Student Name

Mathematics Stage 5 Core

PROBABILITY A

Book 2 Solve problems for multistage chance experiments

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-PRO-C-01 solves problems involving probabilities in multistage chance experiments and simulations

Solve problems for multistage chance experiments

- Record all possible outcomes for multistage chance experiments
- Determine the probabilities of outcomes for multistage independent events using $P(A \text{ and } B) = P(A) \times P(B)$, where necessary
- Determine the probabilities of outcomes for multistage dependent events
- Associate complementary events with probabilities in multistage chance experiments

LIST OUTCOMES USING TABLES

Fundamental Counting Principle **For Independent Events**
If there are $\boldsymbol{p}$ ways to do one thing, and $\boldsymbol{q}$ ways to do another thing, then there are $\boldsymbol{p} \times \boldsymbol{q}$ ways to do both things.

- Use the **fundamental counting principle** to check you've found all the possible outcomes.
- A two-way table helps find all possible outcomes in a two-step experiment.

Example Record all possible outcomes for a two-step experiment.

Tim has three pairs of trousers (blue, brown and grey) and two shirts (one long-sleeved and one short - sleeved) in his wardrobe.
Find all the possible combinations of trousers and shirts he can wear by using a table.

		Trousers		
		Blue	Brown	Grey
Shirts	Long	(L, Bl)	(L, Br)	(L, Gr)
	Short	(S, Bl)	(S, Br)	(S, Gr)

How many outcomes are there? ……

Does this match the fundamental counting principle? ……

The probability of randomly picking a short-sleeved shirt and grey trousers is: ……

A two-step experiment involves flipping a coin then rolling a die.
Fill in the table.

	1	2	3	4	5	6
H						
T						

What is the total number of outcomes?

12

What is $P(\text{tail})$?

0.5

What is $P(\text{head}, 4)$?

$\frac{1}{12}$

What is $P(\text{head and odd number})$?

$\frac{1}{4}$

Example Record all possible outcomes for a two-step experiment

Two letters are taken from the word TREE without replacement.

List the outcomes in a table.

		1st			
		T	R	E	E
2nd	T	×	(R, T)	(E, T)	(E, T)
	R	(T, R)	×	(E, R)	(E, R)
	E	(T, E)	(R, E)	×	(E, E)
	E	(T, E)	(R, E)	(E, E)	×

Why does the table have X in some cells?

…… …… *means it is not possible to get an outcome with two of the same letter.*

Find the probability that both letters chosen are E.

……

Fill in these tables to show all the outcomes then count the number of outcomes.

a Picking cards numbered 1, 2, 3 out of a bag with replacement.

		1st		
		1	2	3
2nd	1	(1, 1)	(2, 1)	
	2			
	3			

Total outcomes: ……

9

b Rolling a die and flipping a coin.

		Die					
		1	2	3	4	5	6
Coin	H	(1, H)					
	T			(3, T)			

Total outcomes: ……

12

c Picking cards numbered A, B, C out of a bag without replacement.

		1st		
		A	B	C
2nd	A	×	(B, A)	(C, A)
	B		×	
	C			×

Total outcomes: ……

6

d Picking marbles out of a bag with 1 black and 2 white marbles without replacement.

		1st		
		•	∘	∘
2nd	•	×	(∘, •)	
	∘		×	(∘, ∘)
	∘			×

Total outcomes: ……

6

Key ideas

1. To list all possible outcomes of a two-step experiment, we can draw up a ……-…… table.

1 Complete these tables.

a

	H	T
H	H,H	H,T
T	T,H	

T, T

b

	1	2	3	4	5
B	B,1		B,3	B,4	B,5
R	R,1	R,2	R,3	R,4	R,5

B, 2

a

	1	2	3	4	5	6	7
1	1,1	1,2	1,3	1,4	1,5	1,6	1,7
2	2,1	2,2		2,4	2,5	2,6	2,7

2, 3

2 This is a two-way table for rolling two dice and adding the results.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a How many possibilities are there in total?

$6 \times 6 = 36$

b How many of these add up to 8?

5

c What it the probability of scoring an 8?

$\frac{5}{36}$

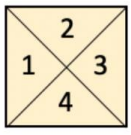
d What is the most likely result?

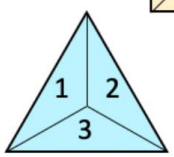
Getting a sum of 7

e What is the probability of this result?

$\frac{6}{36} = \frac{1}{6}$

2 Complete this two-way table for rolling these two spinners and then adding their results.
What is the probability of scoring





	1	2	3	4
1				
2				
3				

a 6?

$\frac{2}{12} = \frac{1}{6}$

b 7?

$\frac{1}{12}$

c 6 or 7?

$\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$

3 These two tables list the outcomes for the selection of two letters from the word MAT.

Table A

	M	A	T
M	(M, M)	(A, M)	(T, M)
A	(M, A)	(A, A)	(T, A)
T	(M, T)	(A, T)	(T, T)

Table B

	M	A	T
M	×	(A, M)	(T, M)
A	(M, A)	×	(T, A)
T	(M, T)	(A, T)	×

a Which table shows selection where replacement is allowed (with replacement)?

Table A

b Which table shows selection where replacement is not allowed (without replacement)?

Table B

c What is the probability of choosing the outcome (T, M) from:

i Table A

$\frac{1}{9}$

ii Table B

$\frac{1}{6}$

4 Two four-sided dice (numbered 1, 2, 3, 4) are rolled.

		1st			
		1	2	3	4
2nd	1	(1, 1)	(2, 1)		
	2				
	3				
	4				

a Complete the table to list the outcomes.

b Find the probability of obtaining (2, 3).

c Find P(double). A double is an outcome with two of the same number.

5 A six-sided die is rolled twice.

First die \ Second die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)			
2	(2, 1)					
3	(3, 1)					
4						
5						
6						

a How many possible outcomes are there?

36

b What is the probability of rolling a double?

$\frac{1}{6}$

c What is the probability of not rolling a double?

$\frac{5}{6}$

d What is the probability of rolling a 2 and a 3, in any order?

$\frac{1}{18}$

e What is the probability of rolling two odd numbers?

$\frac{1}{4}$

f What is the probability of rolling a sum of 5?

$\frac{1}{4}$

6 A coin is tossed and a six-sided die is rolled. List all the outcomes using a table.

Find the probability of:

a a head and a 6

$\frac{1}{12}$

b A tail and a 3

$\frac{1}{12}$

c A head and an even number

$\frac{1}{4}$

d A tail and a number less than 3

$\frac{1}{6}$

7 Jill guesses the answers to two multiple-choice questions with options A, B, C, D or E. Fill in the table.
The answers were A for the first question and D for the second.
What is the probability that Jill will get:

		Guess 1				
		A	B	C	D	E
Guess 2	A	(A, A)	(B, A)			
	B					
	C					
	D					
	E					

a Both answers correct

$\frac{1}{25}$

b Only one answer correct

$\frac{8}{25}$

8 Two fair dice are rolled. The numbers of the dice are added together.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2						
3						
4						
5						
6						

a What is the probability of rolling a score of 8?

$\frac{5}{36}$

b What is the probability of rolling a score greater than or equal to 10?

$\frac{1}{6}$

c Which is more likely, rolling a score of 7 or rolling a score less than 4?

$P(7) = \frac{6}{36}, P(< 4) = \frac{3}{36}$

Rolling 7 is more likely

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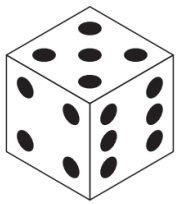
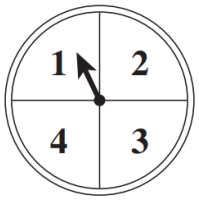
A game involves throwing a die and spinning a spinner.

The sum of the two numbers obtained is the score.

The table of scores below is partially completed.

What is the probability of getting a score 7 or more?





		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

$\frac{10}{24} = \frac{5}{12}$

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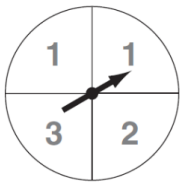
The two spinners shown are used in a game.

Each arrow is spun once. The score is the total of the two numbers shown by the arrows.
A table is drawn to show all scores that can be obtained.

Spinner A



Spinner B

		Spinner B			
		1	**1**	**2**	**3**
Spinner A	**1**	2	2	3	4
	1	2	2	3	4
	3	4	4	X	6

a What is the value of X in the table?

5

b What is the probability of obtaining a score less than 4?

$\frac{6}{12} = \frac{1}{2}$

c On Spinner *B*, a 2 is obtained. What is the probability of obtaining a score of 3?

$\frac{2}{3}$

LIST OUTCOMES USING TREE DIAGRAMS

MULTI-STAGE EXPERIMENTS

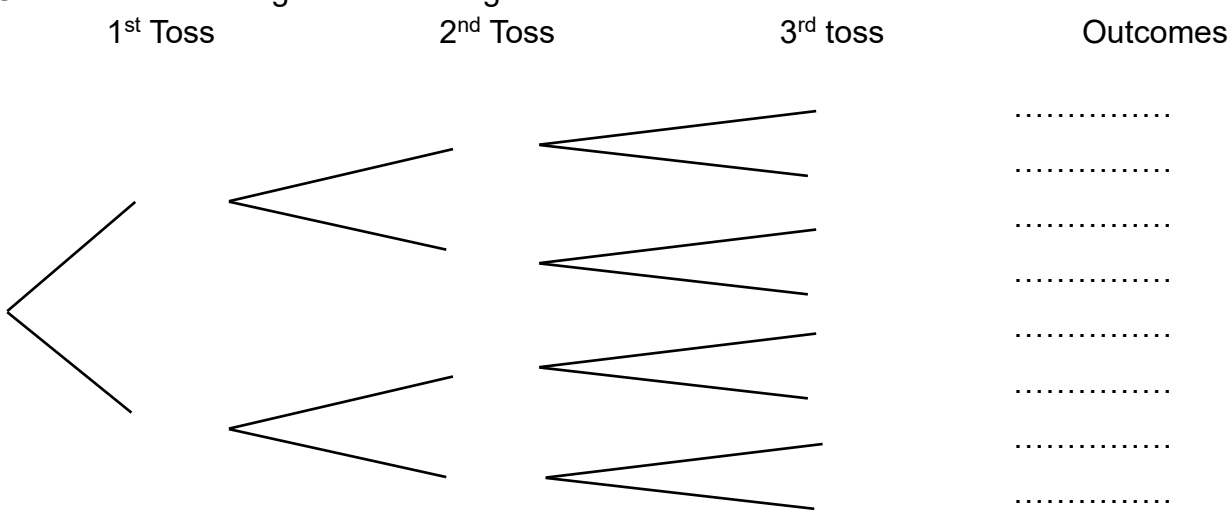
- A **multi-stage experiment** consists of two or more steps.
- The steps can be independent or dependent events.

TREE DIAGRAMS

- Two-way tables are unsuitable for multi-stage experiments.
- **Tree diagrams** are more useful for multi-stage experiments.
- It shows each event as a branch of a tree.
- All possible outcomes are listed at the tree's end.

Investigation Tossing a coin three times.

Construct a tree diagram for tossing a fair coin three times.

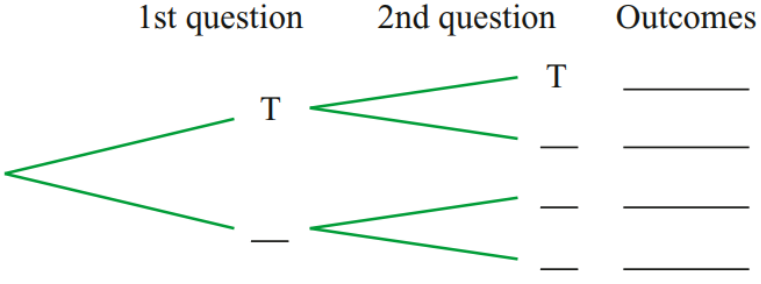


There are ..... possible outcomes, which agrees with the fundamental counting principle, which says there are ..... × ..... × ..... = ... outcomes.

The probability of tossing two heads and one tail, **in that order**, is: .....

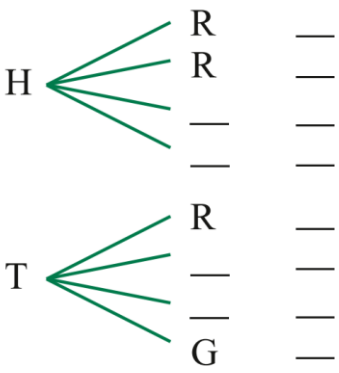
The probability of tossing two heads and one tail, **in any order**, is: .....

a Complete this tree diagram to find all the possible ways you can answer two questions on a True or False test.



b A bag contains 3 red discs and 1 green disc. A coin is tossed and a disc is chosen at random from the bag. Complete the tree diagram and answer the following questions.

Find the probability of getting tails and a red disc.



$\frac{3}{8}$

Find the probability of getting heads and a green disc.

$\frac{1}{8}$

c A bag contains 2 blue, 3 red and 2 white counters. A coin is tossed and a counter is chosen at random from the bag. Complete the tree diagram and answer the following questions.

Find the probability of getting tails and a red counter.

$\frac{3}{14}$

Find the probability of getting tails and a blue counter.

$\frac{1}{7}$

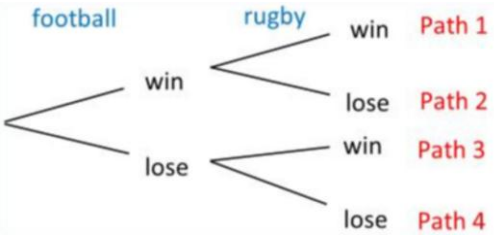
Key ideas

1. A …… is not a good way to represent a multi-stage experiment, because it can be too large or complex to show all the possible outcomes
2. A …… diagram is a better way to represent a multi-stage experiment, it shows each possible event as a …… of a tree

Check your understanding

1 The tree diagram below shows the different outcomes when Sophie plays a game of football and a game of rugby.

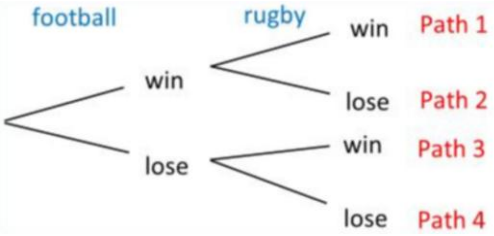
Which path(s) show the probability of her winning both games?



a Path 1 **b** Path 3 **c** Paths 1 and 3 **d** Paths 1, 2, 3

2 The tree diagram below shows the different outcomes when Sophie plays a game of football and a game of rugby.

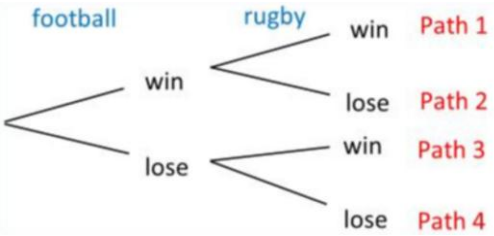
Which path(s) show the probability of her winning at least one game?



a Path 1 **b** Path 3 **c** Paths 1 and 3 **d** Paths 1, 2, 3

3 The tree diagram below shows the different outcomes when Sophie plays a game of football and a game of rugby.

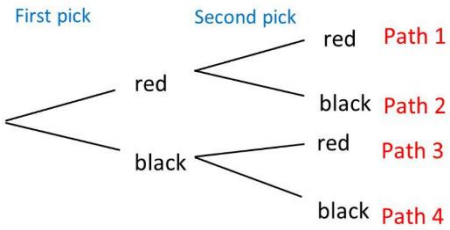
Which path(s) show the probability of her winning exactly one game?



a Paths 2 and 3 **b** Path 3 **c** Paths 1 and 3 **d** Paths 1, 2, 3

4 The tree diagram below shows the different outcomes when picking two counters out of a bag of red and black counters.

Which path(s) show the probability picking two counters of the same colour?

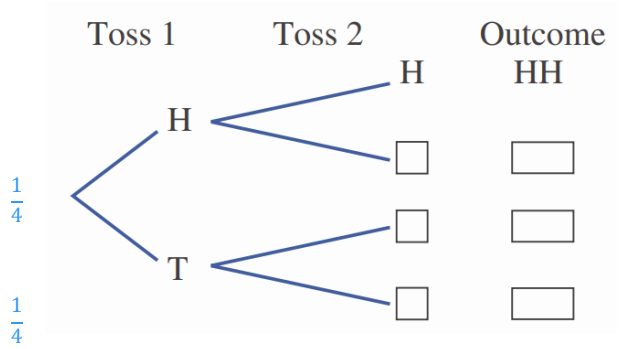


a Path 1 **b** Paths 1 and 4 **c** Paths 1 and 3 **d** Paths 2 and 3

1 Two coins are tossed and the results recorded.

a List all outcomes by completing the diagram.

b Find the probability of the following results:



i Two tails

ii Head then a tail

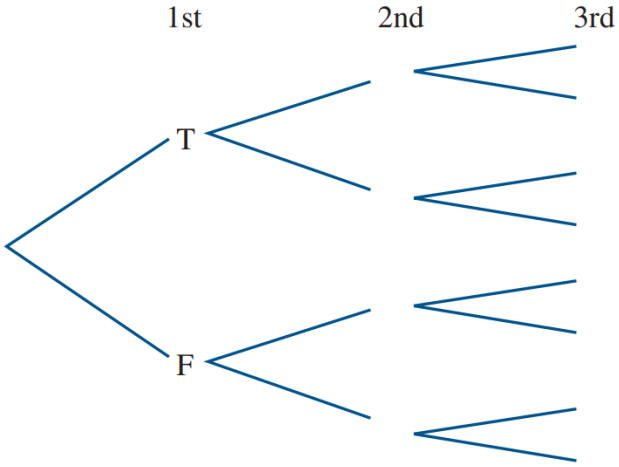
iii One head and one tail

1/2

iv Explain why the answer to parts **ii** and **iii** are different.

ii) specifies an order and iii) does not

2 There are three questions in a True or False test.



a Fill in the partially drawn tree diagram and list the outcomes to the right of the branches.

b How many outcomes are there?

8

c Find the probability of the following random selections:

i All of the selections are false.

1/8

ii Two true selections and one false selection.

3/8

iii At least one of the selections is false.

7/8

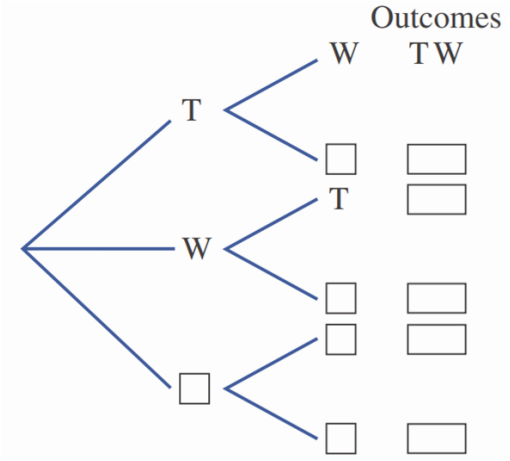
3 Explain why this is incorrect:

"When two coins are tossed together, there are three events: two heads, two tails and one of each. Hence the probability of getting one of each is $\frac{1}{3}$"

There are two ways to get one of each: HT and TH, so it is $\frac{2}{4} = \frac{1}{2}$

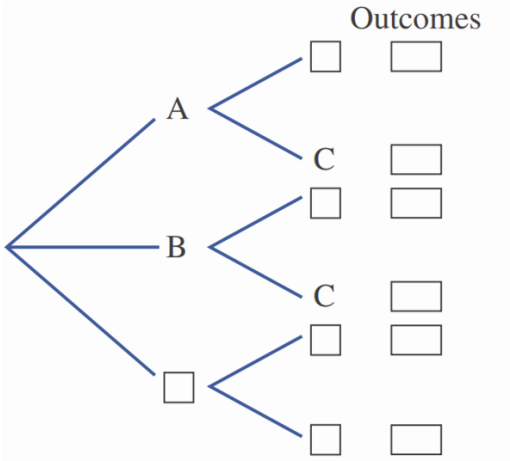
4 Complete each tree diagram for the experiments *without replacement*.

a Selecting two letters from the word TWO



TW, TO, WT, WO, OT, OW

b Selecting two people from a group of three (A, B and C).



AB, AC, BA, BC, CA, CB

5 A drawer contains 2 red socks (R), 1 blue sock (B) and 1 yellow sock (Y).
Two socks are selected at random without replacement. Complete the tree diagram.

a Find the probability of obtaining:

i a red sock then a blue sock.

$\frac{2}{12} = \frac{1}{6}$

ii a red sock and a blue sock, in any order.

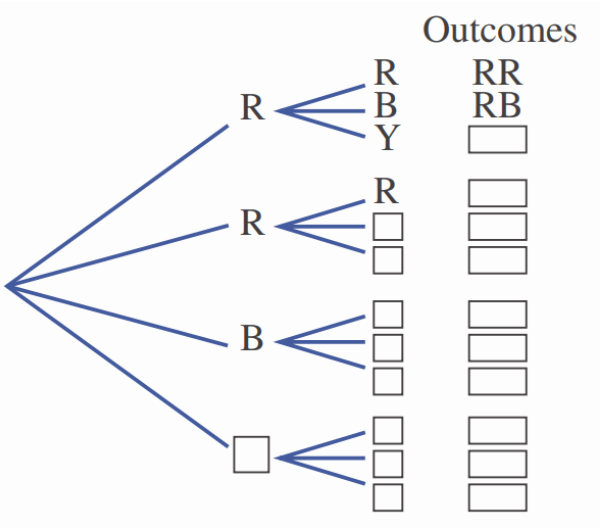
$\frac{4}{12} = \frac{1}{3}$

iii two red socks

$\frac{2}{12} = \frac{1}{6}$

iv any pair of socks of the same colour

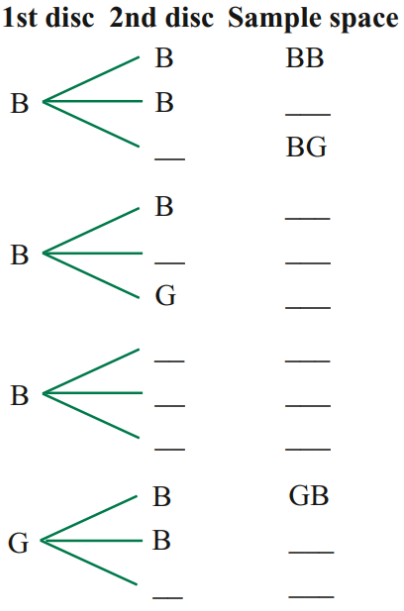
$\frac{1}{6}$



6 A bag contains 3 black discs and 1 green disc.
Two discs are selected at random from the bag.
The first disc is not replaced before the second disc is selected.

a Complete the tree diagram.

b Calculate the probability that the discs are different colours.



$\frac{1}{2}$

7 A bag contains one red, one blue and one white marble.
One marble is chosen at random from the bag, and then **replaced** into the bag.
A second marble is chosen.

a Draw a tree diagram of the situation.

b List the outcomes to the right of the branches.

c Find the probability that:

i	both marbles are red $\frac{1}{9}$	**ii**	both marbles are the same colour $\frac{1}{3}$	**iii**	both marbles are different colours $\frac{2}{3}$	**iv**	the first marble is red $\frac{1}{3}$
v	there is one blue and one white marble $\frac{2}{9}$	**vi**	exactly one marble is red $\frac{4}{9}$	**vii**	at least one marble is red $\frac{5}{9}$	**viii**	no marbles are blue $\frac{4}{9}$

8 A bag contains one red, one blue and one white marble.
One marble is chosen at random from the bag, and **NOT replaced** into the bag.
A second marble is chosen.

a Draw a tree diagram of the situation.

b List the outcomes to the right of the branches.

c Find the probability that:

i	both marbles are red 0	**ii**	both marbles are the same colour 0	**iii**	both marbles are different colours 1	**iv**	the first marble is red $\frac{1}{3}$
v	there is one blue and one white marble $\frac{2}{6}=\frac{1}{3}$	**vi**	exactly one marble is red $\frac{4}{6}=\frac{2}{3}$	**vii**	at least one marble is red $\frac{4}{6}=\frac{2}{3}$	**viii**	no marbles are blue $\frac{2}{6}=\frac{1}{3}$

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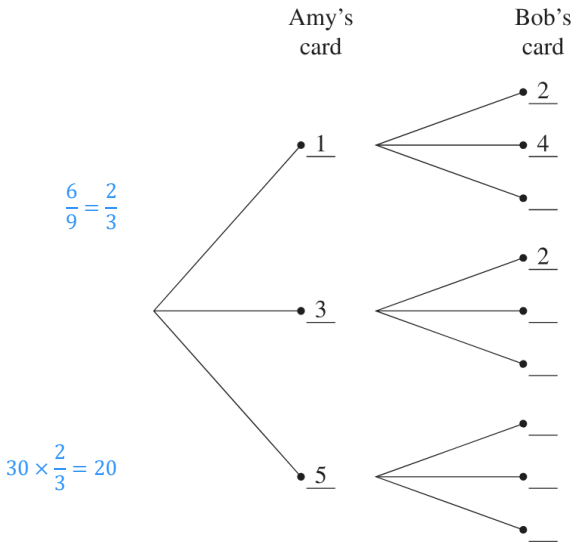
The numbers 1, 2, 3, 4, 5 and 6 are each written on separate cards.

Amy has cards 1, 3 and 5 and Bob has cards 2, 4 and 6.

They play a game in which each person randomly chooses one of their own cards and compares it with the other person's card. The person with the higher card wins.

a Complete the tree diagram and find the probability that Bob wins.



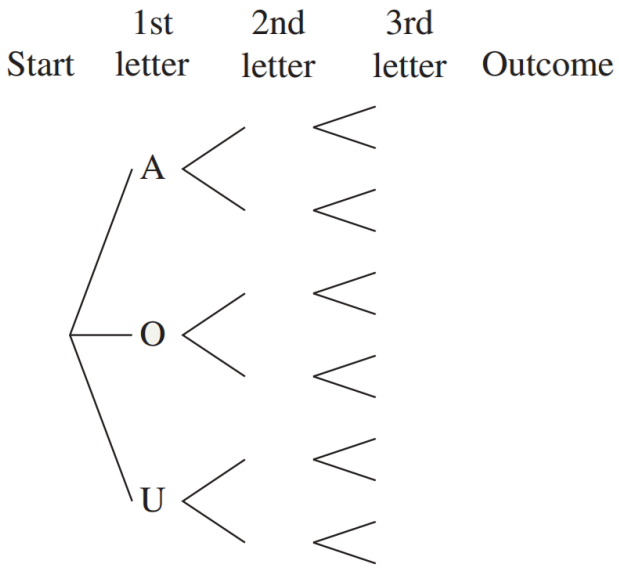
$\frac{6}{9} = \frac{2}{3}$

b Suppose Amy and Bob play this game 30 times.
How many times would Bob be expected to win?

$30 \times \frac{2}{3} = 20$

10 A three-letter word is chosen in the following way:
The first and last letters are chosen from the three vowels 'A', 'O' and 'U', with no repetition, and the middle letter is chosen from 'L' and 'M'.

Fill in the tree diagram,



a Find $P(\text{alphabetical order})$

$\frac{1}{2}$

b Find $P(\text{no 'O"})$

$\frac{1}{3}$

COMPLEMENTARY EVENTS IN MULTISTAGE PROBABILITY

AT LEAST ONE

- The complement of "**at least one**" is "**none**"

Example Identify the complement of “at least one”.

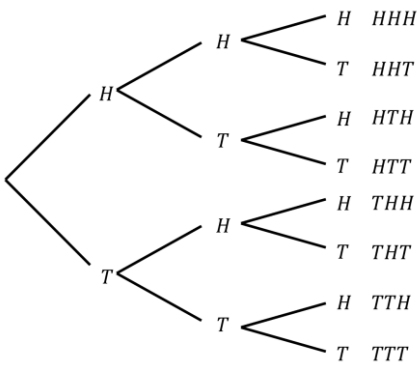
A coin is flipped four times. What is the complementary event to “at least one head”?	You roll a fair six-sided die three times. What is the complementary event to "at least one roll is a 6"?
A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles. Three marbles are drawn. What is the complementary event to "at least one marble drawn is red"?	You flip a fair coin five times. What is the complementary event to "at least one flip results in tails"?
A standard deck of 52 playing cards is shuffled and six cards are drawn. What is the complementary event to "at least one card drawn is a spade"?	In a class of 30 students, 15 are male and 15 are female. If three students are randomly selected, what is the complementary event to "at least one student selected is male"?

Probability of at Least One
$P(\text{At least 1}) = 1 - P(\text{none})$

1 3 coins are flipped. The probability of no tails is $\frac{1}{8}$.

What is the probability of at least one tail? ……

Check that this is true by using the tree diagram.



2 2 coins are flipped.

a What is the probability of no heads? ……

b What is the probability of at least one head? ……

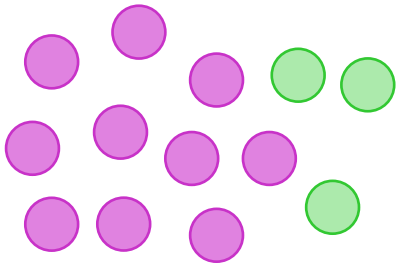
PROBABILITY TREES WITH INDEPENDENT EVENTS

- A **probability tree** is a tree diagram with probabilities on its branches.
- We use probability trees when each event has a different probability, or when there are too many outcomes to list.
- Calculate an event's probability by **multiplying** the probabilities along the branches.

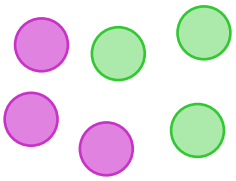
Investigation Tree diagram with probability.

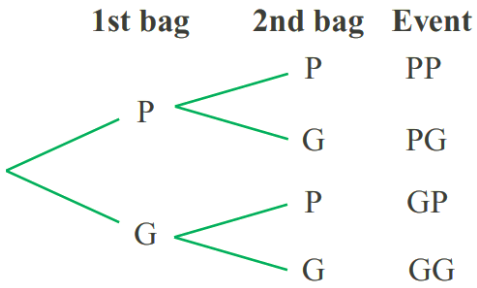
A bag contains 10 purple and 3 green marbles, and a second bag contains 3 purple and 3 green marbles.
A marble is picked out of each bag.

Bag 1



Bag 2





a Why is the tree diagram **NOT** sufficient to represent this scenario?

This tree diagram suggests that the chance of getting P and G are ……., which is not true.

b Label the tree diagram with the probability of choosing the marble at each stage.

c Calculate the probability of selecting one green and one purple marble, in that order.

$P(GP) =$

PROBABILITY TREES WITH INDEPENDENT EVENTS

1. Draw a tree diagram with each single-stage event as a branch of the tree.
2. Label the branches with the probability of each single-stage event.
3. Calculate probability of each multi-stage event by multiplying the probabilities along branches.
4. If the question asks for more the probability of than one event, either:
    a. Add the probabilities of the events.
    b. Use $1 - P(E')$ if the question asks for "at least one."

Example Calculate probabilities using probability trees.

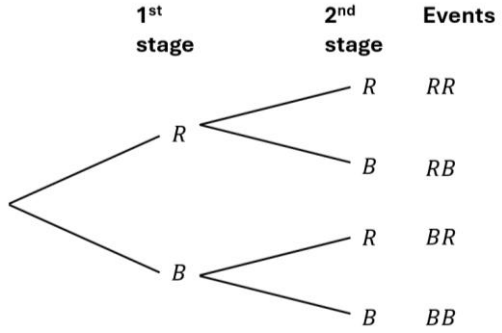
A bag contains 7 red and 5 blue marbles.

A second bag has 4 red and 3 blue.

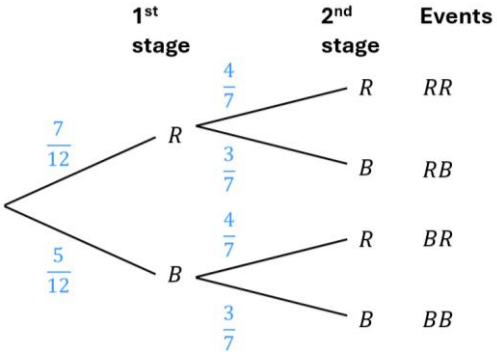
a What is the probability of drawing a red marble and a blue marble, in any order?

b What is the probability of drawing at least one red marble?

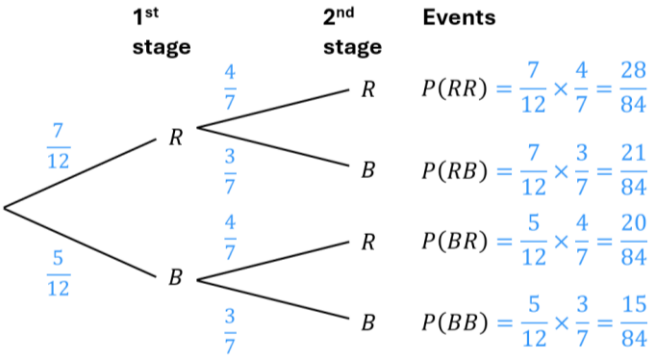
1. Draw a tree diagram



2. Label branches



3. Calculate probabilities

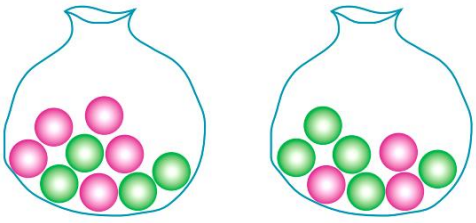


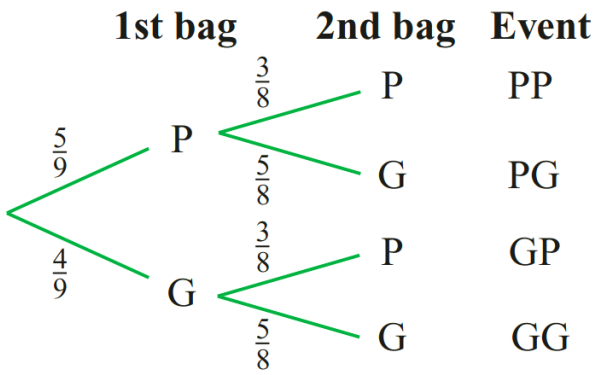
4. Answer questions

a. $P(RB) + P(RB) = \frac{21}{84} + \frac{20}{84} = \frac{41}{84}$

b. $1 - P(BB) = 1 - \frac{15}{84} = \frac{69}{84}$

a A bag contains five purple and four green marbles, and a second bag contains three purple and five green marbles. A marble is selected at random from each bag,





What is the probability of selecting:

two purple marbles? $P(PP) =$ $\frac{15}{72} = \frac{5}{24}$

two green marbles? $P(GG) =$ $\frac{20}{72} = \frac{5}{18}$

a purple marble then a green marble? $P(PG) =$ $\frac{25}{72}$

a green marble then a purple marble? $P(GP) =$ $\frac{12}{72} = \frac{1}{6}$

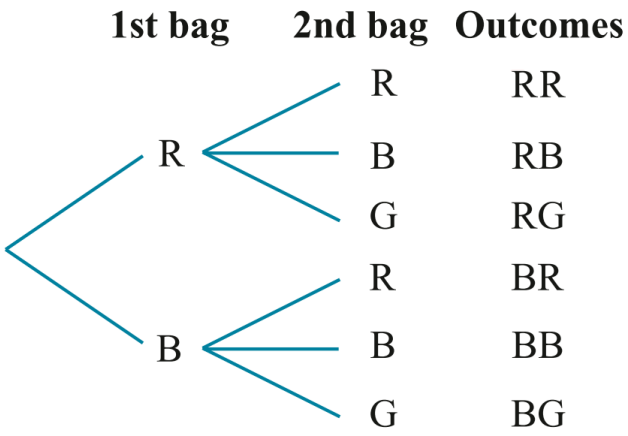
a green marble and a purple marble, in any order? $P(PG) + P(GP) =$ $\frac{37}{72}$

at least one green marble? $1 - P(PP) =$ $\frac{57}{72} = \frac{19}{24}$

b A bag contains two red and one blue marble, and another bag contains two red, one blue and one green marble. One marble is drawn from each bag.

What is the probability of drawing:

two red marbles? $\frac{1}{3}$



a blue marble, then a red marble? $\frac{1}{6}$

a red marble, then a blue marble? $\frac{1}{6}$

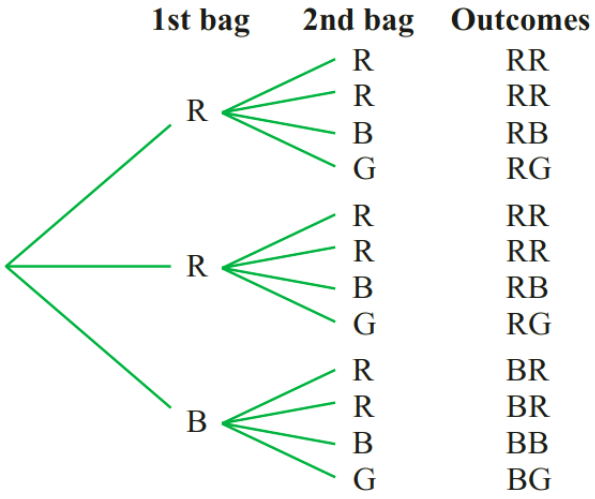
One green marble, in any order? $\frac{1}{4}$

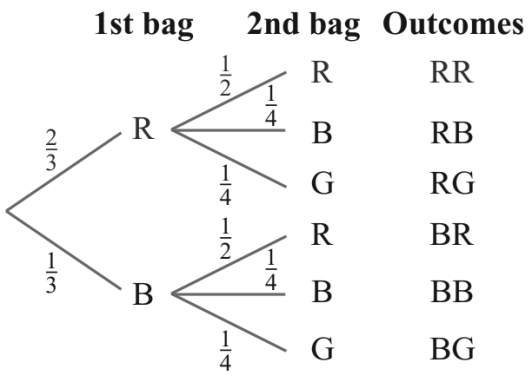
Key ideas

1. We write the probabilities of each event on the …… of the tree.
2. A probability tree allows us to find the probability of an outcome by …… the probabilities along the branches that lead to it

1 A bag contains two red and one blue marble, and another bag contains two red, one blue and one green marble. One marble is drawn from each bag.

The left diagram shows a tree diagram without probabilities.
The right diagram shows a probability tree. Both trees represent the scenario.





Compare the two trees and state the advantages of using a probability tree.

There are less branches to draw

2 A jar contains 7 red discs and 3 blue discs.
Another jar contains 4 red discs and 5 blue discs.
A disc is chosen from each jar.

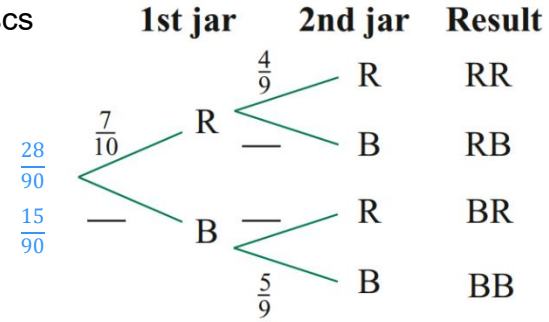
a Complete the probability tree diagram.

b Complete to calculate the probability that the discs chosen are the same colours.

$$P(\text{RR}) = \frac{7}{10} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$P(\text{BB}) = \frac{\square}{\square} \times \frac{5}{9} = \frac{\square}{\square}$$

$$P(\text{same colour}) =$$



28/90
15/90
43/90

3 The probability of Jakub winning a hockey match is $\frac{1}{3}$

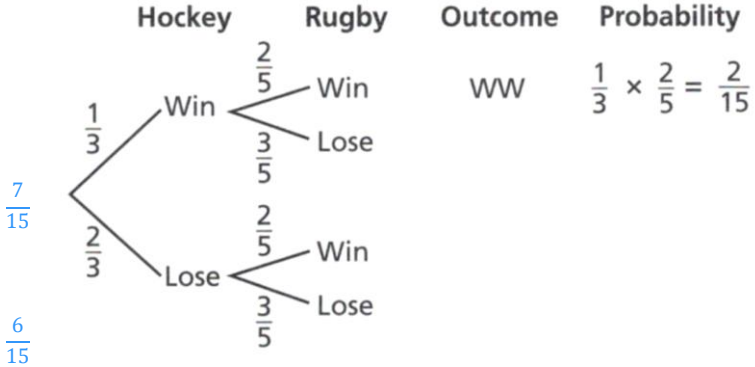
The probability of Jakub winning a rugby match is $\frac{2}{5}$

Complete the probability tree diagram.

Find the probability that Jakub:

a wins exactly one game.

b does not win either game.



7/15
6/15

4 Two light bulbs are selected at random from a large batch of bulbs in which 5% are defective.

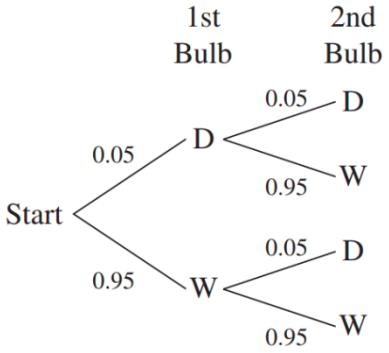
a By multiplying along the branches of the tree, find as a percentage to 2 d.p.

i $P(\text{both work})$ 90.25%

ii $P(\text{first works, second defective})$ 4.75%

iii $P(\text{first defective, second works})$ 4.75%

iv $P(\text{both defective})$ 0.25%



b Hence find the probability that at least one bulb works. 99.75%

5 One bag has three red and two blue balls. Another bag has two red and three blue balls.
A ball is drawn at random from each bag.

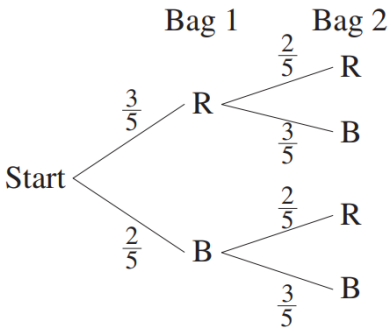
a By multiplying along the branches, find:

i $P(RR)$ $\frac{6}{25}$

ii $P(RB)$ $\frac{9}{25}$

iii $P(BR)$ $\frac{4}{25}$

iv $P(BB)$ $\frac{6}{25}$



b By adding, find the probability that:

i The balls are the same colour. $\frac{12}{25}$

ii The balls are different colours. $\frac{13}{25}$

6 A bag contains 3 white and 4 red balls. Another bag contains 2 white and 3 red balls.
A ball is chosen from each bag.
Find the probability of getting:

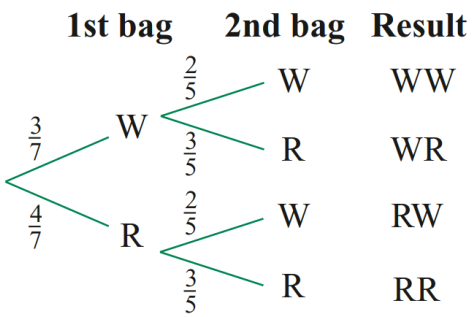
a 2 white balls $\frac{6}{35}$

b 2 red balls $\frac{12}{35}$

c a white and a red ball, in that order $\frac{9}{35}$

d a white and a red ball, in any order $\frac{17}{35}$

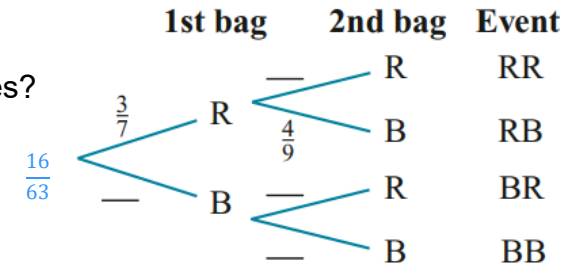
e at least 1 red ball. $\frac{29}{35}$



7 A bag contains three red and four blue marbles, and a second bag contains 5 red and 4 blue. A marble is selected from each bag.

a Complete the probability tree diagram.

b What is the probability of getting two blue marbles?



8 When a child is born, the probability of a boy is 0.51 and the probability of a girl is 0.49. Draw a probability tree diagram for a family's two children.

a Calculate the probability, as a decimal:

i Two boys

0.2601

ii A boy then a girl, in that order.

0.2499

iii A girl and a boy, in any order.

0.4998

iv At least one girl.

0.7399

9 There is an 80% chance that Garry will pass his driving test and a 90% chance that Emma will pass hers. Draw a probability tree diagram, and find the probability as a decimal:

a Garry passes and Emma fails.

0.08

b Garry fails and Emma passes.

0.18

c only one of Garry and Emma passes.

0.26

d at least one fails.

0.28

10 A student enters the 100 m and 400 m events at the school swimming carnival.
Her chance of winning the 100 m event is 0.8 and her chance of winning the 400 m event is 0.6.
Draw a probability tree showing the possible results for each event and the combined results.

a Calculate the probability that she will (answer as a decimal):

i win both events.

0.48

ii not win either event.

0.08

iii win the 100 m but not win the 400 m.

0.32

iv win at least one event.

0.92

11 A council is running two raffles. The probability of winning in each raffle is 1 in 100.

Jared buys one ticket for each raffle. Answer these questions without drawing a tree diagram.

a What is the probability that he wins both?

$\frac{1}{100} \times \frac{1}{100} = \frac{1}{10000}$

b What is the probability he wins at least one?

$1 - \left(\frac{99}{100} \times \frac{99}{100}\right) = \frac{199}{10000}$

12 A magician deals cards from two different decks for a trick.
The probability of dealing a red card from deck 1 is 0.3.
The probability of a red card from deck 2 is three times the probability of a red from deck 1.

a What is the probability of picking two reds?

0.27

b What is the probability of picking a red from deck 1
and a black from deck 2?

0.03

c What is the probability of picking a black from deck 1
and a red from deck 2?

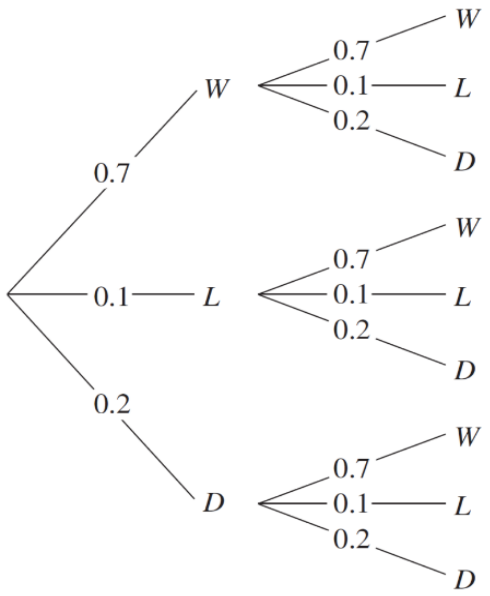
0.63

d What is the probability of picking no reds?

0.07

13 2016 HSC Standard 2 Band 5

A cricket team is about to play two matches. The probability of the team having a win, a loss or a draw is 0.7, 0.1 and 0.2 respectively in each match. The possible results in the two matches are displayed in the probability tree diagram.



a What is the probability of the team having a win and a draw, in any order?

0.28

b Paul claims that 1.4 is the probability of the team winning both matches.
Give one reason why this is NOT correct.

Probability cannot be greater than one.

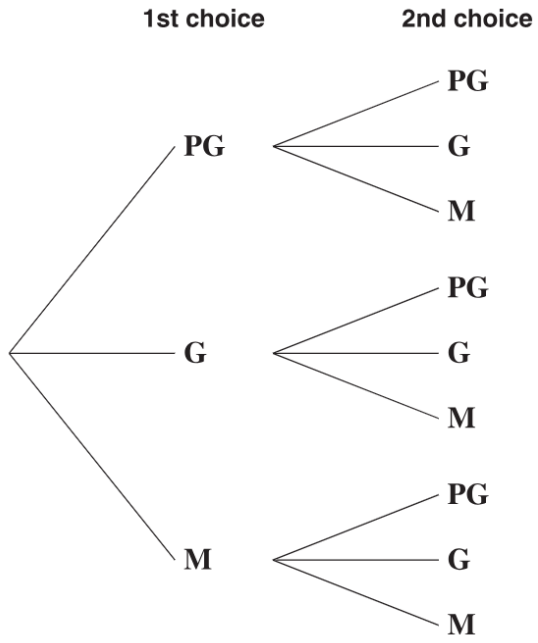
He added instead of multiplying along the branches.

14 2007 HSC Standard 2 Band 5

In a stack of 10 DVDs, there are 5 rated PG, 3 rated G and 2 rated M

Grant chooses two DVDs at random from the stack.

Calculate the probability that Grant chooses two DVDs with the same rating.



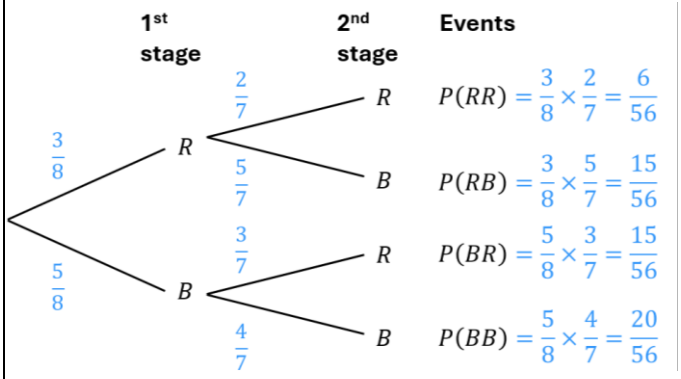
$\frac{14}{45}$

PROBABILITY TREES WITH DEPENDENT EVENTS

- **Dependent events** occur when there's **no replacement**.
  1. Draw tree diagram.
  2. Label branches, **the probability of the second event depends on the first event.**
  3. Calculate probabilities.
  4. Answer questions.

Example Probability trees with dependent events.

A bag contains 3 red marbles and 5 blue marbles. Two are taken from the bag without replacement.



What is the probability of drawing a red marble and a blue marble, in any order?

$P(R,B) + P(R,B) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56}$

What is the probability of drawing at least one red marble?

$1 - P(B,B) = 1 - \frac{20}{56} = \frac{36}{56}$

Why is the probability of picking the first red counter $\frac{3}{8}$?

The bag has …… red marbles and …… marbles total.

Why is the probability of picking the second red counter $\frac{2}{7}$?

Because the first red counter is not ……., there are …… red counters left out of ……

A box contains 2 black balls and 7 white balls. Two balls are drawn at random.
Balls are not replaced after being picked.
Draw and label a tree diagram, then calculate the probability of:

a two white balls?

$\frac{42}{72} = \frac{7}{12}$

b two balls of different colour?

$\frac{28}{72} = \frac{7}{18}$

c two balls of the same colour?

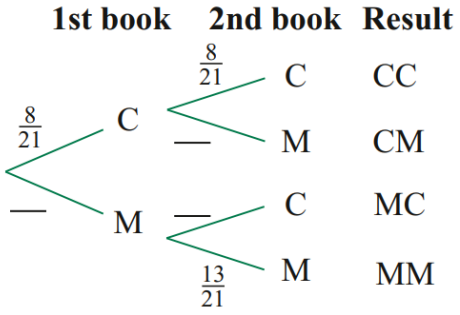
$\frac{44}{72} = \frac{11}{18}$

d at least one white ball?

$\frac{70}{72} = \frac{35}{36}$

1 There are 8 cookbooks and 13 maths books on a bookshelf.
Two books are selected at random from the bookshelf.

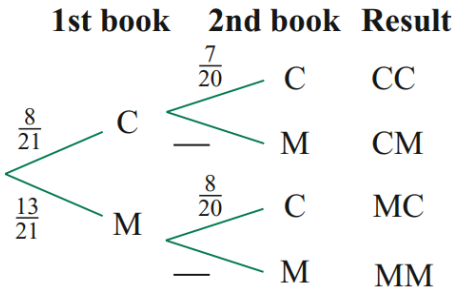
a The first book **is replaced** before the second book is selected.
Complete to find the probability that the books are the same type.



$$P(\text{CC}) + P(\text{MM}) = \frac{8}{21} \times \frac{8}{21} + \frac{\square}{\square} \times \frac{\square}{\square}$$
$$= \frac{\quad}{\quad}$$

$\frac{233}{441}$

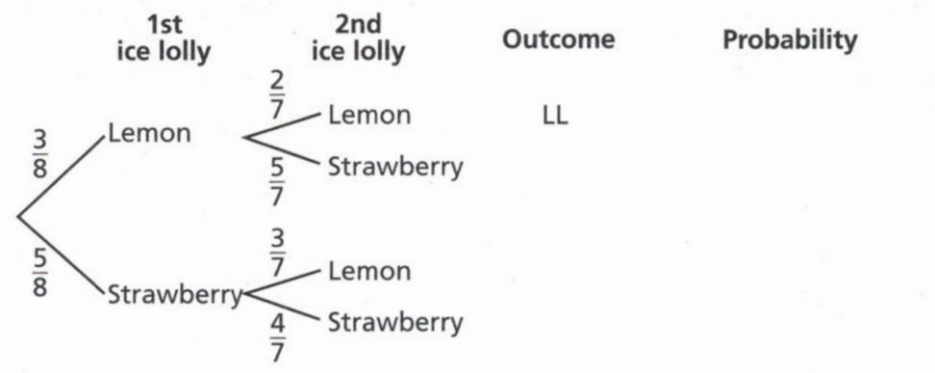
b The first book **is not replaced** before the second book is selected.
Complete to find the probability that the books are the same type.



$$P(\text{CC}) + P(\text{MM}) = \frac{\square}{\square} \times \frac{\square}{\square} + \frac{13}{21} \times \frac{\square}{\square}$$
$$= \frac{\quad}{\quad}$$

$\frac{53}{105}$

2 A freezer contains 3 lemon ice lollies and 5 strawberry ice lollies.
Two ice lollies are selected at random without replacement.
Complete the probability tree diagram



Find the probability that

a both ice lollies are strawberry

$\frac{5}{14}$

b at least one ice lolly is strawberry.

$\frac{25}{28}$

3 A bag contains 3 red and 4 white counters. Two are taken without replacement.

a Draw a tree diagram and label the branches with the corresponding probability.

b What is the probability of picking exactly one white counter?

$\frac{4}{7}$

c What is the probability of picking exactly one red counter?

$\frac{4}{7}$

d What is the probability of picking no red balls?

$\frac{2}{7}$

e What is the probability of picking at least one red?

$\frac{5}{7}$

4 There are 12 novels and 9 science books on a bookshelf. Two books are selected at random from the bookshelf.

a The first book is replaced before the second book is selected.
Draw a tree diagram

Find the probability of choosing:

i novel and then a science book.

$\frac{12}{49}$

ii 2 novels.

$\frac{16}{49}$

iii at least one science book.

$\frac{33}{49}$

b The first book is **NOT** replaced before the second book is selected.
Draw a tree diagram.

Find the probability of choosing:

i novel and then a science book.

$\frac{9}{35}$

ii 2 novels.

$\frac{11}{35}$

iii at least one science book.

$\frac{24}{35}$

5 There are 4 green and 7 pink counters in a bag. Two are picked at random.
Counters are not replaced after each pick. What is the probability of picking at least one pink?

$\frac{49}{55}$

6 I have a 6-sided die and a coin.

a You roll the die twice. Draw a tree diagram to calculate the probability you get exactly 1 six.

$\frac{5}{18}$

b If you roll the die 72 times, how many sixes would you expect to get?

12

c You flip the coin and roll the die.
Draw a two-way table to find the probability you get tails and an even number.

	1	2	3	4	5	6
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

$\frac{1}{4}$

d You flip the coin 3 times.
List all the possible outcomes and find the probability of getting at least 1 head.

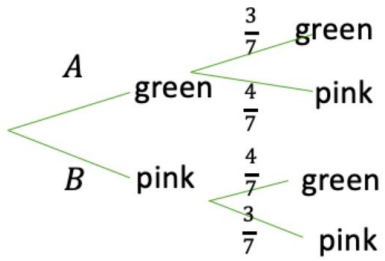
$\frac{7}{8}$

e How could you use complementary events to calculate the answer to part **d** quickly?

$1 - P(\text{no heads}) = 1 - P(\text{all tails})$

$\frac{7}{8}$

7 There are x green and y pink counters in a bag. Two are picked at random.
Counters are not replaced after each pick.
What probability should be written in space A and B on the tree diagram?



$A = \frac{4}{8}, B = \frac{4}{8}$

8 There are 3 yellow and 5 white counters in a bag.
Three are picked at random. Counters are not replaced after each pick.
What is the probability of picking three of the same colour?

$\frac{11}{56}$

9 There are 3 grey, 2 yellow and 5 white counters in a bag.
Three are picked at random. Counters are not replaced after each pick.
What is the probability of picking two greys and a white?

$\frac{1}{8}$

10 2022 HSC Standard 1

A jar contains 12 red, 10 black and 13 white lollies.
Alex picks out a red lolly and eats it. He then randomly picks a second lolly.
What is the probability that the second lolly is also red?

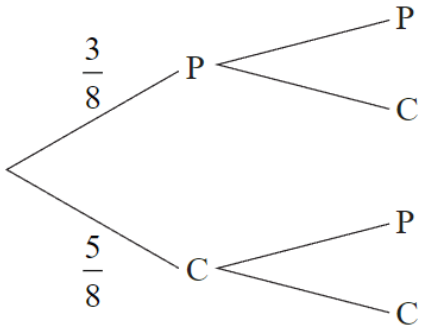
$\frac{11}{34}$

11 2021 HSC Standard 2 Band 5

A box has 8 chocolates. 3 have peppermint centres (P) and 5 have caramel centres (C).
Kim randomly chooses a chocolate from the box and eats it.
Sam then randomly chooses and eats one of the remaining chocolates.

A partially completed probability tree is shown.

What is the probability that Kim and Sam choose chocolates with different centres?



$\frac{15}{28}$

12 2019 HSC Standard 2 Band 5

A bowl of fruit contains 17 apples of which 9 are red and 8 are green.

Dennis takes one apple at random and eats it.
Margaret also takes an apple at random and eats it.

By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour.

$\frac{8}{17}$

13 2023 HSC Standard 2 Band 4

One hundred tickets are sold in a raffle which offers two prizes.
Hazel buys five of the tickets.

A ticket is drawn at random for the first prize.
A second ticket is drawn from the remaining tickets for the other prize.

What is the probability that Hazel wins both prizes?

$\frac{1}{495}$

14 2017 HSC Standard 2 Band 6

A deck of 52 playing cards contains 12 picture cards.
Two cards from the deck are drawn at random and placed on a table.
What is the probability, correct to four decimal places, that exactly one picture card is on the table?

A. 0.0498

B. 0.1810

C. 0.3550

D. 0.3620

D

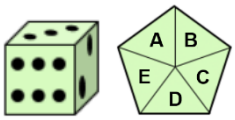
THE MULTIPLICATION RULE

- The **multiplication rule** quickly calculates probabilities for multistage **independent** events without a tree diagram.

Multiplication Rule for Probability
$P(A \text{ and } B) = P(A) \times P(B)$

Example Determine probabilities for independent events $P(A \text{ and } B) = P(A) \times P(B)$

Alex is rolling a dice and spinning a spinner.



What is the probability that he gets:
2 on the die and B on the spinner?

$P(2, B) = P(2) \times P(\text{B})$

$= \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$

An odd number and a vowel?

$P(\text{odd, vowel}) = P(\text{odd}) \times P(\text{vowel})$

$= \frac{3}{6} \times \frac{2}{5} = \frac{6}{30}$

What does $P(2, B)$ mean?

Probability of getting …… in the first stage and …… in the second stage.

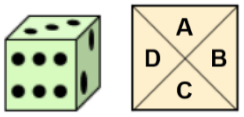
Where did the fractions $\frac{1}{6}$ and $\frac{1}{5}$ come from?

The probability of getting …… is $\frac{1}{6}$

The probability of getting …… is $\frac{1}{5}$

When do we multiply probabilities?
We multiply the probabilities of multistage …… events.

Allie is rolling a die and spinning a spinner.



What is the probability she gets:

a 1 on the dice and C on the spinner.

$\frac{1}{24}$

b An even number and a consonant.

$\frac{3}{8}$

c A number more than 4 and a vowel.

$\frac{1}{12}$

Key ideas

1. The multiplication rule for multistage …… events is

   $P(A \text{ and } B) = \ldots\ldots \times \ldots\ldots$

1

a I roll a fair 6-sided die. What is the probability of scoring a 5?

$\frac{1}{6}$

b I flip a fair coin. What is the probability of getting Tails?

$\frac{1}{2}$

c I do both of the above. What is the probability of getting both a 5 and Tails?

$\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$

2 If I roll two fair, 6-sided dice, what is the probability that both show even numbers?

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

3 Alex has 2 red, 4 blue, 1 yellow and 3 green pens in his pencil case.
He picks a pen at random, draws a circle, then replaces the pen.
Then he does the same for a second circle.
What is the probability that both circles are green?

$\left(\frac{3}{10}\right)\left(\frac{3}{10}\right) = \frac{9}{100}$

4 There is a $\frac{3}{5}$ probability that I sleep through my alarm.
There is a $\frac{3}{7}$ probability that my bus is late.
What is the probability that I sleep through my alarm and my bus is **not** late?

$\left(\frac{3}{5}\right)\left(\frac{4}{7}\right) = \frac{12}{35}$

5 A, B, C and D are independent events, with $P(A) = \frac{1}{8}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{4}$, $P(D) = \frac{2}{7}$.
Use the product rule to find:

a $P(AB)$

$\left(\frac{1}{8}\right)\left(\frac{1}{3}\right) = \frac{1}{24}$

b $P(AD)$

$\left(\frac{1}{8}\right)\left(\frac{2}{7}\right) = \frac{1}{28}$

c $P(BC)$

$\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{12}$

d $P(ABC)$

$\left(\frac{1}{8}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{96}$

e $P(BCD)$

$\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{2}{7}\right) = \frac{1}{42}$

f $P(ABCD)$

$\left(\frac{1}{8}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{2}{7}\right) = \frac{1}{336}$

6 A coin and a die are tossed. Use the product rule to find the probability of obtaining:

a A three and a head

$\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)=\frac{1}{12}$

b A six and a tail

$\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)=\frac{1}{12}$

c An even number and a tail

$\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$

d less than 5 and a head

$\left(\frac{4}{6}\right)\left(\frac{1}{2}\right)=\frac{1}{3}$

7 Two marbles are picked at random, one from a bag containing three red and four blue marbles, and the other from a bag containing five red and two blue. Find the probability of:

a Two red marbles

$\left(\frac{3}{7}\right)\left(\frac{5}{7}\right)=\frac{15}{49}$

b Two blue marbles

$\left(\frac{4}{7}\right)\left(\frac{2}{7}\right)=\frac{8}{49}$

c A red from the first bag and blue from second

$\left(\frac{3}{7}\right)\left(\frac{2}{7}\right)=\frac{6}{49}$

8 A die is rolled twice. Find the probability of:

a A six then a five

$\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)=\frac{1}{36}$

b A one then an odd number

$\left(\frac{1}{6}\right)\left(\frac{3}{6}\right)=\frac{1}{12}$

c A double six

$\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)=\frac{1}{36}$

d Two numbers greater than four

$\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)=\frac{1}{9}$

e A number greater than four and then a number less than four.

$\left(\frac{2}{6}\right)\left(\frac{3}{6}\right)=\frac{1}{6}$

9 A box contains five light bulbs, two of which are faulty.
Two bulbs are selected, one at a time without replacement. Find the probability that:

a Both bulbs are faulty.

$\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)=\frac{1}{10}$

b Neither bulb is faulty.

$\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)=\frac{3}{10}$

c First bulb is faulty and the second not.

$\left(\frac{2}{5}\right)\left(\frac{3}{4}\right)=\frac{3}{10}$

d The second bulb is faulty and the first is not.

$\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)=\frac{3}{10}$

e One of the bulbs is faulty.

$\frac{3}{10}+\frac{3}{10}=\frac{3}{5}$

f At least one bulb is faulty.

$1-\frac{3}{10}=\frac{7}{10}$

10 A box contains 12 red and 10 green discs.
Three discs are selected, one at a time with replacement.
What is the probability that the discs selected are red, green, red in that order?

$\left(\frac{12}{22}\right)\left(\frac{10}{22}\right)\left(\frac{12}{22}\right)=\frac{180}{1331}$

11 From a standard deck of 52 cards, two cards are drawn at random with replacement.
Find the probability of drawing:

a A spade then a heart

$\left(\frac{13}{52}\right)\left(\frac{13}{52}\right)=\frac{1}{16}$

b Two clubs

$\left(\frac{13}{52}\right)\left(\frac{13}{52}\right)=\frac{1}{16}$

c A jack then a queen

$\left(\frac{4}{52}\right)\left(\frac{4}{52}\right)=\frac{1}{169}$

d The king of diamonds then the ace of clubs.

$\left(\frac{1}{52}\right)\left(\frac{1}{52}\right)=\frac{1}{2704}$

12 I roll two 6-sided dice, and then add the results.

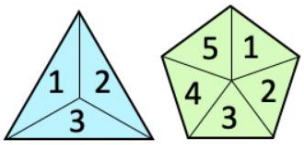
a How many ways can I get a total of 4? List them.

(1,3), (2,2), (3,1)
Three ways

b What is the probability of getting a total of 4?

$\frac{3}{36}=\frac{1}{12}$
Or
$\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)=\frac{3}{36}=\frac{1}{12}$

13 I spin both these two spinners.



What is the probability that:

a I get 1 on both spinners?

$\left(\frac{1}{3}\right)\left(\frac{1}{5}\right)=\frac{1}{15}$

b I get 2 on both spinners?

$\left(\frac{1}{3}\right)\left(\frac{1}{5}\right)=\frac{1}{15}$

c I get 1 on both spinners or 2 on both spinners?

$\frac{1}{15}+\frac{1}{15}=\frac{2}{15}$

d I get the same number on both spinners?

(1,1) (2,2) or (3,3)
$\frac{3}{15}=\frac{1}{5}$

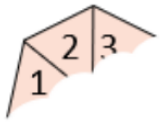
e I get a sum of 3?

2 ways: (1, 2) or (2,1)
$\frac{2}{15}$

f I get a sum of 4?

3 ways: (1,3), (2,2), (3,1)
$\frac{3}{15}$

14 I roll a fair 6-sided die and spin a spinner. The probability that I get a 6 on the die and 3 on the spinner is $\frac{1}{42}$. What is the smallest number of sides my spinner could have?



$\frac{1}{6} \times \frac{1}{x} = \frac{1}{42}$

7

15 The probability that on any given day it will rain is ⅓.
The probability that on any given day I will wear my coat is ½.

a What is the probability of it raining and I wear my coat, assuming these are independent events?

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

b Explain why, in real life, there is **not** a probability of ⅙ that it rains and I wear my coat.

These are not independent events. I am more liekly to wear my coat if it rains.

16

a I flip three fair coins. What is the probability that I get a head on each?

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$

b I have already flipped two coins and both are heads.
Explain why the probability that the next coin I flip will also be heads is $\frac{1}{2}$.

These are independent events. Probability of heads is always $\frac{1}{2}$

17 **2007 HSC Standard 2 Band 4**

Each time she throws a dart, the probability that Mary hits the dartboard is $\frac{2}{7}$
She throws two darts, one after the other.
What is the probability that she hits the dartboard with both darts?

$\left(\frac{2}{7}\right)\left(\frac{2}{7}\right) = \frac{4}{49}$

18 2012 HSC Standard 2 Band 5

Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5, 6, are rolled.

What is the probability of a 6 appearing on at least one of the dice?

$1-\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)=\frac{11}{36}$

19 2004 HSC Standard 2 Band 5

Two dice are rolled. What is the probability that only one of the dice shows a six?

$P(6, \text{not } 6)+P(\text{not } 6, 6)$

$\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)=\frac{5}{18}$

20 2020 HSC Standard 2 Band 5

The top of a rectangular table is divided into 8 equal sections as shown.

1	2	3	4
5	6	7	8

A standard die with faces labelled 1 to 6 is rolled onto the table.
The die is equally likely to land in any of the 8 sections of the table.
If the die does not land entirely in one section of the table, it is rolled again.
A score is calculated by multiplying the value shown on the top face of the die by the number shown in the section of the table where the die lands.
What is the probability of getting a score of 6?

Combinations that score 6: (1,6) (2,3) (3,2) (6,1)

Each combination has $\left(\frac{1}{8}\right)\left(\frac{1}{6}\right)$ chance

$\therefore \frac{4}{48}=\frac{1}{12}$

15 2013 HSC Advanced Band 4

A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random without replacement.

What is the probability that at least one of the marbles selected is red?

$1-P(\text{no red})$

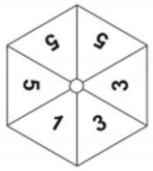
$1-P(BBB)$

$1-\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)=\frac{5}{6}$

CHECK YOUR UNDERSTANDING

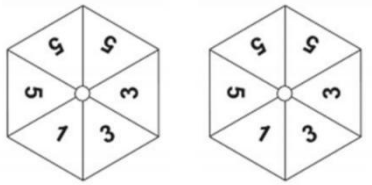
List outcomes using tables

1 This is a fair spinner.
How many possible outcomes are there if it is spun once?
(Hint: the number of outcomes, not the size of the sample space)



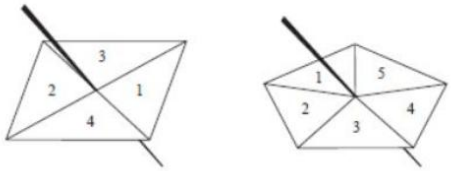
a 6 **b** 3 **c** 1 **d** 1, 3, 5

2 These are fair spinners.
How many possible outcomes are there if
they are spin together and both scores are recorded?



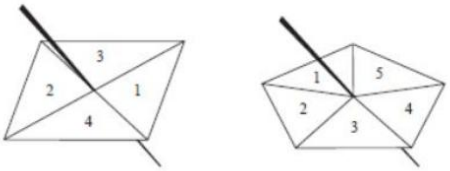
a 12 **b** 6 **c** 9 **d** 36

3 These are fair spinners.
How many possible outcomes are there if
they are spin together and both scores are recorded?



a 20 **b** 9 **c** 5 **d** 2

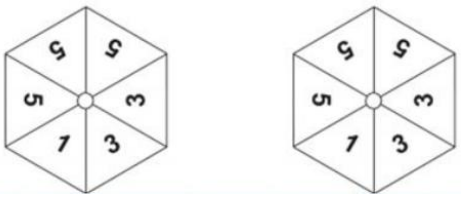
4 These spinners are spin and the scores are added.
What should go in the highlighted cell of the two-way table?



	1	2	3	4	5
1					
2					?
3					
4					

a 52 **b** 7 **c** 10 **d** 3

5 These spinners are spin and the scores are added.
How many cells would have an outcome of 4?



	1	3	3	5	5	5
1						
3						
3						
5						
5						
5						

a 1 **b** 2 **c** 4 **d** 0

Multiplication rule

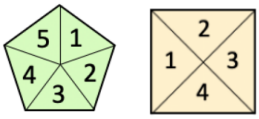
1 You toss two fair coins. What is the probability of getting a heads, then a tail?

a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{1}{4}$

2 You toss two fair coins. What is the probability of getting a heads and a tail, in any order?

a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{1}{4}$

3 You spin the above fair spinners. What is the probability of getting a 3 on both spinners?



a $\frac{1}{9}$ **b** $\frac{1}{20}$ **c** $\frac{2}{9}$ **d** $\frac{2}{20}$

4 The probability of Shane going to the cinema is 0.2.

The probability of Niamh going to bowling is 0.6.

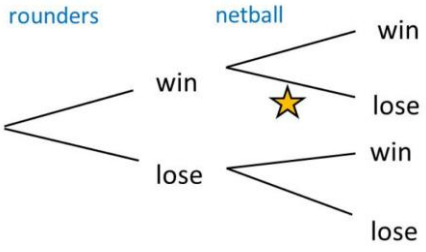
These two events are independent.

What is the probability that Shane goes to the cinema and Niamh goes bowling?

a 0.12 **b** 0.5 **c** 0.8 **d** 1.2

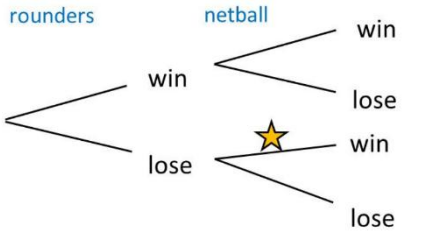
Probability trees with independent events

1 Ruby is going to play a game of rounders (a game similar to baseball) and a game of netball.
The probability of winning at rounders is 0.2.
The probability of winning at netball is 0.7
What number should replace the star in the diagram?



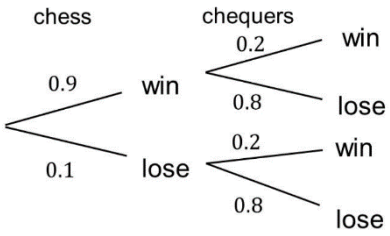
a 0.3 **b** 0.8 **c** 0.7 **d** 0.2

2 Exact same scenario as Q1.
What number should replace the star in the diagram?



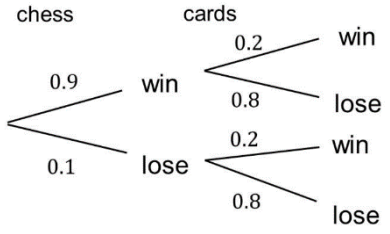
a 0.3 **b** 0.8 **c** 0.7 **d** 0.2

3 What is the probability of losing both chess and chequers?
The results of both games are independent.



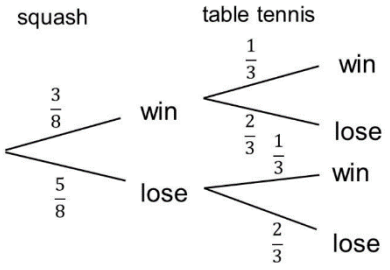
a 0.18 **b** 0.8 **c** 0.9 **d** 0.08

4 What is the probability of winning only one of the games?
The results of both games are independent.



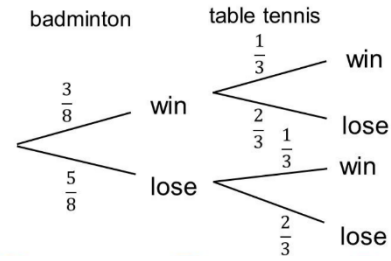
a 0.74 **b** 0.0144 **c** 2 **d** 0.72

5 What is the probability of losing both games?
The results of both games are independent.



a $\frac{10}{24}$ **b** $\frac{15}{24}$ **c** $\frac{52}{83}$ **d** $\frac{7}{11}$

6 What is the probability of winning exactly one game?
The results of both games are independent.

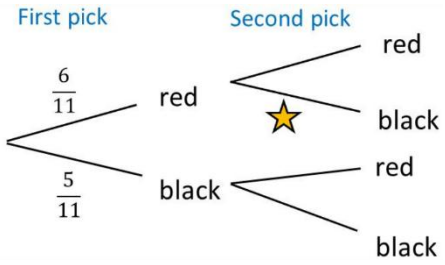


a $\frac{48}{24}$ **b** $\frac{11}{24}$ **c** $\frac{6}{24}$ **d** $\frac{14}{24}$

Probability trees with dependent events

1 A bag contains 6 red counters and 5 black counters. The counters are not replaced each time one is picked.

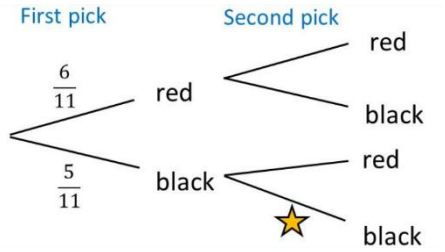
What fraction should replace the star?



a $\frac{4}{10}$ **b** $\frac{5}{11}$ **c** $\frac{4}{11}$ **d** $\frac{5}{10}$

2 Same scenario as Q1.

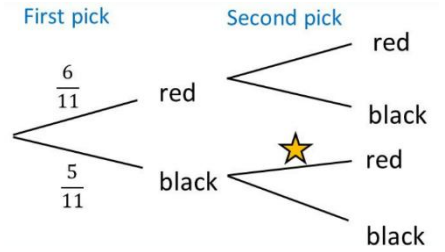
What fraction should replace the star?



a $\frac{4}{10}$ **b** $\frac{5}{11}$ **c** $\frac{4}{11}$ **d** $\frac{5}{10}$

3 Same scenario as Q1.

What fraction should replace the star?

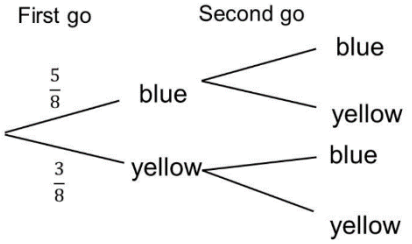


a $\frac{5}{10}$ **b** $\frac{6}{11}$ **c** $\frac{5}{11}$ **d** $\frac{6}{10}$

4 A bag contains 5 blue counters and 3 yellow counters.

The counters are not replaced each time one is picked.

What is the probability of picking two blue?

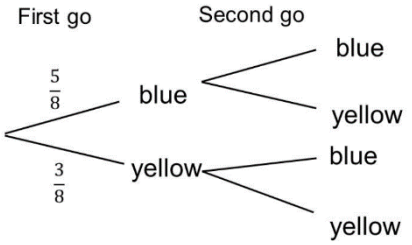


a $\frac{25}{56}$ **b** $\frac{25}{64}$ **c** $\frac{20}{56}$ **d** $\frac{20}{64}$

5 A bag contains 5 blue counters and 3 yellow counters.

The counters are not replaced each time one is picked.

What is the probability of picking two of the same colour?



a $\frac{20}{56}$ **b** $\frac{26}{56}$ **c** $\frac{34}{64}$ **d** $\frac{26}{64}$