Student Name

Mathematics Stage 5 Paths

VOLUME

Book 1 Solve problems involving volumes

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-VOL-P-01 applies knowledge of the volume of right pyramids, cones and spheres to solve problems involving related composite solids

Solve problems involving volumes

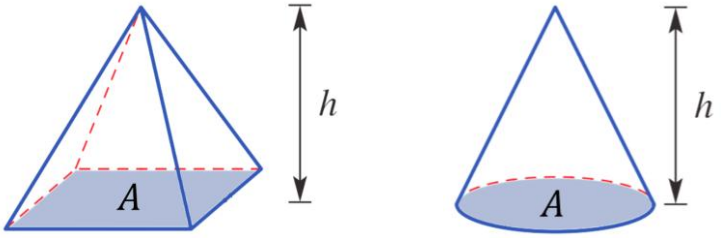
- Find the volume of right pyramids and right cones by using an appropriate formula: volume of pyramid or cone $V = \frac{1}{3}Ah$, where A is the base area and h is the perpendicular height
- Find the volume of spheres by using an appropriate formula: volume of sphere $V = \frac{4}{3}\pi r^3$, where r is the length of the radius
- Apply knowledge of right pyramids, right cones and hemispheres to solve problems involving composite solids
- Solve practical problems related to the volumes and capacities of solids including right pyramids, right cones and spheres

REVIEW OF PRIOR KNOWLEDGE

VOLUME OF PYRAMIDS AND CONES PART 1

- A **pyramid** is a 3D solid with a flat **base** and triangular sides that meet at a point called the **apex**.
- The **perpendicular height** is the vertical distance from apex to base.
- A **cone** is like a pyramid, but with a circular base.

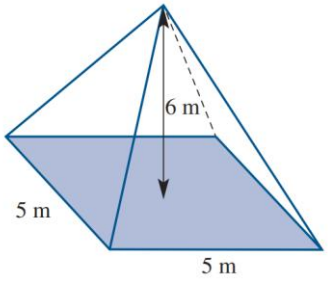
Volume of Pyramids and Cones



$$V = \frac{1}{3}Ah \quad \text{or} \quad V = \frac{Ah}{3}$$

Example Calculate volumes of pyramids and cones.

Calculate the volumes to the nearest m^3.



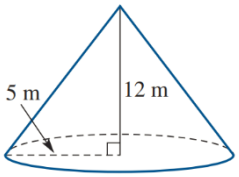
$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times (5 \times 5) \times 6$$
$$= 50 \text{ m}^3$$

What is the formula for the volume of pyramids and cones?

.....

Where did the 5×5 come from?

5×5 *is the ..... of the square base.*



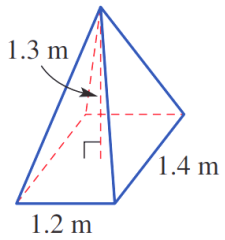
$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times \pi(5)^2 \times 12$$
$$\approx 314 \text{ m}^3$$

Where did $\pi(5)^2$ come from?

$\pi(5)^2$ *is the ..... of the ..... base.*

Calculate the volumes of these pyramids and cones. Round your answer to 2 d.p.

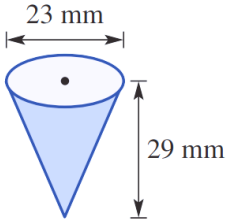
a



$$V = \frac{1}{3}Ah$$
$$=$$
$$=$$

0.73 m^3

b



$$V = \frac{1}{3}Ah$$
$$=$$
$$=$$

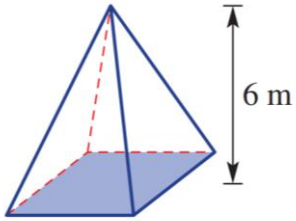
4016.26 mm^3

Key ideas

1. The volume of a pyramid or a cone is one third times the .... and the height.
2. The height of a pyramid or a cone is the ..... distance between the apex and the base.

1 Find the volume of these solids with the given base areas.
Round your answer to 2 decimal places.

a



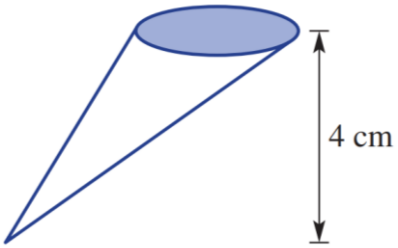
$A = 5\ \text{m}^2$

$V = \frac{1}{3}Ah$

$=$

10 m³

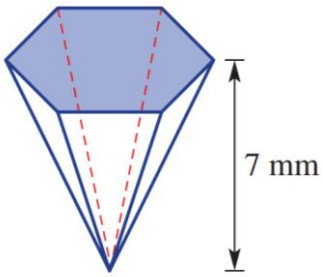
b



$A = 2\ \text{cm}^2$

2.67 cm³

c

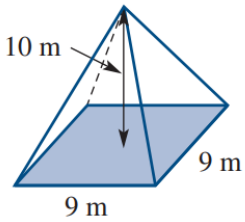


$A = 25\ \text{mm}^2$

58.33 mm³

2 Find the volume of these solids. Round your answer to 2 decimal places.

a

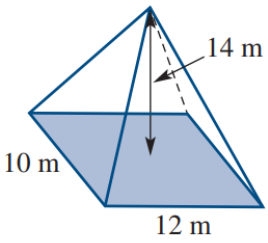


$V = \frac{1}{3}Ah$

$=$

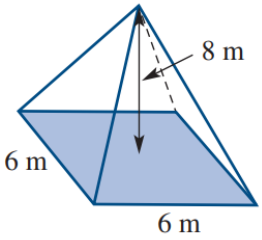
270 m³

b



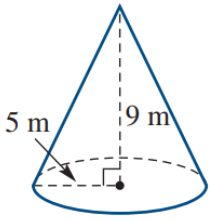
560 m³

c



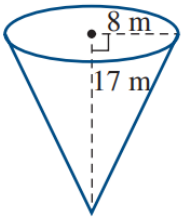
96 m³

d



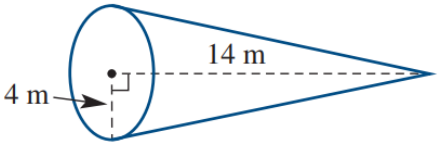
235.62 m³

e



1139.35 m³

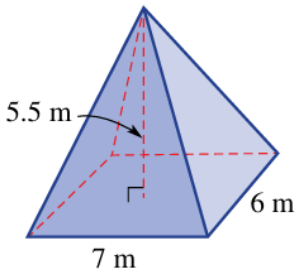
f



234.57 m³

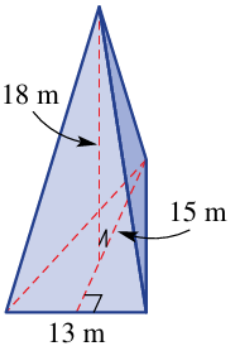
3 Find the volumes of these pyramids.

a



77 m^3

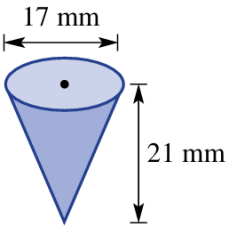
b



585 m^3

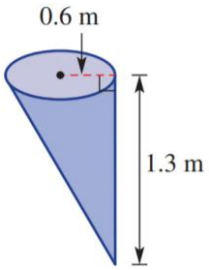
4 Find the volumes of these cones, round your answer to 2 decimal places.

a



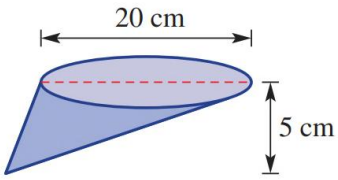
1588.86 mm^3

b



0.49 m^3

c



523.60 m^3

5 The Red Pyramid in Egypt is the third tallest pyramid ever built.

It has a square base with a side length of 220 m and a perpendicular height of 104 m.

a What is the area of the square base? ……

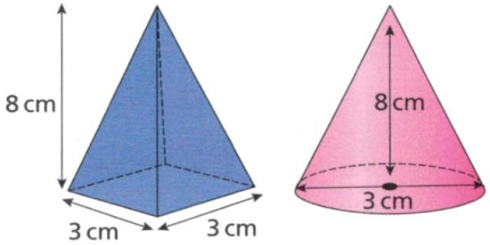
b Determine the volume of the Red Pyramid. Answer to the nearest cubic metre.

1 677 867 m^3

6 An ice-cream cone measures 4.6 cm in diameter and is 9 cm high.
Calculate the volume of the cone. Answer correct to two decimal places.

49.86 m^3

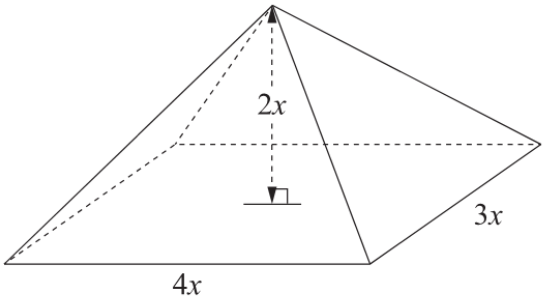
7 What is the difference between the volumes of these two solids?



5.15 cm^3

8 **2018 HSC Standard 2 Band 4**

What is the volume of the pyramid in cubic metres?



A. $8x^3$

B. $9x^3$

C. $12x^3$

D. $24x^3$

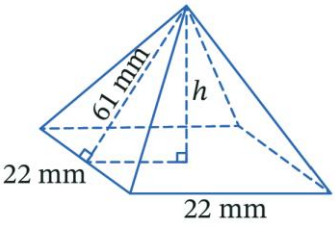
A

VOLUME OF PYRAMIDS AND CONES PART 2

- For some questions, you will need to find the perpendicular height before applying the formula.
  1. Find perpendicular height using Pythagoras: $a = \sqrt{c^2 - b^2}$
     Do not round your answer.
  2. Calculate volume using $V = \frac{Ah}{3}$.

Example Calculate volumes of pyramids and cones by finding perpendicular height.

Calculate the volumes of these pyramids and cones to one decimal place.



1.

$h = \sqrt{61^2 - 11^2}$

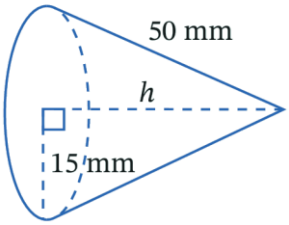
$= 60$

2.

$V = \frac{1}{3}Ah$

$= \frac{1}{3}(22 \times 22)(60)$

$= 9680 \text{ mm}^3$



1.

$h = \sqrt{50^2 - 15^2}$

$= \sqrt{2275}$

2.

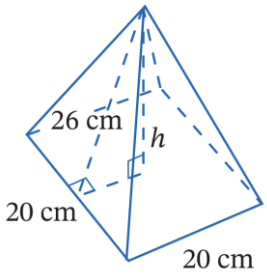
$V = \frac{1}{3}Ah$

$= \frac{1}{3}(\pi \times 15^2) \times \sqrt{2275}$

$= 11238.3 \text{ mm}^3$

Calculate the volumes of these pyramids and cones. Answer to the nearest whole.

a

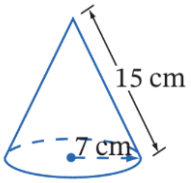


1.

2.

3200 cm³

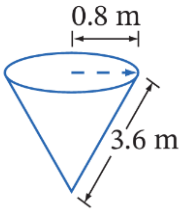
b



681 cm³

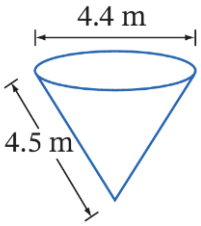
1 Find the volume of these pyramids and cones. Answer to 1 d.p.

a



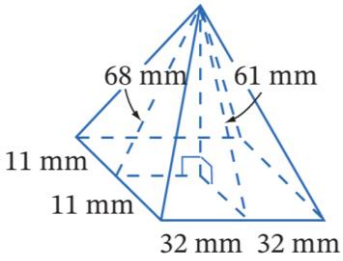
2.4 m^3

b



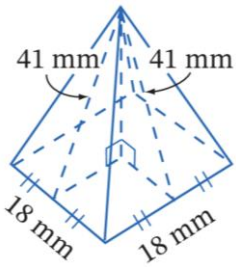
19.9 cm^3

c



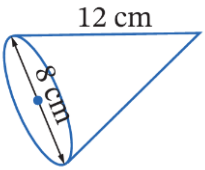
28 160 mm^3

d



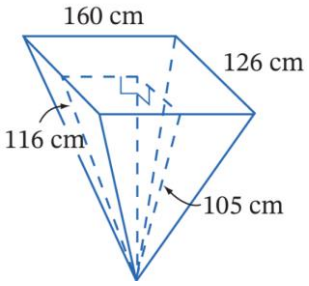
4320 mm^3

e



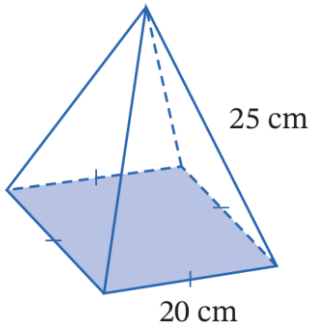
189.6 cm^3

f



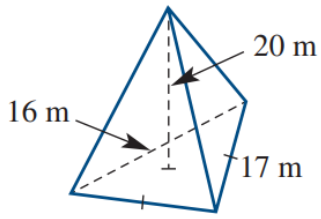
564 480 mm^3

2 Find the volume of this square pyramid. Answer to 1 d.p.
(Hint: you are given the *edge height*; to make a right-angled triangle you need to use the diagonal of the base.)



2748.7 cm^3

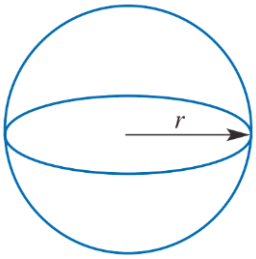
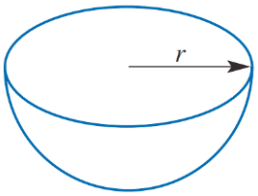
3 A triangular pyramid has a base with sides 16 m, 17 m and 17 m.
The height of the pyramid is 20 m. What is the volume of the triangular pyramid?
(Hint: you need to use Pythagoras' Theorem to find the height of the base triangle)



800 m^3

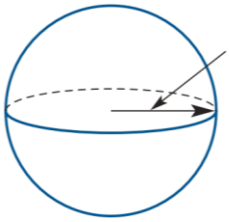
VOLUME OF SPHERES AND HEMISPHERES

- A **sphere** is a perfectly round 3D solid.
- A **hemisphere** is half a sphere.

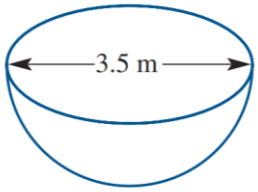
Sphere	Hemisphere
 $V = \frac{4}{3}\pi r^3$	 $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

Example Calculate volumes of spheres and hemispheres.

Calculate the volumes of these spheres and hemispheres to one decimal place.



$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 7^3$$
$$\approx 1436.8 \text{ m}^3$$

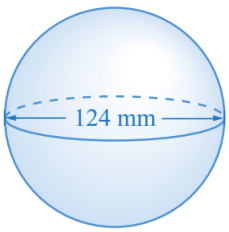


$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 1.75^3$$
$$\approx 11.2 \text{ m}^3$$

Calculate the volumes of these spheres and hemispheres. Answer to 1 d.p.

a

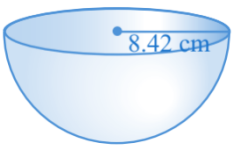
$$V = \frac{4}{3}\pi r^3$$
$$=$$
$$=$$



998 306.0 mm³

b

$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$=$$
$$=$$



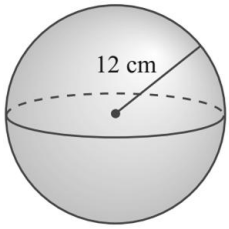
1250.2 cm³

Key ideas

1. The volume of a sphere is ……
2. The volume of a hemisphere is ……

1 Calculate the volume of these spheres. Round your answer to 1 decimal place.

a



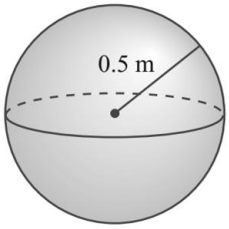
$V = \frac{4}{3}\pi r^3$

$=$

$=$

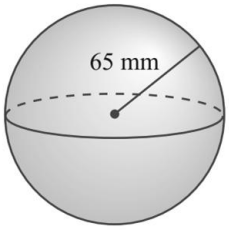
7238.2 cm^3

b



0.5 m^3

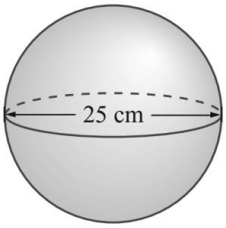
c



1 150 346.5 mm^3

2 Calculate the volume of these spheres. Round your answer to 1 decimal place.

a



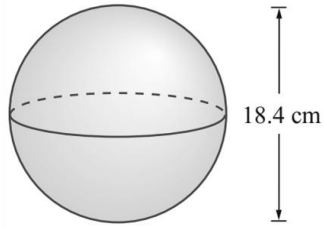
$V = \frac{4}{3}\pi r^3$

$=$

$=$

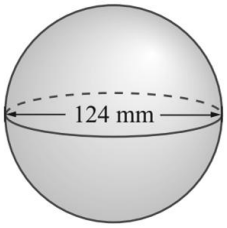
8181.2 cm^3

b



3261.8 cm^3

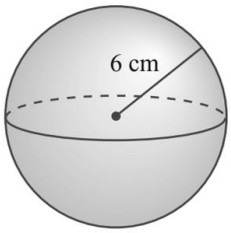
c



998 306.0 mm^3

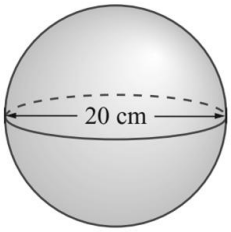
3 Calculate the volume of these spheres. Round your answer to 1 decimal place.

a



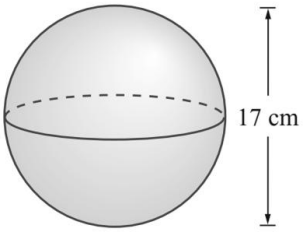
904.8 cm^3

b



4188.8 cm^3

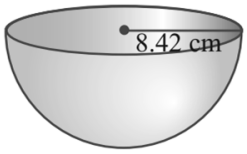
c



2572.4 cm^3

4 Calculate the volume of these hemispheres. Round your answer to 2 decimal places.

a



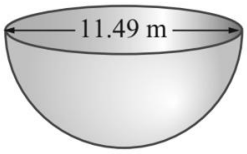
$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$=$

$=$

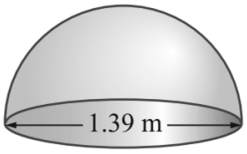
1250.24 cm³

b



397.13 m³

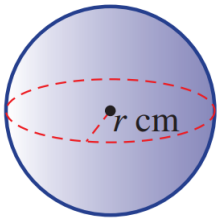
c



0.70 m³

5 Find the radius of these spheres with the given volumes, correct to 2 decimal places.

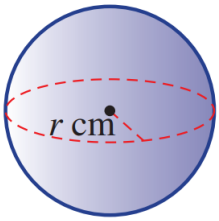
a



$V = 15$ cm³

1.53 cm

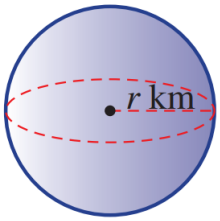
b



$V = 180$ cm³

3.50 cm

c

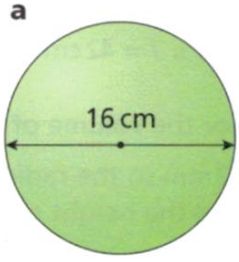


$V = 0.52$ km³

0.50 km

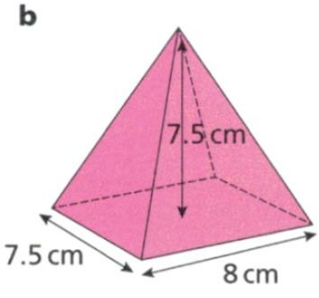
6 Work out the volume of each shape to 1 d.p.

a



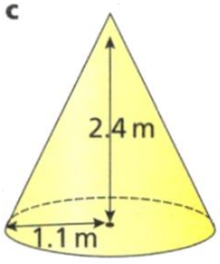
2144.7 cm³

b



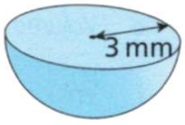
150.0 cm³

c



3.0 m³

d



56.5 mm³

7 Water drops falling from a tap are spheres of diameter 0.4 cm.
15 drops per minute leak from the tap.
How much water, to the nearest mL, is wasted in one day? (1 mL = 1 cm^3)

724 mL

8 A ball's radius is doubled. How many times as large is the new ball?

8 times as larger

9 A box 30 cm long, 30 cm wide and 30 cm high holds 50 tennis balls of radius 3 cm. Find:

a the volume of one tennis ball, correct to 3 decimal places.

113.097 cm^3

b the volume of 50 tennis balls, correct to 1 decimal place.

5654.9 cm^3

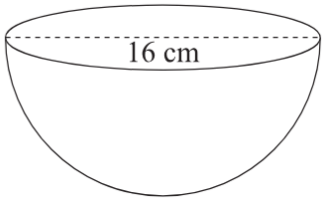
c the volume of the box not taken up by the tennis balls, correct to 1 decimal place.

21345.1 cm^3

10 2019 HSC Standard 2 Band 4

A bowl is in the shape of a hemisphere with a diameter of 16 cm.

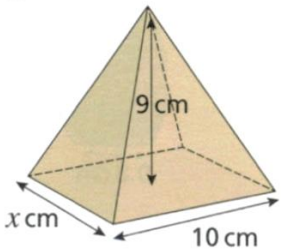
What is the volume of the bowl, correct to the nearest cubic centimetre?



1072 cm^3

11 Work out the value of x in each of these shapes.

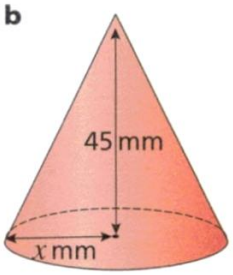
a



$V = 240\ \text{cm}^3$

8

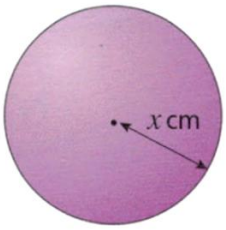
b



$V = 10\,603\ \text{mm}^3$

15

c



$V = 1150\ \text{cm}^3$

6.5

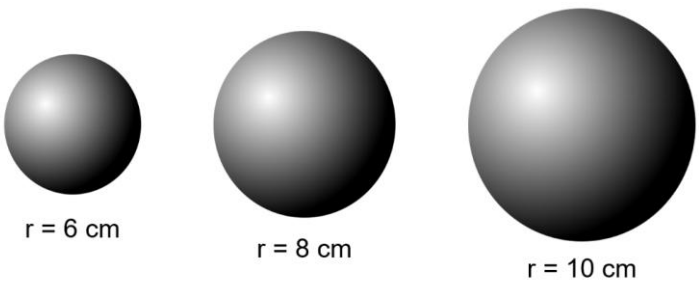
12 An expanding spherical storage bag has 800 cm^3 of water pumped into it.
Find the diameter of the bag after all the water has been pumped in. Answer to 2 d.p.

11.52 cm

13 When a balloon is blown up it roughly forms a sphere.
By taking a large breath, a teenager can expel 1.2 L of air from their lungs.
How many of these breaths would be needed to make a balloon of diameter of 26 cm?

8 breaths

14 Three metal spheres are melted down and made into one bigger sphere.
What the radius of the bigger sphere?



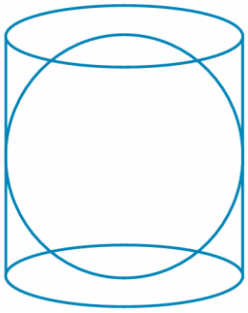
12 cm

15 Archimedes found that for a cylinder with the same height as its base, a cone and a sphere can fit perfectly inside (if you melted them down).

$$V_{\text{Cylinder}} = V_{\text{Cone}} + V_{\text{Sphere}}$$

By substituting the volume of a cylinder $V = \pi r^2 h$ and volume of a cone $V = \frac{1}{3}\pi r^2 h$, derive the formula for the volume of a sphere.
(Hint: what is another expression for h in this scenario?)

16 Archimedes also found that, for a sphere that fits perfectly inside a cylinder:



$$\frac{V_{\text{cylinder}}}{V_{\text{sphere}}} = \frac{SA_{\text{cylinder}}}{SA_{\text{sphere}}}$$

Show that this is true.

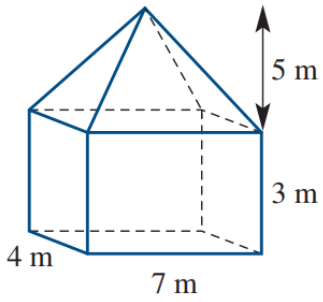
VOLUME OF COMPOSITE SOLIDS

- A **composite solid** is made up of multiple simple solids.
- To find the volume of composite figures, add or subtract the volumes of simple solids.

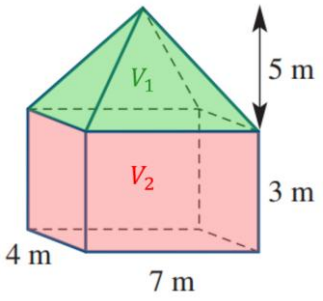
1. Split the composite solid into simple solids.
2. Calculate the volume of each simple solid.
3. Add or subtract the volumes.

Example Calculate the volume of a composite solid.

Calculate the volumes of these solids to the nearest two decimal places.

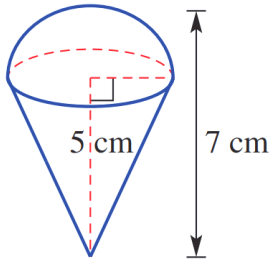


$$V_1 = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times (4 \times 7) \times 5$$
$$= \frac{140}{3}$$

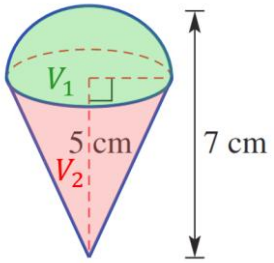


$$V_2 = Ah$$
$$= (4 \times 7) \times 3$$
$$= 84$$

$$V_{\text{tot}} = V_1 + V_2$$
$$= 130.67 \text{ m}^3$$



$$V_1 = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi(2)^3$$
$$= 16.75516082 \ldots$$

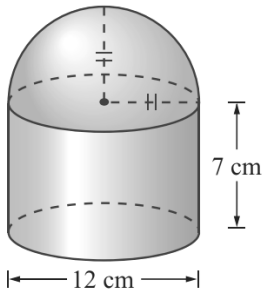


$$V_2 = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times \pi(2)^2 \times 5$$
$$= 20.94385102 \ldots$$

$$V_{\text{tot}} = V_1 + V_2$$
$$= 37.70 \text{ cm}^3$$

Calculate the volumes of these solids to the nearest two decimal places.

a



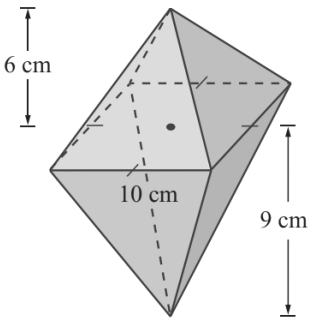
$V_1 =$

$V_2 =$

$V_{\text{tot}} =$

1244.07 cm³

b



$V_1 =$

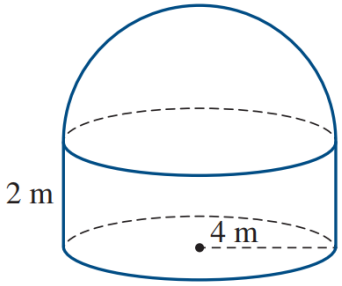
$V_2 =$

$V_{\text{tot}} =$

500 cm³

1 Calculate the volume of these composite solids. Round your answer to 1 decimal place.

a



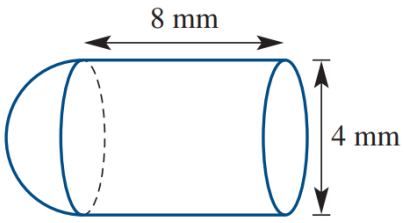
$V_1 =$

$V_2 =$

$V_{tot} =$

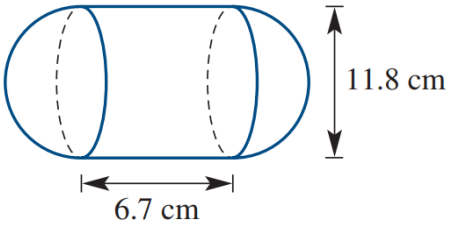
234.6 m^3

b



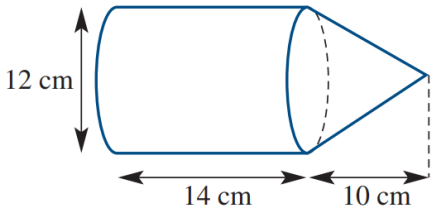
117.3 mm^3

c



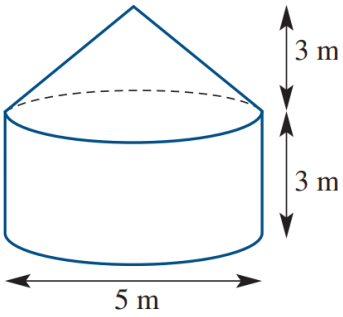
1593 cm^3

d



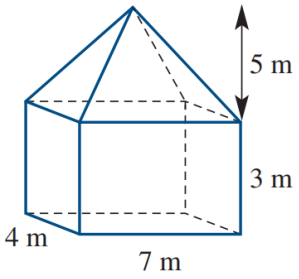
1960.4 cm^3

e



78.5 m^3

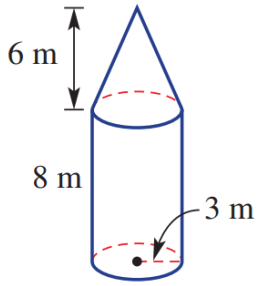
f



130.7 m^3

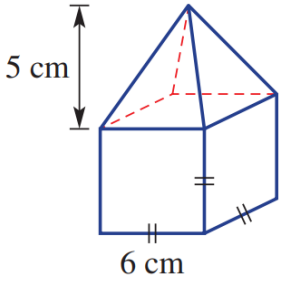
2 Calculate the volume of these composite solids. Round your answer to 2 decimal places.

a



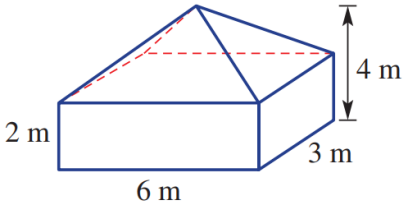
282.74 m^3

b



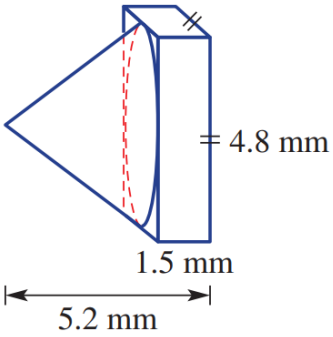
276 cm^3

c



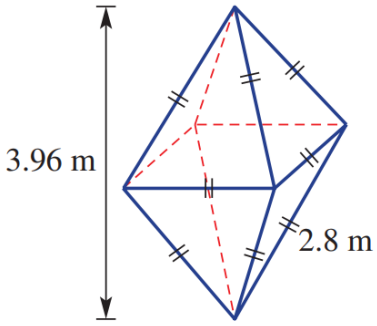
48 m^3

d



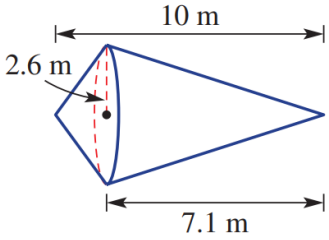
56.88 mm^3

e



10.35 m^3

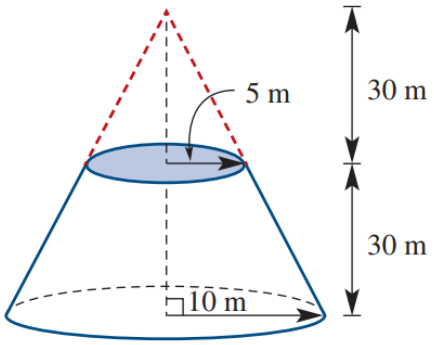
f



70.79 m^3

3 A frustum is a portion of a solid. The frustum shown opposite is formed from a cone.

a What is the volume of the missing top cone? Answer correct to the nearest cubic metre.



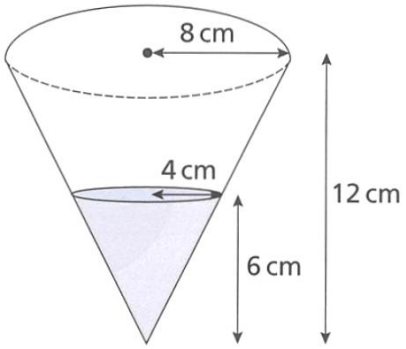
b What was the volume of the entire cone?

c What is the volume of the frustum?

5498 m^3

4

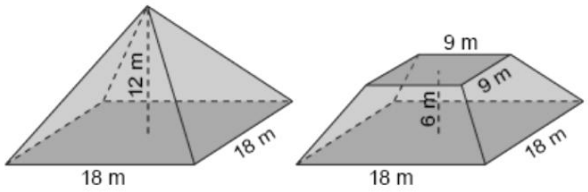
a Explain why this cone is NOT half full.



b Work out the fraction of the cone that is filled with water.

$\frac{1}{8}$

5 A frustum is a cone or pyramid with its top cut off.



Volume of complete pyramid:

1296 m³

Volume of removed pyramid:

162 m³

Volume of frustum:

1134 m³

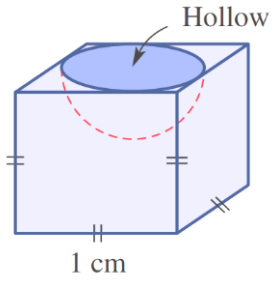
A formula for a square pyramid frustum was found on a papyrus scroll from ≈1800 BC:

$$V = \frac{1}{3}h(a^2 + ab + b^2)$$

where a is the base length, b is the top side length of the truncated pyramid, and h is the perpendicular height of the truncated pyramid.
Calculate volume using this formula and comment on its accuracy.

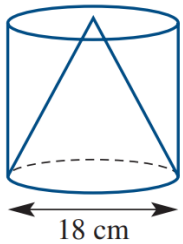
very accurate

6 A hemispherical scoop of ice-cream is taken out of a cubic tub.
Find the volume that remains, as a percentage.



74%

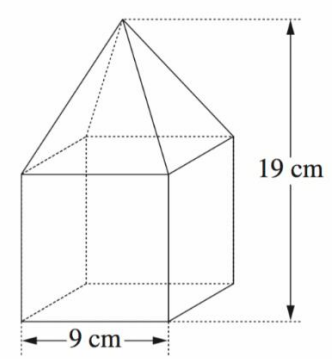
7 A cone sits exactly inside a cylinder as shown. The height of the cone is 16 cm.
What fraction of the volume of the cylinder is the cone?



$\frac{1}{3}$

8 2013 HSC Standard 2 Band 4

A square pyramid fits exactly on top of a cube to form a solid. What is the volume of the solid?



(A) 513 cm^3

(B) 999 cm^3

(C) 1242 cm^3

(D) 1539 cm^3

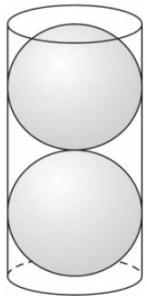
B

9 2009 HSC Standard 2 Band 4

Two identical spheres fit exactly inside a cylindrical container, as shown.

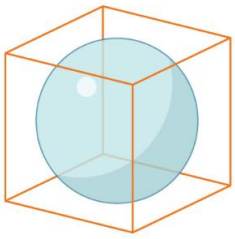
The diameter of each sphere is 12 cm.

What is the volume of the cylindrical container, to the nearest cubic centimetre?



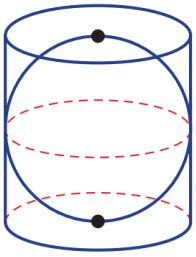
2714 cm^3

10 A sphere just sits inside a cube. What is the volume of the sphere as a percentage of the volume of the cube? Round your answer to the nearest whole percentage.



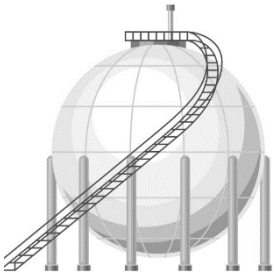
52%

11 A ball just fits inside a cylinder. What percentage of the volume of the cylinder is taken up by the ball? Round your answer to the nearest whole percentage.



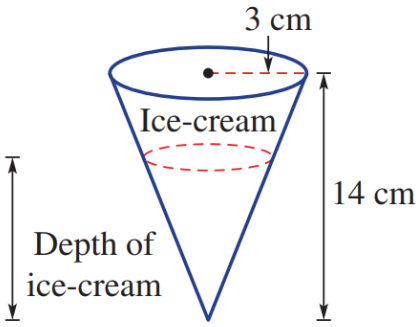
67%

12 A spherical container has an outer radius of 8 m and a steel shell that is 40 mm thick.
What is the volume of the shell?



32 m^3

13 The volume of ice-cream in the cone is half the volume of the cone.
The cone has a 3 cm radius and height of 14 cm, as shown.
What is the depth of the ice-cream, correct to 2 decimal places?



11.11 cm