Student Name

Mathematics Stage 5 Paths

AREA AND SURFACE AREA B

Book 1 Solve problems involving surface areas

Version: 260207

Report mistakes & suggest improvements: https://MrDingMaths.com

CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-ARE-P-01 applies knowledge of the surface area of right pyramids and cones, spheres and composite solids to solve problems

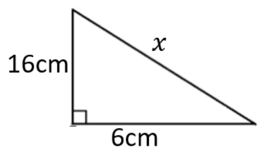
Solve problems involving surface areas

- Identify the perpendicular heights and slant heights of right pyramids and right cones
- Apply Pythagoras' theorem to find the slant heights, base lengths and perpendicular heights of right pyramids and right cones
- Solve problems involving the surface areas of right pyramids and compare methods of solution
- Apply the formula to find the curved surface area of right cones: $A = \pi rl$, where r is the length of the radius and l is the slant height
- Apply the formula to find the surface area of spheres: $A = 4\pi r^2$, where r is the length of the radius
- Solve problems involving the surface area of solids in a variety of contexts including composite solids

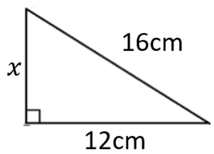
REVIEW OF PRIOR KNOWLEDGE

1 Find the length of the missing side using Pythagoras' Theorem

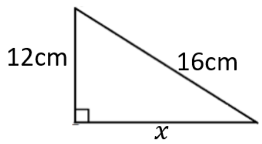
a



b

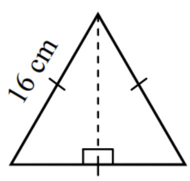


c

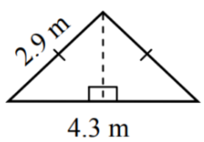


2 Find the heights of these triangles.

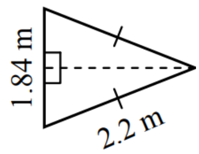
a



b



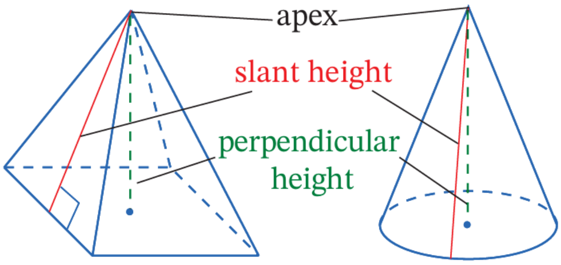
c



PERPENDICULAR HEIGHTS AND SLANT HEIGHTS

HEIGHTS

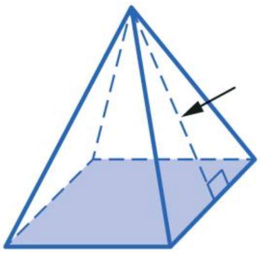
- A **pyramid** is a solid with a **flat base** and **triangular faces** that join at a point called the **apex.**
- A **cone** is like a pyramid but has a **circular base**.
- The **slant height** (l) is the sloped distance from the centre of the base length to the apex.
- The **perpendicular height** (h) is the vertical distance from the base to the apex.



Example Identify slant height or perpendicular height.

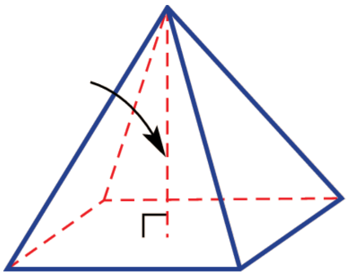
Determine whether the arrow is indicating a slant height or perpendicular height.

a



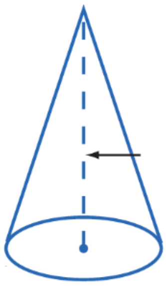
slant

b



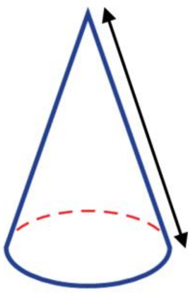
perpendicular

c



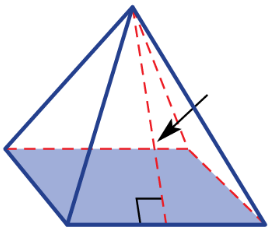
perpendicular

d



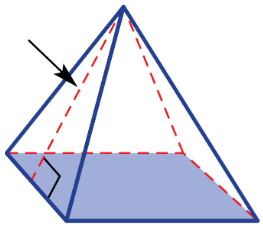
slant

e



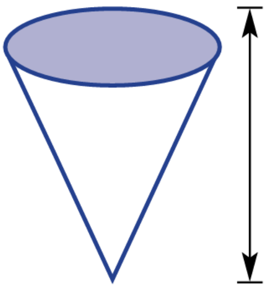
slant

f



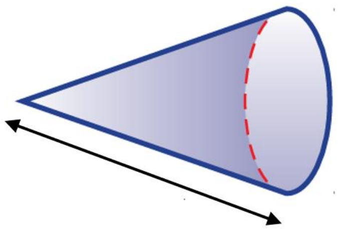
slant

g



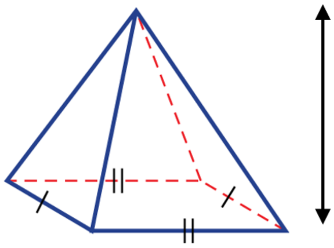
perpendicular

h



slant

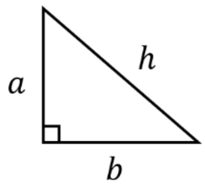
i



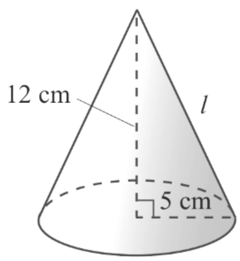
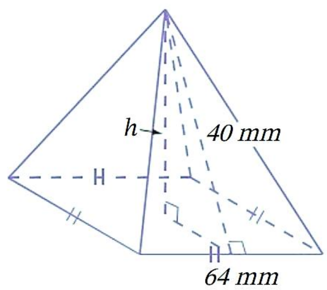
perpendicular

CALCULATING HEIGHTS

- Using a **right-angled triangle**, we can find the **slant height** or **perpendicular height** with Pythagoras' Theorem.

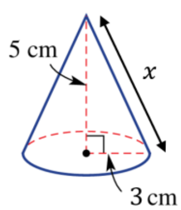
Pythagoras' Theorem
For every right-angled triangle: $a^2 + b^2 = h^2$ two shorter sides hypotenuse 

Example Calculate slant height or perpendicular height.

Calculate the unknown value in these solids.	Where did we get the relationship $l^2 = 12^2 + 5^2$?
 $l^2 = 12^2 + 5^2$ $l^2 = 169$ $l = 13$ cm	*………………… theorem* What happened from $l^2 = 169$ to $l = 13$? *We …………… ………… both sides.* Why did we not consider the negative solution when square rooting? *We can't have a negative ………… …………*
 $h^2 + 32^2 = 40^2$ $h^2 = 40^2 - 32^2$ $h^2 = 576$ $h = 24$ mm	Where did we get 32 from? *The base is a ……………* *Half of the ……… length is 32.* Where did we get the relationship: $h^2 + 32^2 = 40^2$? *………………… theorem*

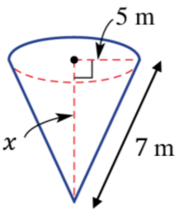
Calculate the unknown height in each diagram. Answer in exact surd form.

a



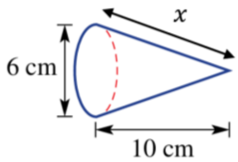
$x = \sqrt{34}$ cm

b



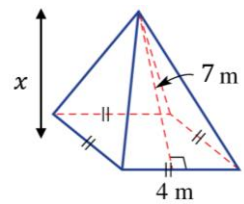
$x = \sqrt{24}$ m

c



$x = \sqrt{109}$ cm

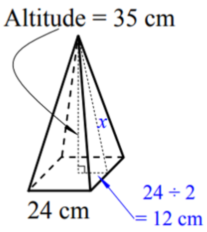
d



$x = \sqrt{45}$ m

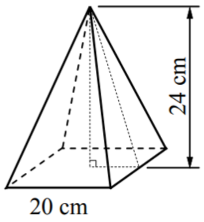
1 You are given the perpendicular height (or altitude).
Calculate the slant height using Pythagoras' Theorem. Answer in exact surd form if necessary.

a



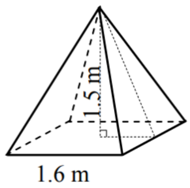
37 cm

b



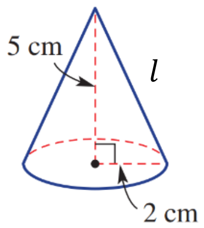
26 cm

c



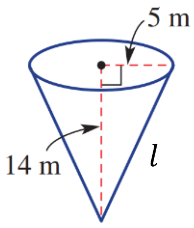
1.7 m

d



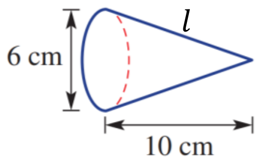
$\sqrt{29}$ cm

e



$\sqrt{221}$ m

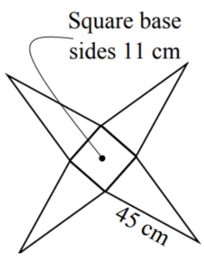
f



$\sqrt{109}$ cm

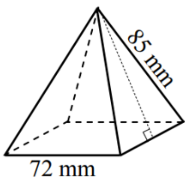
2 You are given a side edge. Calculate the slant height using Pythagoras' Theorem.

a



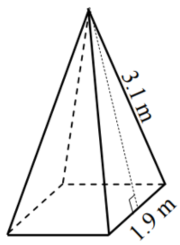
$\sqrt{1994.75}$ cm

b



77 mm

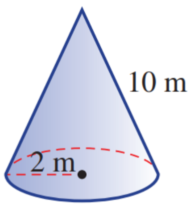
c



$\sqrt{8.7075}$ m

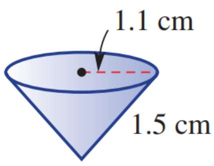
3 Find the perpendicular height using Pythagoras' theorem. Answer in exact surd form.

a



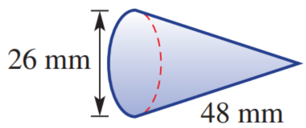
$\sqrt{96}$ m

b



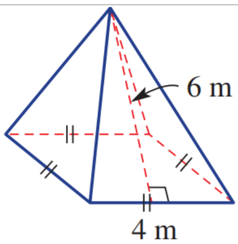
$\sqrt{1.04}$ cm

c



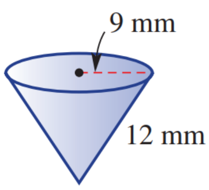
$\sqrt{2135}$ mm

d



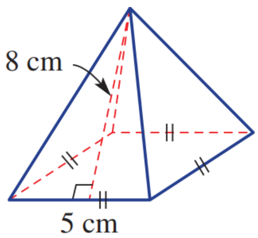
$\sqrt{32}$ m

e



$\sqrt{63}$ mm

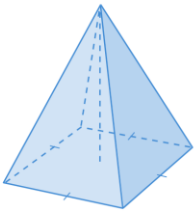
f

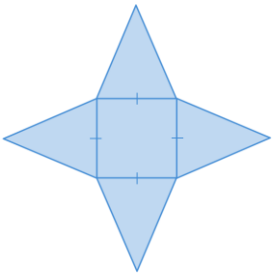


$\sqrt{57.75}$ cm

SURFACE AREA OF PYRAMIDS: SLANT HEIGHT GIVEN

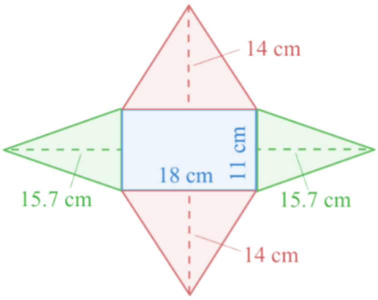
- To calculate the surface area of a pyramid, we must consider its net.
- A pyramid is made up of the base and triangles.





Example Calculate the surface area of a pyramid from its net.

Calculate the surface area of this pyramid.



Area of base:

$$18 \times 11 = 198$$

Area of front and back triangles:

$$2 \times \left(\frac{1}{2} \times 18 \times 14\right) = 252$$

Area of side triangles:

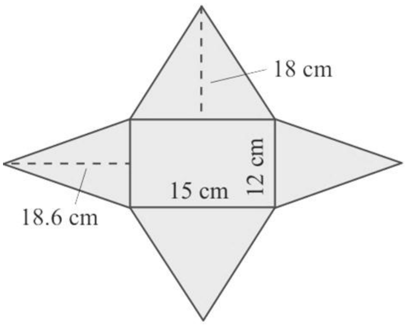
$$2 \times \left(\frac{1}{2} \times 11 \times 15.7\right) = 172.8$$

Total surface area:

$$198 + 252 + 172.8 = 622.8 \text{ cm}^2$$

Calculate the surface area of this pyramid. Round your answer to the nearest whole.

a



Area of base:

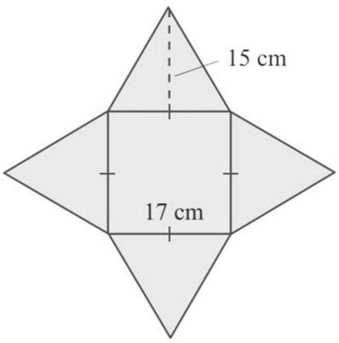
Area of front and back triangles:

Area of side triangles:

Total surface area:

673.2 cm^2

b



Area of base:

Area of front and back triangles:

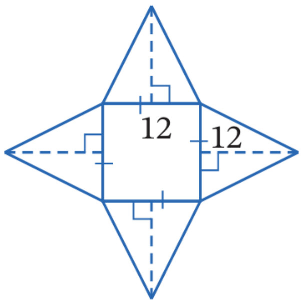
Area of side triangles:

Total surface area:

799 cm^2

1 Calculate the surface area of these pyramids. Round your answer to the nearest whole.

a



Area of base:

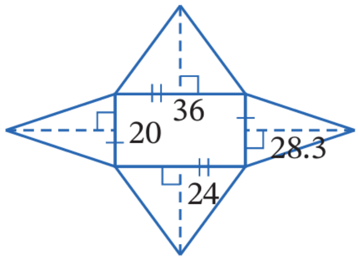
Area of front and back triangles:

Area of side triangles:

Total surface area:

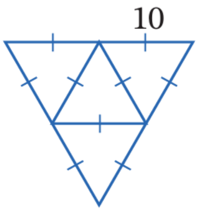
432 cm²

b



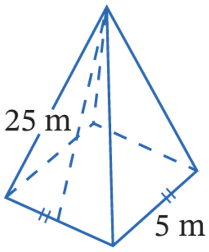
2150 cm²

c



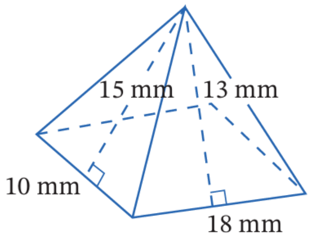
173 cm²

d



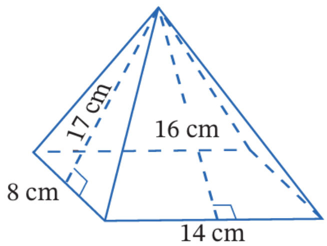
275 m²

e



564 mm²

f



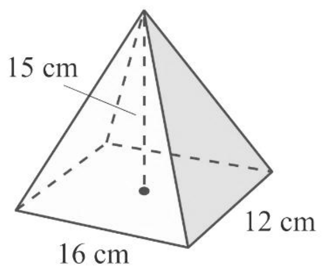
472 cm²

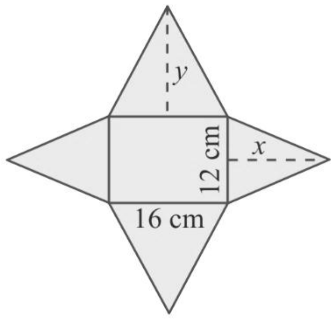
SURFACE AREA OF PYRAMIDS: FIND SLANT HEIGHT

- To calculate the surface area of a pyramid:
  1. Calculate the **slant height** for each triangular face.
  2. Calculate the area of each face.

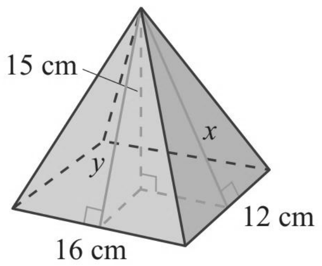
Example Calculate the surface area of a pyramid with unknown sides.

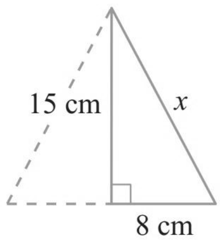
Calculate the surface area of this pyramid. Round your answer to the nearest whole number.

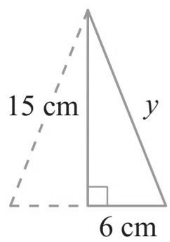




1. Calculate slant heights







$x^2 = 15^2 + 8^2$

$x^2 = 289$

$x = 17 \text{ cm}$

$y^2 = 15^2 + 6^2$

$y^2 = 261$

$y = 16.16 \text{ cm}$

2. Calculate area of each face

Area of base: $16 \times 12 = 192$

Area of front and back triangles:

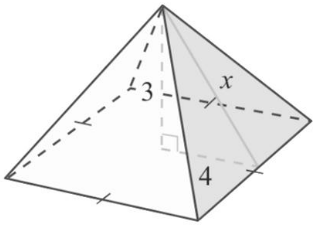
$$2 \times \left(\frac{1}{2} \times 16 \times 16.16\right) = 258.56$$

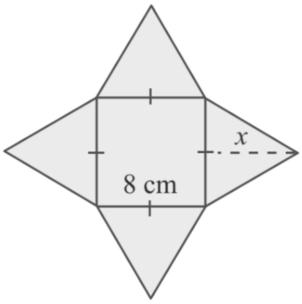
Area of side triangles:

$$2 \times \left(\frac{1}{2} \times 12 \times 17\right) = 204$$

Total surface area: $192 + 258.56 + 204 \approx 255 \text{ cm}^2$

a Calculate the surface area of this pyramid.





Calculate slant height:

Area of base:

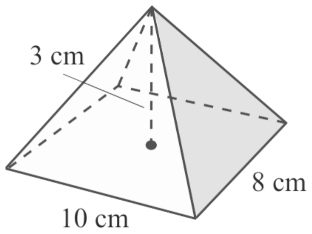
Area of front and back triangles:

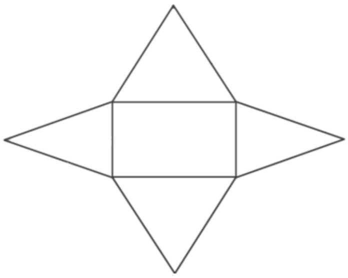
Area of side triangles:

Total surface area:

144 cm^2

b Calculate the surface area of this pyramid. Answer to the nearest whole.





Calculate slant heights:

Area of base:

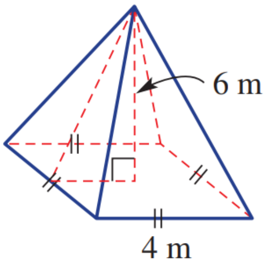
Area of front and back triangles:

Area of side triangles:

Total surface area:

177 cm^2

2 This right square pyramid has base side length 4 m and vertical height 6 m.



a Find the slant height in exact form.

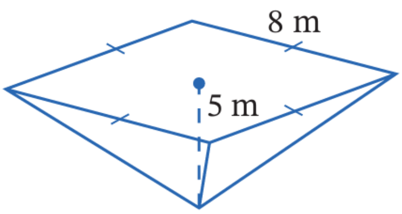
$\sqrt{40}$ m

b Find the surface area, correct to 1 d.p.

66.6 m²

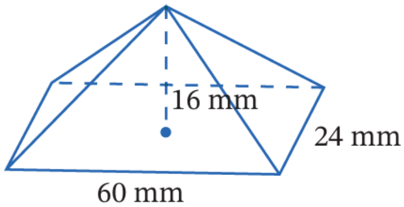
3 Calculate the surface area of these pyramids. You will need to find the slant height. Round your answer to 1 decimal place, where appropriate.

a



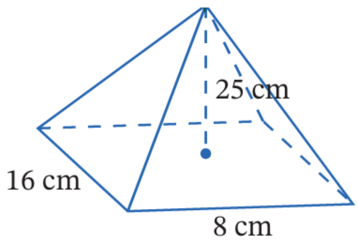
166.4 m²

b



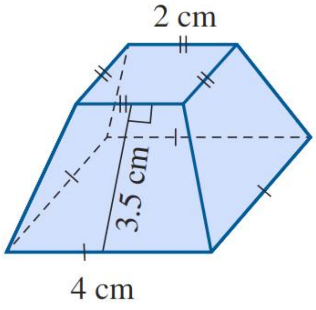
3456 mm²

c



743.1 cm²

4 The solid opposite is called a frustum. It is the portion of a square pyramid that remains after the upper part has been cut off by a plane parallel to its base.
What is the surface area of the frustum? Answer correct to one decimal place.



Top: 2^2

Bottom: 4^2

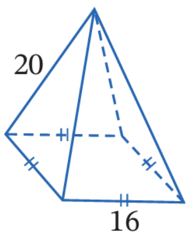
Sides (4 trapeziums): $4\left(\frac{3.5(2+4)}{2}\right)$

Total = 62 cm²

5 Calculate the surface area of these pyramids.

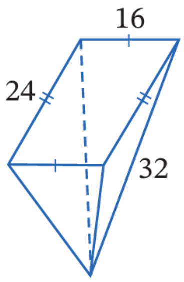
The given lengths are the base lengths and the edge length.

a



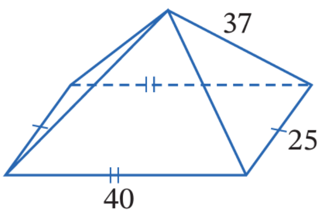
843 cm²

b



1592 cm²

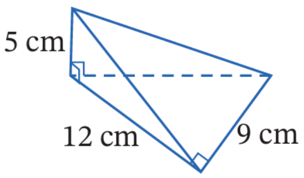
c



3116 cm²

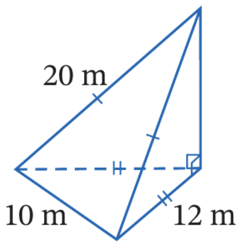
6 Find the surface area of these solids. Round your answer to 1 d.p. where appropriate.

a All faces are right-angled triangles



80 cm²

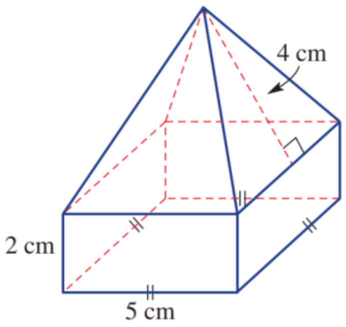
b



343.4 m²

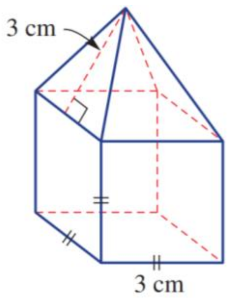
7 Calculate the surface area of these composite solids.
Round your answer to 1 d.p. where appropriate.

a



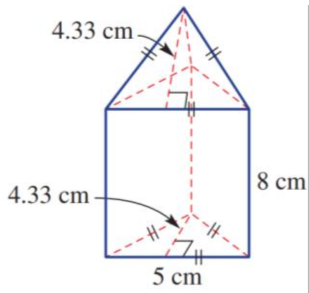
Base: $(5)(5)$
Side rectangles: $(5)(2) \times 4$
Side triangles: $\frac{(5)(4)}{2} \times 4$
105 cm²

b



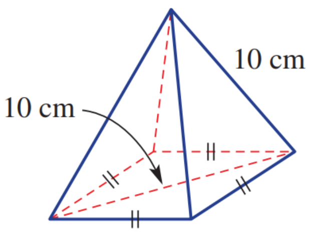
63 cm²

c



163.3 cm²

8 Calculate the surface area of this pyramid.



Side length of base: $\sqrt{50}$

Slant height: $\sqrt{10^2 - \left(\frac{\sqrt{50}}{2}\right)^2} = \sqrt{\frac{175}{2}}$

$SA = (\sqrt{50} \times \sqrt{50}) + 4\left(\frac{\sqrt{50} \times \sqrt{\frac{175}{2}}}{2}\right)$

= 182.3 cm²

9 A square pyramid has a surface area of 4704 m^2 and a base area of 1764 m^2.

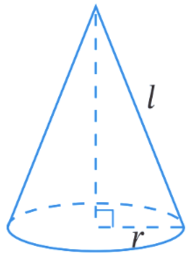
Find its perpendicular height.

Base length: 42 m
Area of triangular face: 735 m²
Slant height of triangular face: 35 m
Perpendicular height: 28 m

SURFACE AREA OF CONES

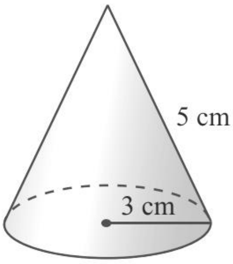
Surface Area of a Cone

$$A = \pi \underset{\substack{\uparrow \\ \text{Radius}}}{r}^2 + \pi r \underset{\substack{\uparrow \\ \text{Slant height}}}{l}$$



Example Calculate surface area of a cone.

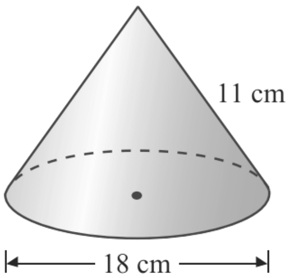
Calculate the surface area of these cones. Round your answer to 1 d.p.



$A = \pi r^2 + \pi r l$

$= \pi(3)^2 + \pi(3)(5)$

$\approx 47.1 \text{ cm}^2$



$A = \pi r^2 + \pi r l$

$= \pi(9)^2 + \pi(9)(11)$

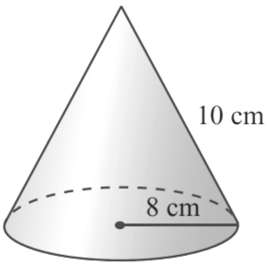
$\approx 565.5 \text{ cm}^2$

In the second example, why is $r = 9$?

The ………………… is 18, so the radius is 9.

Calculate the surface area of these cones. Round your answer to 1 d.p.

a



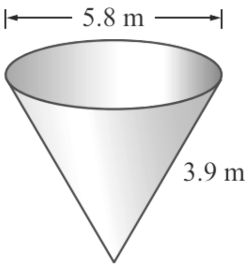
$A = \pi r^2 + \pi r l$

$=$ …………………

$=$ …………………

452.4 cm^2

b



$A = \pi r^2 + \pi r l$

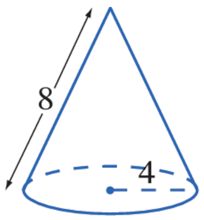
$=$ …………………

$=$ …………………

62.0 cm^2

1 Calculate the surface area of these solids. Round your answer to 2 decimal places.

a



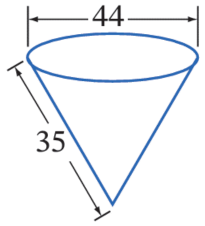
$A = \pi r^2 + \pi r l$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

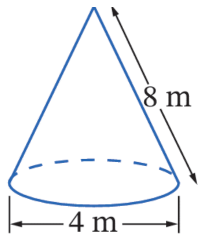
150.80 cm²

b



3939.56 cm²

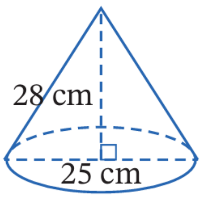
c



62.83 m²

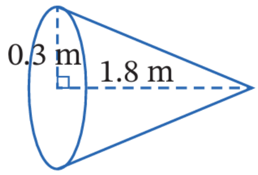
2 Calculate the surface area of these solids. You will need to find the slant height.
Round your answer to 1 decimal place.

a



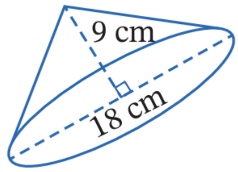
1695.0 cm²

b



2.0 m²

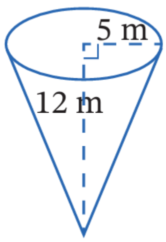
c



614.3 cm²

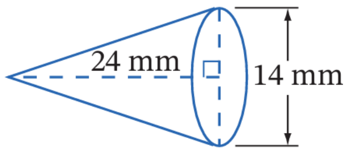
3 Find the surface area of these cones. Leave your answer in exact form in terms of π.

a



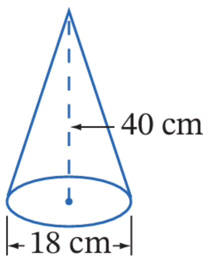
90π m²

b



224π mm²

c



450π cm²

4 A cone with radius 5 cm has a curved surface area of 400 cm^2.

a Find the slant height of the cone, correct to 1 decimal place.

25.5 cm

b Find the height of the cone, correct to 1 decimal place.

25.0 cm

5 The curved surface area of a cone is 150 cm^2 and the radius of the base is 5 cm.
Give answers correct to one decimal place.

a What is the slant height?

9.5 cm

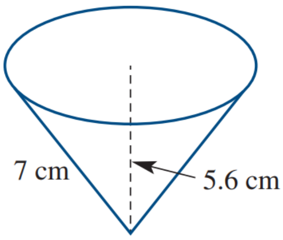
b What is the perpendicular height?

8.1 cm

c What is the surface area of the cone?

228.5 cm²

6 Calculate the surface area of the cone. Answer to 2 decimal places.



$r = 4.2$ cm

$SA = 147.78$ cm

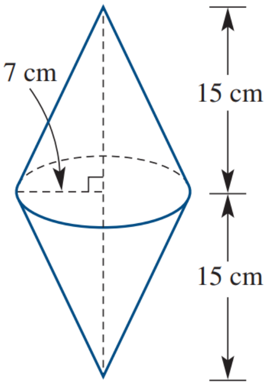
7 A cone with a radius of 5 cm has a total surface area of 200π cm^2.
What is the perpendicular height of the cone? Answer in exact form.

l = 35 cm

$\therefore\ h = \sqrt{1200}$ cm

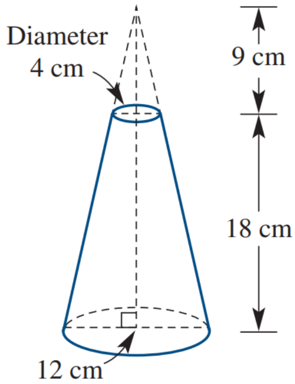
8 Calculate the surface area of these composite solids. Answer to 1 d.p.

a



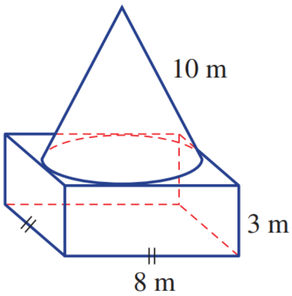
$l = \sqrt{274}$ cm
SA= 728.0 cm²

b



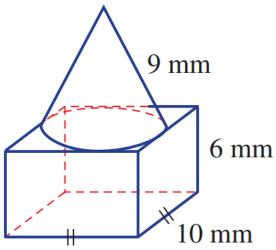
Top circle: $\pi(2)^2$
Bottom circle: $\pi(6)^2$
Curved surface: $\pi(6)\sqrt{765} - \pi(2)\sqrt{85}$
SA = 589.1 cm²

c



299.4 m²

d



502.8 mm²

9 A cone with a base area of 150 cm^2 has a total surface area of 2000 cm^3.

a What is the radius of the base? Answer correct to 1 decimal place.

6.9 cm

b What is the slant height? Answer correct to 1 decimal place.

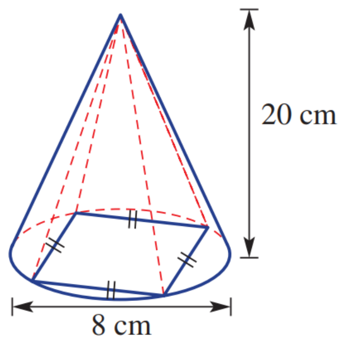
85.3 cm

c What is the perpendicular height? Answer correct to nearest whole number

85 cm

10 A woodworker uses a rotating lathe to produce a cone with radius 4 cm and height 20 cm. From that cone the woodworker then cut slices off the sides of the cone to produce a square pyramid of the same height.

a Find the exact slant height of the cone.



$4\sqrt{26}$ cm

b Find the exact side length of the base of the square pyramid.

$4\sqrt{2}$ cm

c Find the slant height of the pyramid, correct to 3 decimal places.

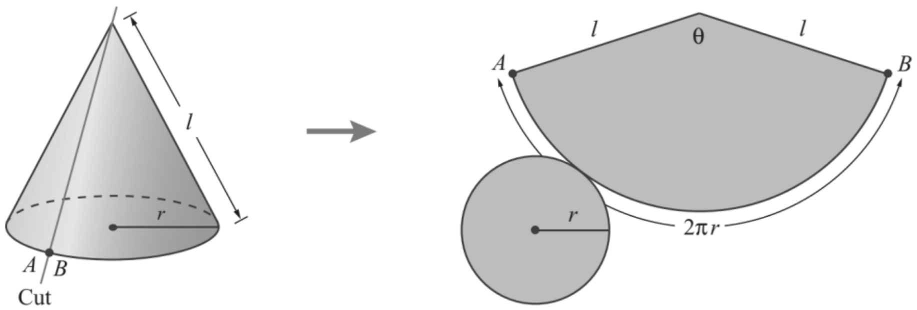
20.199 cm

d Find the surface area of the pyramid, correct to 2 decimal places.

260.53 cm²

Investigation Deriving the formula for the surface area of a cone.

The net of a cone can be obtained by cutting the conical section and unrolling it as shown.



Write an expression for the area of the circular base. ……………………………………………

The sector is part of a larger circle with radius l.

What is the circumference and area of the larger circle?

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

The sector has arc length $2\pi r$.

What is the ratio of the sector arc length to the circumference of the larger circle?

………………………………………………………………………………………………………

Hence find a simplified expression for the area of the sector using

$$\text{Sector area} = \text{Fraction of circle} \times \text{Area of circle}$$

………………………………………………………………………………………………………

………………………………………………………………………………………………………

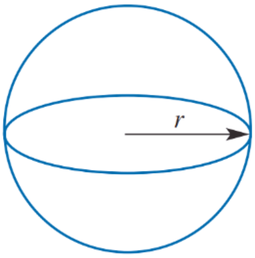
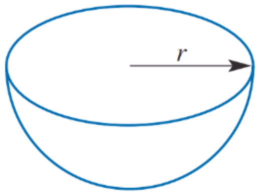
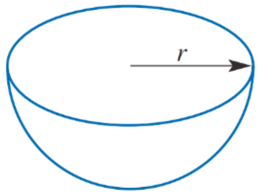
………………………………………………………………………………………………………

Therefore, what is the surface area of the cone?

………………………………………………………………………………………………………

SURFACE AREA OF SPHERES

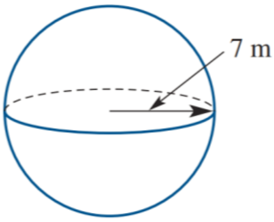
- A **sphere** is a perfectly round solid.
- A **hemisphere** is half a sphere.
  - An **open** hemisphere is half a hollow sphere. It has no circular face.
  - A **closed** hemisphere is half a solid sphere. It has a circular face.

Sphere	Hemisphere (open)	Hemisphere (closed)
		
$A = 4\pi r^2$	Half a sphere $A = \frac{1}{2} \times 4\pi r^2$ $= 2\pi r^2$	Half a sphere + top face $A = 2\pi r^2 + \pi r^2$ $= 3\pi r^2$

- The proof of the sphere formula is outside the scope of high school.

Example Calculate surface area of a sphere.

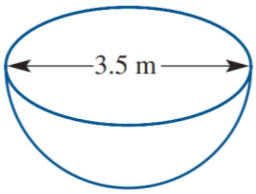
Calculate the surface areas of these spheres and open hemispheres to two decimal places.



$A = 4\pi r^2$

$= 4\pi(7)^2$

$\approx 615.75 \text{ m}^2$



$A = \frac{1}{2} \times 4\pi r^2$

$= \frac{1}{2} \times 4(\pi)(1.75)^2$

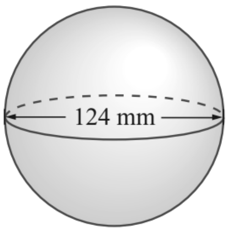
$\approx 19.24 \text{ m}^2$

In the second example, why is $r = 1.75$?

The ………………… is 3.5, so the radius is 1.75.

Calculate the surface areas of these spheres and open hemispheres to 1 d.p.

a



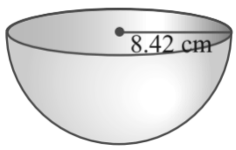
$A = 4\pi r^2$

$=$ …………………………

$=$ …………………………

48305.1 mm^2

b



$A = \frac{1}{2} \times 4\pi r^2$

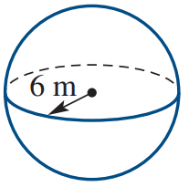
$=$ …………………………

$=$ …………………………

445.5 cm^2

1 Calculate the surface area of these spheres, correct to 2 decimal places.

a



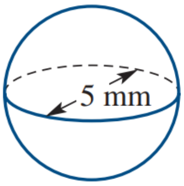
$A = 4\pi r^2$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

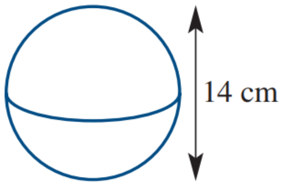
452.39 m²

b



78.54 mm²

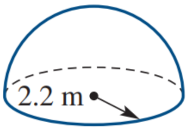
c



615.75 cm²

2 Calculate the surface area of these open hemispheres, correct to 2 decimal places.

a



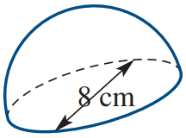
$A = \frac{1}{2} \times 4\pi r^2$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

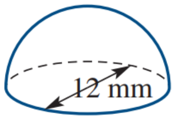
30.41 m²

b



100.53 cm²

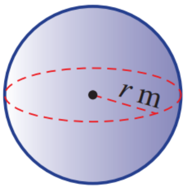
c



226.19 mm²

3 Find the radius of these spheres with the given surface area, correct to 2 decimal places.

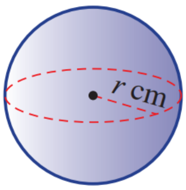
a



$A = 10 \text{ m}^2$

0.89 m

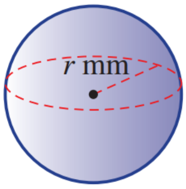
b



$A = 120 \text{ cm}^2$

3.09 cm

c



$A = 0.43 \text{ mm}^2$

0.18 mm

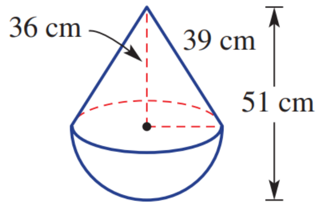
4 How many spheres of 15 cm diameter can be painted with 1 L of glitter paint if the paint covers 10 m^2?

1 sphere = $4\pi(7.5)^2 = 225\pi$ cm²

$10 \text{ m}^2 \times 100^2 = 100\ 000 \text{ cm}^2$

$100\ 000 \div 225\pi = 141$ spheres

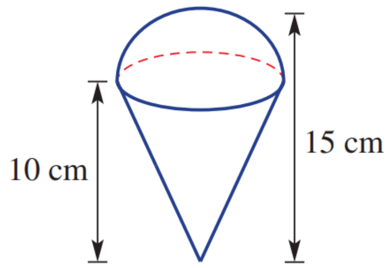
5 A water buoy is in the shape shown. Find the surface area, in exact form.



Radius = 15 cm

SA = 1035π cm²

6 A hemisphere sits on a cone and two height measurements are given as shown. Find the surface area of the solid, correct to 1 decimal place.



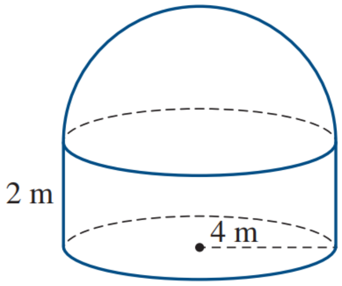
Radius of hemisphere: 5 cm

Slant height: $\sqrt{125}$ cm

SA = 332.7 cm²

7 Calculate the surface area of these composite solids. Round to the nearest whole.

a



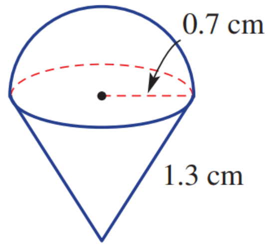
Top hemisphere: $2\pi(4)^2$

Curved side: $2\pi(4)(2)$

Bottom: $\pi(4)^2$

Total: 201 m²

b

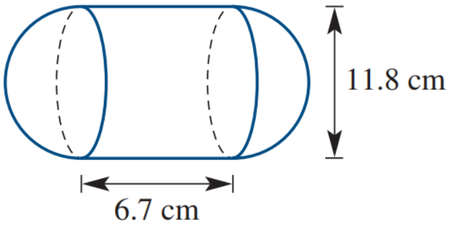


Top hemisphere: $2\pi(0.7)^2$

Curved side: $\pi(0.7)(1.3)$

Total: 6 cm²

c

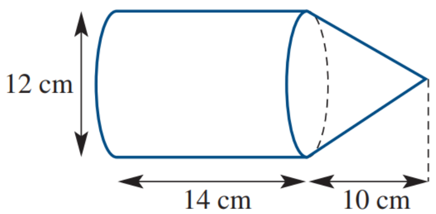


Side hemispheres: $2\pi(5.9)^2 \times 2$

Curved middle: $2\pi(5.9)(6.7)$

Total: 686 cm²

d



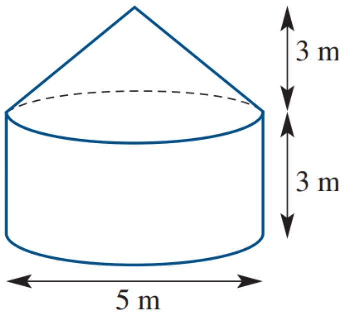
Right cone: $\pi(6)(\sqrt{136})$

Curved middle: $2\pi(6)(14)$

Left circle: $\pi(6)^2$

Total: 861 cm²

e



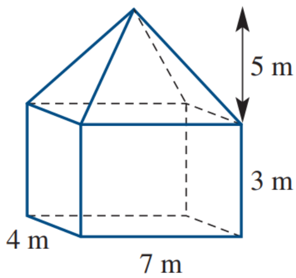
Top cone: $\pi(2.5)(\sqrt{15.25})$

Curved middle: $2\pi(2.5)(3)$

Bottom circle: $\pi(2.5)^2$

Total: 97 m²

f



Top triangles: $2\left(\frac{4 \times \sqrt{37.25}}{2}\right) + 2\left(\frac{7 \times \sqrt{29}}{2}\right)$

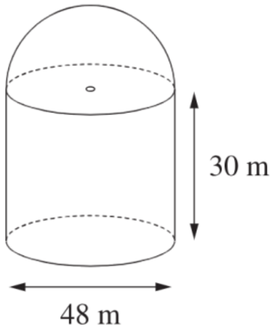
Sides: $2(3)(4)+2(3)(7)$

Bottom: $4(7)$

Total: 156 m²

8 **2014 HSC Standard 2 Band 5**

A grain silo is made up of a cylinder with a hemisphere (half a sphere) on top.
The outside of the silo is to be painted. What is the area to be painted, to 1 decimal place?



Curved surface: $2\pi(24)(30)$

Hemisphere: $2\pi(24)^2$

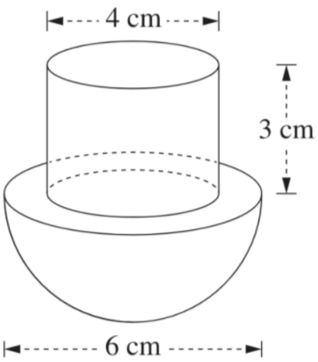
Total: 8143 m²

9 **2022 HSC Standard 2 Band 5**

Consider the solid below.

The top section is a cylinder with a height of 3 cm and a diameter of 4 cm.
The bottom section is a hemisphere with a diameter of 6 cm.
The cylinder is centred on the flat surface of the hemisphere.

Find the total surface area of the composite solid in cm², correct to 1 decimal place.



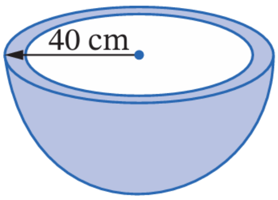
Top-facing surface: $\pi(3)^2$

Curved surface of cylinder: $2\pi(2)(3)$

Bottom hemisphere: $2\pi(3)^2$

Total: 122.5 cm²

10 Find the surface area of this hemispherical bird bath with outer radius 40 cm and 1 cm thick walls. Round your answer to 1 decimal place.



Outer hemisphere: $2\pi(40)^2$

Inner hemisphere: $2\pi(39)^2$

Top: $\pi(40)^2 - \pi(39)^2$

Total: 19 858.0 cm²