Student Name

Mathematics Stage 4

RATIOS AND RATES

Book 4 Solve problems involving rates

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA4-RAT-C-01 solves problems involving ratios and rates, and analyses distance–time graphs

Solve problems involving rates

- Solve a variety of real-life problems involving rates
- Examine financial applications of rates, such as best buys

SUMMARY

Solve Problems involving Rates

Best Buys

Finding the option that has the best value for money.

1. Convert both rates to the same units if the units are different.
2. Simplify both rates to the unit rate.
3. The best buy has the **lowest price per unit**, or the **highest units per dollar**

Which is better value?

Option 1: \$2.90 for 2 cans

Option 2: \$8.46 for 6 cans

Option 1: $\frac{\$2.90}{2 \text{ cans}} = \frac{\$1.45}{1 \text{ can}}$

Option 2: $\frac{\$8.46}{6 \text{ cans}} = \frac{\$1.41}{1 \text{ can}}$

lowest price per unit

Which is better value?

Option 1: A box of 10 pens for \$6.80

Option 2: A pack of 15 pens for \$9.60

Option 1: $\frac{10 \text{ pens}}{\$6.80} = \frac{1.47 \text{ pens}}{\$1}$

Option 2: $\frac{15 \text{ pens}}{\$9.80} = \frac{1.53 \text{ pens}}{\$1}$

highest units per dollar

Rates Problems

1. Write an equation using the given information.
2. Divide to find the unit rate.
3. Multiply to find the rate asked by the question.

A car travels 360 km on 30 L of petrol.
How far does it travel on 7 L?

$$\begin{aligned} 360 \text{ km} &= 30 \text{ L} \\ \div 30 &\quad \div 30 \\ 12 \text{ km} &= 1 \text{ L} \\ \times 7 &\quad \times 7 \\ 84 \text{ km} &= 7 \text{ L} \end{aligned}$$

REVIEW OF PRIOR KNOWLEDGE

1 Find the cost of 1 kg if:

a	2 kg costs $8	**b**	5 kg costs $15	**c**	4 kg costs $10

2 A car is travelling at a speed of 80 km/h

How far would it travel in:

a	3 hours	**b**	6 hours	**c**	30 minutes

3 $300 is to be divided into the ratio 2 : 3. Find the value of the larger portion using the unitary method.

4 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

5 If a prize of $6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

6 Circle which is the best value for money.

Option 1	Option 2
1 can of drink for $1.50	1 can of drink for $1
$1.50 for 1 can of drink	$4 for 3 cans of drink
500 mL of drink for $1	550 mL of drink for $1
550 mL of drink for $5	125 mL of drink for $1

7 Simplify these rates.

a 120 km in 3 hours	**b** 5000 revolutions in 2.5 minutes	**c** 150 km in 10 hours
d 56 metres in 4 seconds	**e** 207 heartbeats in $2\frac{1}{4}$ minutes	**f** 180 mL in 22.5 hours

BEST BUYS

- The **best buy** offers the best value for money.
  1. Convert both rates to the same units if different.
  2. Simplify to unit rates using **$** per **quantity**.
  3. The **lowest price per unit** is the best buy.

Example Determine the best buy.

Which is better value?	Which is better value?
Option 1: \$2.90 for 2 cans	Option 1: A box of 10 pens for \$6.80
Option 2: \$8.46 for 6 cans	Option 2: A pack of 15 pens for \$9.60
Option 1: $\frac{\$2.90}{2 \text{ cans}} = \frac{\$1.45}{1 \text{ can}}$ Option 2: $\frac{\$8.46}{6 \text{ cans}} = \frac{\$1.41}{1 \text{ can}}$ Option 2 is the best buy.	Option 1: $\frac{\$9.60}{15 \text{ pens}} = \frac{\$0.64}{1 \text{ pen}}$ Option 2: $\frac{\$6.80}{10 \text{ pens}} = \frac{\$0.68}{1 \text{ pen}}$ Option 1 is the best buy.

Determine the best buy.

	Option 1	Option 2
a	\$2.55 for 6 eggs. $\frac{\$2.55}{6 \text{ eggs}} = \frac{\quad}{1 \text{ egg}}$	8 eggs for \$3.50. $\frac{\$3.50}{8 \text{ eggs}} = \frac{\quad}{1 \text{ egg}}$
	best buy	
b	\$3.55 for 300 mL of orange juice.	\$5.75 for 500 mL of orange juice.
		Best buy
c	200 g jar of coffee for \$1.55.	0.5 kg jar of coffee for \$3.70.
		Best buy
d	200 g pack of biscuits for \$0.95.	\$1.25 for 300 g pack of biscuits.
		Best buy

Key ideas

1. The best buy is the option with the lowest …… per unit of item.

1 Look at these offers.
Offer A: 9 oranges for $3.20 Offer B: 4 oranges for $1.35

What is the cost of one orange in:

a offer A

$0.35

b offer B

$0.3375

c Which offer is better value for money?

Offer B

2 At a wholesaler, onions cost $8.50 for an 8 kg bag.

In the supermarket, a 2 kg bag costs $2.10

What is the cost of 1 kg of onions at:

a the wholesaler

$1.0625

b the supermarket

$1.05

c In which shop are the onions better value for money

supermarket

3 Which is better value?

a $5 for 230 g $8.60 for 500 g

Allens Snakes

b $3.85 for 1 L $11.40 for 12×300 mL

12L bottle

c $6.20 for 280 g $9.40 for 460 g $7.30 for 350 g

460g jar

4 **2019 HSC Standard 2 Band 3**

Sugar is sold in four different sized packets. Which is the best buy?

A. 100 g for $0.40

B. 500 g for $1.65

C. 1 kg for $3.50

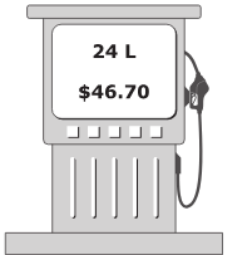
D. 2 kg for $6.90

B

5 **2023 HSC Standard 2 Band 4**

Four petrol pumps are shown, each with the amount of petrol purchased and its cost.
Which one represents the best value?

A.



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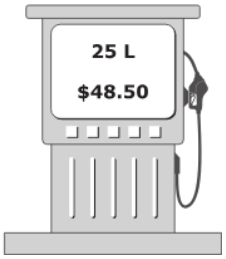
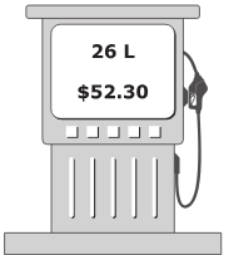
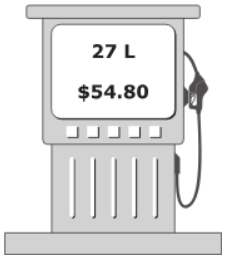


C.



D.



B

6 In a shop, potatoes are sold in 2 kg bags for $3.50.
In a wholesale market, potatoes are sold in 7 kg bags for $12.50.
For each option calculate:

a The cost of 1 kg of potatoes.

Option 1	Option 2
$1.75/kg	$1.79/kg

b The number of kilograms that can be bought for $1.

0.57 kg/$1	0.56 kg/$1

c The cost of 14 kg of potatoes.

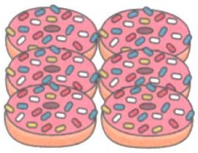
$24.50/14kg	$25/14kg

d Determine the best buy.
Which method (a, b, or c) do you prefer to use for this question?

Option 1 is the best buy.

7 A bakery sells donuts in three packages:

4 for $1.55



6 for $2.30

10 for $3.75

Determine the best buy by:

a Calculating the price of 1 donut for each option. **Do not round your answer**.

$0.3875 per donut | 0.38\dot{3}$ per donut | $0.375 per donut

10 for $3.75 is the best buy

b Calculating the cost of 60 donuts for each option.

$23.25 | $23 | $22.50

c Why do you think 60 donuts was the number in part **b**?

It is the LCM of 4, 6, and 10

8 **HSC Sample Question Band 4**

A brand of pasta is sold in two different packets.

What is the difference in price per kg?

$1.80 $9.00

Option 1: $\frac{\$1.80}{200\text{g}} \frac{\times 5}{\times 5} = \frac{\$9}{1\text{ kg}}$

Option 2: $\frac{\$9}{750\text{ g}} \frac{\div 3}{\div 3} = \frac{\$3}{250\text{ g}} \frac{\times 4}{\times 4} = \frac{\$12}{1\text{ kg}}$

9 The best buy is also the option with the **highest units per dollar.**

a A 2 kg box of sultanas for \$8.85 or a 1 kg box for \$4.65

Option 1: $\frac{2\text{ kg}}{\$8.85} = \frac{0.2259887...\text{kg}}{\$1}$

Option 2: $\frac{1\text{ kg}}{\$4.65} = \frac{0.2150576...\text{kg}}{\$1}$

Option 1 is the best buy, you get more per dollar.

b 45 g chips for \$2.10 or 150 g for \$6.50

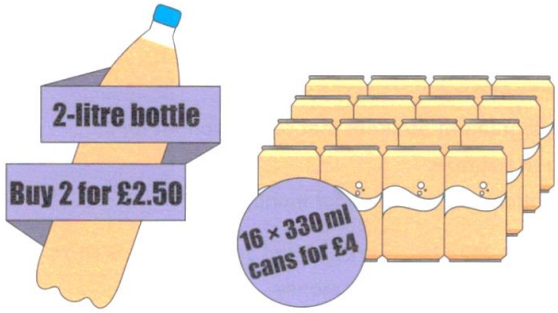
Option 2

10 Mario is doing his Maths homework. He needs to decide which is better value:
200 g of pasta for 80c or
500 g of pasta for \$1.95
His working out is shown:

Explain why he is incorrect.

He did $\frac{\text{Quantity}}{\text{Cost}}$, so he should choose the higher value.

11 A shop has these special offers. Which is the best buy? Explain your reasoning.



Bottles: 1.6 L per £1

Cans: 1.32 L per £1

Bottles are the best buy.

12 Video games tend to use a secondary currency.
Round each option then determine the best buy.

	Robux Packages ⓘ
\$8.49	400
\$16.99	800
\$34.99	1,700
\$90.99	4,500
\$184.99	10,000
\$369.99	22,500

Best buy is 22 500 for \$369.99

Explain why you shouldn't always choose the option that is the best buy.

You are spending more money overall.

RATES PROBLEMS

- We will solve rates problems using **equivalent rates**.
- Solve problems involving rates using algebraic fractions:
  1. Write the given information as an equation. Let the quantity you want to find be x.
  2. Solve for the unknown.
     If the unknown is in the denominator, take the reciprocal of both sides.

Example Solve rates problems.

A car travels 360 km on 30 L of petrol.
How far does it travel on 120 L?

1.
$$\frac{360\text{ km}}{30\text{ L}} = \frac{x\text{ km}}{120\text{ L}}$$
2. $\times 120 \quad \times 120$
$$1440\text{ km} = x$$

Bella can type 240 words every 3 minutes.
How long does it take to write 2800 words?

1.
$$\frac{240\text{ words}}{3\text{ min}} = \frac{2800\text{ words}}{x\text{ min}}$$
2. reciprocal both sides
$$\frac{3\text{ min}}{240\text{ words}} = \frac{x\text{ min}}{2800\text{ words}}$$
$\times 2800 \quad \times 2800$
$$35\text{ min} = x$$

Solve these rates problems using the equivalent rates method.

a $72 can buy 8 kg of chicken.
How much would 3 kg cost?

$$\frac{\$72}{8\text{ kg}} = \frac{\$\,x}{3\text{ kg}}$$

$27

b $5.50 can buy 0.5 kg of mince.
How much would 5 kg cost?

$$\frac{\$5.50}{0.5\text{ kg}} = \text{\_\_\_\_}$$

$55

c One dozen (12) tennis balls cost $9.60.
How much would 22 balls cost?

$$\frac{12\text{ balls}}{\$9.60} = \frac{22\text{ balls}}{\$\,x}$$

$17.60

d 3 pairs of socks cost $12.99.
How much would 10 pairs cost?

$$\frac{3\text{ pairs}}{\$12.99} = \text{\_\_\_\_}$$

$43.30

Key ideas

1. We solve rates problems using …… rates.
2. If the unknown is in the denominator, we take the …… of both sides.

1

a A truck uses 9 L of fuel per 100 km.
How many litres are needed for the truck to travel 28 km?

$$\frac{9\text{ L}}{100\text{ km}} = \frac{x\text{ L}}{28\text{ km}}$$

2.52 L

b Perfume costs $157.90 for 50 mL.
How much does it cost for 80 mL?

$$\frac{\$157.90}{50\text{ mL}} = \frac{\$x}{80\text{ mL}}$$

$252.64

c Phillipa types at 75 words per minute.
How long would it take her to type a 500-word essay?

$$\frac{75\text{ w}}{1\text{ min}} = \frac{500\text{ w}}{x\text{ min}}$$

6.ȯ min

d 2 litres of paint cover 9 m^2.
How much paint is needed for 104 m^2?

$$\frac{2\text{ L}}{9\text{ m}^2} = \frac{x\text{ L}}{104\text{ m}^2}$$

23.İ L

e A tap is dripping at a rate of 200 mL every 5 minutes.
How much water drips in 13 minutes?

$$\frac{200\text{ mL}}{5\text{ min}} = \frac{x\text{ mL}}{13\text{ min}}$$

520 mL

f A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

$$\frac{400\text{ mm}}{8\text{ min}} = \frac{x\text{ mm}}{7\text{ min}}$$

350 mm

g A footballer scores an average of 4 goals every 6 games.
How many games would it take for him to score 20 goals?

$$\frac{4\text{ goals}}{6\text{ games}} = \frac{20\text{ goals}}{x\text{ games}}$$

30 games

h A computer can process 500 megabytes of information in 4 seconds.
How long would it take to process 3700 megabytes?

$$\frac{500\text{ MB}}{4\text{ s}} = \frac{3700\text{ MB}}{x\text{ s}}$$

29.6 s

2 A car uses petrol at an average rate of 7.4 L/100 km. How much petrol will it consume in 58 km?

$$\frac{7.4\text{ L}}{100\text{ km}} = \frac{x\text{ L}}{58\text{ km}}$$

4.292 L

3 A dripping tap fills a 9-litre bucket in 3 hours. How long would it take to fill a 21-litre bucket?

$\frac{9\text{ L}}{3\text{ h}} = \frac{21\text{ L}}{x\text{ h}}$

7 h

4 Gemma runs 70 metres in 8.4 seconds.
If she maintains the same speed, in what time will she run 100 metres?

$\frac{70\text{ m}}{8.4\text{ s}} = \frac{100\text{ m}}{x\text{ s}}$

12 s

5 A factory produces 40 plastic bottles/minute.
How many bottles can the factory produce in 8 hours?

$\frac{40\text{ bottles}}{1\text{ min}} = \frac{x\text{ bottles}}{480\text{ min}}$

19 200 bottles

6 Daniel practises his guitar for 48 minutes every day.
How many days will it take him to log 10 000 hours of practice?

$\frac{48\text{ min}}{1\text{ day}} = \frac{10\,000\text{ h}}{x\text{ days}}$

$x = 12500$ days

7 Mr Ding is making cheesecake.

The recipe requires: 300 g cream cheese, 125 mL thickened cream.

Cream cheese only comes in 250 g packs so he bought 2 packs: 500 grams of cream cheese.

Mr Ding thought "If I'm increasing the amount of cream cheese by 200 g, then I should increase the amount of thickened cream by 200 mL, so I need 325 mL"

a Describe what would happen to his cheesecake if he did this.

He is less-than-doubling the cream cheese but more-than-doubling the cream. It would be too runny.

b How much thickened cream should he use for 500g of cream cheese?

$\frac{300\text{ g cheese}}{125\text{ mL cream}} = \frac{500\text{ g cheese}}{x\text{ mL cream}}$

x = 208.33 mL

8 Fertiliser is to be spread at the rate of 0.2 kg/m^2

a How much fertiliser would be needed for an area of 600 m^2?

$\frac{0.2 \text{ kg}}{1 \text{ m}^2} = \frac{x \text{ kg}}{600 \text{ m}^2}$
x = 120 kg

b If the fertiliser is sold in 20 kg bags, how many bags are needed for an area of 600 m^2?

120 kg ÷ 20 kg/bag = 6 bags

c What area could be fertilised with 1 t of fertiliser?

$\frac{0.2 \text{ kg}}{1 \text{ m}^2} = \frac{1000 \text{ kg}}{x \text{ m}^2}$
$x = 5000 \text{ m}^2$

9 A car travels 800 km on a full tank of petrol.
The petrol tank holds 63 litres of petrol.

a Calculate the rate at which the car uses petrol, giving your answer in km per litre.

$\frac{800 \text{ km}}{63 \text{ L}} = 12.7 \text{ km/L}$

b Write the rate in L per 100 km.

$\frac{64 \text{ L}}{800 \text{ km}} = \frac{x \text{ L}}{100 \text{ km}}$
7.875 L per 100 km

10 A car uses 9 L of petrol to travel 100 km. Answer to 1 d.p.

a How much petrol is needed to travel 50 km?

$\frac{9 \text{ L}}{100 \text{ km}} = \frac{x \text{ L}}{50 \text{ km}}$
x = 4.5 L

b How far can it travel using 50 L?

$\frac{9 \text{ L}}{100 \text{ km}} = \frac{50 \text{ L}}{x \text{ km}}$
x = 555.6 km

11 A car uses 14 L of petrol to travel 147 km.

a How much petrol is needed to travel 100 km?

$\frac{14 \text{ L}}{147 \text{ km}} = \frac{x \text{ L}}{100 \text{ km}}$
x = 9.52 L

b How far can it travel using 100 L of petrol?

$\frac{14 \text{ L}}{147 \text{ km}} = \frac{100 \text{ L}}{x \text{ km}}$
x = 1050 km

12 A car used 30 litres of fuel to travel 105 km. The fuel tank of the car has a capacity of 50 litres. Can the car complete a 180 km journey without refuelling?

$\frac{30\text{ L}}{105\text{ km}} = \frac{x\text{ L}}{180\text{ km}}$

x = 51.4 L

No, needs 51.4 L but tank only holds 50 L

13 A person's heart beats at an average rate of 70 beats per minute.

a How many times will a heart beat in a lifetime of 80 years?

$\frac{70\text{ beats}}{1\text{ min}} = \frac{70\text{ beats}}{1 \div 60 \div 24 \div 365\text{ year}} = \frac{36\,792\,000\text{ beats}}{1\text{ year}}$

$\frac{36\,792\,000\text{ beats}}{1\text{ year}} = \frac{x\text{ beats}}{80\text{ years}}$

= 2 943 360 000 beats

b How long will it take for a heart to beat 1 million times?
Give your answer in days, to 1 decimal place.

$\frac{70\text{ beats}}{1\text{ min}} = \frac{1{,}000{,}000\text{ beats}}{x\text{ min}}$

x = 14,285.7 min = 238.1 h = 9.9 days

14 Anna is paid at the rate of \$23.60/ h.

a How much would she be paid for working 15 h?

$\frac{\$23.60}{1\text{ h}} = \frac{\$x}{15\text{ h}}$

\$354

b How long would she need to work to earn \$472?

$\frac{\$23.60}{1\text{ h}} = \frac{\$472}{x\text{ h}}$

20 h

c Anna wants to save for a trip. If she works 28 h in week 1, and 31 h in the next week, how many more hours does she need to work to earn \$2000?

Current earnings: (28 + 31) × \$23.60 = \$1392.40

Still needs: \$2000 - \$1392.40 = \$607.60

Additional hours: \$607.60 ÷ \$23.60 = 25.7 h

15 On a tin of paint, it states that you can cover 16 m^2 per litre. How many litres of paint do you need to cover a rectangular wall 5.2 m by 2.3 m if you need to apply three coats?

Wall area: 5.2 × 2.3 = 11.96 m²

Three coats: 11.96 × 3 = 35.88 m²

$\frac{16\text{ m}^2}{1\text{ L}} = \frac{35.88\text{ m}^2}{x\text{ L}}$

x = 2.24 L

16 NAPLAN A+

One litre of soft drink contains 90 grams of sugar.

How many millilitres of soft drink contain 4.5 grams of sugar?

$\frac{90\text{ g}}{1000\text{ mL}} = \frac{4.5\text{ g}}{x\text{ mL}}$

x = 50 mL

17 2023 HSC Standard 1

A tap is dripping at the rate of 4 mL per minute.

Which expression shows how many litres this would amount to in one year?

A. $\frac{4 \times 1000}{60 \times 24 \times 365}$

B. $\frac{4 \times 60 \times 24 \times 365}{1000}$

C. $\frac{60 \times 24 \times 365}{4 \times 1000}$

D. $\frac{1000}{4 \times 60 \times 24 \times 365}$

$\frac{4 \div 1000\text{ L}}{1 \div 60 \div 24 \div 365\text{ year}} = \frac{4 \times 60 \times 24 \times 365}{1 \times 1000}$

18 2023 HSC Standard 1

	Petrol car	*Electric car*
Fuel consumption	8.6 L / 100 km	18 kWh / 100 km
Cost of fuel	\$1.87 / L	\$0.25 / kWh

Jun travels on average 35 000 km per year.

How much will he save on fuel costs in a year by using an electric car?

Petrol: $\frac{8.6\text{ L}}{100\text{ km}} = \frac{x\text{ L}}{35000\text{ km}}, x = 3010\text{ L}$

Cost: 3010 × \$1.87 = \$5628.70

Electric: $\frac{18\text{ kWh}}{100\text{ km}} = \frac{x\text{ kWh}}{35000\text{ km}}, x = 6300\text{ kWh}$

Cost: 6300 × \$0.25 = \$1575

Savings: \$5628.70 - \$1575 = \$4053.70

19 The volume of water in a reservoir falls from 430 000 litres to 180 000 litres over 5 days.

a Calculate the rate at which the reservoir is emptying, giving your answer in litres per day.

$$\frac{(430000 - 180000)\text{ L}}{5\text{ days}} = 50000\text{ L/day}$$

b Assume that the reservoir continues to empty at the same rate.
How many more days will it take for the reservoir to empty completely?

$$\frac{50000\text{ L}}{1\text{ day}} = \frac{180000\text{ L}}{x\text{ days}}$$

$$x = 3.6\text{ days}$$

20 A patient in hospital is given an antibiotic solution at the rate of 80 mL/ h.

a How much antibiotic solution will the patient receive in $6\frac{1}{2}$ h?

$$\frac{80\text{ mL}}{1\text{ h}} = \frac{x\text{ mL}}{6.5\text{ h}}$$

x = 520 mL

b How often would 600 mL containers of the antibiotic solution need to be changed?

$$\frac{80\text{ mL}}{1\text{ h}} = \frac{600\text{ mL}}{x\text{ hours}}$$

Change every 7.5 hours

c If 1 mL of this solution contains 15 drops, calculate the rate at which the patient receives the antibiotic in drops/minute.

$$\frac{x\text{ drops}}{1\text{ min}} = \frac{15\text{ drops}}{1\text{ mL}} \times \frac{80\text{ mL}}{60\text{ min}}$$

20 drops/min

21 The conversion rate for US dollars (US$) to Australian dollars (A$) is $0.65\ \text{US\$/ A\$}$.

a How many US dollars would I receive for A$2750?

$$\frac{0.65\text{ US}}{1\text{ A}} = \frac{\$x\text{ US}}{2750\text{ A}}$$

$1785.71 US

b How many Australian dollars would I receive for US$1300?

$$\frac{0.65\text{ US}}{1\text{ A}} = \frac{1300\text{ US}}{x\text{ A}}$$

$2000 A

c Convert US$1800 into A$

$$\frac{0.65\text{ US}}{1\text{ A}} = \frac{1800\text{ US}}{x\text{ A}}$$

$2769.23 A

22 Ruth's tropical fish are breeding at a rate of 8 new fry every 2 days per female fish.
If Ruth has a total of 9 female fish, then how long will it take them to birth a total of 90 new fry?

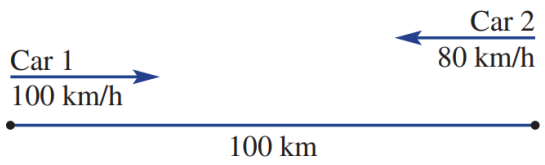
8 new fry/ 2 days = 4 new fry / day

For 9 female fish, 36 new fry / day

$$\frac{36 \text{ fry}}{1 \text{ day}} = \frac{90 \text{ fry}}{x \text{ days}}$$

2.5 days

23 Two cars travel toward each other on a 100 km straight stretch of road.
They leave at opposite ends of the road at the same time.
The cars' speeds are 100 km/h and 80 km/h.
How long does it take for the cars to pass each other?



We will use a physical principle called relative velocity to solve this.

Consider being the driver in Car 2. From your perspective, car 1 is travelling towards you at 180 km/h

You will meet after:

$$t = \frac{d}{s} = \frac{100 \text{ km}}{180 \text{ km/h}} \approx 0.555 \ldots \text{h} = 33 \text{ min } 20 \text{ sec}$$

24 A river is flowing downstream at a rate of 1 km/h.
In still water Portia can swim at a rate of 2 km/h.
Portia dives into the river and swims downstream, then turns around and swims back to the starting point, taking 0.5 hours in total.
How far did she swim?

Downstream speed: 2 + 1 = 3 km/h

Upstream speed: 2 - 1 = 1 km/h

Let distance swum = d km

Time downstream + Time upstream = 0.5 h

Using $t = \frac{d}{s}$

$$\frac{d}{3} + \frac{d}{1} = 0.5$$

$$\frac{4d}{3} = 0.5$$

$$d = 0.375 \text{ km}$$

Total distance swum = 2d = 0.75 km = 750 m

25 Electric cars use on average 17 kWh per 100 km. Electricity costs $0.28 per kWh.
Petrol cars use on average 9 L per 100 km. Petrol costs $1.98 per litre.
The electric car costs on average $10 000 more than a petrol car.
How far do you have to drive the electric car until you start to save money?

Electric cost per 100 km: 17 × $0.28 = $4.76

Petrol cost per 100 km: 9 × $1.98 = $17.82

Savings per 100 km: $17.82 - $4.76 = $13.06

Distance needed: $10000 ÷ $13.06 per 100 km = 76,570 km

COMBINED RATES

FOUNDATION

1 Read the information on combined rates, then answer the questions.

$$\text{If rate 1} = \frac{\text{amount 1}}{\text{units of time}} \text{ and rate 2} = \frac{\text{amount 2}}{\text{units of time}}$$

$$\text{then rate 1} + \text{rate 2} = \text{combined rate}$$

$$\therefore \text{combined rate} = \frac{\text{amount 1}}{\text{units of time}} + \frac{\text{amount 2}}{\text{units of time}}$$

Worked Solution	Explanation
It takes Sean 20 hours to paint a house, while it would take Marty 25 hours to paint the same house. How many hours will it take Sean and Marty to paint the house together, to the nearest hour?	The fraction addition was performed using the calculator. Explain why it is important we show the units.
$\text{Combined Rate} = \text{Sean} + \text{Marty}$ $= \frac{1 \text{ house}}{20 \text{ hours}} + \frac{1 \text{ house}}{25 \text{ hours}}$ $= \frac{9 \text{ houses}}{100 \text{ hours}}$ $= \frac{1 \text{ house}}{11.\dot{1} \text{ hours}}$	To interpret what the answer represents, is it houses/hour or hours/house? Explain why we changed the numerator to 1 instead of finding the unit rate.
	The unit rate finds houses/hour, not hours/house Explain how we can use the calculator to convert hours to minutes and seconds.
$\therefore 11.\dot{1}$ hours, or 11 hours 6 minutes 40 sec	The °′ ″ button converts between decimal and base 60.

2 It takes two movers 6 and 5 hours respectively to move a one-bedroom apartment.
How long will it take both of them to move the apartment together, to the nearest minute?

2 hours 44 minutes

3 A cold-water tap can fill a bathtub in half an hour. A hot water tap can fill a bathtub in one hour. Stephen takes a guess:

> Well, the average of 30 minutes and 60 minutes is 45. So, I reckon both taps would take 45 minutes.

Explain, without any calculations, why Stephen must be incorrect.

Stephen averaged the times instead of the rates. You add the rates of work, not average the times.

Intuitively, both taps must take less than 30 minutes.

Find how long it would take to fill the bathtub.

20 minutes

4 Three water hoses are used to fill a large swimming pool.

The first hose alone would take 200 hours.

The second hose takes 120 hours and the third hose 50 hours.

How long would it take to fill the pool?

30 hours

5 Michelle can complete a landscaping job in 6 days and Danielle can complete the job in 4 days.

Working together, in how many days could they complete the job?

2 days 9 hours 36 minutes

6 Mr Ding, Ms Fu and Mrs Falconer are organising the maths textbooks in the 8 bookshelves in the staffroom. Mr Ding by himself would take 25 hours to do all 8 bookshelves. Ms Fu can organise 2 bookshelves in 7 hours. Mrs Falconer can take 11 hours to organise 3 bookshelves. How long will it take if they work together? Answer to the nearest minute.

9 hours 6 minutes

RELATED RATES

- Usually when working with rates, we only use two units.
- Sometimes a question will give a *third* unit (or more).

- To solve these problems:
  1. Identify the three quantities in the question.
  2. Set up a **related rate equation**.

$$\frac{\text{Quantity 1}}{\text{Quantity 2}} = \frac{\text{Quantity 1}}{\text{Quantity 3}} \times \frac{\text{Quantity 3}}{\text{Quantity 2}}$$

  3. Substitute given information.
     The scenario will always give two of the rates.
  4. Convert units to be the same, and to the unit wanted by the question.
  5. Simplify and solve the equation.

- A common application is drip rates for medication, important for nurses.

Example Solve problems involving related rates.

Worked Example	Your Turn
A patient must have 1.2 L of fluid over 4 hours through an intravenous drip. There are 15 drops/mL. How many drops per minute are required? 1. volume, time, drops. 2. $\frac{\text{Drops}}{\text{Time}} = \frac{\text{Drops}}{\text{Volume}} \times \frac{\text{Volume}}{\text{Time}}$ 3. $\frac{\text{Drops}}{\text{Time}} = \frac{15 \text{ drops}}{1 \text{ mL}} \times \frac{1.2 \text{ L}}{4 \text{ hours}}$ 4. $\frac{\text{Drops}}{\text{Time}} = \frac{15 \text{ drops}}{1 \text{ mL}} \times \frac{1200 \text{ mL}}{240 \text{ min}}$ 5. $\frac{\text{Drops}}{\text{Time}} = \frac{75 \text{ drops}}{1 \text{ min}}$ 75 drops/min	What is the drop rate per minute if 1.3 L of fluid is to be delivered over 6 h with 12 drops/mL? $43.3\dot{3}$ drops/min

Worked Example	Your Turn
A drip is delivering 30 drops/min. There are 20 drops/mL and 900 mL of liquid to be delivered. How long will the drip take to deliver the required dose? 1. volume, time, drops. 2. $$\frac{\text{Time}}{\text{Volume}} = \frac{\text{Time}}{\text{Drops}} \times \frac{\text{Drops}}{\text{Volume}}$$ 3. $$\frac{x \text{ min}}{900 \text{ mL}} = \frac{1 \text{ min}}{30 \text{ drops}} \times \frac{20 \text{ drops}}{1 \text{ mL}}$$ $$\frac{x \text{ min}}{900 \text{ mL}} = \frac{20 \text{ min}}{30 \text{ mL}}$$ 5. $$x = 600 \text{ minutes}$$	Calculate how long it will take to deliver 800 mL at 20 drops/min if there are 25 drops/mL. 1000 minutes
Worked Example	**Your Turn**
A coffee shop buys coffee beans in 5 kg bags for $45 each. Each cup of coffee uses 12 g of beans. What is the cost of beans per cup of coffee? 1. cost, cup, beans. 2. $$\frac{\text{Cost}}{\text{Cup}} = \frac{\text{Cost}}{\text{Bean}} \times \frac{\text{Bean}}{\text{Cup}}$$ 3. $$\frac{\text{Cost}}{\text{Cup}} = \frac{\$45}{5000 \text{ g}} \times \frac{12 \text{ g}}{1 \text{ cup}}$$ 5. $$\frac{\text{cost}}{\text{cup}} = \$0.108/\text{cup}$$	A car uses 8.2 L of fuel per 100 km. Fuel costs $1.85 per litre. What is the cost per kilometre? $0.152/ km

2 Calculate the number of drops per minute required at a rate of 15 drops/mL in these situations.

a volume of 1.4 L over 7 h

50 drops/min

b volume of 1.5 L over 6 h

63 drops/min

c volume of 800 mL over 4 h

50 drops/min

d volume of 900 mL over 6 h

38 drops/min

3 Calculate the time it will take to deliver intravenous liquid at these rates.

a 600 mL delivered at 15 drops/min and there are 10 drops/mL

6 h 40 min

b 500 mL delivered at 10 drops/min and there are 12 drops/mL

10 h

c 1.2 L delivered at 25 drops/min and there are 20 drops/mL

16 h

d 1.8 L delivered at 20 drops/min and there are 15 drops/mL

22 h 30 min

4 What is the drip rate per minute for 850 mL of fluid over 5 h with a drip size giving 8 drops/mL?

23 drops/min

5 What is the drip rate per minute for 2.6 L of fluid over 6 h with a drip size giving 12 drops/mL? Answer to the nearest whole number of drops.

87 drops/min

6 A patient is to receive 800 mL of liquid through an intravenous drip delivering 25 drops/min. If there are 16 drops/mL, how long will it take to deliver the required dose?

8 h 32 min

7 A patient is to receive 1.5 L of fluid over 6 h through an intravenous drip. If there are 12 drops/mL, how many drops per minute are required to deliver the dose?

50 drops/min

8 A patient is to receive 600 mL of saline solution.
An intravenous drip delivers 30 drops/min and there are 12 drops/mL.
How long will it take to deliver the required dose?

4 h

9 What is the drip rate per minute for 1.25 L of fluid over 8 h, with a drip size giving 14 drops/mL?

36 drops/min

10 A patient is to receive 1.1 L of fluid over 9 h.
Find the drip rate in drops/min if there are 18 drops/mL.

37 drops/min

11 A patient needs 0.75 g of medicine per day. Pills contain 125 mg each and cost 75 c per pill.
What is the annual cost of medicine, in dollars?

$$\frac{\text{Cost}}{\text{Time}} = \frac{\text{Medicine}}{\text{Time}} \times \frac{\text{Pills}}{\text{Medicine}} \times \frac{\text{Cost}}{\text{Pills}}$$

$$\frac{\$x}{1 \text{ year}} = \frac{0.75 \text{ g}}{1 \text{ day}} \times \frac{1 \text{ pill}}{125 \text{ mg}} \times \frac{75 \text{ c}}{1 \text{ pill}}$$

$$\frac{\$x}{1 \text{ year}} = \frac{0.75 \text{ g}}{1 \div 365 \text{ years}} \times \frac{1 \text{ pill}}{125 \div 1000 \text{ g}} \times \frac{\$0.75}{1 \text{ pill}}$$

$$\therefore \$1642.50/\text{yr}$$

12 A factory produces 25 widgets in 4 minutes. Each widget requires 15 g of plastic.
Plastic costs $3.20 per kg. What is the cost per hour, in dollars?

$$\frac{\text{cost}}{\text{time}} = \frac{\text{widgets}}{\text{time}} \times \frac{\text{plastic}}{\text{widget}} \times \frac{\text{cost}}{\text{plastic}}$$

$$\frac{\$x}{1 \text{ hour}} = \frac{25 \text{ w}}{4 \text{ min}} \times \frac{15 \text{ g}}{1 \text{ w}} \times \frac{\$3.20}{1 \text{ kg}}$$

$$\frac{\$x}{1 \text{ hour}} = \frac{25 \text{ w}}{4 \div 60 \text{ hr}} \times \frac{15 \div 1000 \text{ kg}}{1 \text{ w}} \times \frac{\$3.20}{1 \text{ kg}}$$

$$x = \$18/\text{hr}$$

25 A large bag of flour weights 24 kg and costs \$21.50.
A sponge cake uses 150 g of flour per cake.

What **questions** do these calculations find out?

a	**b**	**c**	**d**
$\frac{24000}{150}$	$\frac{24}{21.50}$	$\frac{2150}{24}$	$\frac{21.50}{24000} \times 150$

a .....

Units are $\frac{\text{g}}{\text{g/cake}}$, so how many cakes can be made

b .....

Units are $\frac{\text{kg}}{\$21.50}$, so kilograms of flour per dollar

c .....

Units are $\frac{\text{cents}}{\text{kg}}$, so cents per kilogram

d .....

Units are $\frac{\$}{\text{g}} \times \frac{\text{g}}{\text{cake}}$ so dollars per cake

13 Mr Ding makes maths booklets that are on average 25 sheets of paper.
Each ream of printing paper is 500 sheets.
A single sustainably farmed tree can produce approximately 10 000 sheets of paper.
Each ream of paper is \$9.
He teaches 4 classes and prints a new booklet for each student once every two weeks.
Each class has on average 20 students.
How many trees does he use in a 38-week academic year, and how much do these booklets cost in paper?

First find how many sheets used per year:

$$\frac{20 \text{ students}}{\text{class}} \times 4 \text{ classes} \times \frac{25 \text{ sheets}}{\text{book}} \times \frac{0.5 \text{ books}}{\text{student} \cdot \text{week}} \times 38 \text{ weeks} = 38\,000 \text{ sheets}$$

Therefore, trees used per year:

$$38\,000 \text{ sheets} \div \frac{10\,000 \text{ sheets}}{\text{tree}} = 3.8 \text{ trees}$$

Cost per year:

$$38\,000 \text{ sheets} \div \frac{500 \text{ sheets}}{\text{ream}} \times \frac{\$9}{\text{ream}} = \$684$$

CHECK YOUR UNDERSTANDING

Applications of rates

1 6 pencils cost $1.50. How much do 3 pencils cost?

a 65 c **b** 0.75 c **c** 75c **d** 25c

2 6 pizzas cost $42. How much do 2 pizzas cost?

a $7 **b** $14 **c** $84 **d** $1.40

3 90 g of brown sugar is needed to make 4 cakes. How many grams are needed for 6 people?

a 540 **b** 90 **c** 135 **d** 100

4 4 cupcakes cost $2.80. How much do 7 cupcakes cost?

a $19.60 **b** $4.90 **c** $11.20 **d** $1.60

5 5 pens cost $7.50. How much do 9 pens cost?

a $19.60 **b** $4.90 **c** $11.20 **d** $1.60

6 It takes $\frac{3}{4}$ of an hour to lie 24 carpet tiles. How many carpet tiles can be laid in 6 hours?

a 32 **b** 144 **c** 108 **d** 192

Calculate best buys

1 Which is the best buy?

a	$0.43 for 100g	**b**	$0.45 for 100g	**c**	$0.50 for 100g	**d**	$0.41 for 100g

2 Which is the best buy?

a	520g for $1	**b**	499g for $1	**c**	1.2kg for $1	**d**	952g for $1

3 Sugar is sold in four different sized packets.
Which is the best buy?

a	100 g for $0.40	**b**	500 g for $1.65	**c**	1 kg for $3.50	**d**	2 kg for $6.90

4 A refreshment stand sells iced tea.
Which is the best buy?

a	150 mL for $1.50	**b**	750 mL for $6.10	**c**	375 mL for $3.00	**d**	200 mL for $1.80

5 There are four sizes of sunscreen bottles at a pharmacy.
Which is the best buy?

a	$10.90 for 1200 mL	**b**	$5.40 for 600 mL	**c**	$4.75 for 500 mL	**d**	$7.40 for 750 mL