Student Name

Mathematics Stage 5 Paths

LINEAR RELATIONSHIPS C

Book 4 Solve problems by applying coordinate geometry formulas

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-LIN-P-01 describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems

Solve problems by applying coordinate geometry formulas

- Solve problems including those involving geometrical figures by applying coordinate geometry formulas

REVIEW OF PRIOR KNOWLEDGE

1 Classify the triangles with these side lengths as equilateral, right-angled, scalene, or isosceles.

a 4 cm, 4 cm, 4 cm	**b** 8 cm, 8 cm, 6 cm	**c** 6 cm, 8 cm, 10 cm.

2 Classify quadrilaterals:

a One pair of parallel sides	**b** Two pairs of parallel sides	**c** Diagonals bisect at right angles
d Two pairs of parallel sides and adjacent sides meet at right angles.	**e** All sides equal	

3 Calculate the length, gradient, and midpoint of these intervals.

a (1,2) and (5,4)	**b** $(-3,-2)$ and (2,3)	**c** $(-5,3)$ and $(7,-10)$

COORDINATE GEOMETRY PROBLEMS: TRIANGLES

Type of Triangle	Standard Definition
Equilateral Triangle	• Three sides of equal length.
Isosceles Triangle	• two sides of equal length.
Right-angled Triangle	• side lengths satisfy Pythagoras' Theorem **or** • gradients of two sides are negative reciprocals.
Scalene Triangle	• All side lengths different.

Distance	Gradient	Area of a Triangle
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$A = \dfrac{1}{2}bh$ Where b and h are perpendicular to each other.

Example Solve coordinate geometry problems involving triangles.

A triangle has vertices at $A(1, -2), B(-4,2), C(5, -7)$.
Find the lengths of the sides AB and AC and hence show it is isosceles.

$$AB = \sqrt{(-4-1)^2 + \left(2-(-2)\right)^2}$$
$$= \sqrt{41}$$

$$AC = \sqrt{(5-1)^2 + \left(-7-(-2)\right)^2}$$
$$= \sqrt{41}$$

$AB = AC$, so ΔABC is isosceles.

What formula do we use to calculate the distance between two points?

$d =$ ....

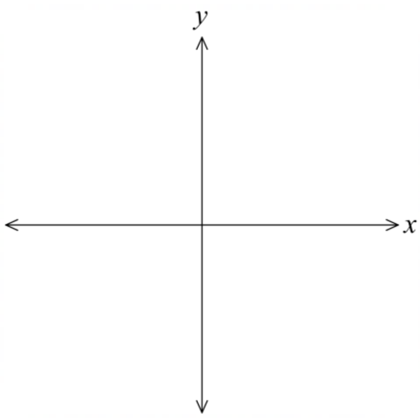
Why does $AB = AC$ mean the triangle is isosceles?

There are .... equal sides, so the triangle is isosceles.

The vertices of the ΔABC are given by $A(0, 0), B(9, 12)$ and $C(25, 0)$.

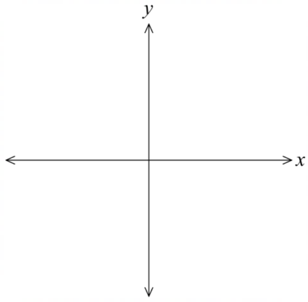
a Find the lengths of the three sides AB, BC, AC.

b By checking that $AB^2 + BC^2 = AC^2$, show that ΔABC is right-angled.



1 The triangle ABC has vertices A(−1, 0), B(3, 2) and C(4, 0).

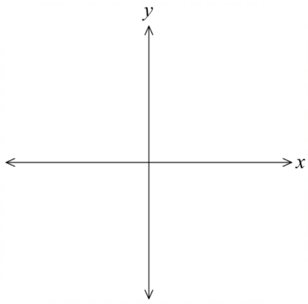
Calculate the gradient of each side, and hence show that ΔABC is a right-angled triangle.



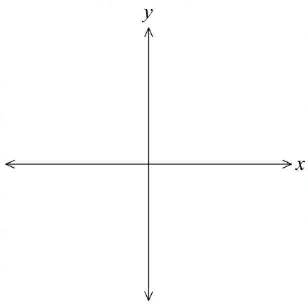
2

a Find the length of each side of the triangle formed by $P(0,3), Q(1,7)$ and $R(5,8)$.

b Hence show that ΔPQR is isosceles.



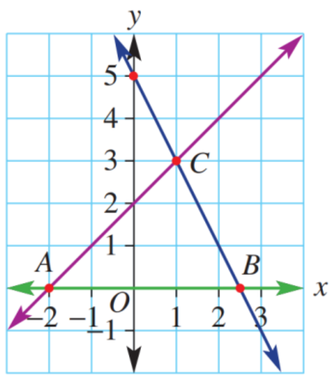
3 Prove that $A(1,-1), B(-1,1), C(-\sqrt{3},-\sqrt{3})$ form an equilateral triangle.



4 The three lines with equations $y = 0,\ y = x + 2$ and $y = -2x + 5$ are illustrated here.

a State the coordinates of the intersection point of $y = x + 2$ and $y = -2x + 5$.

b Use the formula for the area of a triangle to calculate the area of the enclosed region ΔACB

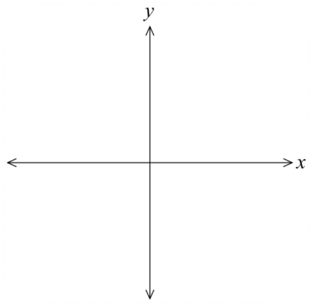


5 Consider a triangle ΔXYZ with vertices $X(0, -4), Y(4, 2)$ and $Z(-2, 6)$.

a Find the side lengths.

b Show that ΔXYZ is a right-angled isosceles triangle by showing that its side lengths satisfy Pythagoras' theorem.

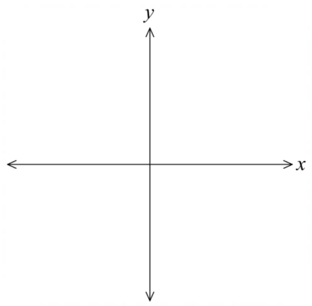
c Hence find the area of ΔXYZ.



6 The triangle ABC has vertices $A(-1, 0), B(3, 2)$ and $C(4, 0)$.

a Calculate the gradient of each side,

b Explain why ΔABC is a right-angled triangle.



7 Show that the triangle with vertices A(1, 0), B(6, 5) and C(0, 2) is right-angled by:

a Using Pythagoras' Theorem.

b Using the gradients of two sides.

8 The vertices of ΔABC are $A(0,0), B(3,4)$ and $C\left(\frac{25}{3}, 0\right)$

a Find the gradient of:

i AB	**ii** BC	**iii** CA

b What type of triangle is ΔABC? Why?

……

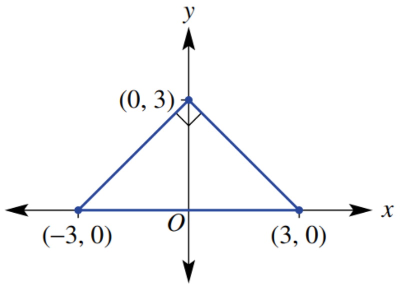
c Find the perimeter of ΔABC.

9 Each set of three points given below forms a triangle that is either isosceles, equilateral, right-angled, or none of the above.
Find the side lengths of each triangle and hence determine its type.

a $A(-1,0),\ B(1,0),\ C(0,\sqrt{3})$ $AB =$ $BC =$ $AC =$	**b** $P(-1,1),\ Q(0,-1),\ R(3,3)$
c $D(1,1),\ E(2,-2),\ F(-3,0)$	**d** $X(-3,-1),\ Y(0,0),\ Z(-2,2)$

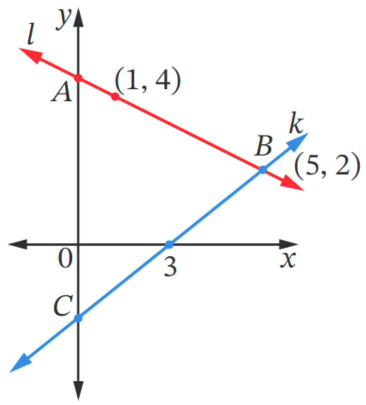
10 Find the gradients of the three lines $5x - 7y + 5 = 0,\ 2x - 5y + 7 = 0$ and $7x + 5y + 2 = 0$.
Show that they enclose a right-angled triangle.

11 A right-angled isosceles triangle has vertices at $A(0,3)$, $B(3,0)$ and $C(-3,0)$.
Find the equation of each side.



1 The lines k and l are shown in the diagram. Find:

a The equation of line k, in general form.



$x - y - 3 = 0$

b The equation of line l, in general form.

$x + 2y - 9 = 0$

c The coordinates of point A.

(0,4.5)

d The coordinates of point C.

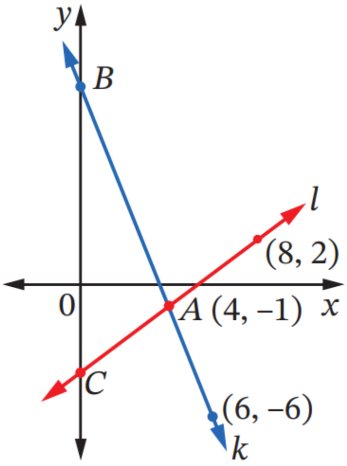
$(0, -3)$

e The area of ΔABC.

18.75 square units

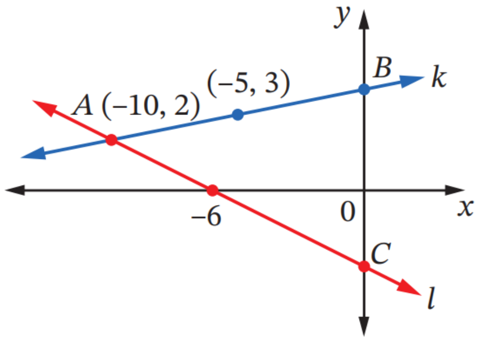
12 Find the area of ΔABC.

a



26 square units

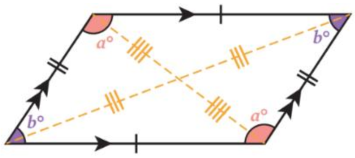
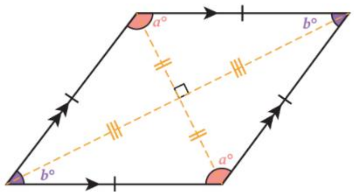
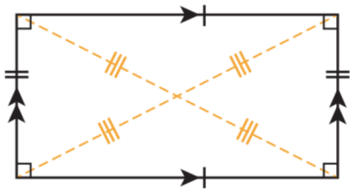
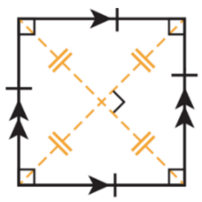
b



35 square units

COORDINATE GEOMETRY PROBLEMS: QUADRILATERALS

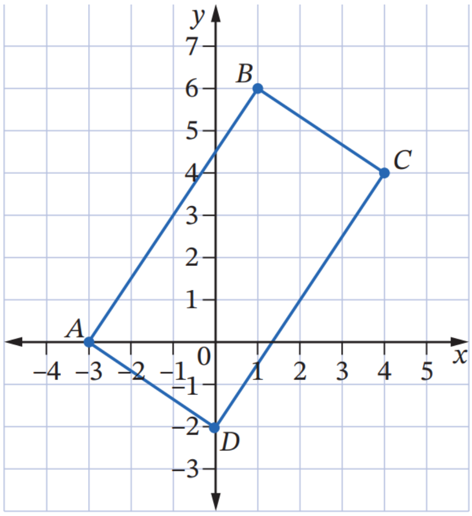
- The special quadrilaterals and their properties are shown below.
- **A sketch is often essential.**
- To show that two lines bisect each other, i.e. cut each other in half, show that the midpoints are the same.

	Standard Definition	Alternate Definitions
Parallelogram 	• two pairs of opposite sides parallel	• the opposite sides are equal, **or** • one pair of opposite sides are equal and parallel, **or** • the diagonals bisect each other
Rhombus 	• two pairs of opposite sides parallel **and** • one pair of adjacent sides equal	• all sides are equal, **or** • the diagonals bisect each other at right angles
Rectangle 	• two pairs of opposite sides parallel **and** • one angle a right angle	• the diagonals are equal and bisect each other
Square 	• two pairs of opposite sides parallel **and** • one pair of adjacent sides equal **and** • one angle a right angle	

Midpoint
$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

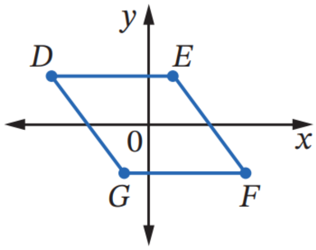
1 $A(-3,0), B(1,6), C(4,4)$ and $D(0,-2)$ are the vertices of a rectangle.

- **a** Find the lengths of AC and BD and show that the diagonals are equal.
- **b** Find the midpoints of the diagonals AC and BD.
- **c** What property of rectangles have you shown?



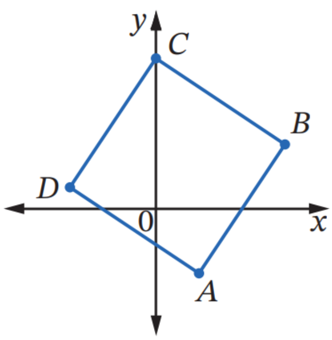
2 The vertices of a rhombus are $D(-4,2), E(1,2), F(4,-2)$ and $G(-1,-2)$

- **a** Show that all side lengths are equal.
- **b** By finding the gradients, show that the opposite sides are parallel.
- **c** Show that the diagonals DF and GE are perpendicular.
- **d** Show that the midpoints of DF and GE intersect at the same point (diagonals bisect).



3 A square has vertices $A(2,-3), B(6,3), C(0,7)$ and $D(-4,1)$

- **a** Show that the diagonals are equal.
- **b** Show that its diagonals meet at right angles.
- **c** Show that the diagonals bisect each other.
- **d** Hence explain why $ABCD$ must be a square.



4 Consider a quadrilateral $ABCD$. $A(4,9), B(0,4), C(-2,3)$ and $D(2,8)$.

a Find the midpoint of the interval joining $A(4,9)$ and $C(-2,3)$.

b Find the midpoint of the interval joining $B(0,4)$ and $D(2,8)$.

c What can you conclude about the diagonals of the quadrilateral?

d Explain why $ABCD$ must be a parallelogram.

5 The points $A(3,1)$, $B(10,2)$, $C(5,7)$ and $D(-2,6)$ are the vertices of a quadrilateral.

a Find the lengths of all four sides.

b Explain why ABCD is a rhombus.

6 Show that $A(2,5)$, $B(3,7)$, $C(-4,-1)$ and $D(-5,2)$ do not form a parallelogram.

7 The quadrilateral ABCD has vertices $A(-1,1), B(3,-1), C(5,3)$ and $D(1,5)$.

a Show that the opposite sides are parallel, and hence that ABCD is a parallelogram.

b Show that AB ⊥ BC, and hence that ABCD is a rectangle.

c Show that AB = BC, and hence that ABCD is a square.

8 A quadrilateral, $ABCD$, has vertices $A(2,7)$, $B(4,9)$, $C(8,5)$ and $D(6,3)$.

a Find the gradient of:

i AB	**ii** BC	**iii** CD	**iv** DA

b What type of quadrilateral is $ABCD$? …………………………………………………………….

c Explain why.

……………………………………………………………………………………………………

9 Find the gradients of these four lines and explain why the quadrilateral formed by the four lines must be a rectangle.

$4x - 3y + 10 = 0, \quad 3x + 4y + 7 = 0, \quad 4x - 3y - 7 = 0, \quad 3x + 4y + 1 = 0$

10 A quadrilateral has vertices $P(-7,2)$, $Q(2,-7)$, $R(5,-4)$ and $S(-4,5)$.

a Find the lengths of PR and QS in surd form.

b Find the midpoints of PR and QS.

c Is PR perpendicular to QS? Explain why.

d What type of quadrilateral is $PQRS$? Explain.

11 A quadrilateral has vertices $C(2,6)$, $D(-5,2)$, $E(-1,-5)$ and $F(6,-1)$

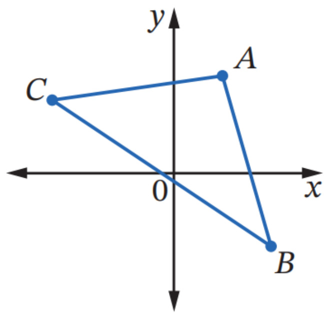
a Find the length of each diagonal.

b Find the midpoint of each diagonal.

c Show that the diagonals are perpendicular.

d What type of quadrilateral is $PQRS$? Explain.

12 $A(2,4), B(4,-3)$, and $C(-5,3)$ are the vertices of a triangle.



a X and Y are midpoints of AB and AC respectively.
Find the coordinates of X and Y.

b Find the gradients of XY and CB.

c What type of quadrilateral is $XYBC$?

d Find the lengths of XY and CB. Hence show that $CB = 2XY$.

13 The quadrilateral $ABCD$ has vertices $A(1,-4), B(3,2), C(-5,6)$ and $D(-1,-2)$.

a Find the midpoints P of AB, Q of BC, R of CD, and S of DA.

b Prove that PQRS is a parallelogram by showing that PQ ∥ RS and PS ∥ QR.

14 Explain why the four lines $l_1: y = x + 1, l_2: y = x - 3, l_3: y = 3x + 5$ and $l_4: y = 3x - 5$ enclose a parallelogram. Then find the vertices of this parallelogram.