve word problems using quadratic equations in various contexts
- Solve equations that are reducible to quadratics

REVIEW OF PRIOR KNOWLEDGE

1 Factorise these expressions.

a $2x^2 + 3x - 2$	**b** $4x^2 + 5x + 1$	**c** $13x^2 - 7 - 6$
d $9x^2 + 9x - 10$	**e** $8x^2 + 18x + 4$	**f** $6x^2 + 38x + 40$

2 Factorise these expressions.

a $x^2 + 8x + 16$	**b** $x^2 - 10x + 25$	**c** $x^2 - 4x + 4$
d $x^2 + 24x + 144$	**e** $x^2 + x + \frac{1}{4}$	**f** $x^2 + 3x + \frac{9}{4}$

NULL FACTOR LAW WITH NON-MONICS

- **Non-monic** means the coefficient of x^2 is not 1.

Null Factor Law	
If $a \times b = 0$ Then $a = 0$ or $b = 0$	If $(ax + b)(cx + d) = 0$ Then $ax + b = 0$ or $cx + d = 0$

Example Identify solution using null factor law.

a $(x-6)(x-2)=0$ $x-6=0$ or $x-2=0$ $x = \ldots\ldots$ $x = \ldots\ldots$ $x = 6 \text{ or } 2$	**e** $(x+6)(2x-3)=0$ $x = -6 \text{ or } \frac{3}{2}$
b $(2x-6)(x-2)=0$ $2x-6=0$ or $x-2=0$ $x = 3 \text{ or } 2$	**f** $(x+6)(2x-5)=0$ $x = -6 \text{ or } \frac{5}{2}$
c $(2x+6)(x-2)=0$ $x = -3 \text{ or } 2$	**g** $(x-6)(5x-2)=0$ $x = 6 \text{ or } \frac{2}{5}$
d $(x+6)(2x-2)=0$ $x = -6 \text{ or } 1$	**h** $(7x+6)(5x-2)=0$ $x = -\frac{6}{7} \text{ or } \frac{2}{5}$

SOLVING NON-MONIC QUADRATIC EQUATIONS

- To solve quadratic equations that are monic quadratic trinomials:
  1. Write equation in the form $x^2 + bx + c = 0$, if it isn't already.
  2. Factorise the expression using the PSF method.
  3. Use null factor law to solve for x.

Example Solve non-monic quadratic equations by factorising.

$2x^2 + 11x + 12 = 0$	$10x^2 - 13x = 3$
2. $2x^2 + 3x + 8x + 12 = 0$ P: 24 $x(2x+3) + 4(2x+3) = 0$ S: 11 $(x+4)(2x+3) = 0$ F: 3,8 3. $x + 4 = 0$ or $2x + 3 = 0$ $x = -4$ $2x = -3$ $x = -\frac{3}{2}$	1. $10x^2 - 13x - 3 = 0$ 2. $10x^2 - 15x + 2x - 3 = 0$ P: -30 $5x(2x-3) + (2x-3) = 0$ S: -13 $(5x+1)(2x-3) = 0$ F: $-15,2$ 3. $5x + 1 = 0$ or $2x - 3 = 0$ $5x = -1$ $2x = 3$ $x = -\frac{1}{5}$ $x = \frac{3}{2}$

Solve for x.

a $4x^2 + 16x + 7 = 0$

P: 28

S: 16

F: ....

$x = -\frac{7}{2}$ or $-\frac{1}{2}$

b $2x^2 - 17x + 35 = 5$

P: ....

S: ....

F: ....

$x = 6$ or $\frac{5}{2}$

c $2x^2 - 23x + 11 = 0$

$x = 11$ or $\frac{1}{2}$

d $18x^2 + 12x = 16$

$x = \frac{2}{3}$ or $-\frac{4}{3}$

1 Solve these equations.

a $2x^2 + 11x + 12 = 0$

b $4x^2 + 16x + 7 = 0$

c $2x^2 - 17x + 35 = 0$

d $2x^2 - 23x + 11 = 0$

e $3x^2 - 4x - 15 = 0$

f $5x^2 - 7x - 6 = 0$

g $6x^2 + 7x - 20 = 0$

h $7x^2 + 25x - 12 = 0$

2 Solve these equations.

a $2x^2 + 4x + 2 = 0$

b $2x^2 + 6x + 4 = 0$

c $2x^2 - 2x - 4 = 0$

d $2x^2 - 6x + 4 = 0$

e $3x^2 - 8x + 4 = 0$

f $6x^2 - 10x + 4 = 0$

g $6x^2 + 2x - 4 = 0$

h $6x^2 + 10x + 4 = 0$

3 Using the null factor law, write down the solutions to these equations.

a $(x-1)(x+2)=0$

b $2(x-1)(x+2)=0$

c With reference to the null factor law, explain why a common factor of 2 does not change the answer in the two examples above.

....

....

d Solve $x^2-5x-6=0$

....

....

e Solve $3x^2-15x-18=0$

....

....

4 Solve these quadratic equations. First rearrange to the form $ax^2+bx+c=0$

a $2x^2=15-7x$

b $4x^2=12x-9$

c $x=3x^2-14$

d $17x=12-5x^2$

e $25=-9x^2-30x$

f $17x-6x^2=5$

5 This question has a mix of linear and quadratic equations.
Solve each equation using an appropriate technique.

a $3x + 3 = 2x + 9$

b $5x^2 = 3x$

c $50 - x^2 = 25 - x - x^2$

d $x^2 - 11x - 12 = 0$

e $x^2 + 10x = 0$

f $3(3x + 5) + 6 = 3$

g $(y + 2)^2 = y^2 + 13$

h $8x^2 - 2x - 3 = 0$

i $3x^2 + 27x + 42 = 0$

j $2x - 9 = \frac{x}{4}$

k $(x - 3)(x + 5) = -16$

l $6x^2 = x + 2$

m $(b + 5)^2 = (b + 1)^2$

n $x^3 - 6x^2 + 8x = 0$

o $4x(x + 1) = 3$

p $(a + 2)(x - 4) = (a + 3)^2$

q $(2y + 3)^2 = (y + 1)(y - 3) + 3y^2$

6 Solve these quadratic equations by first removing the decimals or fractions.
You can do this by multiplying both sides by some common factor.

a $0.1x^2 - 0.3x - 2.8 = 0$

b $0.01x^2 + 0.12x + 0.2 = 0$

c $0.25x^2 + x + 1 = 0$

d $\frac{1}{3}x^2 - 2x + 3 = 0$

e $0.4x^2 + x - 2.4 = 0$

f $\frac{1}{2}x^2 + \frac{7}{3}x = \frac{5}{6}$

7 Solve these equations.

a $2x(x-2) = 6$

b $3x(x+6) = 4(x-2)$

c $4x(x+5) = 6x - 4x^2 - 3$

d $\frac{20-3x}{x} = 2x$

e $\frac{6x+8}{5x} = x$

f $\frac{7x+10}{2x} = 3x$

g $\dfrac{4}{x-2} = x + 1$

h $\dfrac{1}{x} = 3 - 2x$

i $\dfrac{x^2+12}{x} = -8$

j $\dfrac{2x^2-12}{x} = 5$

k $\dfrac{x+4}{2} - \dfrac{3}{x-3} = 1$

l $\dfrac{x}{x-2} - \dfrac{x+1}{x+4} = 1$

8 Quartic (x to power of 4) equations can be reduced to quadratic equations by using a substitution $u = x^2$.

Worked Solution	**Explanation**
Solve $x^4 + 5x^2 - 36 = 0$ $x^4 + 5x^2 - 36 = 0$ $u^2 + 5u - 36 = 0$ $(u - 4)(u + 9) = 0$ $\therefore\ u = 4$ or $u = -9$ $x^2 = 4$ or $x^2 = -9$ $x = \pm 2$ No solution to $x^2 = -9$.	What did replacing x^2 with u achieve? *We made a quartic equation into a ………………… equation.* Why isn't $u = 4, u = -9$ our final answer? u represents ……, we want to find the value of x. Why is there no solution to $x^2 = -9$? *We cannot square root a …………… number. (for now)*

Hence use the same method as above to solve these equations.

a $x^4 - 13x^2 + 36 = 0$

$x = \pm 2, \pm 3$

b $x^4 + 13x^2 + 36 = 0$

No solution

9 Solve these quadratics in disguise by using an appropriate substitution.

a $x^4 - 29x^2 + 100 = 0$

$x = \pm 2, \pm 5$

b $x^6 - 14x^3 - 32 = 0$

$x = -\sqrt[3]{3}, 2\sqrt[3]{3}$

c $(x + 2)^2 - 9(x + 2) = 0$

$x = -2, 7$

d $\frac{1}{x^2} + \frac{1}{x} - 30 = 0$

$x = -\frac{1}{6}, \frac{1}{5}$

e $x - 2\sqrt{x} - 8 = 0$

$x = 16$

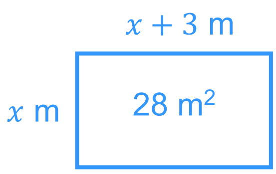
f $9^x - 82 \times 3^x + 81 = 0$

$x = 0, 4$

QUADRATIC MODELLING

- A mathematical model describes a situation involving unknown numbers using an equation.
    1. Let a variable represent the unknown number.
    2. Write an equation to describe the problem. (A diagram helps!)
    3. Solve the equation.
    4. Check your answer makes sense and has correct units.

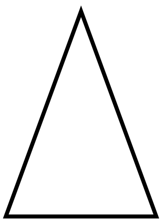
Example Model and solve problems using quadratic equations.

The area of a rectangle is fixed at 28 square metres and its length is 3 m more than its breadth. Find the width of the rectangle. 1. Let x be the width.  	Why did we rearrange the equation to standard form $ax^2 + bx + c = 0$? *We made the RHS = ……… so that we can solve the equation using the ………… ………… ……………*
2. $x(x+3) = 28$ 3. $x^2 + 3x = 28$ $x^2 + 3x - 28 = 0$ $(x+7)(x-4) = 0$ $x = -7$ or 4 4. The width is 4.	Why is the width not -7? *It is not possible for the length to be ……………………*

a A rectangle has area of 24 m^2. Its length is 5 m longer than its breadth.
Find the length of the rectangle.

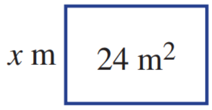
3 m

b A triangle has area of 24 m^2. Its height is 2 cm more than its base.
Calculate the length of the base.



6 m

1 A rectangle has an area of 24 m². Its length is 5 m longer than its breadth.
Let x be the breadth of the rectangle.



a Write an expression for the rectangle's length.

$x + 5$

b Write an equation using the rectangle's area.

$x(x + 5) = 24$

c Write your equation from part **c** in standard form (i.e. $ax^2 + bx + c = 0$) and solve for x.

$x = 3$, (ignore $x = -8$)

d What are the dimensions of the rectangle?

3 m by $3 + 5 = 8$ m

2 Repeat the steps from Q1 to find the dimensions of a rectangle with:

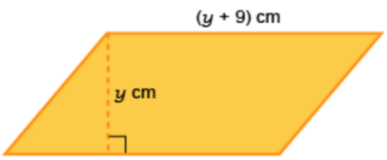
a Area 60 m² and its length is 4 m more than its breadth.

6 m by 10 m

b Area 154 cm² and its length is 3 cm less than its breadth.

14 cm by 11 cm

3 The area of the parallelogram is 22 cm². Find its length and perpendicular height.



length 11 m, height 2 m

4 A triangle has an area of 24 cm² and height 2 cm less than its base.
Find the height and base lengths.

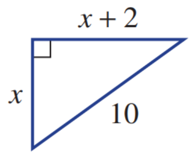
base 8 cm, height 6 cm

5 A triangle that has an area of 7 m² and height 5 m more than its base.
Find the height and base lengths.

base 2 m height 7m

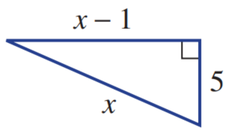
6 Use Pythagoras' theorem to solve for x in these right-angled triangles.

a



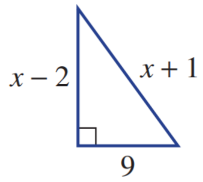
6

b



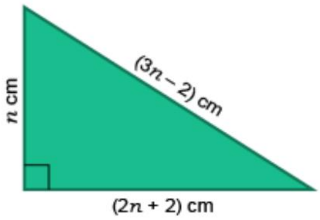
13

c



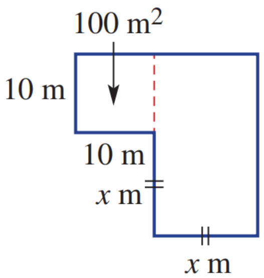
14

7 Determine the lengths of all three sides of this right-angled triangle.



5, 12, 13

8 A 100 m^2 hay shed is to be extended to give 475 m^2 of floor space in total. Find the value of x.



15 m

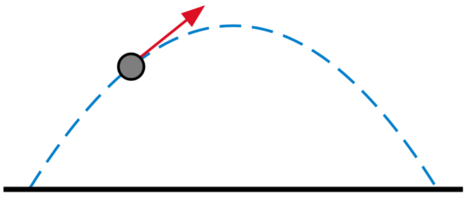
9 Find a positive integer that, when increased by 30, is 12 less than its square.

7

10 Two positive numbers differ by 3 and the sum of their squares is 117.
Find the numbers.

−3 or 5

11 A ball is thrust upwards from a machine on the ground.
The height (h metres) after t seconds is given by $h = t(4 - t)$.



a Find the height after 1.5 seconds.

3.75 m

b Find when the ball is at ground level.

0 s and 4 s

c Why are there two solutions to part **b**?

The ball was on the ground at the start and after it fell back down 4 s later

d Find when the ball is at a height of 3 metres.

3 s and 1 s

e Find when the ball is at a height of 4 metres.

2 s

f Why is there only one solution for part **e**?

This must be the maximum height of the ball

12 If the length of each side of a square is increased by 6, the area is multiplied by 16.
Find the length of one side of the original square.

$(x + 12)^2 = 16x^2$

$x = 4$ cm

13 A square hut with side length 5 metres is to be surrounded by a veranda of width x metres. Find the width of the veranda if the area of the veranda is 24 m^2. (Hint: draw a diagram first)

$(2x + 5)(2x + 5) - 24 = 5 \times 5$

1 m

14 A rectangular swimming pool, 12 m by 8 m, is surrounded by a concrete path of uniform width. If the area of the path alone is 224 m^2, find its width.

$(2x + 12)(2x + 8) - 224 = 12 \times 8$

4 m

15 A rectangular painting is to have a total area (including the frame) of 1200 cm^2. If the painting has length 30 cm and breadth 20 cm, find the width of the frame.

$(2x + 30 + (2x + 20) = 1200$

5 cm

16 There are three positive odd consecutive integers such that the product of the first and third is 15 less than 12 times the middle integer. Find the integers.

Let numbers be $x, (x + 2), (x + 4)$

$x(x + 4) = 12(x + 2) - 15$

9, 11, 13

17 In a trapezium, one of the parallel sides is 4 cm longer than the other.
The perpendicular height is 2 cm longer than the shorter parallel side.
The length of the shorter parallel side is x cm. The area of the trapezium is 49 cm^2.

a Draw a diagram and show that $x^2 + 4x - 45 = 0$

$$A = \frac{h(a+b)}{2}$$

$$49 = \frac{(x+2)(x+x+4)}{2}$$

b Find the value of x.

5 cm

18 The product of two consecutive numbers is 72. Find the numbers using a quadratic equation.
Note that there are two possible sets of numbers.

$$x(x+1) = 72$$

8 and 9 or −9 and −8

19 The perimeter of a rectangle is 40 cm and its area is 84 cm^2. Let its width be x.
Write a quadratic formula using this information and find the rectangle's dimensions.

$$2x + 2y = 40$$

$$\therefore y = 20 - x$$

$$A = xy$$

$$84 = x(20 - x)$$

$$x^2 - 20x + 84 = 0$$

14 cm by 6 cm

20 A small rectangular block of land has a perimeter of 100 m and an area of 225 m^2.
Find the dimensions of the block of land.

$y = 50 - x$

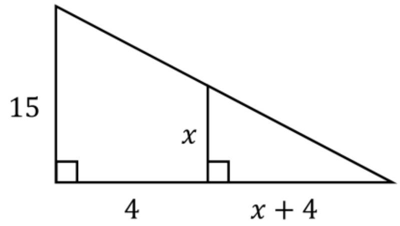
$A = xy$

$225 = x(50 - x)$

5 m and 45 m

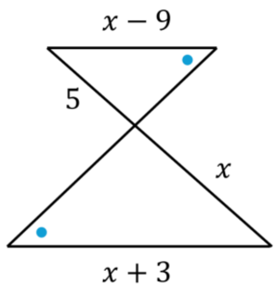
21 The triangles are similar. Find the value of x.

a



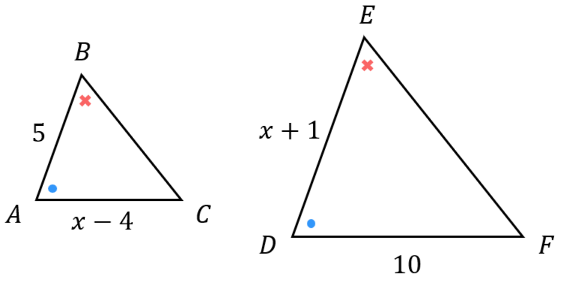
12

b



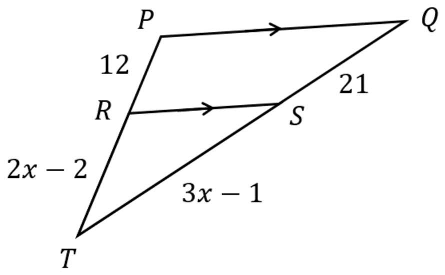
15

c Find the scale factor from ΔABC to ΔDEF



2

d $\Delta TRS \sim \Delta TPQ$. Determine the scale factor.



$\frac{5}{2}$

22 A photograph is 18 cm by 12 cm. It is to be surrounded by a frame of uniform width whose area is equal to the area of the photograph. Find the width of the frame.

$(2x + 18)(2x + 12) = 18 \times 12 + 18 \times 12$

3 cm

23 A cylindrical water tank with a closed top and bottom is being designed.
The height of the cylinder is 4 meters more than its radius.
The total surface area of the tank (including top and bottom) is 140π square meters.
Find the radius of the tank.

$SA = 2\pi r^2 + 2\pi rh$

$140\pi = 2\pi r^2 + 2\pi r(r + 4)$

5 m

24 Doug went to a conference in a city 120 km away. On the way back, due to road construction, he had to drive 10 km/h slower, which resulted in the return trip taking 2 hours longer. How fast did he drive on the way to the conference?

$d = st$

$120 = st$

$120 = (s - 10)(t + 2)$

Solve simultaneously to find s

30 km/h

25 A train travelled 240 km at a certain speed. When the engine was replaced by an improved model, the speed increased by 20 km/hr and the travel time decreased by 1 hr.
What was the speed of each engine?

$d = st$

Let original speed be s

$240 = st$

$240 = (s + 20)(t - 1)$

Solve simultaneously to find s

original speed is 6 km/h, new speed is 8 km/h

26 If a cyclist had travelled 5 km/h faster, she would have needed 1.5 hr less time to travel 150 km.
Find the speed of the cyclist.

$150 = st$

$150 = (s + 5)(t - 1.5)$

Solve simultaneously to find s

20 km/h

27 The product of the ages of Sally and Joey now is 175 more than the product of their ages 5 years prior. If Sally is 20 years older than Joey, what are their current ages?

$SJ = 175 + (S - 5)(J - 5)$

$S = J + 20$

Joey is 10 and Sally is 30.

28 The product of the ages of Cally and Katy is 130 less than the product of their ages in 5 years. If Cally is 3 years older than Katy, what are their current ages?

$C = K + 3$

$CK = (C+5)(K+5) - 130$

Katy is 9, Cally is 12

29 Mark is rowing in a river that flows downstream at 2 km/h. He rows downstream for 30 km, then turns around and returns to his original location. The total trip took 8 hrs.
How fast would Mark row in still water?

Using $t = \frac{d}{s}$

Let s be Mark's speed in still water, in km/h.

$t_{\text{downstream}} = \frac{30}{s+2}$

$t_{\text{upstream}} = \frac{30}{s-2}$

$t_d + t_u = 8$

$\therefore \frac{30}{s+2} + \frac{30}{s-2} = 8$

Solve for s: $s = 8 \text{ or } -0.5$

He rows 8 km/hr

30 A pilot flew at a constant speed for 600 km to its destination. Returning the next day, the pilot flew against a headwind of 50 km/h to return to the starting point. If the plane was in the air for a total of 7 hours, what was the speed of this plane on its outbound flight?

$t_{\text{outbound}} = \frac{600}{s}$

$t_{\text{return}} = \frac{600}{s-50}$

$\frac{600}{s} + \frac{600}{s-50} = 7$

Solving for s, $s = 200$ km/h

($\frac{150}{7}$ is not a valid solion as that would mean the pilot has negative speed on return journey)

31 If the length of each side of a square is increased by 6, the area is multiplied by 16.
Find the length of one side of the original square.

$(x + 12)^2 = 16x^2$

$x = 4$ units

32 Nick and Chloe want to surround their 60 by 80 cm wedding photo with frame of equal width.
The resulting photo and frame is to be covered by a 1 m^2 sheet of glass.
Find the width of the frame.

$(80 + 2x)(60 + 2x) = 10000$

$x = -35 + 5\sqrt{101} \approx 15.25$ cm

33 The length of a room is 8 m greater than its width.
If both the length and the width are increased by 2 m, the area increases by 60 m^2.
Find the dimensions of the room.

$L = W + 8$

$(L + 2)(W + 2) = LW + 60$

Solve simultaneously

$W = 10, L = 18$

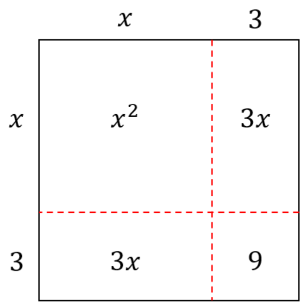
COMPLETING THE SQUARE

- Some quadratic equations cannot be factorised easily, e.g., $x^2 - 2x - 5 = 0$
- They can be solved by **completing the square**.

Investigation Completing the Square

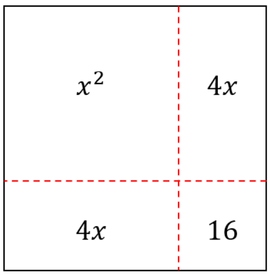
Consider the area model of a binomial expansion.

The binomial perfect square of $(x + 3)^2$ is a square with sides $x + 3$ and area $x^2 + 6x + 9$

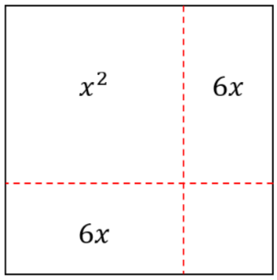


Using these diagrams, make a perfect square by finding the missing number.

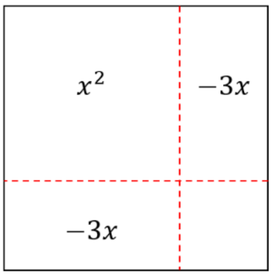
a $x^2 + 8x + 16 = (\qquad)^2$



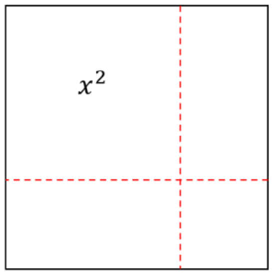
b $x^2 + 12x + \ldots\ldots\ldots = (\qquad)^2$



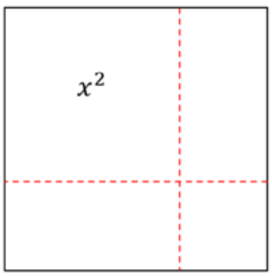
c $x^2 - 6x + \ldots\ldots = (\qquad)^2$



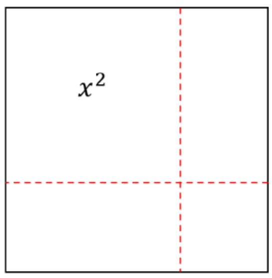
d $x^2 + 24x + \ldots\ldots\ldots = (\qquad)^2$



e $x^2 + 10x + \ldots\ldots = (\qquad)^2$



f $x^2 - 2x + \ldots\ldots\ldots = (\qquad)^2$



COMPLETING THE SQUARE

- A perfect square is of the form $x^2 + 2ax + a^2 = (x + a)^2$
- To make a perfect square from $x^2 + 2ax$:
  1. Halve the coefficient of x.
  2. Square the result.
  3. Add this to the original expression.
  4. Factorise the expression by recognising it as a perfect square.

Example Add a number to an expression to make a perfect square.

Add a number to make this a perfect square. $x^2 + 8x + \underset{\dots\dots\dots}{4^2}$ $= (x + 4)^2$	How did we know to add on 4^2? *The coefficient of x is ....* *Half the coefficient of x is ...., we add 4^2* How do we know what goes inside the brackets when we factorise? *It is the .... of the constant term.*
$x^2 + 5x + \underset{\dots\dots\dots}{\left(\frac{5}{2}\right)^2}$ $= \left(x + \frac{5}{2}\right)^2$	How did we know to add on $\left(\frac{5}{2}\right)^2$? *The coefficient of x is ....* *Half the coefficient of x is ...., we add $\left(\frac{5}{2}\right)^2$* Expand $\left(x + \frac{5}{2}\right)^2$ to check it equals $x^2 + 5x + \frac{25}{4}$

What number must be added to these expressions to make a perfect square?

a $x^2 + 6x + \dots\dots\dots$ $= (x + 3)^2$ $x^2 + 6x + 9 = (x + 3)^2$	**b** $x^2 - 6x + \dots\dots\dots$ $= \dots\dots\dots\dots$ $x^2 - 6x + 9 = (x - 3)^2$	**c** $x^2 + 10x + \dots\dots\dots$ $= \dots\dots\dots\dots$ $x^2 + 10x + 25 = (x + 5)^2$
d $x^2 + 7x + \dots\dots\dots$ $= \left(x + \frac{7}{2}\right)^2$ $x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$	**e** $x^2 - 9x + \dots\dots\dots$ $= \dots\dots\dots\dots$ $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$	**f** $x^2 - 15x + \dots\dots\dots$ $= \dots\dots\dots\dots$ $x^2 - 15x + \frac{225}{4} = \left(x - \frac{15}{2}\right)^2$

Key ideas

1. A perfect square is: $(x + a)^2 = \dots\dots\dots\dots\dots\dots\dots\dots\dots$
2. To create a perfect square from an expression of the form $x^2 + bx$,
   a. .... the coefficient of x
   b. .... the result and add it to the expression.
   c. .... the resulting expression.

1 Add a number to create a perfect square.

a $x^2 + 10x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 10x + 25 = (x+5)^2$

b $x^2 - 10x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 10x + 25 = (x-5)^2$

c $x^2 + 20x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 20x + 100 = (x+10)^2$

d $x^2 - 4x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 4x + 4 = (x-2)^2$

e $x^2 - 8x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 8x + 16 = (x-4)^2$

f $x^2 - 18x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 18x + 81 = (x-9)^2$

g $x^2 + 9x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$

h $x^2 - 5x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$

i $x^2 + 12x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 12x + 36 = (x+6)^2$

j $x^2 + 7x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

k $x^2 + 15x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 15x + \frac{225}{4} = \left(x + \frac{15}{2}\right)^2$

l $x^2 - 3x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

m $x^2 - 9x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

n $x^2 - x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2$

o $x^2 + x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

p $x^2 + 50x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 + 50x + 625 = (x+25)^2$

q $x^2 - 13x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 13x + \frac{169}{4} = \left(x - \frac{13}{2}\right)^2$

r $x^2 - 100x + \ldots\ldots\ldots$

$= (\qquad)^2$

$x^2 - 100x + 2500 = (x-50)^2$

SOLVING QUADRATIC EQUATIONS BY CTS

- Completing the square involves creating a perfect square then factorising it.

$$x^2 + 2ax + a^2 = (x + a)^2$$

1. Halve the coefficient of x, square the result, then add it to both sides.
2. Factorise the perfect square
3. Solve for x. Leave answer in exact form.

Example Solve quadratic equations by completing the square.

$x^2 + 8x = 20$ 1. $+4^2 \quad + 4^2$ $x^2 + 8x + 16 = 36$ 2. $(x + 4)^2 = 36$ 3. $x + 4 = \pm 6$ $x = -4 \pm 6$ $= -10 \text{ or } 2$	How did we know to add 4^2 to both sides? *The coefficient of x is …………* *Half the coefficient of x is ………* *We add 4^2*
$x^2 - 6x + 2 = 0$ 1. $+(-3)^2 \quad + (-3)^2$ $x^2 - 6x + 9 + 2 = 9$ 2. $(x - 3)^2 + 2 = 9$ 3. $(x - 3)^2 = 7$ $x - 3 = \pm\sqrt{7}$ $x = 3 \pm \sqrt{7}$ $\approx 5.65 \text{ or } 0.35$	Why didn't we simplify $9 + 2$ to 11? *The goal was to complete the ……………, if we add $9 + 2 = 11$, we wouldn't be able to recognise it as a perfect square.* When would you use completing the square over the PSF method? *When it is not possible to find …………… factors using the PSF method.*

Solve these equations by completing the square.

a $x^2 + 10x = 0$

$x = 0 \text{ or } -10$

b $x^2 + 6x = 0$

$x = 0 \text{ or } -6$

c $x^2 - 6x + 1 = 0$

$x = 3 \pm 2\sqrt{2}$

d $x^2 + 4x + 1 = 0$

$x = -2 \pm \sqrt{3}$

e $x^2 - 7x + 10 = 0$

$x = 5 \text{ or } 2$

f $2x^2 - 12x - 22 = 0$

$x = 3 \pm 2\sqrt{5}$

1 Solve these quadratic equations by completing the square. Leave answers in exact form.

a $x^2 + 6x + 3 = 0$

$+3^2 \quad +3^2$

$x^2 + 6x + 9 + 3 = 9$

$(x + 3)^2 + 3 = 9$

$(x + 3)^2 = 6$

$x + 3 = \pm\sqrt{6}$

$x = -3 \pm \sqrt{6}$

b $x^2 + 8x + 3 = 0$

$+ \ldots\ldots \quad + \ldots\ldots$

$x^2 + 8x + 16 + 3 = 16$

………………… = ……………………………

$(x + 4)^2 = 13$

……………………………………………………

……………………………………………………

$x = -4 \pm \sqrt{13}$

c $x^2 + 4x + 2 = 0$

$+ \ldots\ldots \quad + \ldots\ldots$

$x = -2 \pm \sqrt{2}$

d $x^2 + 10x + 15 = 0$

$x = -5 \pm \sqrt{10}$

e $x^2 - 12x - 3 = 0$

$x = 6 \pm \sqrt{39}$

f $x^2 - 2x - 16 = 0$

$x = 1 \pm \sqrt{17}$

2 Read the example carefully and answer the question.

Osborne's work:

$(n+2)^2 = 9$

$(n+2)^2 = 9$

$\sqrt{(n+2)^2} = \sqrt{9}$

$n + 2 = 3$

$\quad -2 \quad -2$

$n = 1$

Osborne did not completely solve the problem.
He found the first answer correctly.
What did he need to do to find the second answer?

He forgot …… when square rooting both sides.

Solve: $(n - 5)^2 = 100$

$n = -5$ or 15

3 Solve these quadratic equations by completing the square. Answer using fully simplified surds.

a $x^2 + 8x + 4 = 0$

$x = -4 \pm \sqrt{3}$

b $x^2 - 10x + 5 = 0$

$x = 5 \pm 2\sqrt{5}$

c $x^2 - 4x - 14 = 0$

$x = 2 \pm 3\sqrt{2}$

d $x^2 - 10x - 3 = 0$

$5 \pm 2\sqrt{6}$

e $x^2 + 12x - 18 = 0$

$-6 \pm \sqrt{6}$

f $x^2 + 6x - 41 = 0$

$-3 \pm 5\sqrt{2}$

4 Solve by completing the square.

a $x^2 - 3x + 1 = 0$

$+\left(-\frac{3}{2}\right)^2 \quad +\left(-\frac{3}{2}\right)^2$

$x^2 - 3x + \frac{9}{4} + 1 = \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 + 1 = \frac{9}{4}$

$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

b $x^2 + 7x + 5 = 0$

$x = -\frac{7}{2} \pm \frac{\sqrt{29}}{2}$

c $x^2 - 3x - 2 = 0$

$x = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$

d $x^2 + 5x - 2 = 0$

$x = -\frac{5}{2} \pm \frac{\sqrt{33}}{2}$

e $x^2 - 7x + 2 = 0$

$\frac{7}{2} \pm \frac{\sqrt{41}}{2}$

f $x^2 - 2x = 3$

$x = -1$ or 3

5 Complete the square and solve the equation.

a $x^2 + 4x + 3 = 0$

$x = -3, -1$

b $x^2 + 8x + 3 = 0$

$x = -4 \pm \sqrt{13}$

c $x^2 + 12x + 3 = 0$

$x = -6 \pm \sqrt{33}$

d $x^2 - 4x + 3 = 0$

$x = 1,3$

e $x^2 - 8x + 3 = 0$

$x = 4 \pm \sqrt{13}$

f $x^2 - 12x + 3 = 0$

$6 \pm \sqrt{33}$

g $x^2 + 3x + 1 = 0$

$x = -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

h $x^2 + 5x + 1 = 0$

$x = -\frac{5}{2} \pm \frac{\sqrt{21}}{2}$

6 Solve these equations. The quadratic must be **monic** before completing the square.

a $2x^2 - 12x + 8 = 0$

$x = 3 \pm \sqrt{5}$

b $2x^2 - 10x + 4 = 0$

$x = \frac{5}{2} \pm \frac{\sqrt{17}}{2}$

c $x^2 + 3x = 5$

$x = -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$

d $x^2 + 5x = 9$

$x = -\frac{5}{2} \pm \frac{\sqrt{61}}{2}$

e $3x^2 + 27x + 9 = 0$

$x = -\frac{9}{2} \pm \frac{\sqrt{69}}{2}$

f $x^2 + 12x + 10 = 2x + 5$

$x = -5 \pm 2\sqrt{5}$

7

a Solve $x^2 + 5x - 24 = 0$

$x = -8$ or 3

b Solve $5x^2 + 6x - 2 = 0$

$x = \frac{-3 \pm \sqrt{19}}{5}$

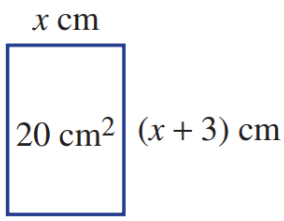
c Write $x^2 - 8x + 6$ in the form of $(x - p)^2 + q$

$(x - 4)^2 - 10$

d Solve $2x^2 + 7x + 5 = 0$

$x = -\frac{5}{2}, -1$

8 A rectangle's length is 3 cm more than its breadth.
Find the dimensions of the rectangle if its area is 20 cm^2.



$x = -\frac{3}{2} \pm \frac{\sqrt{89}}{2}$

9 Complete the square and explain why these quadratic equations have no solutions.

a $x^2 + 4x + 5 = 0$

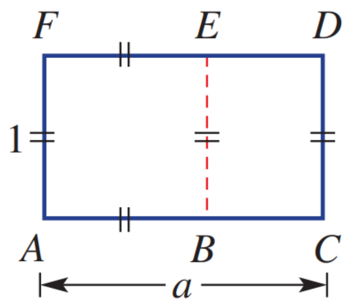
$(x+2)^2 = -1$, can't square root a negative.

b $x^2 - 3x = -3$

$\left(x - \frac{3}{2}\right)^2 = -\frac{3}{4}$, can't square root a negative.

10 This rectangle is a golden rectangle. All metric paper (A4, A3, etc) is constructed like this:

- ABEF is a square.
- Rectangle BCDE is similar to ACDF



a Using similarity, show that $\frac{a}{1} = \frac{1}{a-1}$

b Solve this equation to find a. The value of a is called the golden ratio.

$a = \frac{1}{2} + \frac{\sqrt{5}}{2}$

11 Here's an alternate way of solving after completing the square by using difference of two squares and the null factor law.

Worked Solution	Explanation
Solve: $$(x-1)^2-4=0$$ $$\big((x-1)+2\big)\big((x-1)-2\big)=0$$ $$(x+1)(x-3)=0$$ $$x=-1 \text{ or } 3$$	Which rule do we use to factorise: $$(x-1)^2-4$$ $$=\big((x-1)+2\big)\big((x-1)-2\big)$$ How did we simplify: $$\big((x-1)+2\big)\big((x-1)-2\big)$$ $$=(x+1)(x-3)$$ Why is it important RHS = 0 for using this method?

Now solve these equations using a similar method.

a $(x-2)^2-9=0$

$x=-1,5$

b $(x+5)^2-16=0$

$x=-9,-1$

c $(2x+1)^2-1=0$

$x=-1,0$

d $(5x-3)^2-25=0$

$x=-\frac{2}{5},\frac{8}{5}$

e $(4-x)^2=9$

$x=1,7$

f $(3-7x)^2=100$

$x=-1,\frac{13}{7}$

QUADRATIC FORMULA

- The **quadratic formula** solves any quadratic equation.

Quadratic Formula
For any equation $ax^2 + bx + c = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Rearrange equation to $ax^2 + bx + c = 0$, if not already.
2. Identify a, b, c
3. Substitute into formula.
4. Simplify; do not put into calculator (we usually want the surd in exact form).

Example Solve quadratic equations by the quadratic formula.

$x^2 - x - 2 = 0$

2. $a = 1,\ b = -1,\ c = -2$
3. $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$
4. $= \dfrac{1 \pm \sqrt{9}}{2}$

$= \dfrac{1 \pm 3}{2}$

$= 2$ or 1

$2y^2 - 9y = -3$

1. $2y^2 - 9y + 3 = 0$
2. $a = 2,\ b = -9,\ c = 3$
3. $x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(3)}}{2(2)}$
4. $= \dfrac{9 \pm \sqrt{57}}{4}$

What might be the disadvantages of the quadratic formula?

The quadratic equation can take ……………… than the PSF method.

Why don't we put the entire formula into the calculator?

We usually want the surd in ………… form.

Solve by using the quadratic formula.

a $x^2 + 4x + 3 = 0$

$a = \ldots,\ b = \ldots,\ c = \ldots$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -3 \text{ or } -1$

b $x^2 + 6x + 2 = 0$

$x = -3 \pm \sqrt{7}$

c $2x^2 - x - 1 = 0$

$x = 1 \text{ or } -\frac{1}{2}$

d $-8x^2 + x + 3 = 0$

$\frac{1 \pm \sqrt{97}}{16}$

e $x^2 + x = 4$

$\frac{-1 \pm \sqrt{17}}{2}$

f $16h^2 + 48h = -4$

$\frac{-3 \pm 2\sqrt{2}}{2}$

1 Determine the values of a, b and c in these quadratic equations.
You may need to rearrange into the form $ax^2 + bx + c = 0$

Equation	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$
$3x^2 + 5x + 1 = 0$			
$0 = 3x^2 + 5x + 1$			
$0 = 3x^2 + 5x + 2$			
$0 = 3x^2 + 4x - 2$			
$3x^2 - 4x + 2 = 0$			
$x^2 - 4x + 2 = 0$			
$x^2 + 2 - 4x = 0$			
$x^2 = 2 - 4x$			

2 Find the exact solutions to the following quadratic equations, using the quadratic formula.
To help you remember it, write down the formula every time.

a $x^2 + 3x - 2 = 0$

$a = 1 \quad b = 2 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b $x^2 + 7x - 4 = 0$

c $x^2 - 7x + 5 = 0$

d $x^2 - 8x + 16 = 0$

3 Solve these quadratics using the quadratic formula. Leave answers in exact surd form.

a $2x^2 + 7x + 1 = 0$

b $2x^2 - 7x + 1 = 0$

c $x^2 - 7x + 1 = 0$

d $x^2 - 7x - 1 = 0$

e $2x^2 - 14x - 2 = 0$

f $-2 - 14x + 2x^2 = 0$

g $-14x + 2x^2 = 0$

h $-2 + 2x^2 = 0$

i $2x^2 - 9x = -2$

j $2x^2 = 9x - 2$

k $9x - 2 = 2x^2$

l $2 - 9x = 2x^2$

m $2 - x = 2x^2$

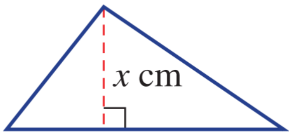
n $-2 + x = -2x^2$

o $-2 + x = -2x^2 + 10$

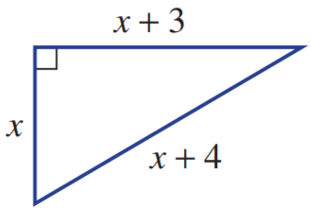
p $-2 + x = -2x^2 + 10 + 4x$

Solve these questions by writing a quadratic equation, then solving it.
You do not necessarily need to use the quadratic formula.

4 A triangle's base is 5 cm more than its height of x cm.
Find its height if the triangle's area is 10 cm^2.

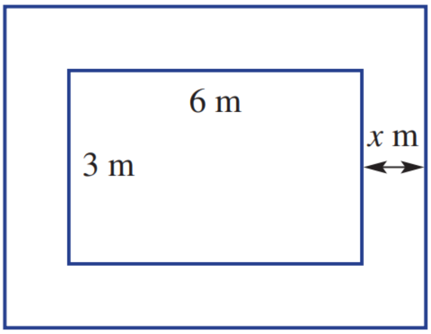


5 Find the exact value of x using Pythagoras' Theorem and the Quadratic Formula.



Hence find the exact perimeter and area using your knowledge of surd laws.

6 A pool measuring 6 m by 3 m is to have a path surrounding it.
If the total area of the pool and path is to be 31 m^2, find the width (x m) of the path, correct to the nearest cm.



7 Use the quadratic formula to find the solutions α, β of each equation, then rewrite each equation in factored form $a(x-\alpha)(x-\beta)=0$.

a $x^2 - 6x + 4 = 0$

$(x-3+\sqrt{5})(x-3-\sqrt{5})$

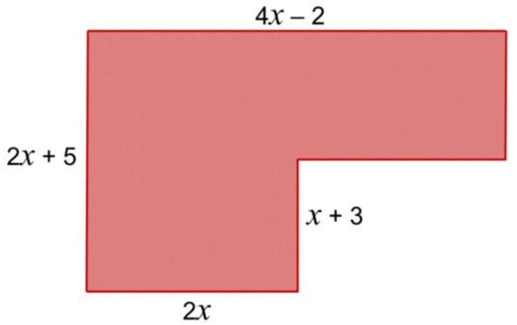
b $x^2 - 3x + 1 = 0$

$\left(x-\frac{3-\sqrt{5}}{2}\right)\left(x-\frac{3+\sqrt{5}}{2}\right)$

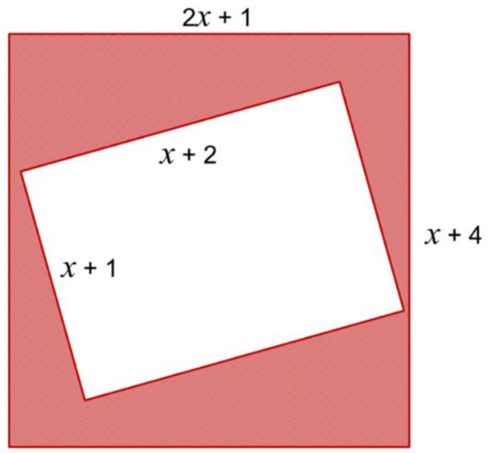
c $3x^2 + 6x + 2 = 0$

$3\left(x+\frac{3+\sqrt{3}}{3}\right)\left(x+\frac{3-\sqrt{3}}{3}\right)$

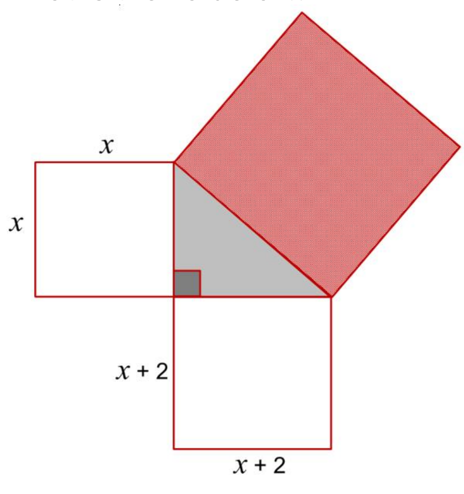
8 The shaded area is 206 cm^2. What is the value of x?



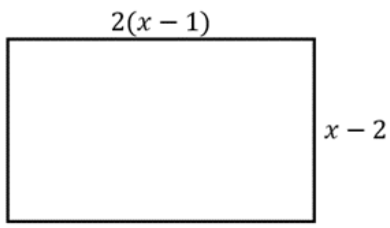
9 The shaded area is 18 cm^2. What is the value of x?



10 Three squares surround a right-angled triangle. The shaded square has area of 34 cm^2. What is the value of x?



11 Consider this rectangle.



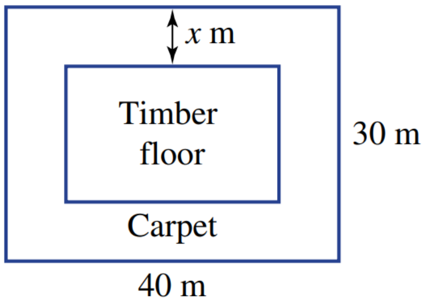
a The perimeter of the rectangle is 19 cm. Find the value of x.

b Write an expanded expression for the area of the rectangle.

c Find the length of x such that the length is 5 times the width.

d The area of the rectangle is 1.5 cm^2. Find the length.

12 A rectangular gallery floor is to be partially carpeted around the edge.
The hall is 30 metres wide and 40 metres long.
The breadth of the carpeted edge is x metres as shown.
Inside the carpeted area is a rectangular timber floor.



Find an expression in terms of x for the following:

a The breadth of the timber floor. ....

b The length of the timber floor. ....

c The area of the timber floor. ....

d Find the value of x if the timber floor area is to be 600 square metres.

13 Solve the following by rewriting as $ax^2 + bx + c = 0$ then using the quadratic formula.

a $3x^2 = 1 + 6x$	**b** $2x^2 = 3 - 4x$	**c** $5x = 2 - 4x^2$
d $2x - 5 = -\frac{1}{x}$	**e** $\frac{3}{x} = 3x + 4$	**f** $-\frac{5}{x} = 2 - x$
g $5x = \frac{2x+2}{x}$	**h** $x = \frac{3x+4}{2x}$	**i** $3x = \frac{10x-1}{2x}$
j $\frac{5k+7}{k-1} = 3k + 2$ $k = -1$ or 3	**k** $\frac{u+3}{2u-7} = \frac{2u-1}{u-3}$ $u = \frac{4}{3}$ or 4	**l** $2(k-1) = \frac{4-5k}{k+1}$ $k = \frac{-5 \pm \sqrt{73}}{4}$
m $\frac{2}{a+3} + \frac{a+3}{2} = \frac{10}{3}$ $a = -\frac{7}{3}$ or 3	**n** $\frac{3t}{t^2 - 6} = \sqrt{3}$ $t = 2\sqrt{3}$ or $-\sqrt{3}$	**o** $\frac{3m+1}{3m-1} - \frac{3m-1}{3m+1} = 2$ $m = \frac{1 \pm \sqrt{2}}{3}$

14 Solve these equations by making a substitution so that they can be considered a quadratic.

a $2^{2x} - 2^x - 12 = 0$

b $3^{2x} - 8 \times 3^{2x} - 9 = 0$

c $3 \times 9^x - 3^{x+1} - 18 = 0$

d $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$

15 A rectangular area can be completely tiled with 200 square tiles.

If the side length of each square is increased by 1 cm, it would take only 128 tiles.

Find the side length of each tile.

(hint: let x represent the side length of each tile)

4 cm

16 Two trains each make a journey of 330 km.

One of the trains travels 5km/h faster than the other and takes 30 minutes less time.

a Show that this scenario can be modelled using the equation $\frac{330}{v} - \frac{330}{v+5} = \frac{1}{2}$

b Hence find the speeds of the trains.

55 km/h and 60 km/h

34 Solve the following by forming a quadratic equation.

a The length of a square prism is 1 cm longer than the height and the width.
The surface area is 66 cm^2.
What are the dimensions of the prism?

b The length, width, and height of a rectangular prism are consecutive numbers.
The surface area is 214 cm^2.
What are the dimensions of the prism?

c A square prism has height 1 cm less than the length (and width).
The surface area is 80 cm^2.
What are the dimensions of the prism?

d The height of a rectangular prism is 1 cm more than the width. The length is double the height.
The surface of the cuboid is 72 cm^2.
What are the dimensions of the prism?

Investigation Deriving Quadratic Formula.

We will derive the quadratic formula by completing the square on this equation:

$$ax^2 + bx + c = 0$$

a Make the quadratic monic by dividing both sides by a.

b Complete the square by halving the coefficient of x, squaring it, and adding to both sides

c Rearrange the expression so that the perfect square is the subject of the LHS.

d Continue to rearrange the equation so that x is the subject.

THE DISCRIMINANT

- The **discriminant** $\Delta = b^2 - 4ac$ is the part of the quadratic formula inside the square root.

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

- The value of the discriminant allows us to distinguish how many solutions a quadratic has.

Discriminant and Number of Solutions		
$\mathbf{\Delta > 0}$ 2 solutions	$\mathbf{\Delta = 0}$ 1 solution	$\mathbf{\Delta < 0}$ No real solutions

Example Interpret the discriminant.

Calculate the discriminant, then confirm the number of solutions by solving the quadratic.

$x^2 + 5x + 4 = 0$	$x^2 + 4x + 4 = 0$	$x^2 + x + 1 = 0$
$\Delta = 5^2 - 4(1)(4)$ $= 9$ There are 2 solutions. Solve: $x^2 + 5x + 4 = 0$ $(x+4)(x+1) = 0$ $x = -4 \text{ or } -1$	$\Delta = 4^2 - 4(1)(4)$ $= 0$ There is 1 solution. Solve: $x^2 + 4x + 4 = 0$ $(x+2)^2 = 0$ $x + 2 = 0$ $x = -2$	$\Delta = 1^2 - 4(1)(1)$ $= -3$ There is no real solution. Solve: Using the quadratic formula, the calculator says MATH ERROR

Explain why $\Delta < 0$ means there are no solutions.

$\Delta = b^2 - 4ac$ is the term inside the ………….. …………..

If $\Delta < 0$, you are square rooting by a ……………., which means there are no real solutions.

Explain why $\Delta > 0$ means there are two solutions.

Explain why $\Delta = 0$ means there is only one solution.

Calculate the discriminant, identify the number of solutions, then solve the quadratic.

a $x^2 + 5x - 3 = 0$

$\Delta = 37$, 2 solutions

$x = \frac{-5 \pm \sqrt{37}}{2}$

b $2x^2 - 3x + 4 = 0$

$\Delta = -23$, no real solutions

c $x^2 + 6x + 9 = 0$

$\Delta = 0$, 1 solution

$x = -3$

REAL OR RATIONAL ROOTS

- If $\boldsymbol{\Delta} > \mathbf{0}$, the solutions are **real** and **distinct** (there are 2 different solutions).
- If $\boldsymbol{\Delta} = \mathbf{0}$, the solutions are **real** and **equal** (there is 1 solution).
- If $\boldsymbol{\Delta} < \mathbf{0}$, the solutions are **not real**. (there is no solution).
- If **$\boldsymbol{\Delta}$ is a perfect square**, the solutions will be **rational**. (i.e., not an irrational surd).
  - Perfect squares are: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, etc.

Example Decide whether the solutions will be real, rational.

The discriminant of a quadratic equation is given.
Decide whether the solutions will be real, not real, or rational or irrational.

a $\Delta = 15$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**b** $\Delta = -36$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**c** $\Delta = 0$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational
d $\Delta = 31$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**e** $\Delta = -25$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**f** $\Delta = 16$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational
g $\Delta = -139$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**h** $\Delta = 1$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational	**i** $\Delta = 169$ ❒ Real ❒ Not real ❒ Distinct ❒ Equal ❒ Rational ❒ Irrational

1 Calculate the discriminant of each equation and state the number of solutions.

a $x^2 - x - 4 = 0$

17, two solutions

b $2x^2 + 3x + 6 = 0$

-39, no real solution

c $x^2 - 9x + 20 = 0$

1, two solutions

d $x^2 + 6x + 9 = 0$

0, one solution

e $2x^2 - 5x - 1 = 0$

33, two solutions

f $-x^2 + 2x - 5 = 0$

-16, no real solution

g $-2x^2 - 5x + 3 = 0$

49, two solutions

h $-5x^2 + 2x - 6 = 0$

-116, no real solution

i $-x^2 + x = 0$

1, two solutions

2 Read the example carefully and answer the questions.

Arabella's work:

$4w^2 - 4w = -1$

$4w^2 - 4w = -1$
$\quad +1 \quad\quad +1$

$4w^2 - 4w + 1 = 0$

$w = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \dfrac{4 \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$

$w = \dfrac{4 \pm \sqrt{16 - 16}}{8}$

$w = \dfrac{4 \pm \sqrt{0}}{8}$

$w = \dfrac{4}{8} = \dfrac{1}{2}$

Why did Arabella add +1 to both sides before applying the quadratic formula?

Quadratic formula requires the equation to be in the form $ax^2 + bx + c = \ldots\ldots$

Why is there only one solution to this equation?

The d…………………… is 0

Solve: $9w^2 + 12w = -4$

$w = -\frac{2}{3}$

3 Find the discriminant of $x^2 + 4x + k = 0$:

$16 - 4k$

By solving the inequality, find values of k where the equation $x^2 + 4x + k = 0$ has:

a No solutions $\Delta < 0$

$k > 4$

b One solution $\Delta = 0$

$k = 4$

c Two solutions $\Delta > 0$

$k < 4$

4 By examining the discriminant, state the number of distinct solutions and if they are rational or irrational.

a $9x^2 - 4 = 0$ 2 distinct rational solutions	**b** $x^2 + 3x + 2 = 0$ 2 distinct rational solutions	**c** $-2x^2 + 5x + 2 = 0$ 2 distinct irrational solutions
d $-4x^2 + 20x - 25 = 0$ 1 rational solution	**e** $3x^2 + 4x + 2 = 0$ No real solutions	**f** $25x^2 - 20x + 4 = 0$ 1 rational solution

5 Find the value of p for which the quadratic equation $x^2 + 2x + p = 0$ has one solution.

$p = 1$

6 Find any values of k for which the quadratic equation $x^2 + kx + 1 = 0$ has one solution.

$k = \pm 2$

7 Find all the values of b for which $2x^2 + x + b + 1 = 0$ has two real solutions.

$b < -\frac{7}{8}$

8 Find the values of p if $px^2 + 4x + 2 = 0$ has no real solutions.

$p > 2$

9 Find all values of k for which $(k + 2)x^2 + x - 3 = 0$ has 2 real solutions.

$k > -\frac{25}{12}$

10 Find the values of k for which these equations have exactly one real solution.

a $kx^2 - 6x + k = 0$

$k = \pm 3$

b $x^2 - (k+4)x + 9 = 0$

$k = -10, 2$

c $2kx^2 + 2(k-2)x - 3 = 0$

Not possible

11 $kx^2 - (2k-1)x + k = 0$. Find the values of k for which this equation has:

a No solutions $\Delta < 0$

$k = \frac{1}{4}$

b One solution $\Delta = 0$

$k < \frac{1}{4}$

c Two solutions $\Delta > 0$

$k > \frac{1}{4}$

12 $4(k+1)x^2 + 4kx + k - 2 = 0$. Find the values of k for which this equation has:

a No solutions $\Delta < 0$

$k = -2$

b One solution $\Delta = 0$

$k > -2$

c Two solutions $\Delta > 0$

$k < -2$