

Student Name

Mathematics Stage 5 Paths

VARIATION & RATES OF CHANGE A

Book 3 Solve problems involving direct and inverse variation and examine the relationship between graphs and equations corresponding to proportionality

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-RAT-P-01 identifies and solves problems involving direct and inverse variation and their graphical representations

Solve problems involving direct and inverse variation and examine the relationship between graphs and equations corresponding to proportionality

- Solve problems involving direct or inverse variation using an equation
- Use linear conversion graphs to convert from one unit to another
- Graph equations representing direct variation, with or without digital tools

REVIEW OF PRIOR KNOWLEDGE

SOLVING DIRECT VARIATION PROBLEMS USING EQUATIONS

- $y \propto x$ indicates a direct variation relationship between the two variables.

Direct Variation using Equations

1. Write $y = kx$
2. Substitute given information.
3. Solve the equation to find the constant of variation k .
4. Rewrite $y = kx$ with the k you calculated.
5. Use your equation to find the required information.

Example Find the equation of a direct variation relationship.

Given that $y \propto x$ and $y = 4.5$ when $x = 13.5$:

a Find y when $x = 42$

b Find x when $y = 7$

1. $y = kx$

2. $4.5 = k(13.5)$

3. $\frac{4.5}{13.5} = k$

$$k = \frac{1}{3}$$

4. $y = \frac{1}{3}x$

5. Find y when $x = 42$:

$$y = \frac{1}{3}(42)$$

$$= 14$$

Find x when $y = 7$:

$$7 = \frac{1}{3}x$$

$$21 = x$$

Given that $y \propto x$ and $y = 42$ when $x = 5$, find:

a y when $x = 7$

b x when $y = 51$

a) 58.5 b) 6.07

Given that $a \propto b$ and $a = 9.6$ when $b = 1.6$, find:

a a when $b = 21$

b b when $a = 11$

a) 126 b) $\frac{11}{6}$

- 1 $y = kx$ and $y = 24$ when $x = 8$. Find:
- a The value of k and write the equation.
 - b y when $x = 4$
 - c x when $y = 15$

a) $y = 3x$ b) $y = 12$ c) $x = 5$

- 2 y varies directly with x and $y = 20$ when $x = 10$. Find:
- a y when $x = 8$
 - b x when $y = 12$

a) $y = 16$ b) $x = 6$

- 3 a varies directly with b and $a = 84$ when $b = 56$. Find:
- a b when $a = 22$
 - b a when $b = 12$

a) $b = 14\frac{2}{3}$ b) $a = 18$

- 4 In the tables below $y \propto x$. Write the equation and find the values of a, b, c .

a

x	2	3	5	b
y	36	54	a	20

Equation:

b

x	4	12	1	b
y	2	6	a	24

Equation:

$y = 18x, a = 90, b = 11\frac{1}{11}$

$y = 0.5x, a = 0.5, b = 48$

c

x	11.1	-12	34.2	c
y	3.7	a	b	17.1

Equation:

d

x	a	5.1	-7.8	c
y	14.4	22.95	b	31.05

Equation:

$y = \frac{1}{3}x, a = -4, b = 11.4, c = 51.3$

$y = 4.5x, a = 3.2, b = -35.1, c = 6.9$

MODELLING DIRECT VARIATION RELATIONSHIPS

- The goal is to find k , then write the equation to solve for the unknowns.

Direct Variation using Equations
1. Write $y = kx$ 2. Substitute given information. 3. Solve the equation to find the constant of variation k. 4. Rewrite $y = kx$ with the k you calculated. 5. Use your equation to find the required information.

Example Model a direct variation relationship.

Garden mulch is sold in bulk, with the cost C proportional the volume V in cubic metres. The shop says 7.5 m³ costs \$270. How much does 12 m³ cost?

1. $C = kV$

2. $270 = k \times 7.5$

3. $\frac{270}{7.5} = k$

$k = 36$

4. $C = 36V$

5. Finding C when $V = 12$:

$C = 36 \times 12$

$C = \$432$

12 m³ would cost \$432.

What does $k = 36$ represent in this scenario? (hint: consider the units)

.....

The petrol P used by a car at a constant speed is directly proportional to distance travelled d . A car uses 6 L of petrol to travel 50 km.

How much petrol is needed to travel 180 km?

Finding k :

$y = kx$

$P = kd$

Write the equation:

Find P when $d = 180$ km:

What does k represent?

21.6 L

The amount of paint P needed to paint a wall is directly proportional to the area covered A . A 4-litre can of paint will cover 12 m².

Find the number of litres of paint needed to paint a 63 m² wall.

Finding k :

Write the equation:

Find P when $A = 63$ m²:

What does k represent?

21 L

- 1 Oscar's pay p is directly proportional to the number of hours h he works. For a 9-hour day he receives \$193.50.

a Calculate the constant of variation k . What does it represent in this scenario?

$$k = \$21.5/\text{hr}$$

b Write an equation connecting p and h .

$$p = 21.5h$$

c What is Oscar's pay if he works for 11 hours?

$$\$236.50$$

d What is Oscar's pay if he works for 6.5 hours?

$$\$139.75$$

e How many hours does Oscar have to work to earn \$365.50?

$$17 \text{ hours}$$

- 2 The cost of a photocopier (C , in dollars) varies directly with its speed (s , in pages per minute). A photocopier with a speed of 50 pages per minute costs \$700.

a What is the constant of variation k ? What does it represent in this scenario?

$$k = \$14/\text{min}$$

b Write an equation connecting C and S .

$$C = 14s$$

c How much does a photocopier with a speed of 60 pages per minute cost?

$$\$840$$

- 3 The amount of paint P , in litres, used to paint a building is directly proportional to the area A , in square metres. A certain building requires 48L to cover an area of 576 m².

a Find the constant of variation and write the equation relating P and A .

$$k = \frac{1}{12}, \quad P = \frac{1}{12}A$$

b A larger building has an area of 668 m² to be painted. How many litres of paint will this require? Round to 2 d.p.

$$55.67 \text{ litres}$$

- 4** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 6 L of petrol to travel 56 km. How much petrol will be used to travel 140 km at the same speed?

15 litres

- 5** The resistance, R ohms, to the flow of electricity in a wire varies directly with the length, l cm, of the wire and, when $l = 20$ cm, $R = 0.08$ Ohms.
- a** Find R when $l = 85$ cm.
- b** Find l when $R = 6$ Ohms.

a) 0.034 ohms b) 1500 cm

- 6** Christopher runs 120 m in 15 s.
- a** Write an equation relating distance d and time t .

 $d = 8t$

- b** What does the constant of variation represent in this scenario?
Include units in your answer.

8 metres per second (the speed)

- c** Use your equation to show that Christopher runs 4800 metres in 10 minutes.

10 minutes = 600 seconds

 $d = 8 \times 600 = 4800$ m

- d** Considering your answer to part **b**, what is a limitation of the linear model?

His speed will not stay constant over 10 minutes as he will slow down when he gets tired; The linear model is not valid for long durations..

7 2020 HSC Standard 2 Band 5

The weight of a bundle of A4 paper (W kg) varies directly with the number of sheets (N) of A4 paper that the bundle contains.

This relationship is modelled by the formula $W = kn$, where k is a constant.

The weight of a bundle containing 500 sheets of A4 paper is 2.5 kilograms.

Show that k is 0.005.

A bundle of A4 paper has a weight of 1.2 kilograms.

Calculate the number of sheets of A4 paper in the bundle.

240 sheets

8 2014 HSC Standard 2 Band 4

The weight of an object on the moon varies directly with its weight on Earth.

An astronaut who weighs 84 kg on Earth weighs only 14 kg on the moon.

A lunar landing craft weighs 2449 kg when on the moon.

Calculate the weight of this landing craft when on Earth.

14 694 kg

9 HSC Standard 2 Sample Question Band 5

The weight of a steel beam, w , varies directly with its length, l .

A 1200 mm steel beam weighs 144 kg.

Calculate the weight of a 750 mm steel beam.

90 kg

MODELLING INVERSE VARIATION USING EQUATIONS

Inverse Variation using Algebra

1. Write $y = \frac{k}{x}$
2. Substitute given information.
3. Solve the equation to find the constant of variation k .
4. Rewrite $y = \frac{k}{x}$ with the k you calculated.
5. Use your equation to find the required information.

Example Find the equation of an inverse variation relationship.

$y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$.

a Find y when $x = 42$

b Find x when $y = 7$

1. $y = \frac{k}{x}$

2. $4 = \frac{k}{3}$

3. $12 = k$

4. $y = \frac{12}{x}$

5. Find y when $x = 42$:

$$y = \frac{12}{42} = \frac{2}{7}$$

Find x when $y = 7$:

$$7 = \frac{12}{x}$$

$$7x = 12$$

$$x = \frac{12}{7}$$

How do you know when to use $y = kx$ and when to use $y = \frac{k}{x}$?

Given $y \propto \frac{1}{x}$ and $y = 18$ when $x = 2$. Find:

a y when $x = 9$

b x when $y = 2$

a) 4 b) 18

Given $y \propto \frac{1}{x}$ and $y = 3$ when $x = 17$. Find:

a y when $x = 102$

b x when $y = 6$

Example Model an inverse variation relationship.

The cost per person of hiring a bus C is inversely proportional to the number of passengers n . When there are 60 passengers, the cost per person is \$8.

a Find the cost per person when 45 people go on the bus.

b How many people need to take the bus for the cost to be \$6?

1. $C = \frac{k}{N}$

2. $8 = \frac{k}{60}$

3. $480 = k$

4. $C = \frac{480}{N}$

5. Find C when $N = 45$:

$$C = \frac{480}{45}$$

$$= \$10.67$$

Find N when $C = 6$:

$$6 = \frac{480}{N}$$

$$6N = 480$$

$$N = \frac{480}{6} = 80 \text{ people}$$

What does k represent in this scenario? (Hint: what does $60 \times \$8$ find?)

The number of painters N is inversely proportional to the amount of time to paint a fence t . 3 painters can paint a fence in 12 hours. Find:

a Find how many painters are needed to paint the fence in 9 hours.

b How long it would take 2 people to paint the fence.

a) 4 painters b) 18 hrs

- 1 y is inversely proportional to x and $y = 42$ when $x = 6$. Find:
- a Write an equation $y = \frac{k}{x}$ and calculate the constant of variation.

$$y = \frac{252}{x}$$

- b What is y when $x = 3$?

84

- c What is x when $y = 21$?

12

- 2 It is known that y varies inversely with x . If $y = 6$ then $x = 8$.

- a Write an equation $y = \frac{k}{x}$ and calculate the constant of variation.

$$y = \frac{48}{x}$$

- b Find the value of y if $x = 24$

2

- c Find the value of x if $y = 4$

12

- 3 Time taken in hours (t) for a trip varies inversely with average speed (s). It takes 5 hours to complete a trip travelling at 68 km/h.

- a Find the constant of variation.

340

- b Write an inverse variation equation to describe this situation.

$$t = \frac{340}{s}$$

- c What is the time taken for a trip with an average speed of 34 km/h?

10 hours

- d What speed is required to complete the trip in 4 hours?

85 km/h

4 The cost per person (c), in dollars, to attend a formal dance varies inversely with the number of people (n) attending. When 90 people attend, the cost is \$60 per person.

a If only 80 people attend the formal dance, how much will each person have to pay?

\$67.50

b The organising committee has suggested a cost per ticket of \$40.
How many people need to attend the formal dance for the tickets to be \$40?

135

5 The number of people (N) who attend a show varies inversely with the amount of floor space (in cm^2) allowed per person (A). A venue can hold 480 people if each person is allowed 2000 cm^2 .

a How many people could the venue hold if each person was allowed 2400 cm^2 ?

400

b What is the space allowed per person to allow 500 to attend the show?

1920

6 4 workers can resurface a road in 6 hours.

a How long would it take to resurface the same road if 6 people worked together?

4 hours

b How many workers are needed if the road is to be resurfaced in 3 hours?

8 workers

- 7** Squeaky Clean Windows can clean all windows of a building in 5 days when it hires 8 people. This year the client wants the job completed in 4 days. How many cleaners are required?

10

- 8** Connor uses a microwave oven to reheat a meal. The time taken (in minutes) for heating is inversely proportional to the power setting. The meal takes 3 minutes at a power setting of 60%. How long would it take at a power setting of 80%?

2 min 15 sec

- 9** 4 people take 1 hour and 20 minutes to water a football pitch. How long would it take to water the same pitch if 5 people worked together? Give your answer in hours and minutes.

1 hr 4 min

- 10** It takes one day for 2 workers to harvest a crop from a field with an area of 40 m^2 . Mario's farm has an area of 320 m^2 . He hires 8 workers and pays each of them \$180 per day. How much will it cost him to harvest his crop?

\$2880

11 2021 HSC Standard 2 Band 5

The time taken to clean a warehouse varies inversely with the number of cleaners employed.

It takes 8 cleaners 60 hours to clean a warehouse.

Working at the same rate, how many hours would it take 10 cleaners to clean the same warehouse.

48 hours

12 HSC Standard 2 Sample Question Band 5

The number of trees that can be planted along the fence line of a paddock varies inversely with the distance between each tree. There will be 108 trees if the distance between them is 5 metres.

How many trees can be planted if the distance between them is 6 metres?

90

What is the distance between the trees if 120 trees are planted.

4.5 m