Student Name

Mathematics Stage 5 Core

PROPERTIES OF GEOMETRICAL FIGURES

Book 2 Solve problems using ratio and scale factors in similar figures

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-GEO-C-01 identifies and applies the properties of similar figures and scale drawings to solve problems

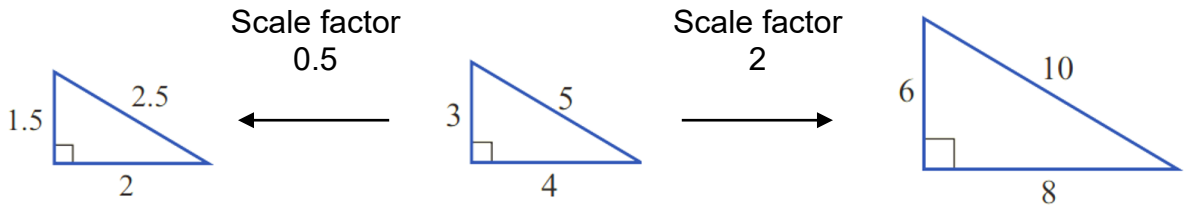
Solve problems using ratio and scale factors in similar figures

- Apply an appropriate scale to enlarge or reduce a diagram
- Determine the scale factor for pairs of similar polygons and circles
- Apply knowledge of scale factor to find unknown sides in similar polygons
- Solve problems involving unknown lengths and scale factors of similar figures and related practical problems
- Solve problems involving scale drawings, with or without digital tools

ENLARGE AND REDUCE DIAGRAMS

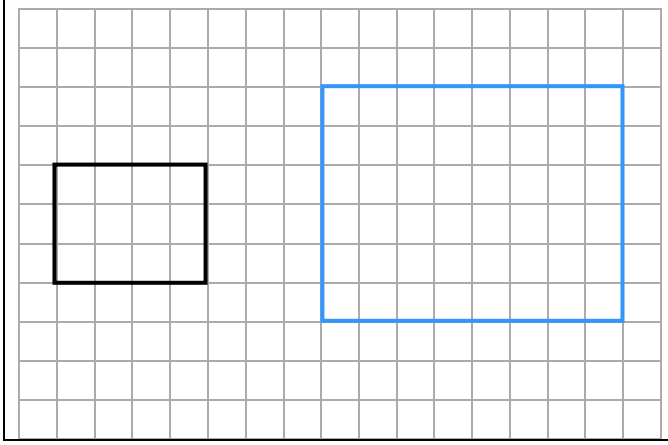
- **Similar figures** have **corresponding** sides in the same ratio.
- This ratio is called the **scale factor**, k.
- The **scale factor** is the number you multiply the original shape by to get the image.

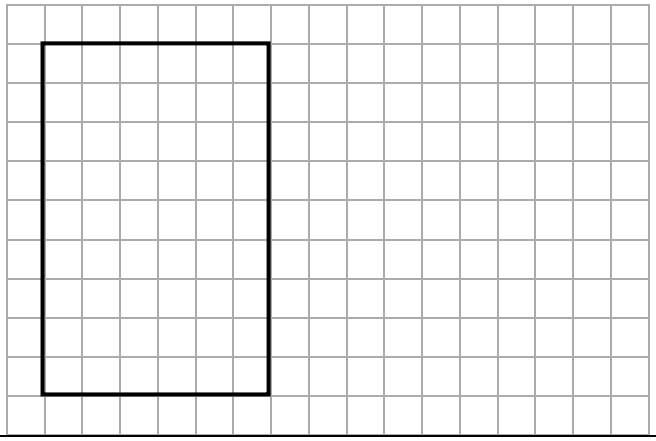
Reduction	Congruent	Enlargement
scale factor < 1 the image is smaller than the original.	scale factor = 1 the image is identical, or congruent, to the original.	scale factor > 1 the image is larger than the original.



Example Enlarge and reduce diagrams

Worked Solution	Your Turn
Enlarge this shape by a scale factor of 2.	Reduce this shape by a factor of 0.5.



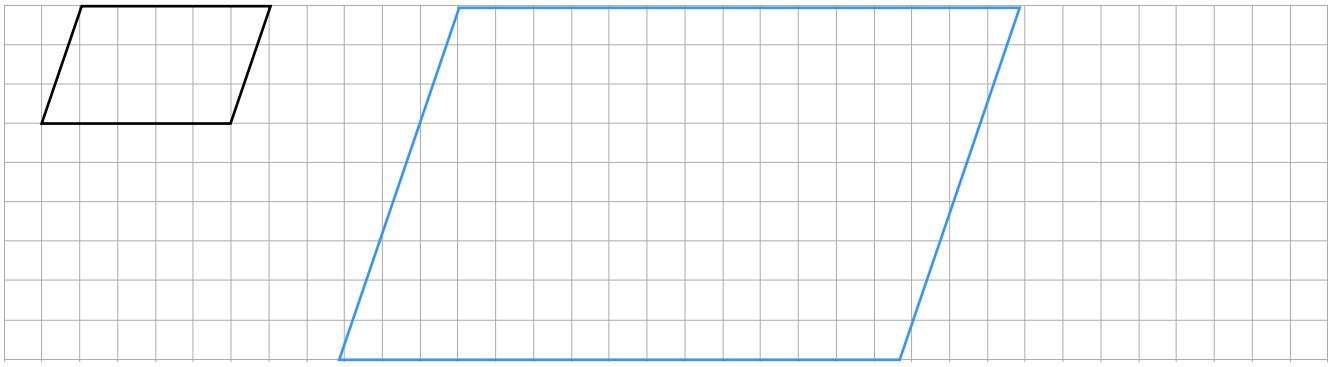


Key ideas

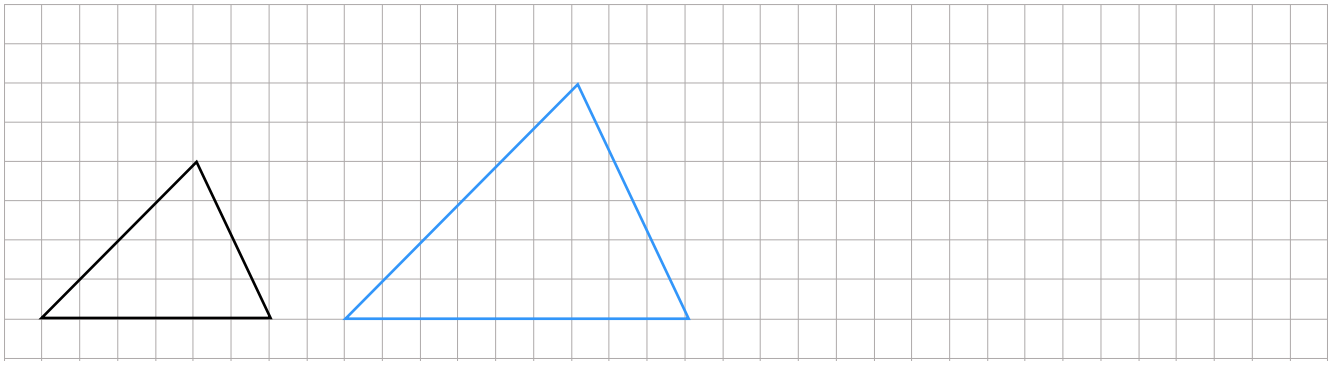
1. A scale factor of k means to …… every side of the original shape by k to get the lengths of the image.

1 Enlarge or reduce these given diagrams by the scale factor.

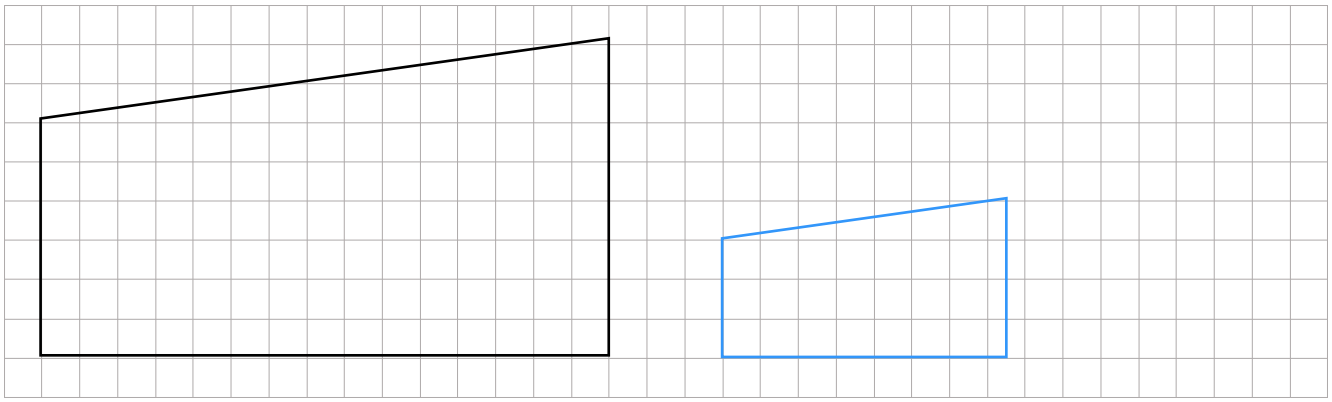
a Scale Factor: 3



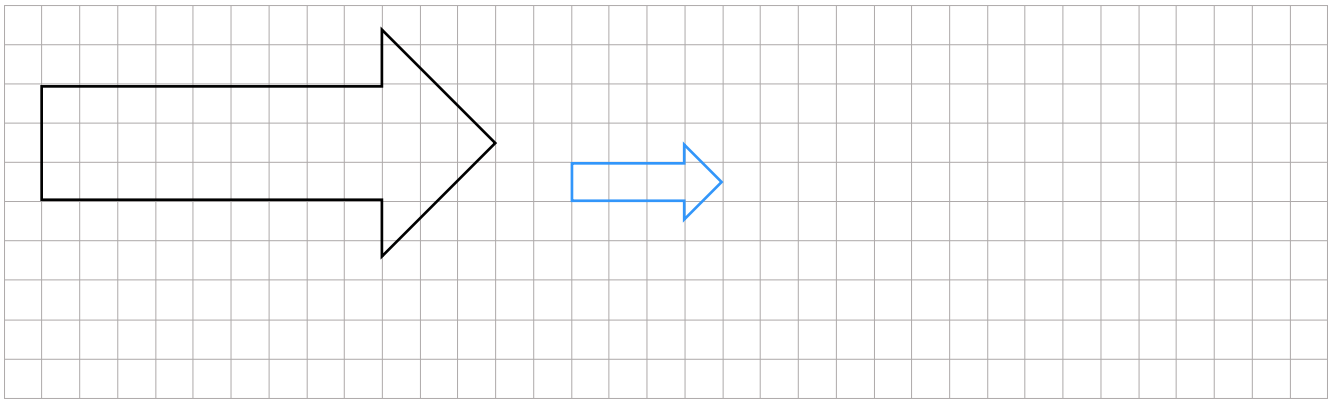
b Scale Factor: 1.5



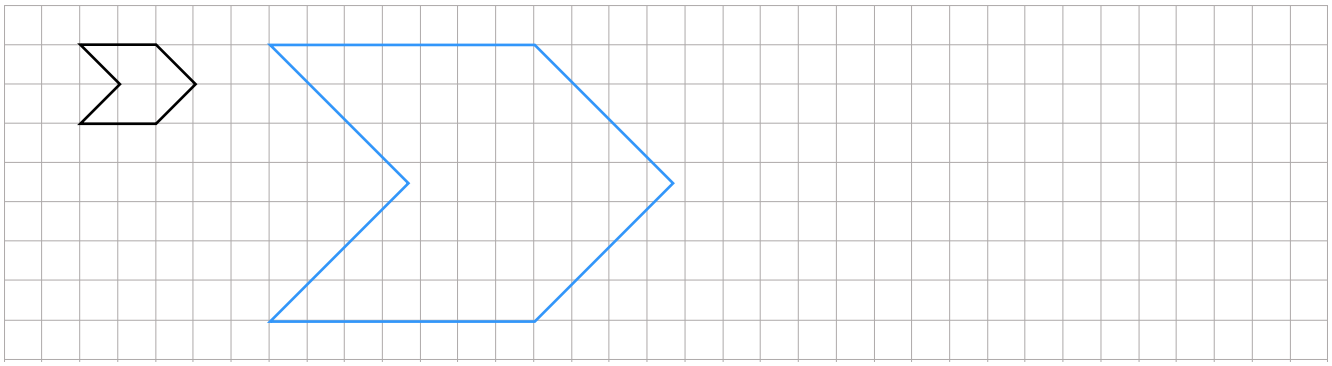
c Scale Factor: 0.5



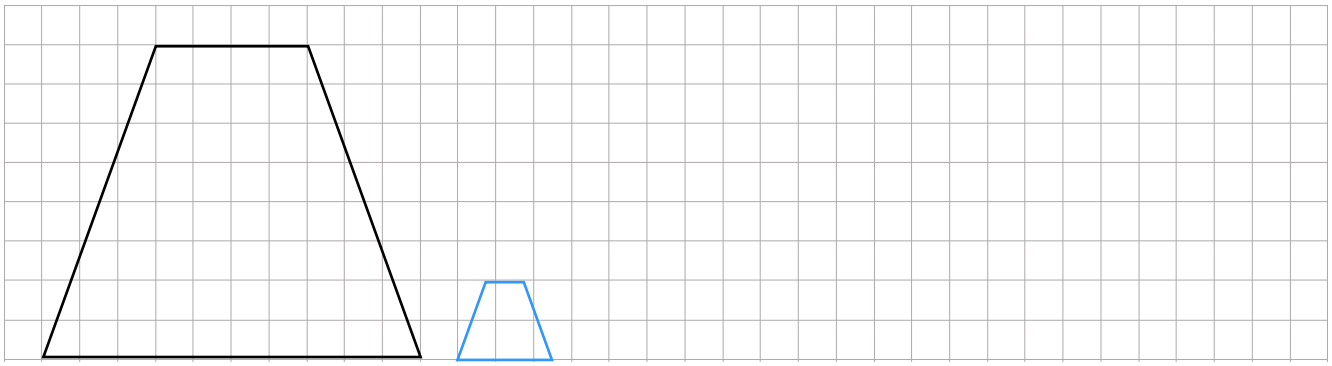
d Scale Factor: $\frac{1}{3}$



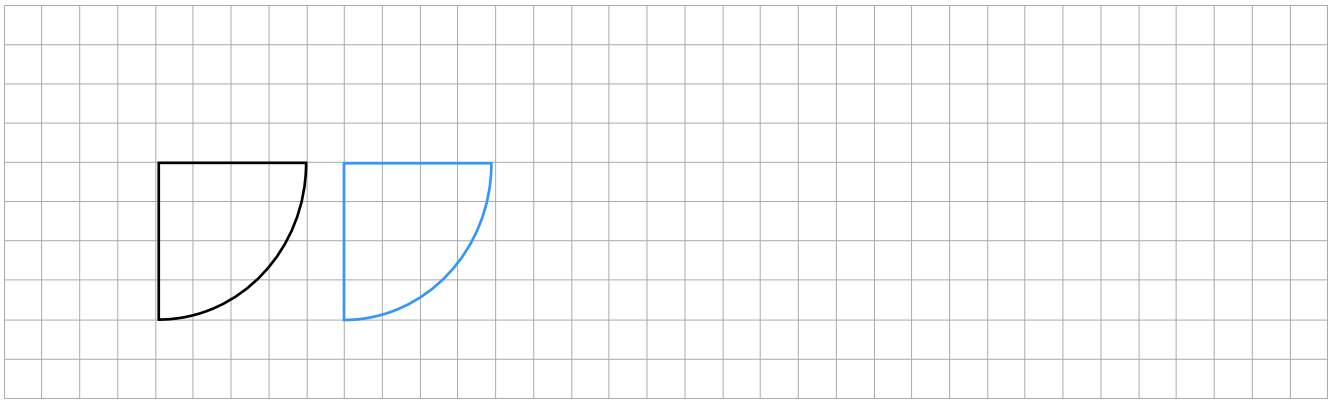
e Scale Factor: 3.5



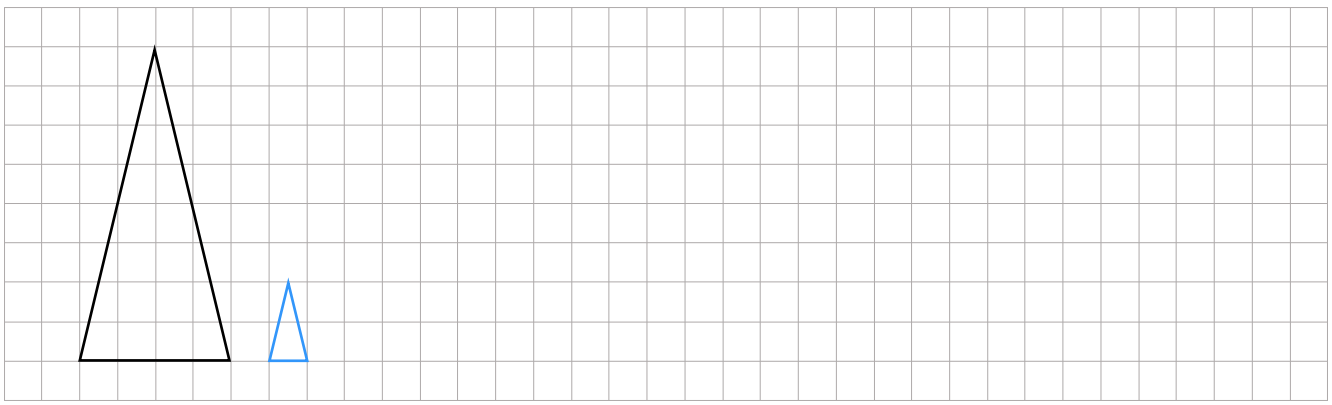
f Scale Factor: 0.25



g Scale Factor: 1



h Scale Factor: $\frac{1}{4}$



CALCULATING UNKNOWN SIDE LENGTHS

Finding Image Length and Original Length

For corresponding sides in similar figures:

× Scale Factor

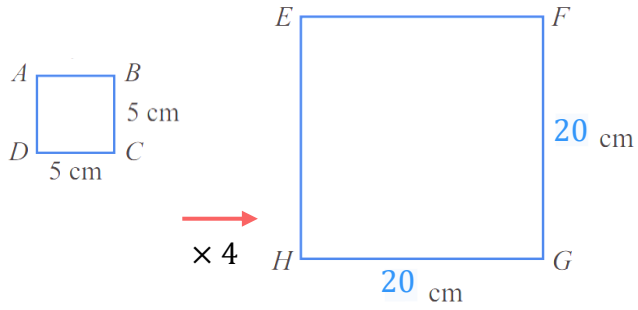
Original : Image

÷ Scale Factor

Example Calculate unknown sides given the scale factor.

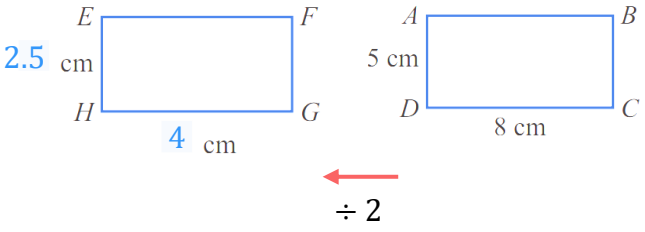
The scale factor is 4.
Calculate the sides of the image.

Original Image



The scale factor is 2.
Calculate the sides of the original.

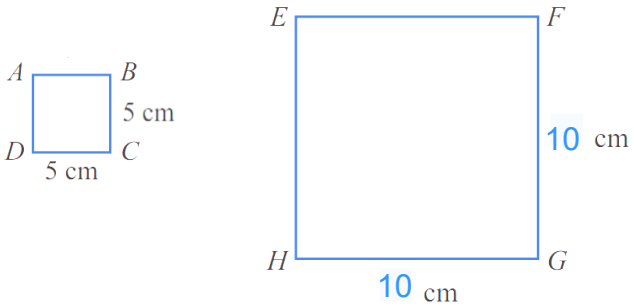
Original Image



Calculate the unknown sides.

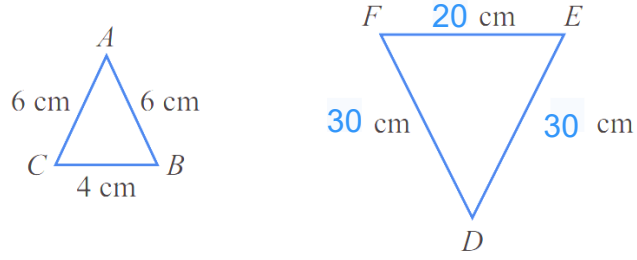
a The scale factor is 2

Original Image



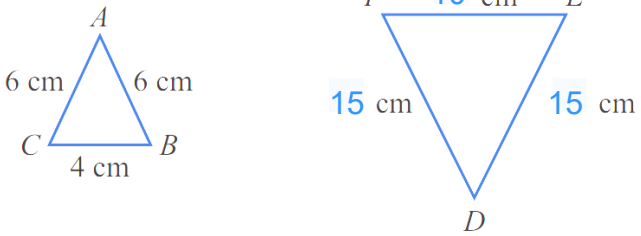
b The scale factor is 5

Original Image



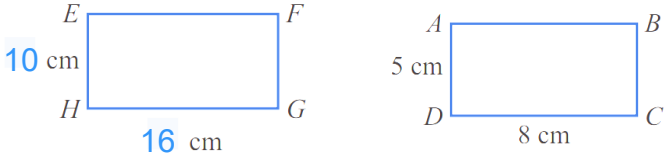
c The scale factor is 2.5

Original Image



d The scale factor is 0.5

Original Image

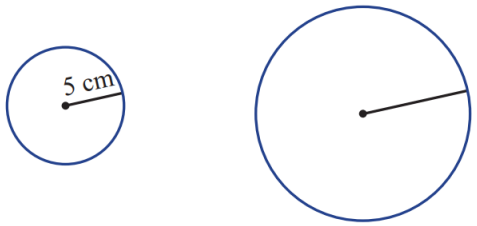


Key ideas

1. To find the length of the image, we …… the original length by the scale factor.
2. To find the length of the original, we …… the image length by the scale factor.

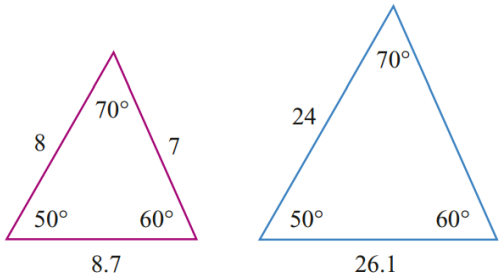
1 Calculate the missing dimensions.

a The scale factor is 2.



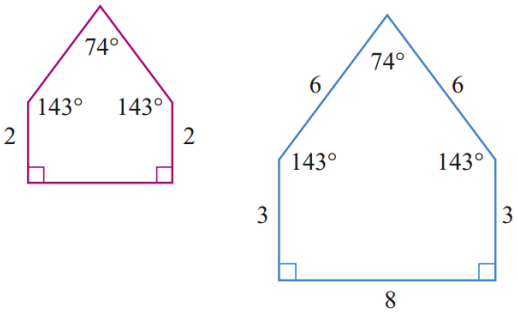
10 cm

b The scale factor is 3.



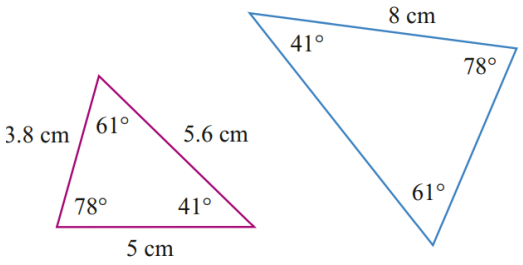
21

c The scale factor is 1.5.



4, 4, 5.3̇

d The scale factor is 1.6.



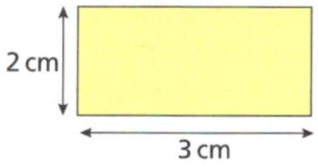
8.96, 6.08

2 A photo of a sun-flower is enlarged by a scale factor of 2.
If the larger flower has a diameter of 7.4 cm, what is the diameter of the smaller flower?

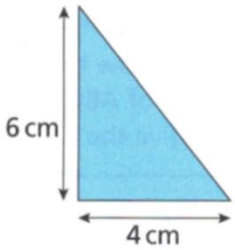
3.7 cm

3 Sketch two shapes that are similar to each of these. Label the dimensions on your diagrams.

a



b



4 A regular hexagon has been enlarged by scale factor 4. The image has a perimeter of 36 cm.
What is the length of each side of the original hexagon?

1.5 cm

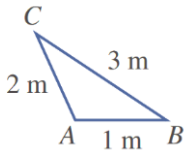
CALCULATING SCALE FACTOR

Scale Factor
$\text{Scale Factor} = \dfrac{\text{Length of Image}}{\text{Length of Original}}$

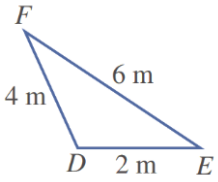
1. Identify a pair of corresponding sides.
2. Calculate scale factor.

Example Determine scale factor.

Original

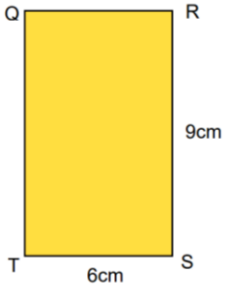


Image

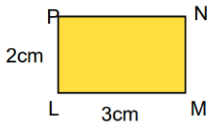


1. CB and FE are matching sides.
2. $\text{Scale Factor} = \dfrac{\text{Image}}{\text{Original}}$
$= \dfrac{FE}{CB} = \dfrac{6}{3} = 2$

Original



Image



1. RS and LM are matching sides.
2. $\text{Scale Factor} = \dfrac{\text{Image}}{\text{Original}}$
$= \dfrac{LM}{RS} = \dfrac{3}{9} = \dfrac{1}{3}$

How do we know which number to put in the numerator and denominator of the fraction?

The denominator is the length of the ……,

the numerator is the corresponding side of the ……

Determine the scale factor:

a Original length: 6 cm
Image length: 3 cm

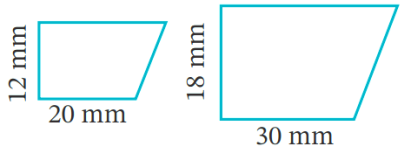
$\text{Scale Factor} = \dfrac{\text{Image}}{\text{Original}} =$

$\frac{1}{2}$

b Original length: 51 cm
Image length: 7 cm

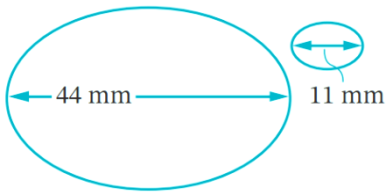
$\frac{7}{51}$

c **Original** **Image**



1.5

d **Original** **Image**



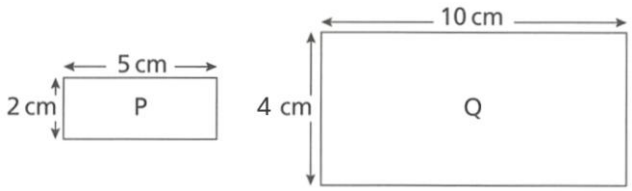
$\frac{1}{4}$

Key ideas

1. Calculate scale factor by …… the image length by the original length for a pair of …… sides.

1 Calculate the scale factor for these similar figures.

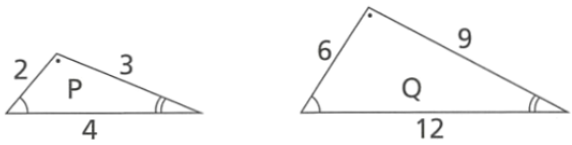
a Original Image



$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} =$

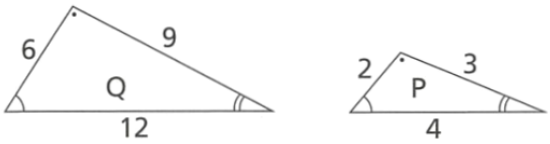
2

b Original Image



3

c Original Image



$\frac{1}{3}$

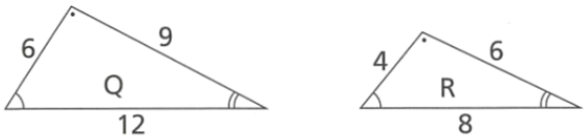
d Original Image

$\frac{3}{2}$

e Original Image

$\frac{1}{2}$

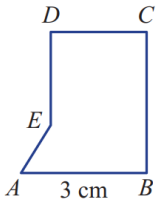
f Original Image



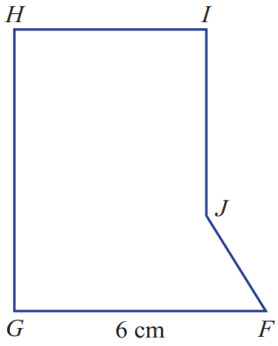
$\frac{2}{3}$

2 The two figures below are similar.

Original



Image



a Name the angle in the larger figure that matches to $\angle A$.

$\angle F$

b Name the angle in the smaller figure that matches to $\angle I$.

$\angle D$

c Name the side in the larger figure that matches to BC.

GH

d Name the side in the smaller figure that matches to FJ.

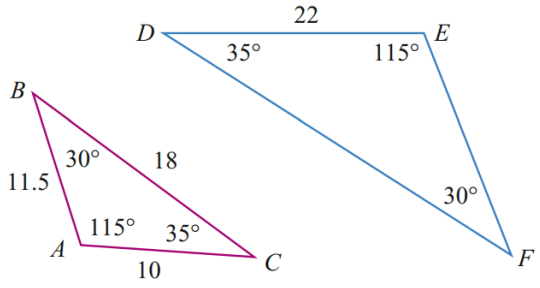
AE

e Use FG and AB to find the scale factor.

2

3 Write a similarity statement for these triangles and calculate the scale factor.
Treat the blue triangle as the image.

a



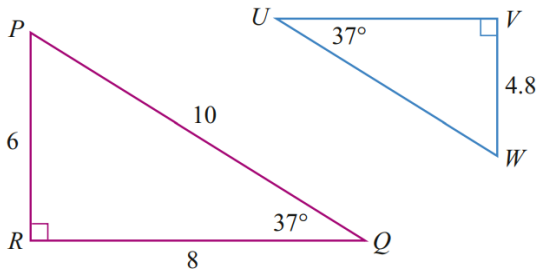
$ABC \sim$ .....

EFD

Scale factor:

2.2

b



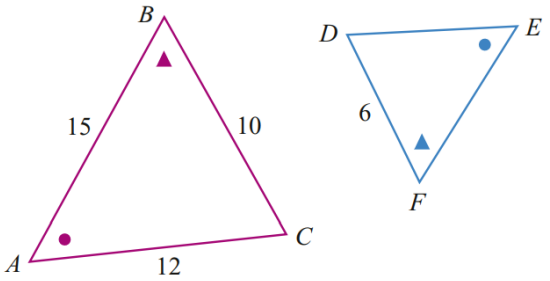
$RPQ \sim$ .....

VWU

Scale factor:

0.8

c



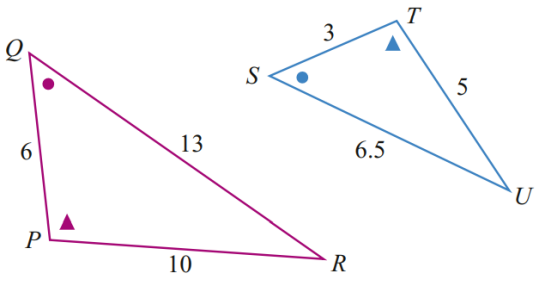
$ABC \sim$ .....

EFD

Scale factor:

0.6

d



$PQR \sim$ .....

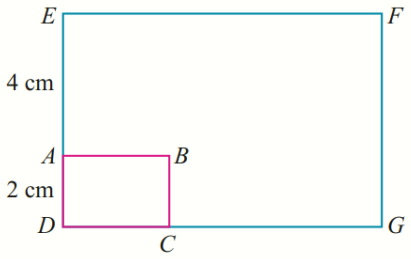
SUT

Scale factor:

0.5

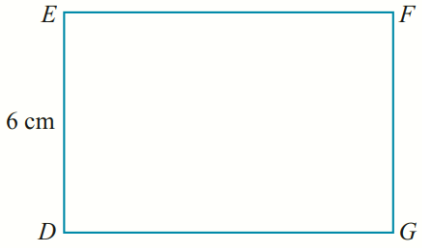
4 Determine the scale factor of the following pairs of similar figures.
It is helpful to redraw as two separate diagrams.

a



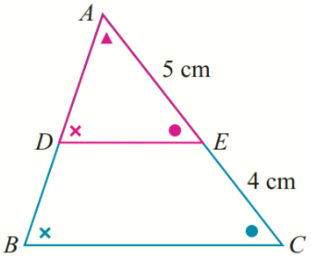
Original

Image

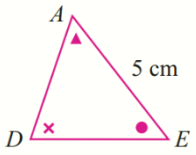


3

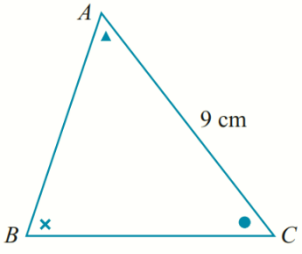
b



Original



Image



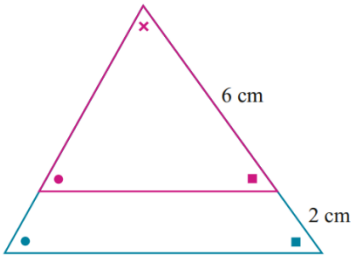
1.8

5 Determine the scale factor for each case. You may need to redraw as two separate diagrams. The purple shape is the original, the blue shape is the image.

a

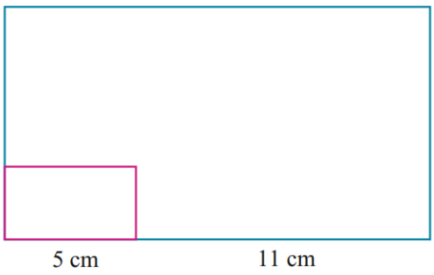
5

b



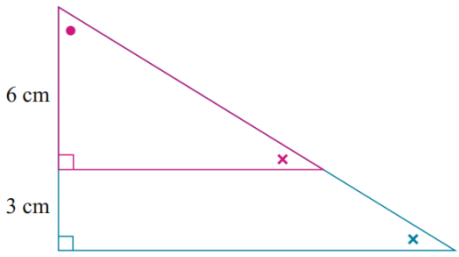
$\frac{4}{3}$

c



3.2

d



1.5

6 A photograph 129 mm by 90 mm is enlarged to 172 mm by 120 mm.
What is the scale factor?

$\frac{4}{3}$

7 A photo is projected onto a screen.
If the height of the photo is 10 cm and the height of its image on the screen is 1.2 m, what is the scale factor?

12

5 An A4 sheet of paper is 210 mm by 297 mm. An A3 sheet of paper is 297 mm by 420 mm.
What is the scale factor from A4 to A3, correct to two decimal places?

1.41

CALCULATING SIDES BY FINDING SCALE FACTOR FIRST

1. Identify corresponding sides.
2. Set up a ratio table *original : image* with corresponding sides for each row:

$\times$ scale factor $\longrightarrow$

Original	:	Image
Side 1	:	Side 1
Side 2	:	Side 2

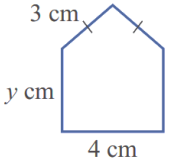
$\div$ scale factor $\longleftarrow$

3. Calculate scale factor using $\frac{\text{image}}{\text{original}}$.
4. Find unknown sides by multiplying or dividing by the scale factor.

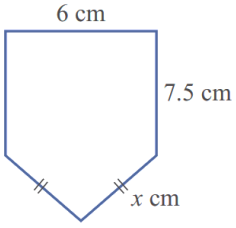
Example Calculate unknown sides given the scale factor.

Find x and y in these similar figures

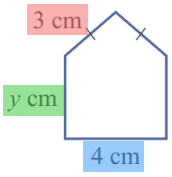
Original

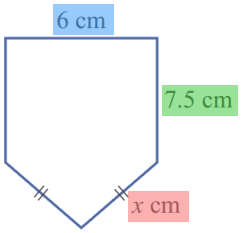


Image



1. Identify corresponding sides





2. Set up ratio table

Original	:	Image
4	:	6
3	:	x
y	:	7.5

3. Calculate scale factor

$$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} = \frac{6}{4} = 1.5$$

4. Find unknowns

Original	:	Image
4	:	6
3	$\times 1.5 \rightarrow$: $\leftarrow \div 1.5$	4.5
5	:	7.5

$x = 4.5$ cm $\qquad$ $y = 5$ cm

What part of the question tells us all sides are in equal ratio?

The question says that two figures are .....

When setting up the ratio table, what do the rows need to be?

The rows are pairs of ..... sides.

How did we know to use the lengths 6 and 4 to calculate the scale factor?

6 and 4 is the only pair of ..... sides where the length is given.

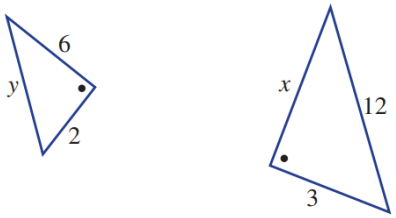
How did we know whether to multiply or divide to find the unknown?

To find the image, we .....

To find the original, we .....

These figures are similar. Find the value of x and y.

a Original Image



1. identify corresponding sides
2. set up ratio table

Original	:	Image
2	:	
6	:	
y	:	

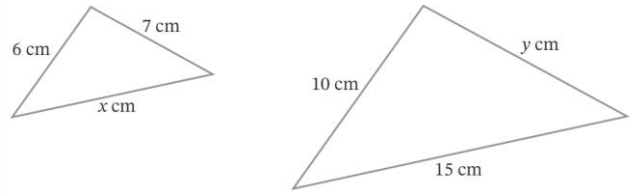
3. calculate scale factor

$$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} =$$

4. find unknowns

$x = 9, y = 8$

b Original Image



Original	:	Image
	:	
	:	
	:	

$$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} =$$

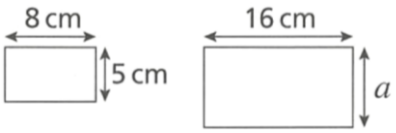
$x = 9, y = 11.\dot{6}$

Key ideas

1. To find the scale factor, we identify two …… sides, then divide the length of the …… by the length of the ……
2. To find the length of the image, we …… the original length by the scale factor.
3. To find the length of the original, we …… the image length by the scale factor.

1 Calculate the missing number.

a



Original	:	Image
8	:	12
5	:	a

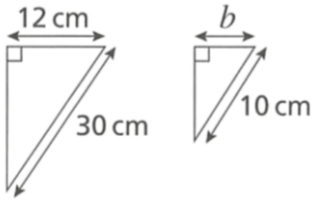
Scale Factor $= \frac{\text{Image}}{\text{Original}} =$

$a = 2 \times \ldots\ldots\ldots = \ldots\ldots\ldots\ldots$

2

10

b



Original	:	Image
30	:	10
	:	

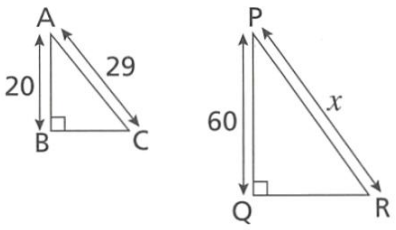
Scale Factor $= \frac{\text{Image}}{\text{Original}} =$

$\frac{1}{3}$

$b = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

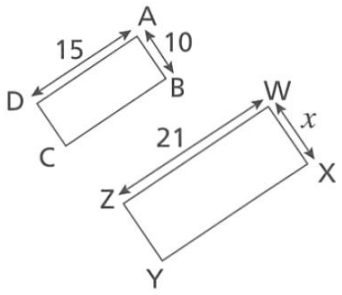
4

c



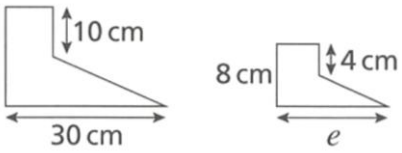
87

d



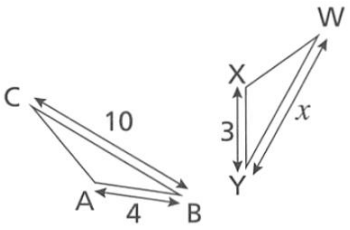
14

e



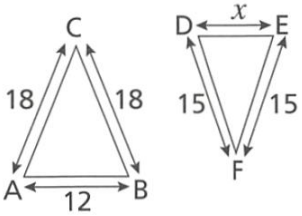
12

f



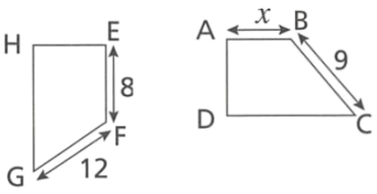
7.5

g



10

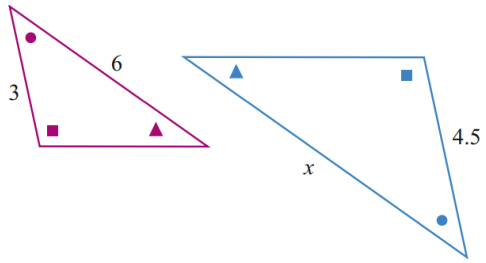
h



6

2 Calculate the scale factor and then determine the unknown length.

a



Original	:	Image
3	:	4.5
6	:	x

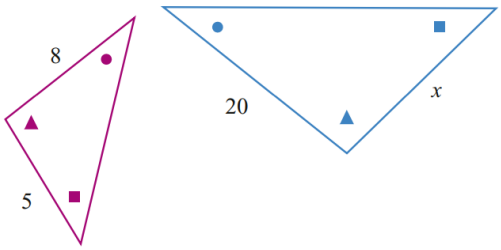
Scale Factor $= \dfrac{\text{Image}}{\text{Original}} =$

$x = 6 \times$ ..... $=$ .....

1.5

9

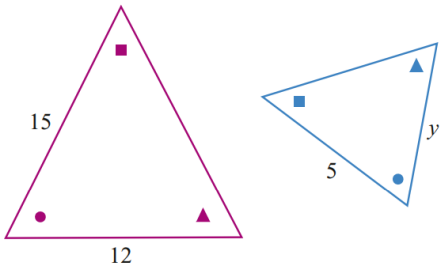
b



Original	:	Image
8	:	20
	:	

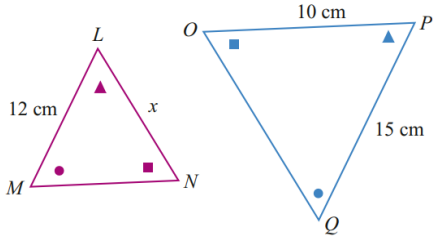
12.5

c



4

d



Original	:	Image
12	:	15
x	:	10

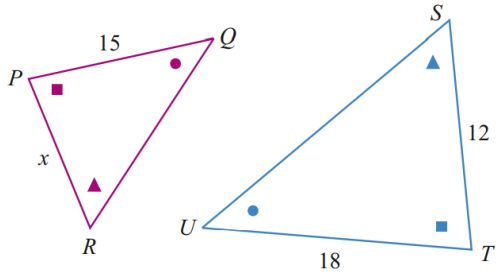
Scale Factor $=$

$x = 10 \div$ ..... $=$ .....

1.25

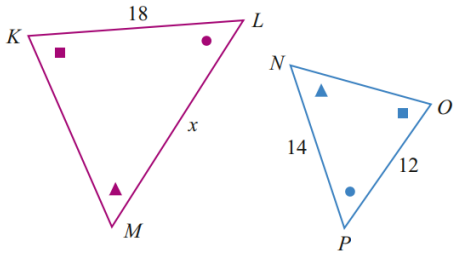
8

e



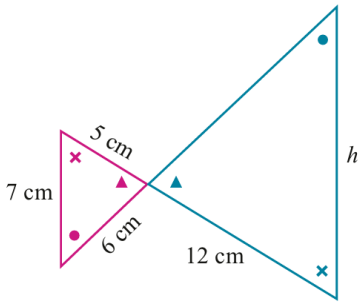
10

f



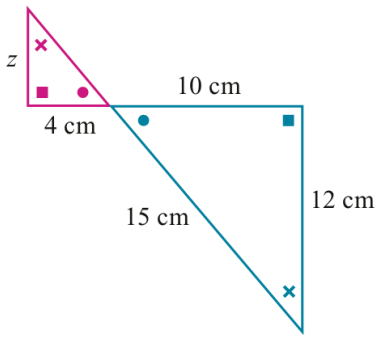
21

g



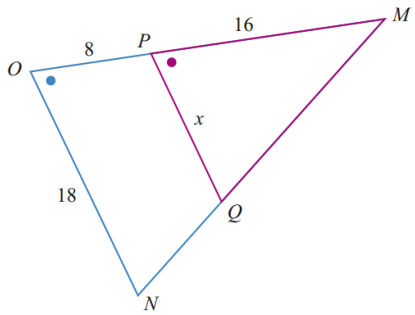
16.8 cm

h



4.8 cm

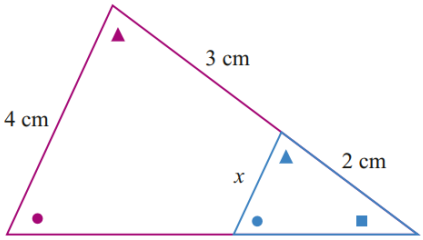
i



Redraw | 2 triangles

12

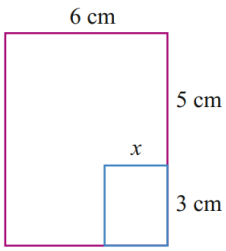
j



Redraw | 2 triangles

1.6 cm

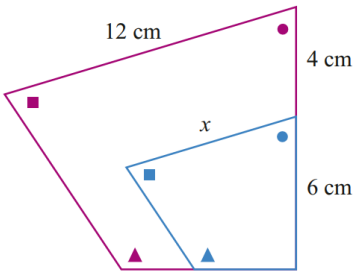
k



Redraw

2.25 cm

l

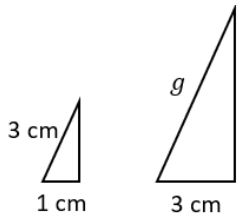


Redraw

7.2 cm

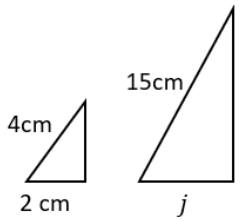
3 Find the length of the unknown sides.

a



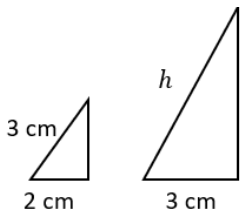
9

b



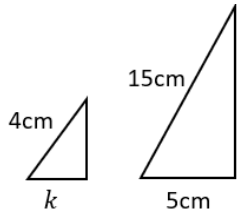
7.5

c



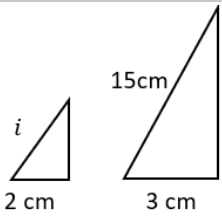
4.5

d



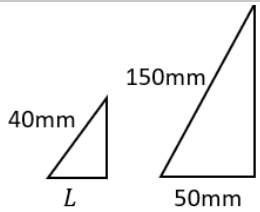
$1.\dot{3}$

e



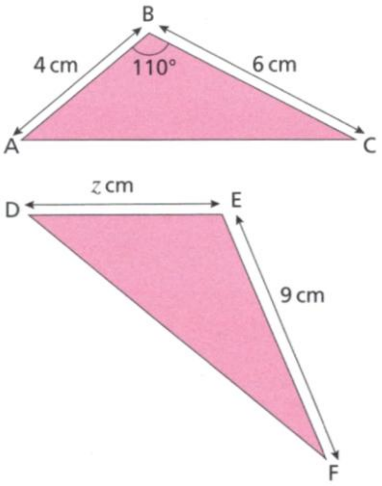
10

f



$13.\dot{3}$

4 These two triangles are similar.



a What is the scale factor of enlargement from ABC to DEF?

1.5

b Find the value of z

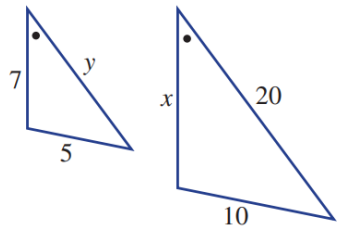
6 cm

c What is the size of angle DEF? Why?

110°, as corresponding angles in similar figures are equal.

5 Find the value of the missing sides.

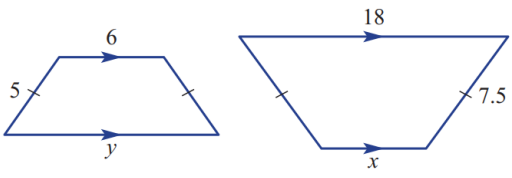
a



Original	:	Image
5	:	10
7	:	x
y	:	20

$x = 14, y = 10$

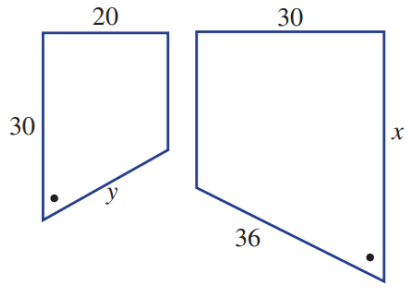
b



Original	:	Image
5	:	7.5
	:	
	:	

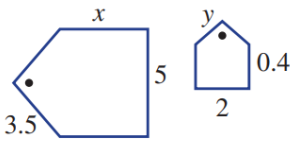
$x = 9, y = 12$

c



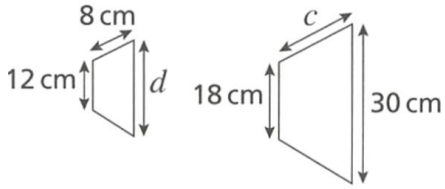
$x = 45, y = 24$

d



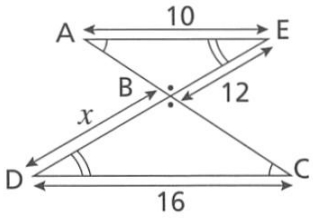
$x = 1, y = 1.4$

e



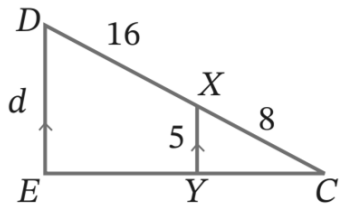
$c = 12, d = 20$

f



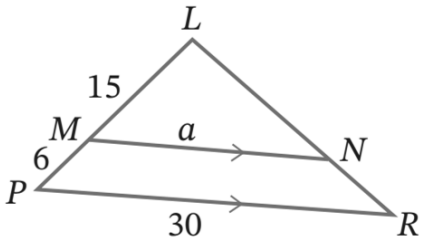
$x = 19.2$

g



$d = 15$

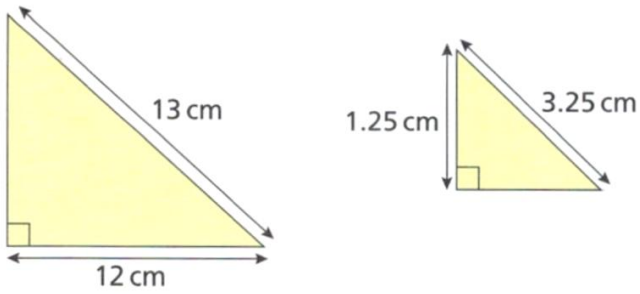
h



$a = 21.43$

6 Here are two similar right-angled triangles.

a Find the missing lengths.



5 on left triangle and 3 on right triangle

b Find the perimeter of each triangle.

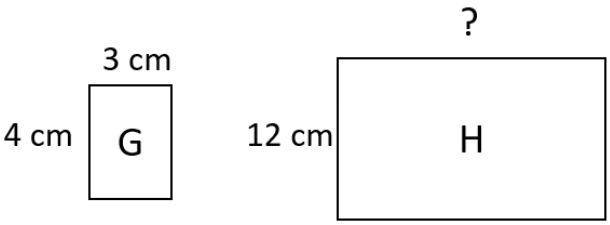
Left: 30 cm, right: 7.5 cm

c Is the ratio of the perimeters the same as the ratio of the sides? Why is this so?

Yes. Each side was scaled by the same amount, so the sum of all the sides is also scaled by the same amount.

7 Rectangles G and H are similar. Eva thinks the missing side length is 6.

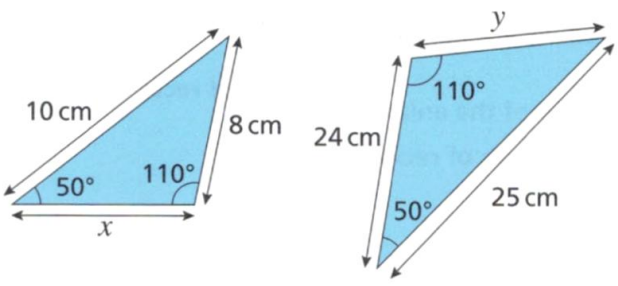
Explain why she is incorrect.



She didn't correctly identify the corresponding sides. The length should be 16 cm.

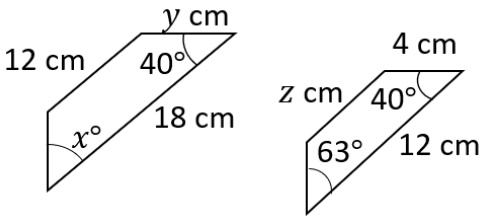
8 Calculate the missing information for these two similar figures.

a



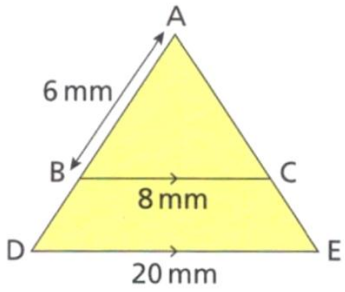
$x = 9.6, y = 20$

b



$x = 63°, y = 6, z = 8$

9 You can prove that two triangles are similar if all corresponding angles are equal.



a Using angle relationships, show that ABC and ADE are similar.

$\angle ABC = \angle ADE$ (corresponding angles on parallel lines)

$\angle ACB = \angle AED$ (corresponding angles on parallel lines)

$\angle DAE$ is common

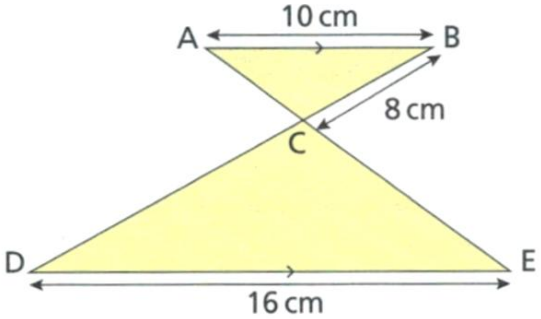
$\therefore \Delta ABC$ is similar to ΔADE (all corresponding angles are equal)

b Find the length BD.

9 cm

10

a Using angle relationships, show that ABC and CDE are similar.



$\angle ACB = \angle DCE$ (vertically opposite)

$\angle BAE = \angle DEA$ (alternate angles on parallel lines)

$\angle BDE = \angle ABD$ (alternate angles on parallel lines)

$\therefore \Delta ABC$ is similar to ΔADE (all corresponding angles are equal)

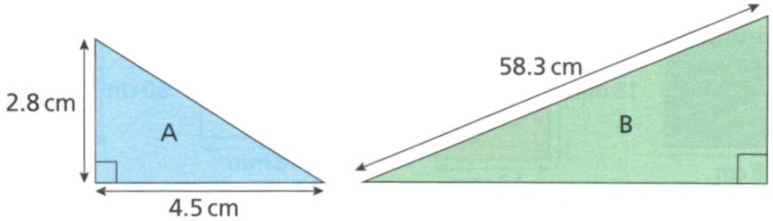
b Four students have calculated the missing lengths. Here are their answers.

CD = 12.8 cm | CD = 20 cm | CE = 12.8 cm | CE = 20 cm

Which answer is correct? Show working to explain your answer.

`CD = 12.8 cm

11 These two triangles are similar. What is the perimeter of triangle B? (Hint: Pythagoras)



138.6 cm

ALGEBRAIC FRACTIONS METHOD

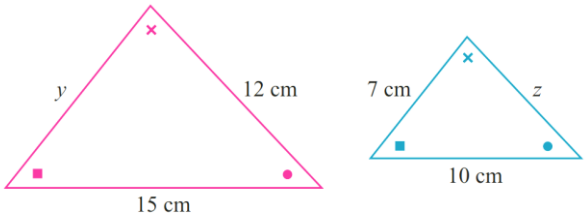
- Corresponding sides of similar figures are in the same ratio:

$$\left(\frac{\text{Image}}{\text{Original}}\right)_{\text{matching sides}} = \left(\frac{\text{Image}}{\text{Original}}\right)_{\text{matching sides}}$$

$$\left(\frac{\text{Original}}{\text{Image}}\right)_{\text{matching sides}} = \left(\frac{\text{Original}}{\text{Image}}\right)_{\text{matching sides}}$$

- Always have the unknown in the numerator to make the equation easier to solve.

Example Solve ratio problems using the algebraic fractions method.



Finding z:

$$\frac{z}{12} = \frac{10}{15}$$
$$\times 12 \quad \times 12$$
$$z = 8$$

Finding y:

$$\frac{y}{7} = \frac{15}{10}$$
$$\times 7 \quad \times 7$$
$$y = 10.5$$

How did we set up the equation $\frac{z}{12} = \frac{10}{15}$?

The ratios of ..... sides are equal.

How did we solve $\frac{z}{12} = \frac{10}{15}$?

..... both sides by the ..... of z.

Why did we write $\frac{y}{7} = \frac{15}{10}$ and not $\frac{7}{y} = \frac{10}{15}$?

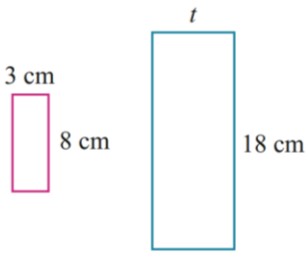
We want the ..... to be in the numerator.

Check the this method works using the ratio table.

Original	:	Image
	:	
	:	
	:	

Use the algebraic fractions method to determine the value of the pronumerals.

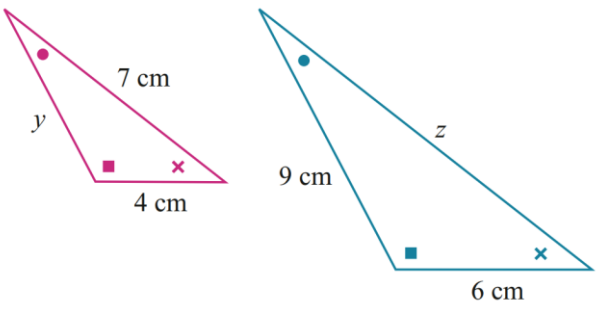
a



$$\frac{t}{3} = \frac{18}{8}$$

$t = 6.75$ cm

b



$$\frac{z}{7} = \frac{6}{4}$$

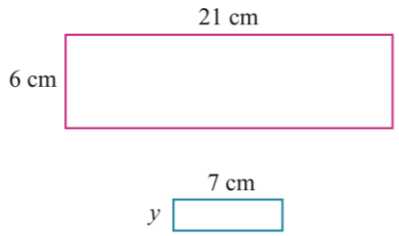
$z = 10.5$ cm

$$\frac{y}{9} = \frac{4}{6}$$

$y = 6$ cm

1 Use the algebraic fractions method to find the unknown values.

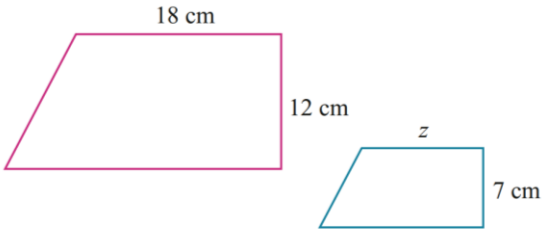
a



$\frac{y}{6} = \frac{\quad}{\quad}$

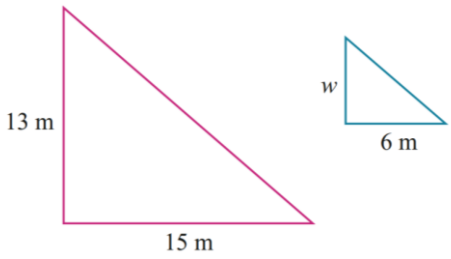
2 cm

b



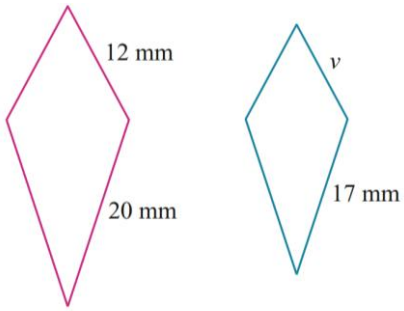
10.5 cm

c



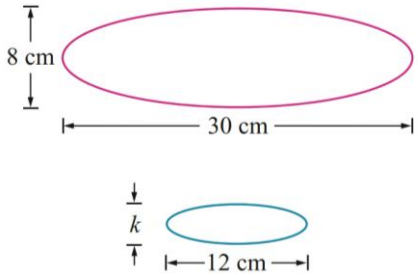
5.2 cm

d



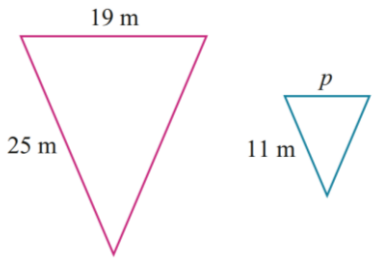
10.2 cm

e



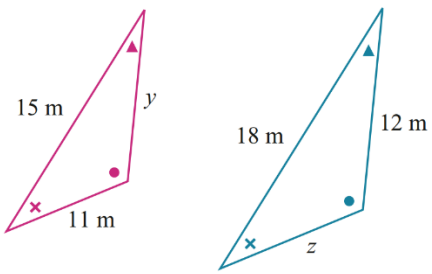
3.2 cm

f



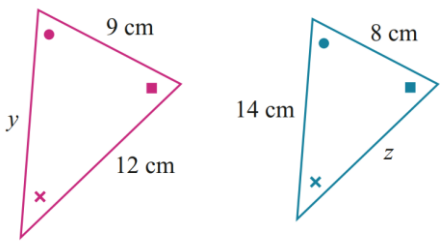
8.36 cm

g



$y = 10$ m, $z = 13.2$ m

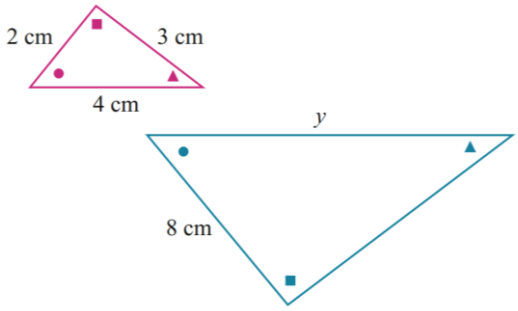
h



$y = 15.75$ cm, $z = 10.7$ cm

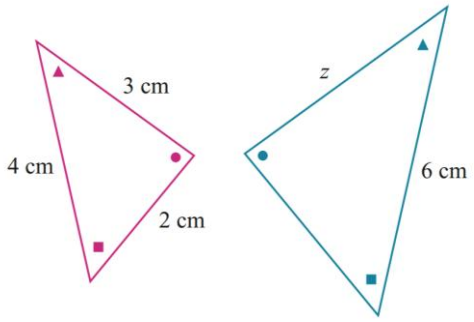
2 Determine the value of the pronumeral. Careful, as these triangles are not oriented the same.

a



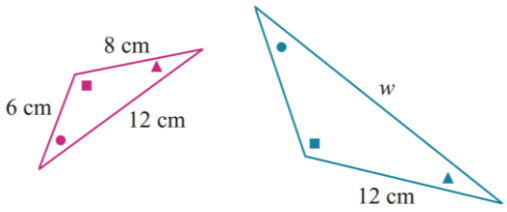
16 cm

b



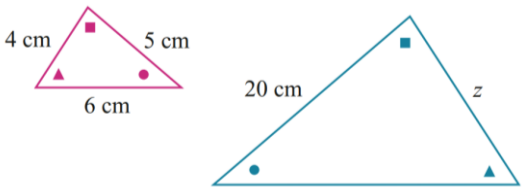
4.5 cm

c



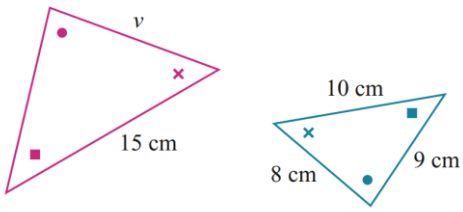
18 cm

d



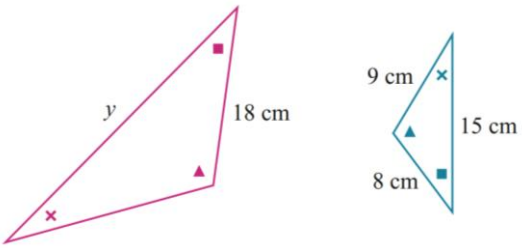
16 cm

e



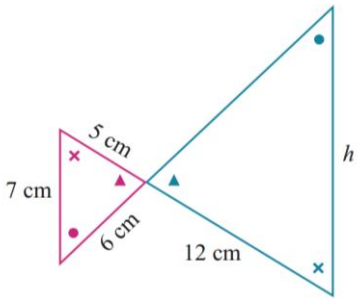
12 cm

f



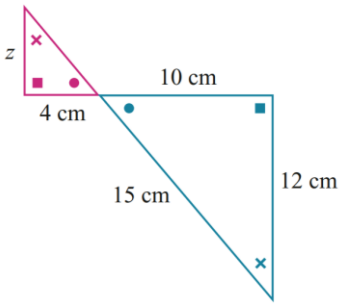
33.75 cm

g



16.8 cm

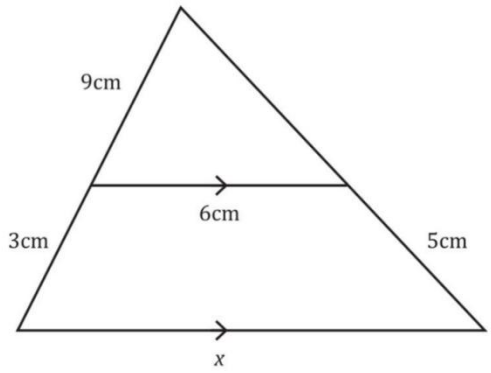
h



4.8 cm

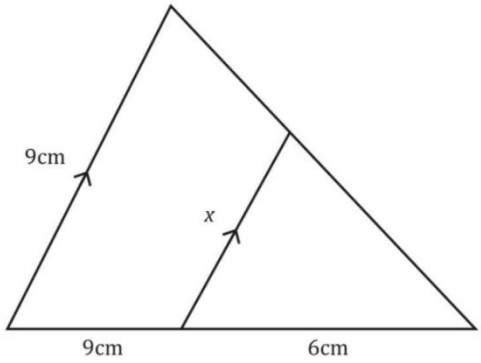
3 Find the value of x in each of these triangles.

a



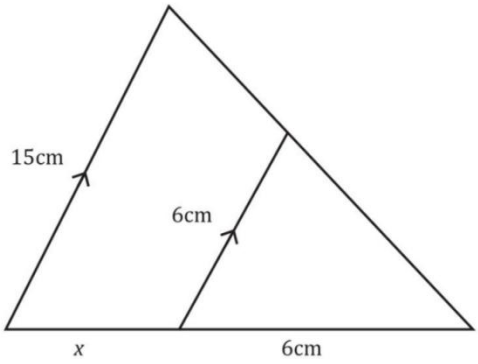
8 cm

b



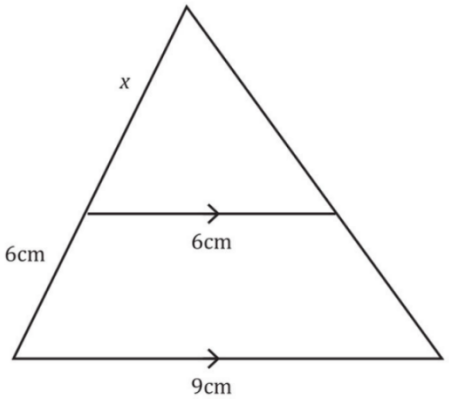
3.6 cm

c



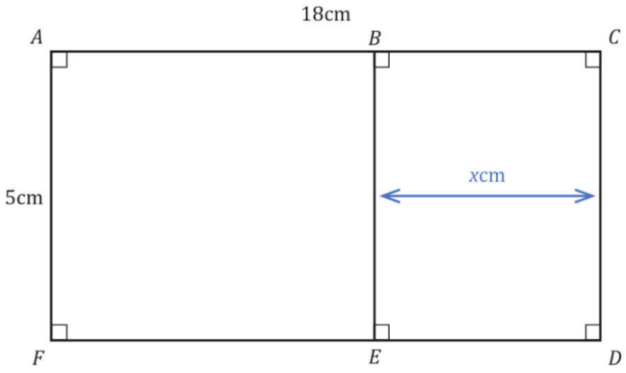
9 cm

d



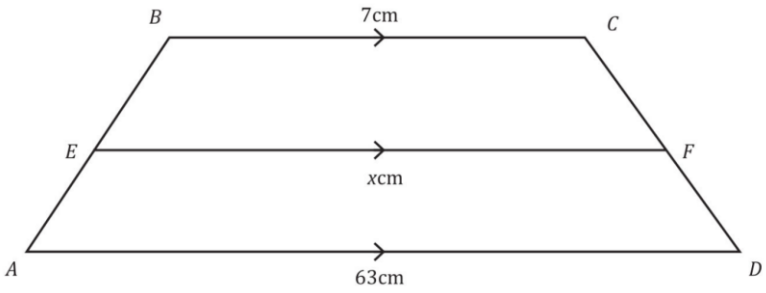
12 cm

4 Rectangles ACDF and EBCD are similar. Find the value of x.



$x = \frac{25}{18}$ cm

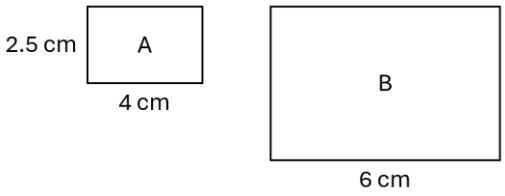
5 Trapezia BCFE and EFDA are similar. Find the value of x.



21 cm

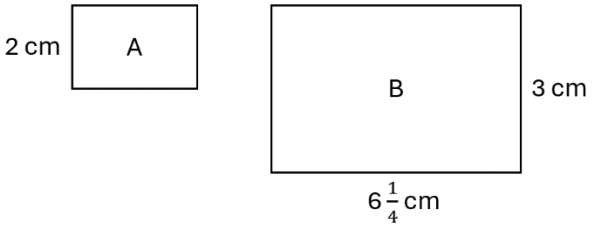
6

a These rectangles are similar.
Find the area of B.



22.5 cm^2

b These rectangles are similar.
Find the length of rectangle A.



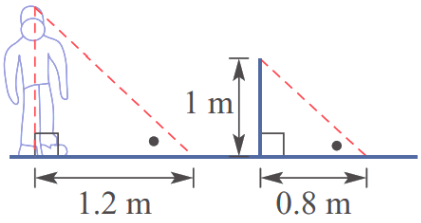
$\frac{25}{6}$

SIMILAR FIGURE PROBLEMS

1. Set up ratio table.
2. Calculate scale factor.
3. Find unknown sides.

Example Solve problems involving scale and similarity.

Chris' shadow is 1.2 m long and a 1 m vertical stick has a shadow that is 0.8 m long. Determine Chris' height.



Ratio table method

Original	:	Image
1.2	:	0.8
1.5	:	1

$\div \frac{2}{3}$

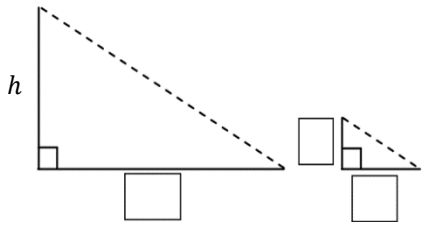
Algebraic fraction method

$$\frac{x}{1} = \frac{1.2}{0.8}$$

$$x = 1.5$$

Chris is 1.5 m tall.

A tree's shadow is 20 m long and a 2 m vertical stick has a shadow that is 1 m long. Find the height of the tree.

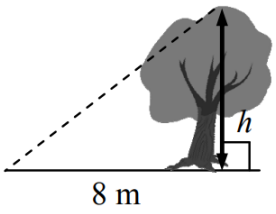


40 m

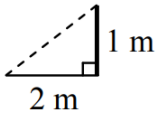
1

a A 1 m high post casts a 2 m long shadow. How tall is the tree that casts an 8 m long shadow?

Original

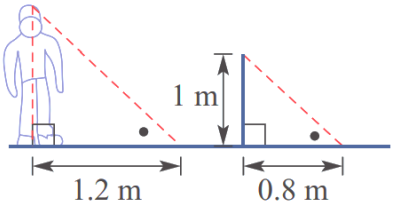


Image



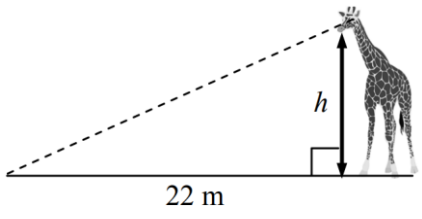
4 m

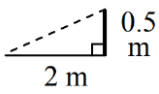
b Chris' shadow is 1.2 m long and a 1 m vertical stick has a shadow that is 0.8 m long. Determine Chris' height



1.5 m

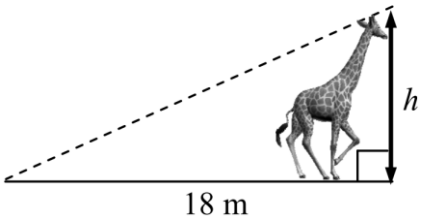
c A 0.5 m high post casts a 2 m long shadow. How tall is the giraffe that casts a 22 m long shadow?

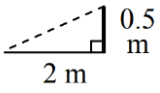




5.5 m

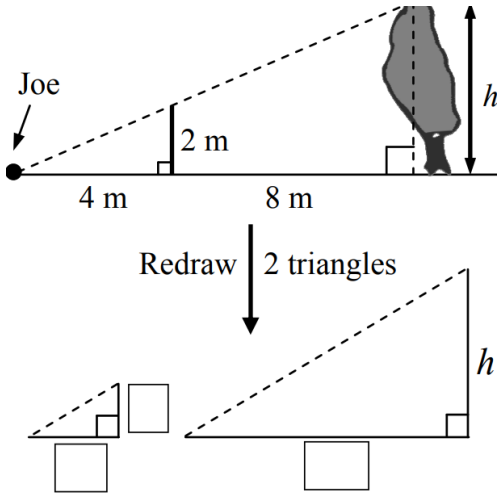
d How tall is the giraffe that casts a 18 m long shadow?





4.5 m

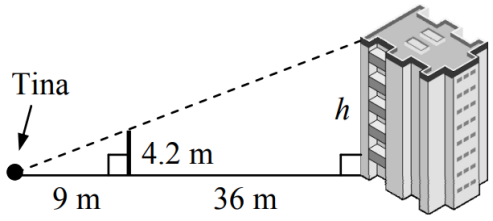
e



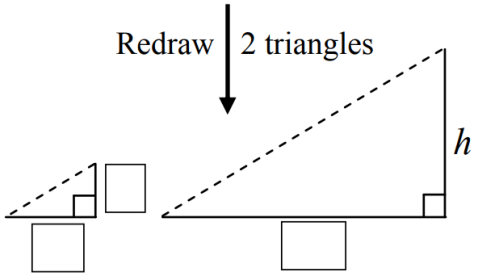
Redraw 2 triangles

6 m

f

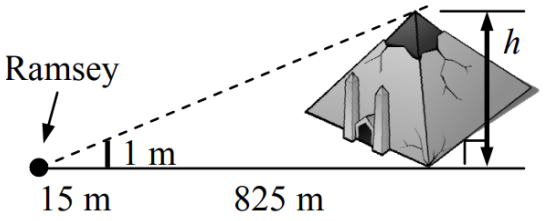


Redraw 2 triangles

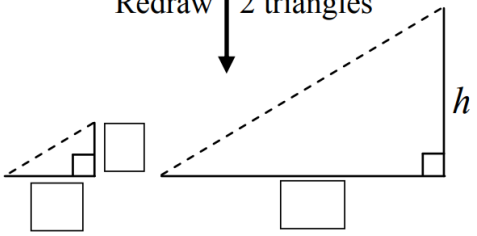


21 m

g

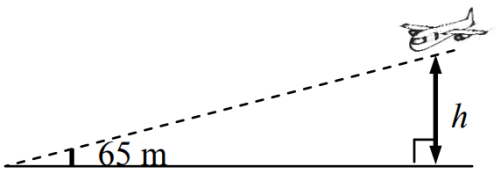


Redraw 2 triangles

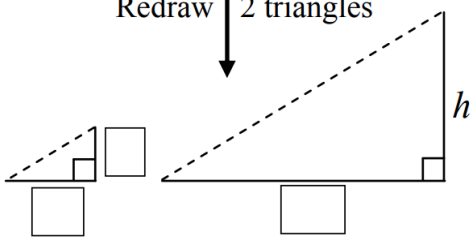


56 m

h

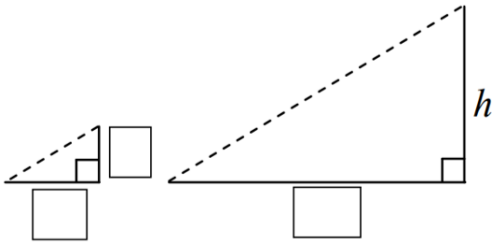


Redraw 2 triangles



0.39 km

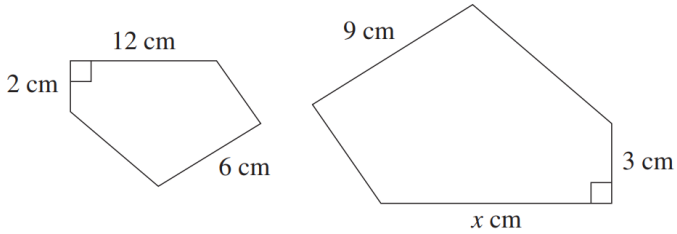
2 On a bright sunny day the shadow cast by a flagpole is 12 m long.
At the same time the shadow cast by a 1 m ruler is 2 m long. Find the height of the flagpole.



6 m

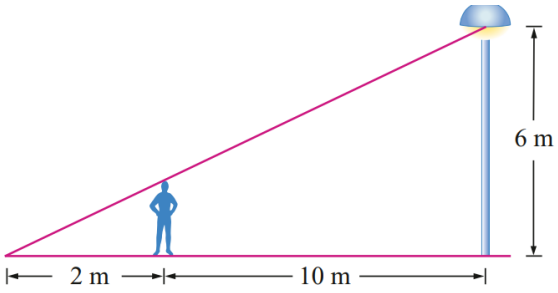
3 **2022 HSC Standard 1**

Two similar figures are shown. What is the value of x?



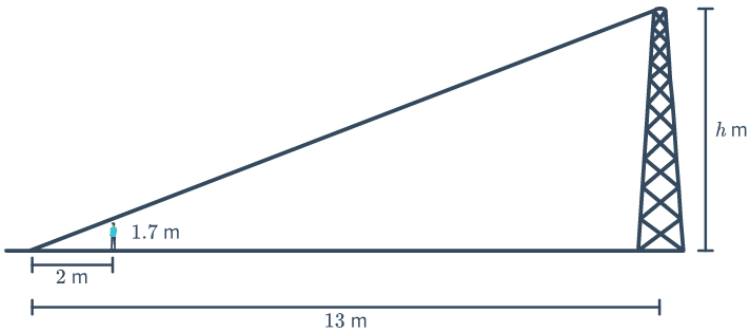
18 cm

4 A light pole has a globe that is 6 m above the ground. How tall is a boy who casts a 2 m long shadow when standing 10 m from the base of the pole?



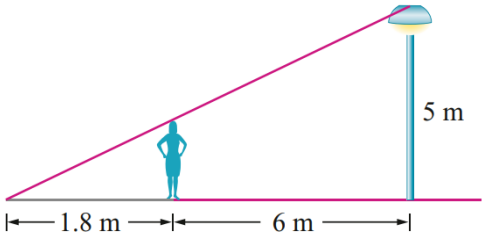
1 m tall

5 James is 1.7 m tall and casts a shadow 2 m long. At the same time, a tower casts a shadow 13 m long. If the tower is *h* metres high, solve for *h*.



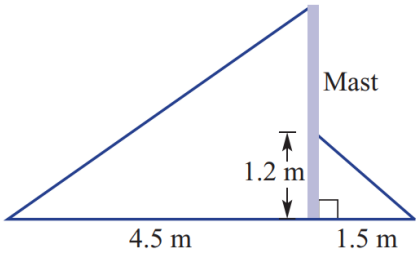
11.05 m

6 A street lamp is 5 m high. When a girl stands 6 m from the base of the lamp her shadow is 1.8 m long. Calculate the height of the girl, to the nearest centimetre.



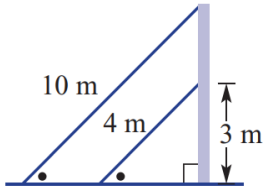
115 cm

7 Two similar triangles are created by cables supporting a yacht's mast.
Solve for h, the height of the mast.



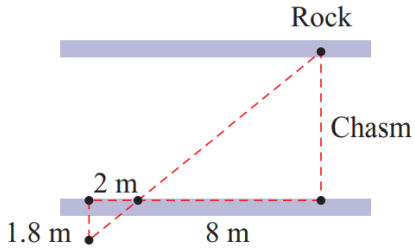
3.6 m

8 Two cables support a pole at the same angle, as shown.
The two cables are 4 m and 10 m in length. The shorter cable reaches 3 m up the pole.
Find the height of the pole.



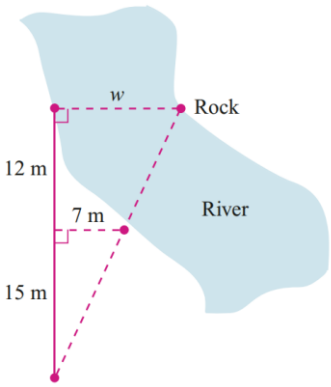
7.5 m

9 A deep chasm has a large rock sitting on its side.
Four pegs are placed on the other side using the rock as a reference point.
Find the width of the chasm.



7.2 m

10 Determine the width of the river.



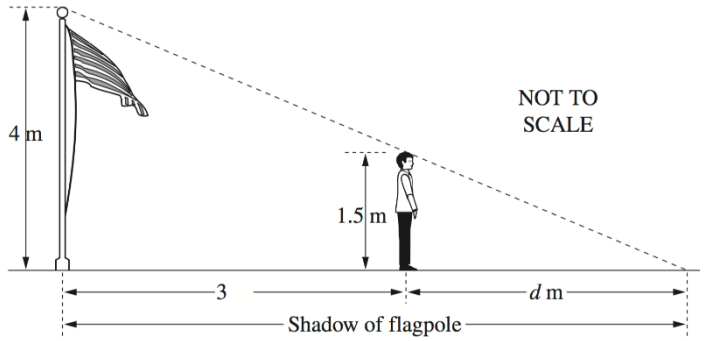
12.6 m

11 Determine the area of the scaled rectangle.

60 m^2

12 **2012 HSC Standard 2 Band 5**

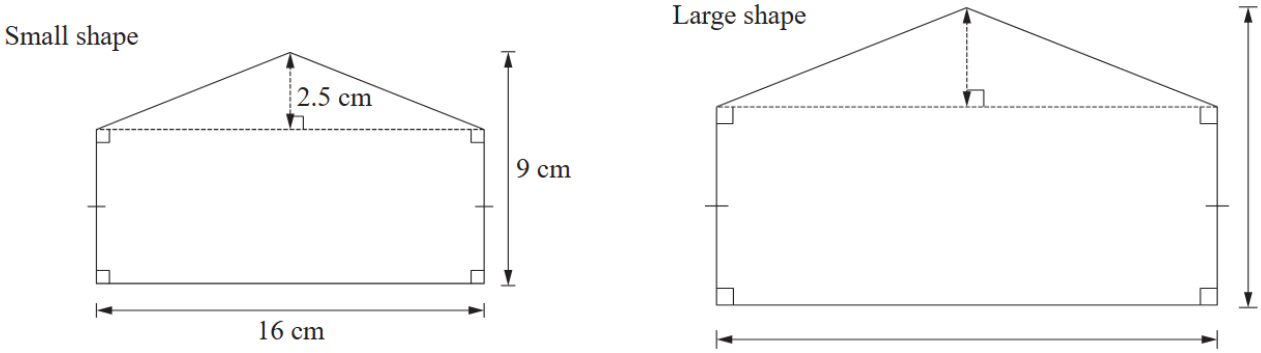
Jacques and a flagpole both cast shadows on the ground.
The difference between the lengths of their shadows is 3 metres.
What is the value of d, the length of Jacques' shadow?



1.8 m

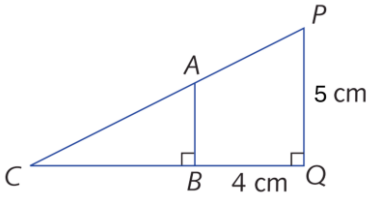
13 **2021 HSC Standard 2 Band 5**

The diagrams show two similar shapes.
The scale factor is 1.5. Calculate the area of the large shape.



279 cm^2

14 Determine the scale factor for these similar triangles if AB is half the length of BC.
Let ΔABC be the original triangle.



Scale factor is $\frac{5}{3}$

FINDING LENGTHS IN SCALE DRAWINGS

- A **scale drawing** is a **similar figure** of a real-life object.
  The **original** is the real-life object, and the **image** is the drawing.
- Examples include maps, house plans, and scientific diagrams.
- Scale factors for scale drawings are written in **simplified ratio notation:** $1 : n$
- A scale of 1:500 means every 1 unit on the drawing is 500 units in real life.

Scale Factor in Scale Drawings
image : original

1. Set up a ratio table ***image : original*** (this is different to original : image in similar figures)
   a. The first row should be the scale factor.
   b. The second row should be the given length and the length you want to find.
2. Find the unknown side by multiplying or dividing.

Example Calculate original length.

A map has a scale of $1:500$.
What is the real-life distance that 28 mm on the map would represent?

1.

Image	:	Original
1	:	500
28	:	

2.

Image	:	Original
1	:	500
28	:	14 000

×500 →

The real-life distance is 14 000 mm, or 14 m.

Why do we set up our ratio boxes as image : original?
(previously, we did original : image)

The scale factor is given as ..... : .....

How did we know 28 is the image length?
The question says 28 mm on a ..... meaning it is the image.

How do we know that it is 14 000 *millimetres?*
The unit given was 28 .....

a A map has a scale of $1:1000$.
What is the real-life distance that 37 mm on the map would represent?

Image	:	Original
	:	
	:	

37 m

b A city map has a scale of 1:10,000.
If the distance between two landmarks on the map is 3 cm, what is the real-life distance between the landmarks?

Image	:	Original
	:	
	:	

300 m

Example Calculate image length.

A map has a scale of $1:500$. What length on the map would represent 85 m in real life?

1.

Image	:	Original
1	:	500
	:	85

2.

Image	:	Original
1	:	500
0.17 ÷500	:	85

The map distance is 0.17 m, or 17 cm.

How did we know to divide by 500 to find x?
Image to original, we multiply,
so, from ..... to ....., we divide.

How do we know that it was 0.17 ***metres***?
The unit given was 85 .....

a A diagram has a scale of 1:200.
What length on the diagram represents 450 metres in real life?
Answer in metres.

Image	:	Original
	:	
	:	

2.25 m

b A map has a scale of $1:10\,000$.
What length on the map would represent 23 km in real life?
Answer in metres.

Image	:	Original
	:	
	:	

2.3 m

1 A map has a scale of 1:50 000.

a How many centimetres in real life does 1 cm on the map equal?

50 000 cm

b Therefore, how many metres does 1 cm represent?

500 m

c What is the scale factor, as a fraction?

$\frac{1}{50000}$

2 A map scale is given as 1 cm = 100 m. Write this as a simplified ratio.

1: 10 000

3

a A map has a scale of 1:100.
How many metres on the map does 300 m in real life equal?

Image	:	Original
	:	
	:	

3 m

b Your science teacher tells you to draw a scale drawing of a tree with scale 1:150. If the tree is 45 m high, how high should it be in your drawing?

Image	:	Original
	:	
	:	

0.3 m or 30 cm

c A map scale is 1:200 000.
Chatswood to Redfern is 4 cm on the map. How far is it in real life?

8 km

d Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1:50 000. How many km apart are the towns in real life?

8.5 km

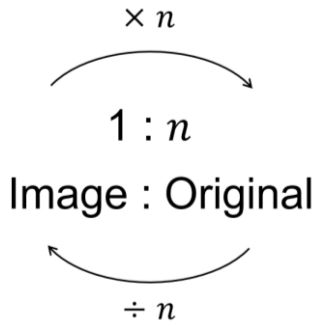
e The town of Gilgandra is 66 km north of Dubbo. On a map with a scale of 1 : 100 000, what is the scaled distance between the 2 towns?

66 cm

f Lord Howe Island is 2.8 km wide.
How long would its scaled width be on a map with a scale of 1 : 50 000?

5.6 cm

A shortcut you can use **only** if scale factor is given in the form $\mathbf{1}:\boldsymbol{n}$



4 Find the **actual distance** for the given scaled distances.
Give your answer in an appropriate unit.

a	1: 5	**i**	44 m $44\text{ m} \times 5 = \ldots\ldots\ldots\ldots$ 220 m	**ii**	310 cm 1550 m
b	1: 10 000	**i**	2 cm 200 m	**ii**	4 mm 40 m
c	1: 20 000	**i**	80 cm 16 km	**ii**	18.5 mm 370 m
d	1: 400	**i**	16 mm 6.4 m	**ii**	0.03 m 12 m
e	1: 0.5	**i**	12 cm 6 cm	**ii**	3.2 mm 1.6 mm
f	100: 1	**i**	3 cm 0.03 cm	**ii**	11.5 km 115 m

5 Find the **scaled length** for the give actual lengths.

a	1: 10	**i**	40 m $40\text{ m} \div 10 = \ldots\ldots\ldots\ldots$ 4 m	**ii**	310 cm 31 cm
b	1: 200	**i**	200 m 1 m	**ii**	4 km 20 m
c	1: 500	**i**	10 000 m 20 m	**ii**	75 cm 0.15 cm
d	1: 10 000	**i**	1350 m 0.135 m	**ii**	45 km 4.5 m
e	1: 0.1	**i**	23 m 230 m	**ii**	6.15 m 61.5 m
f	100: 1	**i**	7.5 cm 750 cm	**ii**	8.2 m 820 m

6 A map scale is a ratio of **scaled distance : real-life distance**.
A map of a park was drawn using the scale ratio of 1 : 500.

a 1 mm on the map represents …… mm in real life.

500

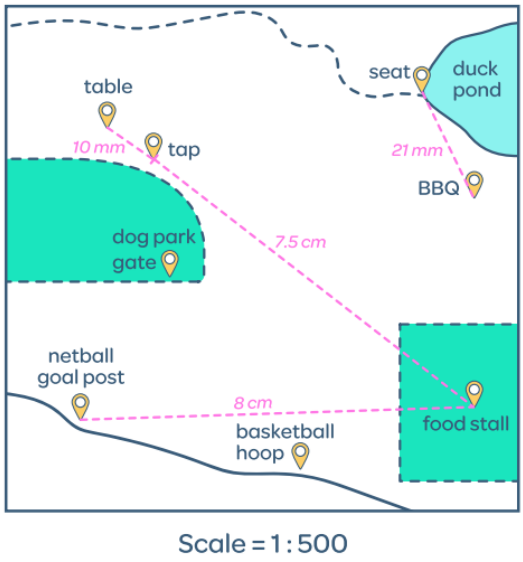
b 1 cm on the map represents …… cm in real life.

500

c Calculate the real-life distance between the table and the tap:

Scaled	:	Real Life
1	:	500
10	:	

5000 cm



d Convert your answer to metres: …… m

5 m

e Calculate the distance between the seat and the BBQ in metres.

10.5 m

f Calculate the real-life distance between the netball goal post and the food stall in metres.

40 m

g In real life, the distance between the dog park and the food stall is 35 000 mm.
How far would it be on the map?

7 cm

7 A model city has a scale ratio of 1 : 1000.

a Find the real-life height of a skyscraper that has a scaled height of 8 cm.

Scaled	:	Real Life
	:	
	:	

80 m

b Find the scaled length of a train platform that is 45 m long in real life.

Scaled	:	Real Life
	:	
	:	

4.5 cm

8 A scientist draws a bacteria using the scale 100:1.
If the drawing is 7 cm long, how long is the bacteria in real life?

Scaled	:	Real Life
100	:	1
	:	

0.07 cm

9 A map is drawn with a scale of 1:125 000.

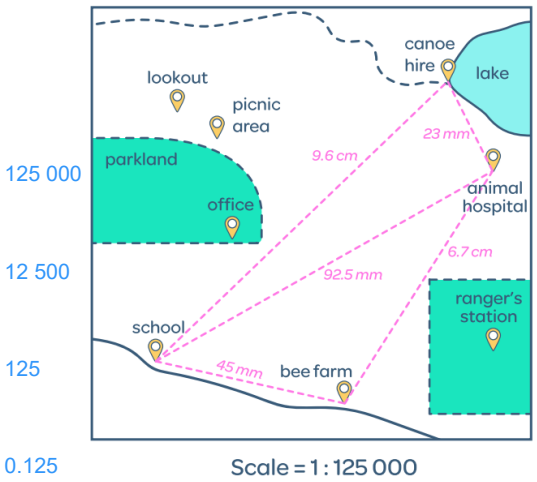
a 1 mm on the map represents:

..... mm in real life.

..... cm in real life.

..... m in real life.

..... km in real life.



b A teacher is planning a class excursion from the school to the bee farm.
What is the real-life distance between the two locations?

5.625 km

c Between the office and the ranger station the real-life distance is 7 km.
How far apart would they be on the map?

56 mm

10 A map maker wants to make a map of an area that is 8 km by 8 km.
She decides to use a scale of 1:100. Explain why this map scale is not appropriate.

A 1:100 scale would make the 8 km × 8 km map 80 meters × 80 meters, which is impractically large for any practical use

11 A scientist shows a photomicrograph image of a 10 cm long cell.
He says the cell is magnified by a scale of 1:100 000 through his microscope.
Explain what is wrong with the scientist's ratio.

The scientist's ratio is backwards - a magnification should be written as 100,000:1, not 1:100,000, since the image is larger than the actual cell.

12 An engineer designs a mobile phone SIM card using a diagram with a scale of 100:1.
If the scaled drawing is 80 cm long, what is its actual length in millimetres?

8 mm

13 Your science teacher asks you to draw a scale diagram of a bacteria cell with a scale of 500:1
If the microscope says the bacteria is 0.01 mm in length, how long should the bacteria in your diagram be?

5 mm

14 2012 HSC Standard 2 Band 5

A map has a scale of 1 : 500 000.

Two mountain peaks are 2 cm apart on the map.
What is the actual distance between the two mountain peaks, in kilometres?

10 km

Two cities are 75 km apart. How far apart are the two cities on the map, in centimetres?

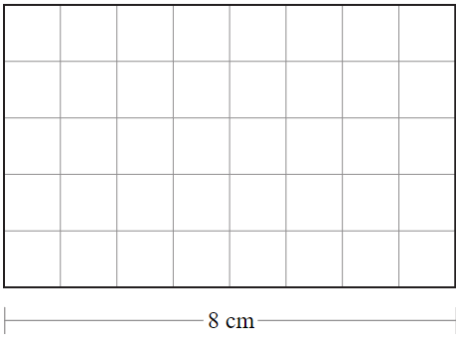
15 cm

15 2021 HSC Standard 2 Band 5

A rectangular sportsground has been drawn to scale on a 1-cm grid as shown.
The scale used is 1: 3000

Kerry took 12 minutes to walk around the perimeter of this sportsground.

What was Kerry's average speed in kilometres per hour?



3.9 km/h

CALCULATING SCALE FACTOR IN SCALE DRAWINGS

Scale Factor
$\frac{\text{Image}}{\text{Original}} = \text{image} : \text{original}$

1. Identify image (scale drawing) and original (real life).
2. Convert units to be the same.
3. Calculate scale factor using $\frac{\text{image}}{\text{original}}$
4. Rewrite scale factor in ratio notation.

Example Calculate the scale factor.

Drawing

50 cm

Real Life

150 m

1. image original
2. 50 15 000
3. scale factor $= \frac{50}{15000} = \frac{1}{300}$
4. scale factor $= 1:300$

Why did we change 150 to 15 000?

Ratios always compare the same ……

Why is the scale factor $\frac{50}{15000}$ and not $\frac{15000}{50}$?

Scale factor as a fraction is ……

divided by ……

Why is $\frac{1}{300} = 1:300$?

A fraction written as a ratio is …… : ……

Calculate the scale factor if:

a the length on the drawing is 32 cm and represents 1600 m in real life.

1. image = original =
2. scale factor $= \frac{\text{image}}{\text{original}} =$
3. scale factor = 1 : ……

1:5000

b 1.5 kilometres in real life is represented by 60 cm on a blueprint.

1.

2.

3.

1:2500

Key ideas

1. Scale drawings are …… figures of real-life objects.
2. Scale factor for scale drawings are written in ratio notation …… : ……

1 Convert the two measurements provided in each scale into the same unit, and then write the scale as a ratio of two numbers in simplest form.

a 2 cm : 200 m

$= 2$ cm : 20 000 cm

$=$ 1 : .....

1:10 000

b 5 mm : 500 cm

1:1000

c 12 mm : 360 cm

1:300

d 4 mm : 600 m

1:150 000

e 4 cm : 5 m

1:125

f 1 cm : 2 km

1:200 000

g 28 mm : 2800 m

1:100 000

h 3 cm : 0.6 mm

50:1

i 1.1 m : 0.11 mm

10 000:1

2 Calculate the scale factor for each of the following. Reminder to convert all units to the same.

a 2 cm on a scale drawing represents an actual distance of 50 cm.

1:25

b 4 cm on a scale drawing represents an actual distance of 2 km.

1:50 000

c 1.2 cm on a scale drawing represents an actual distance of 0.6 km.

1:50 000

d 5 cm on a scale drawing represents an actual distance of 900 m.

1:18 000

e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.

7:1

f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

600:1

3 A scaled length of 5.5 cm is equal to 44 km in real life.

a How many kilometres does a scaled length of 3 cm equal?

24 km

b How many kilometres does a scaled length of 20 cm equal?

160 km

c How many centimetres does an actual distance of 100 km equal?

12.5 cm

4 Complete the table. Answer in the most appropriate unit.

Scale Factor Image : Original	**Image Length**	**Original Length**
1 : 10 000	0.9 m	9 km
1 : 10 000	9 cm	900 m
1 : 10 000	18 cm	1800 m
1 : 5000	36 cm	1.8 km
1 : 5000	72 cm	3600 m
1 : 5000	35 mm	175 m
1 : 5000	70 mm	350 m
1 : 2500	70 mm	175 m
1 : 2500	35 mm	87.5 m
1: 100 000	52 cm	52 km
1:1 000 000	52 cm	520 km
1:500 000	104 cm	520 km
1: 500 000	104 mm	52 km

5 HSC Sample Question Band 5

On a map, the distance between two towns is measured at 54 millimetres.
The actual distance between the two towns is 16.2 kilometres. What is the scale of the map?

1:300 000

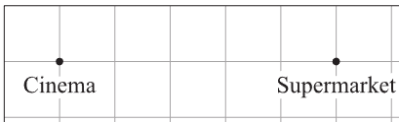
If two other towns on the same map are 9.2 centimetres apart, what is the actual distance between the two towns in kilometres?

27.6 km

6 2019 HSC Standard 2 Band 5

A map is drawn to scale, on 1-cm paper, showing the position of a supermarket and a cinema.

It takes 10 minutes to walk in a straight line from the cinema to the supermarket at a constant speed of 3 km/h. Show that the scale of the map is 1 cm = 100 m.



SCALE DRAWINGS AND MEASUREMENT: PART 1

- To find an unknown side, given the scale factor and scale diagram:
  1. Measure the length you want to find.
  2. Set up ratio table image : original.
  3. Find the unknown by multiplying or dividing.

Example Calculate original length by measuring a scale diagram.

Worked Example	Your Turn
This scale drawing of a building has scale of 1:400. Find its height in real life.	This scale drawing of a field has scale of 1:2500. Find its length in real life. Answer in metres.

1. Measure

$\text{Height}_{\text{image}} = 4$ cm

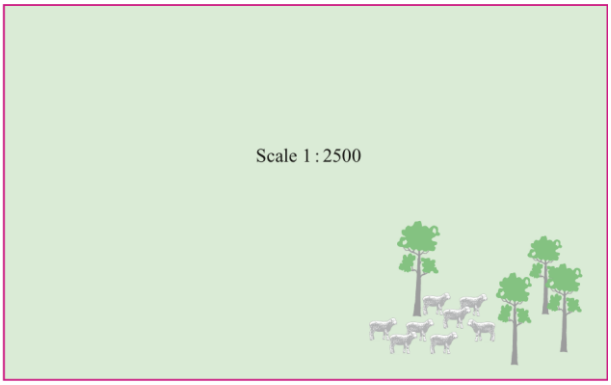
2. Ratio table

Image	:	Original
1	:	400
4	:	

3. Find unknown

Image	:	Original
1	:	400
4	: (×400 →)	1600

The height in real life is 1600 cm or 16 m.



1. Measure

2. Ratio table

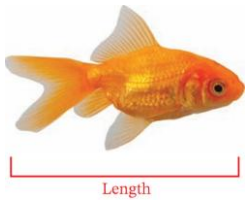
Image	:	Original
	:	
	:	

3. Find unknown

200 m

1 Measure the length of each scaled-down image below, then use the scale to calculate its actual length in centimetres.

a Fish 1 : 2



6 cm

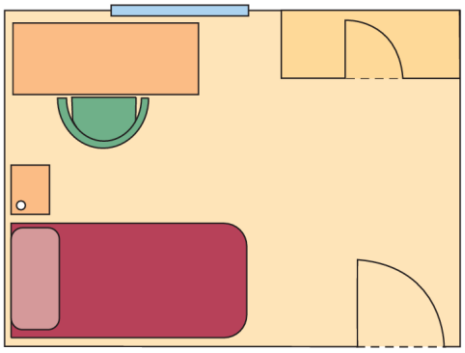
b Pen 1 : 3

12 cm

c Racquet 1 : 16

67.2 cm

2 This diagram of a bedroom has scale 1 : 80.
Find the actual lengths, rounded to nearest 0.1 metre.



a Length of bedroom

4.8 m

b Width of bedroom

3.5 m

c Length of bed

2.5 m

d Length of window

1.4 m

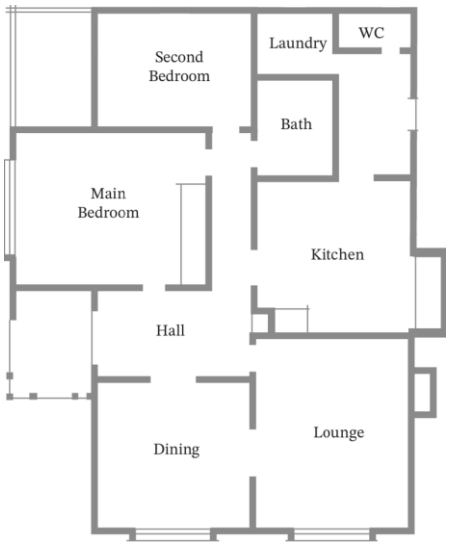
e Length of table

1.9 m

f Area of bedroom

16.8 m^2

3 The house plan is drawn to a scale of 1 : 200
Find the actual lengths, rounded to nearest 0.1 metre.



a Length of main bedroom

5 m

b Length of laundry

2 m

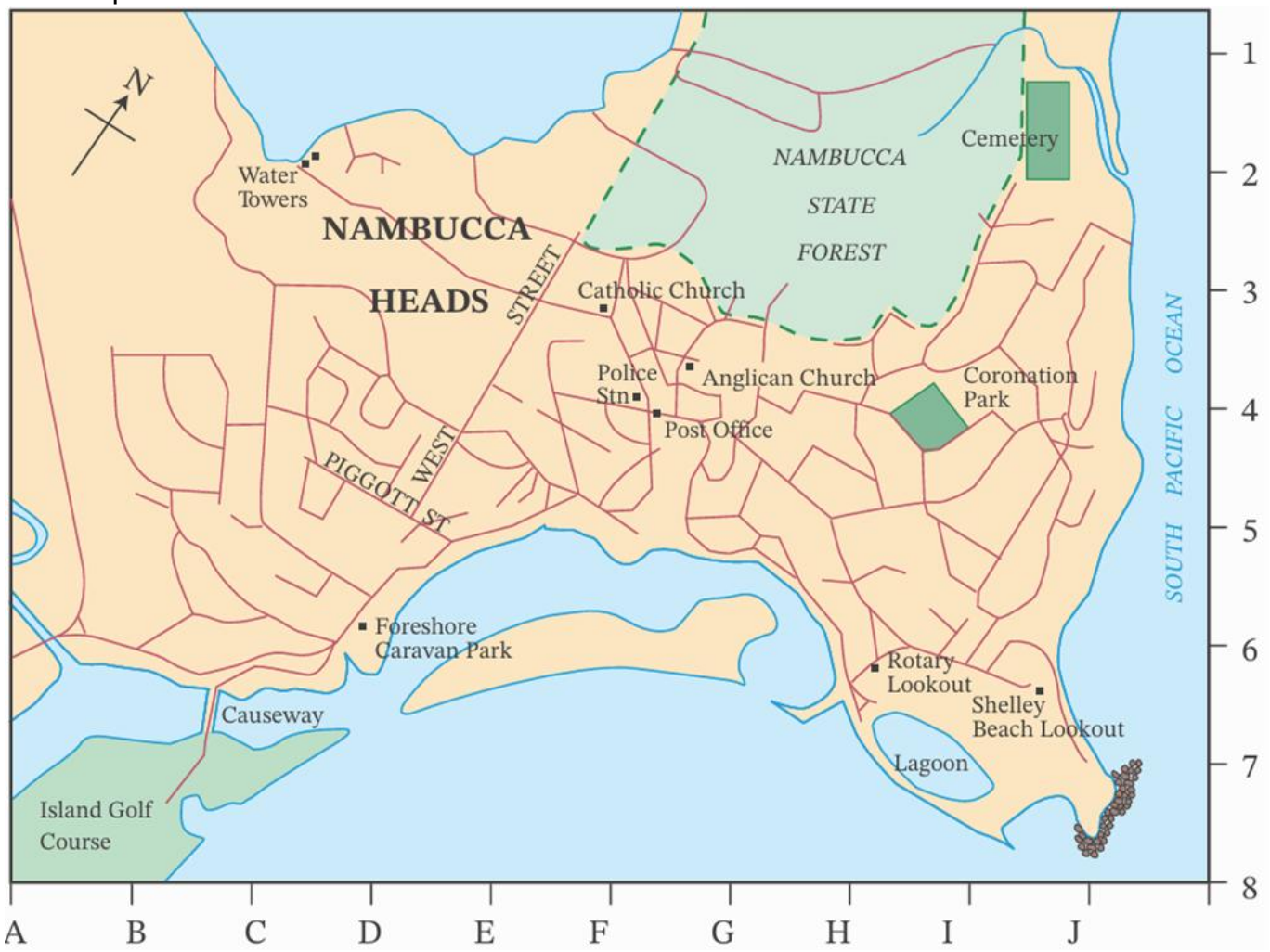
c Area of bathroom

5 m^2

d Area of dining room

15.2 m^2

4 This map of Nambucca Heads has a scale of 1 : 20 000.



Find, to the nearest 10 metres, the distance between:

a the Water Towers (C2) and the Catholic Church (F3)

940 m

b Rotary Lookout (H6) and Shelley Beach Lookout (J6)

480 m

c the post office (F4) and the Foreshore Caravan Park (D6)

1020 m

d the length of the lagoon (I7)?

380 m

e What are the dimensions of the cemetery (J2)?

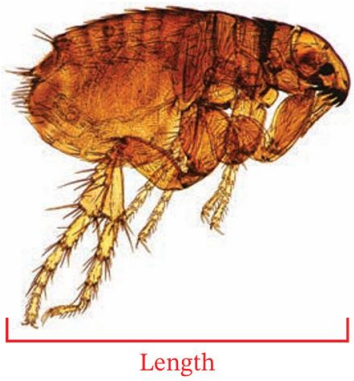
120 m by 280 m

f What is the area of the cemetery?

33 600 m^2

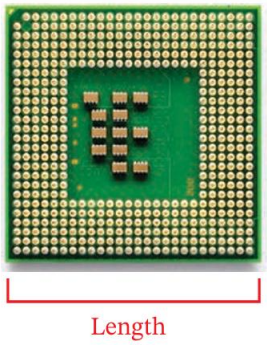
5 Measure the length of each magnified image below,
then use the scale ratio to calculate its actual length in millimetres.

a Flea 100 : 1



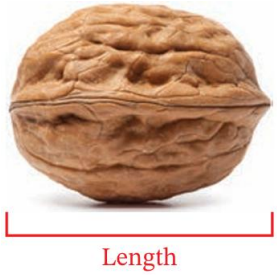
0.45 mm

b Microchip 3 : 1



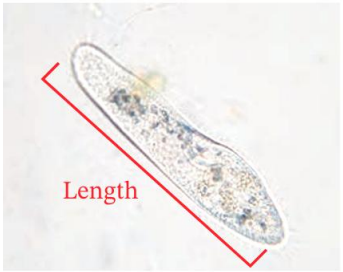
11 mm

c Nut 4 : 3



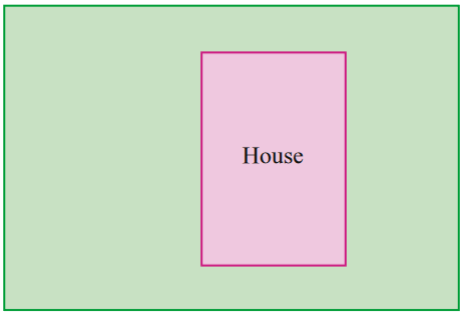
26 mm

d Bacterium 1200 : 1



0.03 mm

6 A top-down view of a property is shown. The scale factor is 1 : 500



a Find the real-life length and width of the house, to the nearest metre.

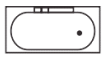
9.5 m by 14 m

b Find the real-life area of the house.

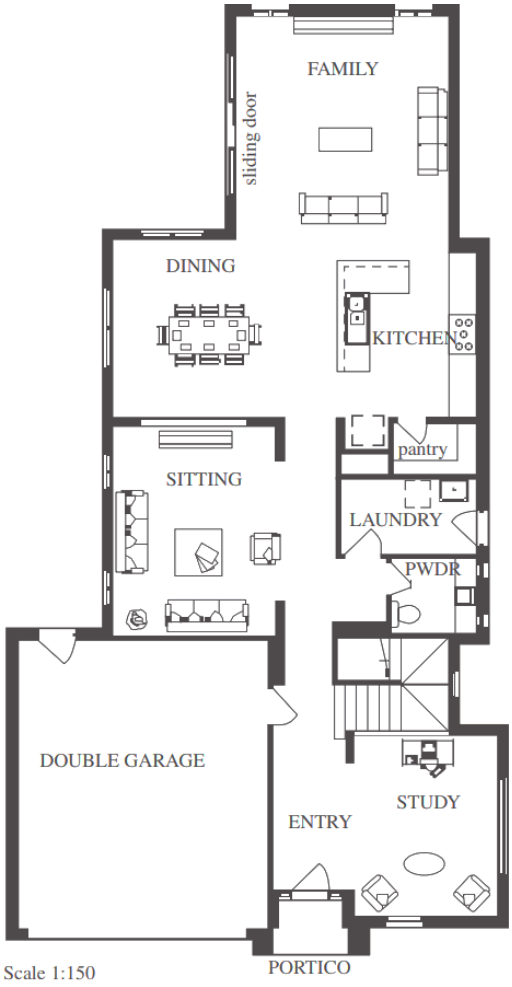
133 m^2

7 A common application of scale drawings is floor plans, which is a top-down view of a building.

Common symbols for floor plans:

Door	Window	Shower	Bathtub	Sink	Toilet
					

Refer to the floor plan to answer the questions.



a How many doors are there?

7

b What is the length of the house, from outer wall to outer wall? Answer to the nearest metre.

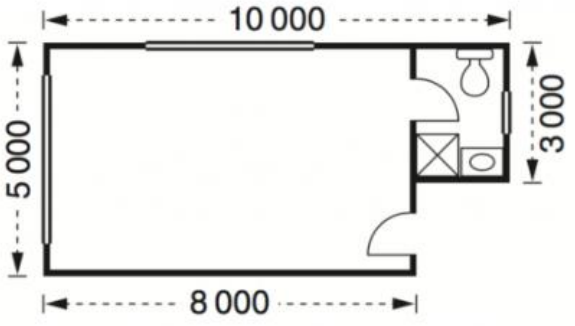
19 m

c What is the area of the double garage? Answer to the nearest square metre.

28 m^2

8 **2010 HSC Standard 2 Band 4**

A floor plan usually gives measurements in millimetres.
Calculate the area of the floor of this building. Answer in square metres.



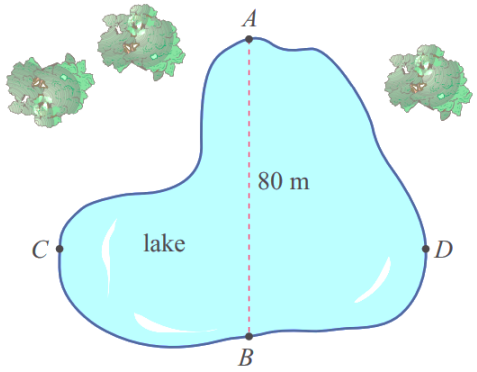
46 m^2

SCALE DRAWINGS AND MEASUREMENT: PART 2

- Sometimes the scale factor is not given. You will need to calculate it yourself.
  1. Measure the side length given in the image and the length you want to find.
  2. Convert units to be the same.
  3. Set up ratio table image : original.
  4. Calculate scale factor and write in ratio notation.
  5. Find the unknown by multiplying or dividing.

Example Calculate original length by measuring a scale diagram.

This is a scale diagram of a park lake.



Calculate the real-life width of lake CD.

1. Measure lengths

$AB_{\text{image}} = 4$ cm

$CD_{\text{image}} = 5$ cm

2. Convert units

$AB_{\text{original}} = 8000$ cm

3. Ratio table

	Image	:	Original
AB	4	:	8 000
CD	5	:	

4. Scale factor

$$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} = \frac{4 \text{ cm}}{8000 \text{ cm}}$$

$$= \frac{1}{2000} \text{ or } 1:2000$$

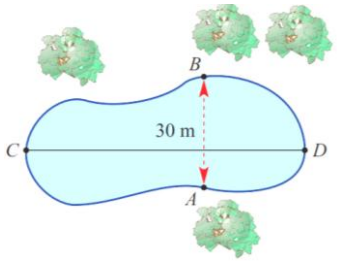
5. Find unknown

	Image	:	Original
AB	4	:	8 000
CD	5	:	10 000

$\times 2000$

The real-life width of the lake is 100 m.

This is a scale diagram of a park lake.



Calculate the width of the lake CD in real life.

1. Measure lengths

$AB_{\text{image}} =$ ..... cm

$CD_{\text{image}} =$ ..... cm

2. Convert units

$AB_{\text{original}} =$ ..... cm

3. Set up ratio table

	Image	:	Original
AB		:	
CD		:	

4. Calculate scale factor

$$\text{Scale Factor} = \frac{\text{Image}}{\text{Original}} =$$

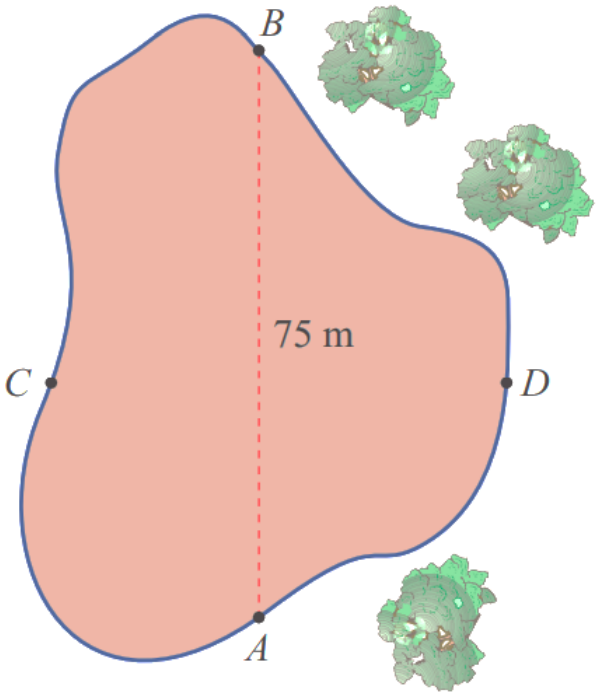
$$= 1 : \text{.....}$$

5. Find unknown:

74 m

1 The given diagram is a map of a children's play area.

a Measure AB and CD in centimetres.



b Convert 75 m to centimetres.

c Fill in the ratio table

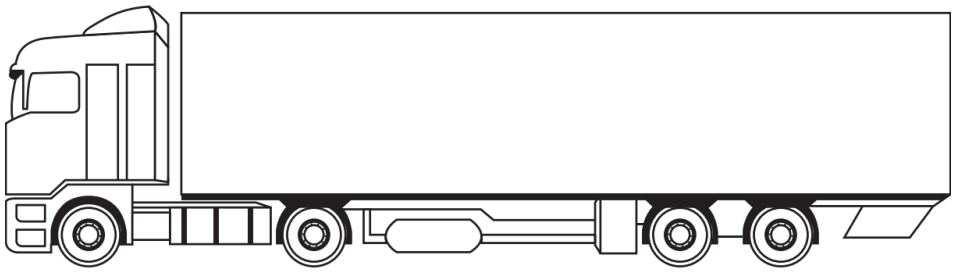
	Image	:	Original
AB		:	
CD		:	

d Determine the scale factor.

e Determine the real-life distance CD.

60 m

2 This is a scale diagram of a 12.5-metre-long truck.



a Measure the length and height of the truck.

12.5 cm long, 3.4 cm high

b Convert 12.5 m to cm

1250 cm

c Use the ratio table to calculate the height of the truck in real-life.

	Image	:	Original
Length		:	
Height		:	

3.4 m

3 What is the length of Drummond Field? 0.25 cm on the map represents 5 m. Answer to the nearest metre.

80 m

4 The bar to the bottom right represents 50 m.

a Calculate the scale factor

1 : 2500

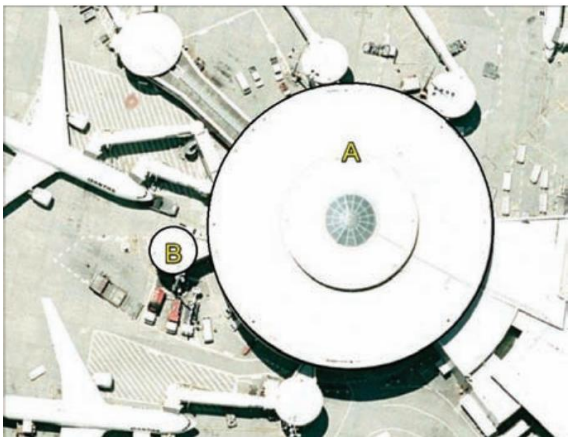
b Find the length and width of the field to the nearest metre.

length 100 m, width 62.5 m

c Hence find the area of the field to the nearest square metre.

6250 m^2

5 Two circular buildings at Sydney airport are shown. The bar to the bottom right represents 25 m. Find the area of the roof of building A to the 1 significant figure.



3100 m^2

6 Consider the satellite image of a 50 m long swimming pool.

a Calculate the scale factor

1 : 2000

b Find the width of the pool to the nearest metre.

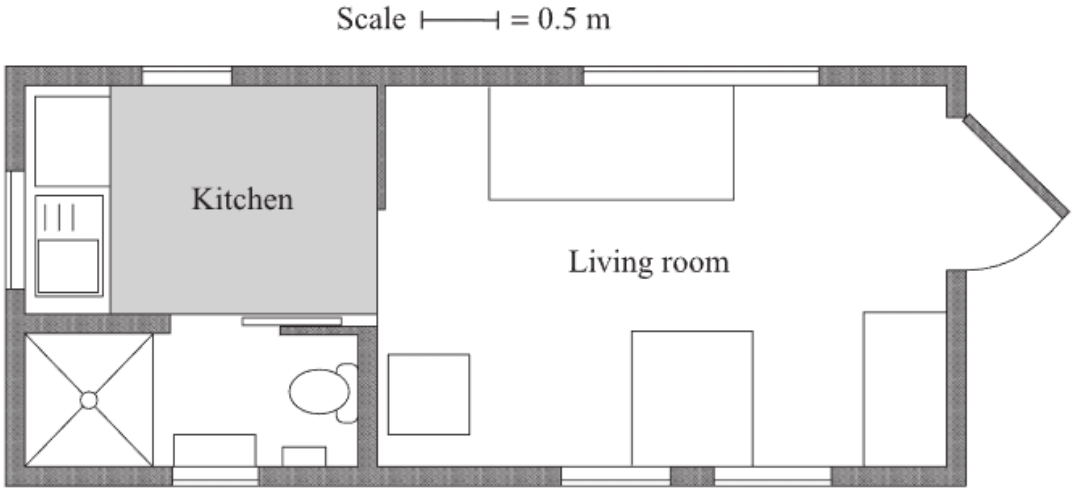
25 m

c Hence find the area of the pool to the nearest square metre.

1250 m^2

9 **2019 HSC Standard 2 Band 4**

The plan of the lower level of a small house is shown.



How many windows are there?

6

What is the scale factor?

1:50

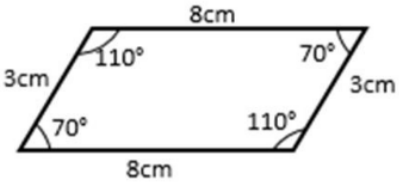
What is the actual perimeter, in metres, of the shaded part of the kitchen?

6.5 m

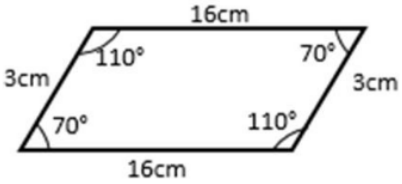
CHECK YOUR UNDERSTANDING

Scale factor

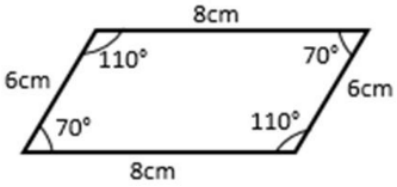
1 Which of these parallelograms A, B, C, or D has been enlarged by a scale factor of 2?



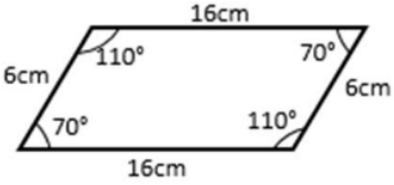
a



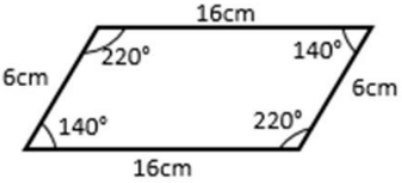
b



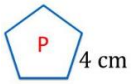
c

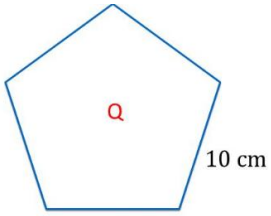


d



2 Which of these calculations find the scale factor to get from P to Q?

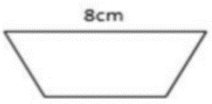




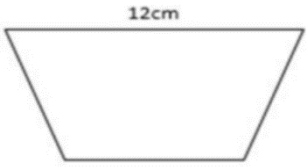
a 4×10 **b** $10 - 4$ **c** $\frac{4}{10}$ **d** $\frac{10}{4}$

3 What is the scale factor?

Original



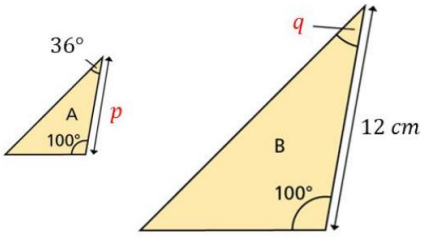
Image



a 96 **b** $0.\dot{6}$ **c** 4 **d** 1.5

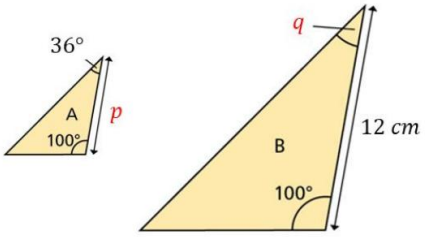
Calculating a similar side length

1 Shape A has been enlarged by a scale factor of 3 to give shape B.
What is the value of p?



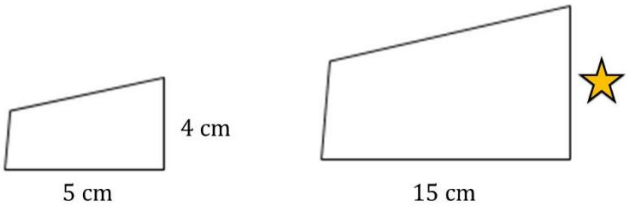
a 6 cm **b** 4 cm **c** 9 cm **d** 12 cm

2 Shape A has been enlarged by a scale factor of 3 to give shape B.
What is the value of q?



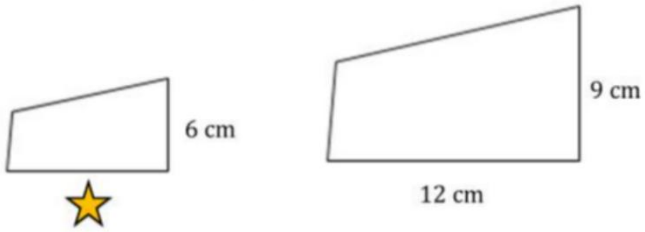
a 39° **b** 138° **c** 12° **d** 36°

3 The two shapes are similar. Find the value of the star.



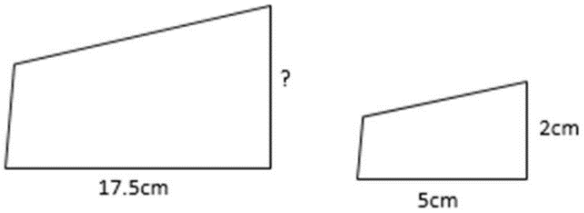
a 12 cm **b** 14 cm **c** 3 cm **d** 10 cm

4 The two shapes are similar. Find the value of the star.



a 9 cm **b** 8 cm **c** 10 cm **d** 7.5 cm

5 The two shapes are similar. Find the missing length.



a 7 cm **b** 8 cm **c** 14.5 cm **d** 3.5 cm