

Student Name

Mathematics Stage 4

ANGLE RELATIONSHIPS

Book 4 Solve numerical problems involving
angles using reasoning

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview	2
Problem-solving involving Angles	4
Writing Formal Reasoning	8
Geometry Problems involving Equations	13

OVERVIEW

SYLLABUS CONTENT

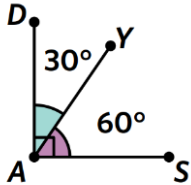
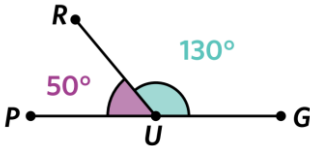
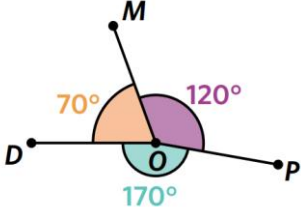
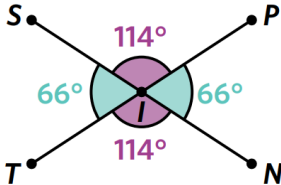
MA4-ANG-C-01 applies angle relationships to solve problems, including those related to transversals on sets of parallel lines

Solve numerical problems involving angles using reasoning

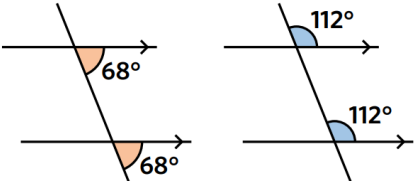
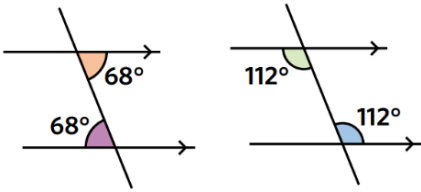
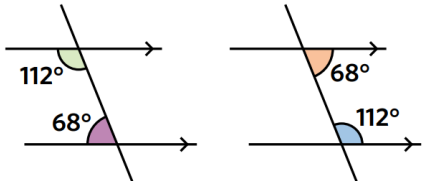
- Apply the knowledge of angle relationships including angles at a point to find the sizes of unknown angles embedded in diagrams and give reasons
- Apply the knowledge of angles associated with parallel lines to find the sizes of unknown angles embedded in related diagrams and give reasons

SUMMARY

- If there are angles at a point:

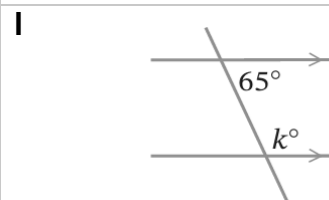
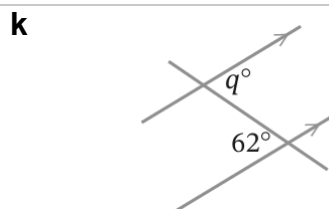
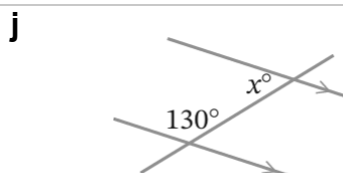
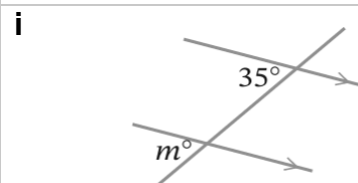
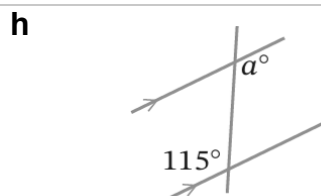
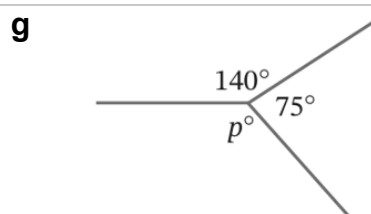
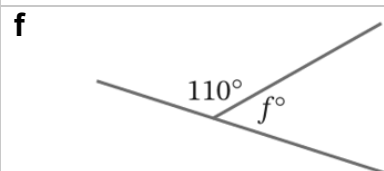
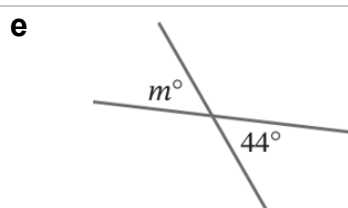
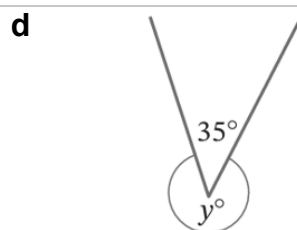
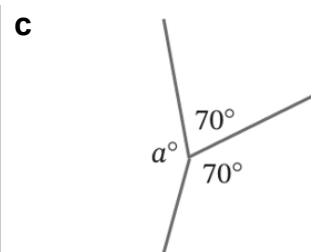
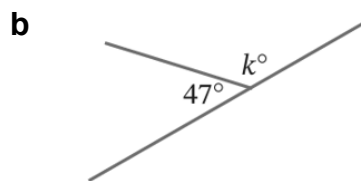
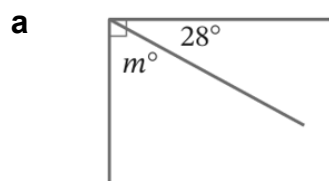
Angles in a right-angle	Angles on a straight line
If angles are within a right angle, then they add up to 90 degrees.	If angles are on a straight-line, then they add up to 180 degrees.
	
Angles at a point	Vertically Opposite Angles
If angles share a common vertex, then they sum to 360°	If angles are vertically opposite, then they are equal
	

- If there are parallel lines:

Corresponding	Alternate	Co-interior
If corresponding angles lie on parallel lines, then they are equal.	If alternate angles lie on parallel lines, then they are equal.	If co-interior angles are between parallel lines, then they sum to 180°.
		

REVIEW OF PRIOR KNOWLEDGE

1 Find the value of each pronumeral, provide reasoning.



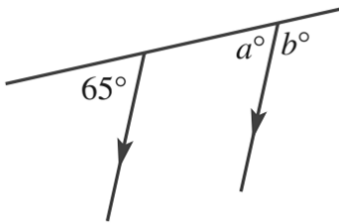
PROBLEM-SOLVING INVOLVING ANGLES

- It is often not possible to find an unknown angle immediately; you need to figure out other information from the diagram.
- You should **never** be guessing or estimating the value of an angle.

FOUNDATION

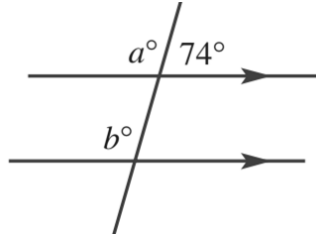
1 Calculate the values of the unknown angles in alphabetical order. Give a brief reason.

a



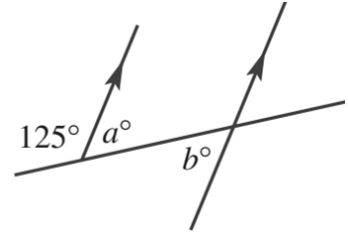
$a = 65$ (corresponding $\angle$ on $\parallel$ lines)
 $b = 115$ (angles on a straight line)

b



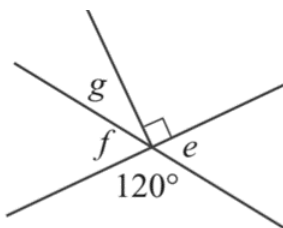
$a = 106$ (angles on a straight line)
 $b = 106$ (corresponding)

c



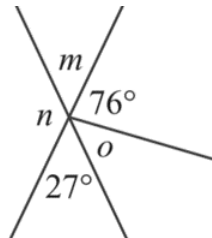
$a = 55$ (angles on a straight line)
 $b = 55$ (alternate $\angle$ on $\parallel$ lines)

d



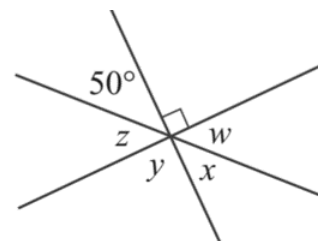
$e = 60$ (angles on a straight line)
 $f = 60$ (vertically opposite)
 $g = 30$ (angles in a right-angle)

e



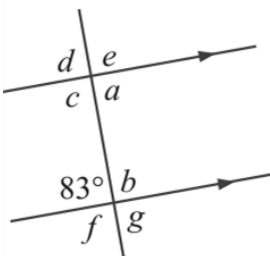
$m = 27$ (vertically opposite)
 $n = 153$ (angles on a straight line)
 $o = 77$ (angles on a straight line)

f



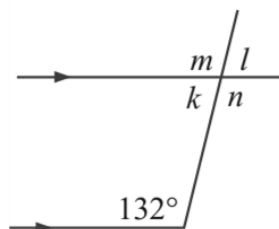
$x = 50$ (vertically opposite)
 $z = 40$ (angles on a straight line)
 $w = 40$ (vertically opposite)
 $y = 90$ (vertically opposite)

g



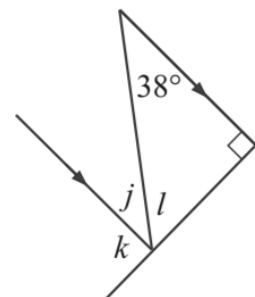
$a = 83$ (alternate $\angle$ on $\parallel$ lines)
 $b = 97$ (angles on a straight line)
 $c = 97$ (alternate to b)
 $d = 83$ (corresponding $\angle$ on $\parallel$ lines)
 $e = 97$ (vertically opposite to c)
 $f = 97$ (vertically opposite to b)
 $g = 83$ (vertically opposite)

h



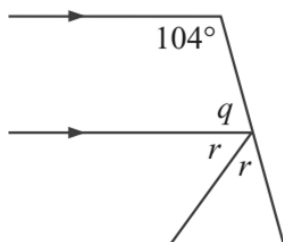
$k = 48$ (co-interior $\angle$ between $\parallel$ lines)
 $l = 48$ (vertically opposite to k)
 $m = 132$ (corresponding $\angle$ on $\parallel$ lines)
 $n = 132$ (vertically opposite to m)

i



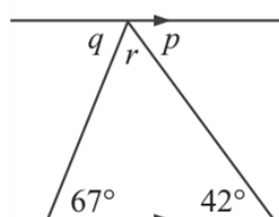
$j = 38$ (alternate $\angle$ on $\parallel$ lines)
 $l = 52$ (angles on a straight line)
 $k = 90$ (corresponding $\angle$ on $\parallel$ lines)

j



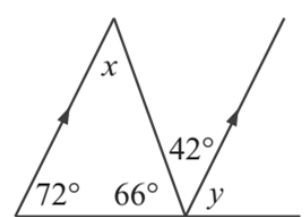
$q = 76$ (co-interior $\angle$ between $\parallel$ lines)
 $2r = 104$ (corresponding $\angle$ on $\parallel$ lines)
 $r = 52$

k



$q = 67$ (alternate $\angle$ on $\parallel$ lines)
 $p = 42$ (alternate $\angle$ on $\parallel$ lines)
 $r = 71$ (angles on a straight line)

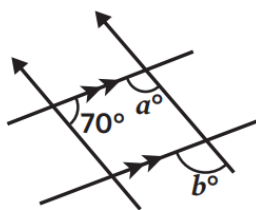
l



$x = 42$ (alternate $\angle$ on $\parallel$ lines)
 $y = 72$ (angles on a straight line)

2 Find the value of the pronumerals. You do not need to give reasoning.

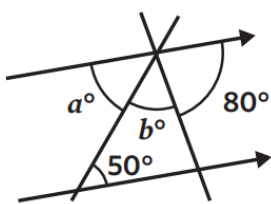
a



$$a = 110$$

$$b = 110$$

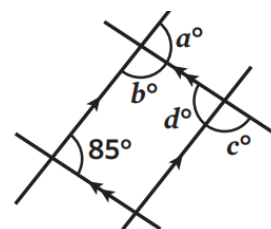
b



$$a = 50$$

$$b = 50$$

c



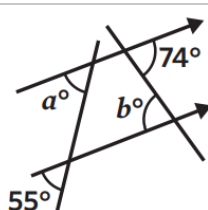
$$b = 95$$

$$a = 85$$

$$d = 85$$

$$c = 95$$

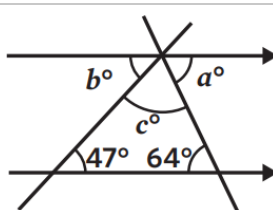
d



$$a = 55$$

$$b = 74$$

e

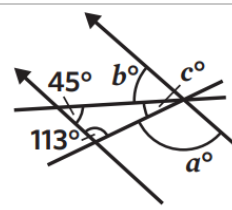


$$b = 47$$

$$a = 64$$

$$c = 69$$

f

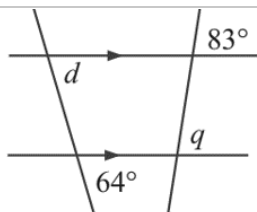


$$a = 113$$

$$b = 45$$

$$c = 22$$

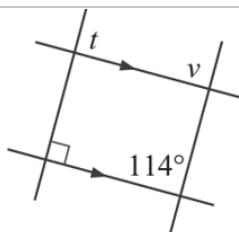
g



$$q = 83$$

$$d = 64$$

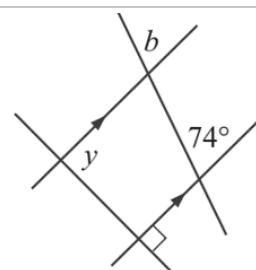
h



$$v = 114$$

$$t = 90$$

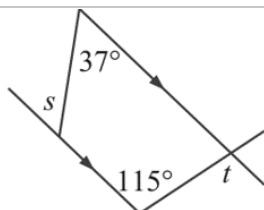
i



$$b = 74$$

$$y = 90$$

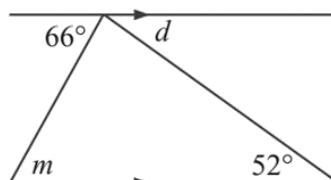
j



$$s = 37$$

$$t = 115$$

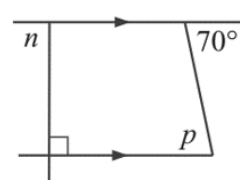
k



$$m = 66$$

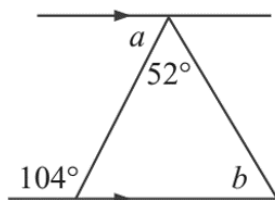
$$d = 52$$

l



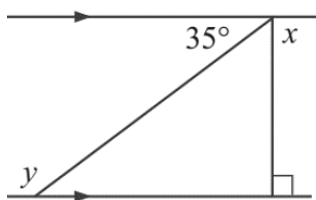
$$p = 70$$

$$n = 90$$

m

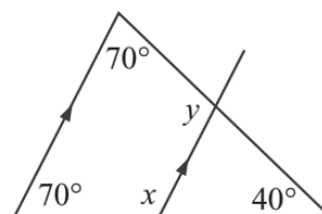
$$a = 76$$

$$b = 52$$

n

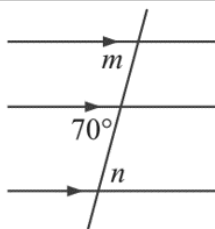
$$x = 90$$

$$y = 145$$

o

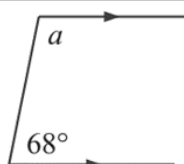
$$y = 110$$

$$x = 110$$

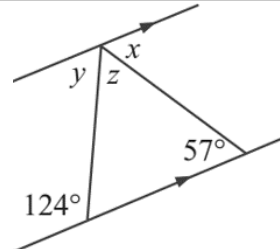
p

$$m = 70$$

$$n = 70$$

q

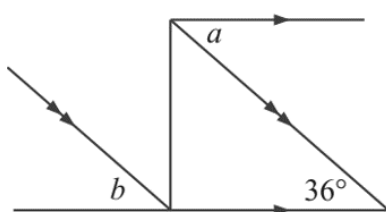
$$a = 112$$

r

$$x = 57$$

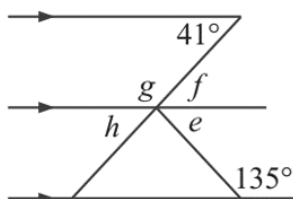
$$y = 56$$

$$z = 67$$

s

$$a = 36$$

$$b = 36$$

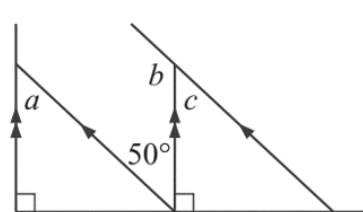
t

$$f = 41$$

$$g = 139$$

$$h = 41$$

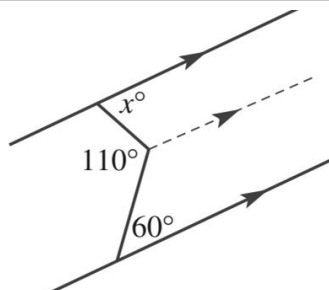
$$e = 45$$

u

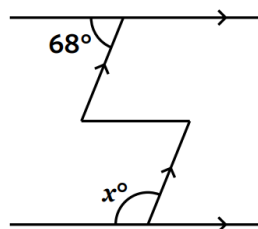
$$a = 50$$

$$b = 130$$

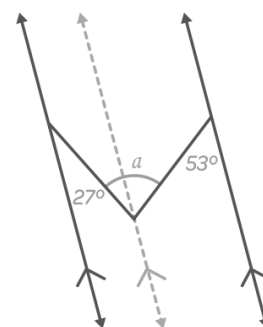
$$c = 50$$

v

$$x = 50$$

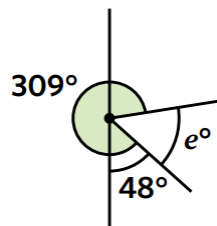
w

$$x = 112$$

x

$$a = 80$$

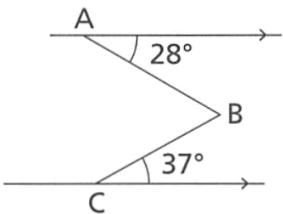
3 Find the value of e . Notice that 309° and 48° are overlapping.



$e = 180 - 48 - 81 = 51$

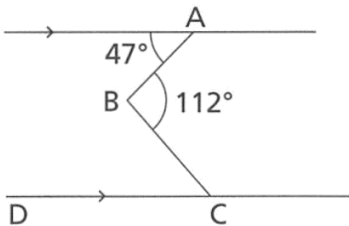
4 Find these angles (hint: construct a parallel line)

a Find the size of $\angle ABC$.



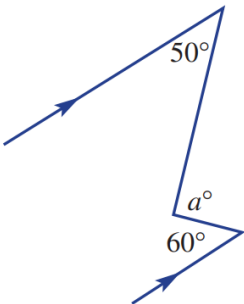
65°

b Find the size of $\angle BCD$.



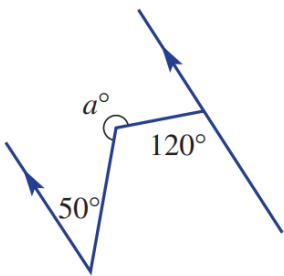
65°

c



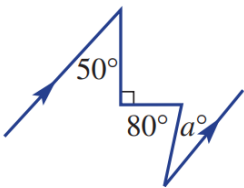
110°

d



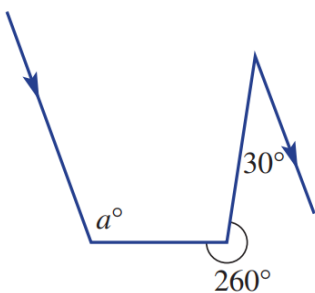
250°

e



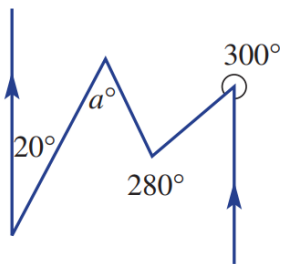
40°

f



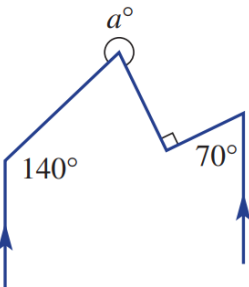
110°

g



40°

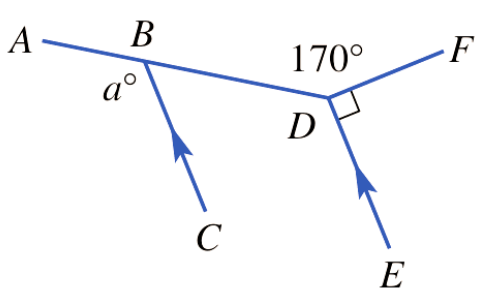
h



300°

WRITING FORMAL REASONING

Example Solve geometry problems and write formal reasoning.



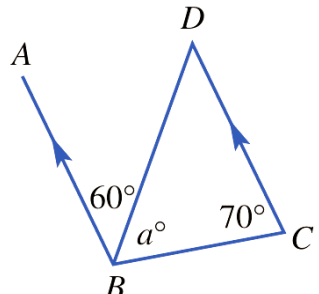
$$\angle BDE + 170 + 90 = 360 \text{ (angles at a point)}$$

$$\angle BDE + 260 = 360$$

$$\angle BDE = 100$$

$$a = 100 \text{ (corresponding angles on } \parallel \text{ lines)}$$

a



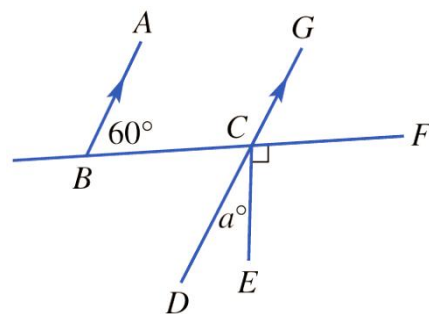
$(60 + a) + 70 = 180^\circ$ (..... angles between $\parallel$ lines)

$$60 + a + 70 = 180^\circ \text{ (co-interior angles between } \parallel \text{ lines)}$$

$$a + 130 = 180$$

$$a = 50^\circ$$

b



$\angle GCF = 60^\circ$ (.....)

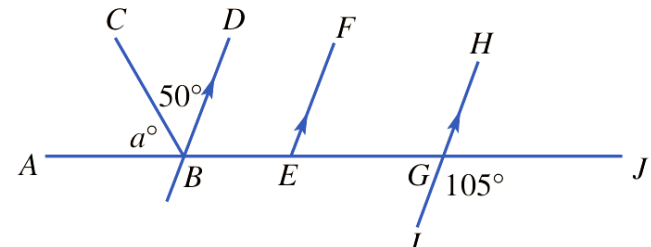
$$\angle GCF = 60^\circ \text{ (corresponding angles on } \parallel \text{ lines)}$$

$$a + 90 + 60 = 180^\circ \text{ (angles on a straight line)}$$

$$a + 150 = 180$$

$$a = 30^\circ$$

c



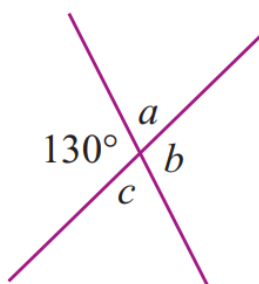
$$\angle HGE = 105^\circ \text{ (vertically opposite)}$$

$$a + 50 = 105^\circ \text{ (corresponding angles on } \parallel \text{ lines)}$$

$$a = 55^\circ$$

- 1 Complete the following to calculate a , b , c .

If you have a choice of reasons, use them to check your answers.

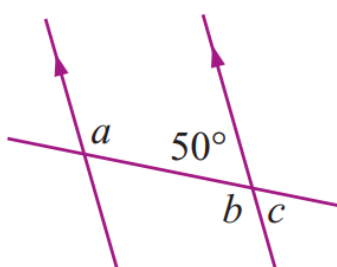


$$\begin{aligned} a + 130^\circ &= \underline{\hspace{2cm}}^\circ && \text{(straight angle)} \\ \therefore a &= \underline{\hspace{2cm}}^\circ \\ b &= 130^\circ && \text{(_____ opposite angles)} \\ c + 130^\circ &= 180^\circ && \text{(_____ angle)} \\ \therefore c &= \underline{\hspace{2cm}}^\circ \\ \text{or } c &= a = \underline{\hspace{2cm}}^\circ && \text{(_____ angles)} \end{aligned}$$

180 | 50 | vertically | straight | 50 | 50 | vertically opposite

- 2 Complete the following to calculate a , b , c .

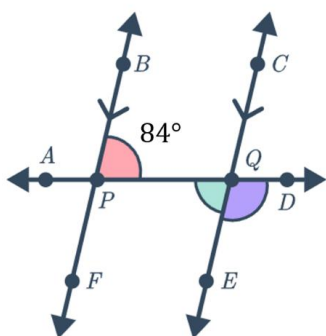
If you have a choice of reasons, use them to check your answers.



$$\begin{aligned} a + 50^\circ &= \underline{\hspace{2cm}}^\circ && \text{(co-interior angles on || lines)} \\ \therefore a &= \underline{\hspace{2cm}}^\circ \\ b + 50^\circ &= \underline{\hspace{2cm}}^\circ && \text{(straight angle)} \\ \therefore b &= \underline{\hspace{2cm}}^\circ \\ \text{or } b &= a = \underline{\hspace{2cm}}^\circ && \text{(_____ angles on || lines)} \\ c &= \underline{\hspace{2cm}}^\circ && \text{(_____ opposite angles)} \\ \text{or } c + b &= 180^\circ && \text{(_____ angle)} \\ \therefore c &= \underline{\hspace{2cm}}^\circ \end{aligned}$$

180 | 130 | 180 | 130 | 130 | alternate | 50 | vertically | straight | 50

- 3 Calculate $\angle AQE$ and $\angle DQE$ giving full reasoning.



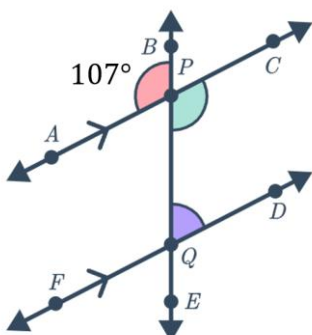
$$\begin{aligned} \angle AQE &= \underline{\hspace{2cm}}^\circ && \text{(..... angles on || lines)} \\ \angle DQE + \underline{\hspace{2cm}}^\circ &= 180^\circ && \text{(angles on a line)} \\ \therefore \angle DQE &= \underline{\hspace{2cm}}^\circ \end{aligned}$$

$\angle AQE = 84^\circ$ (alternate angles on || lines)

$\angle DQE + \angle AQE = 180^\circ$ (angles on a straight line)

$\therefore \angle DQE = 180 - 84 = 96^\circ$

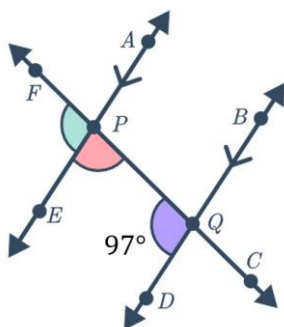
- 4 Calculate $\angle CPE$ and $\angle PQD$ giving full reasoning.



$\angle CPE = 107^\circ$ (vertically opposite)

$\angle PQD = 73^\circ$ (co-interior angles between parallel lines)

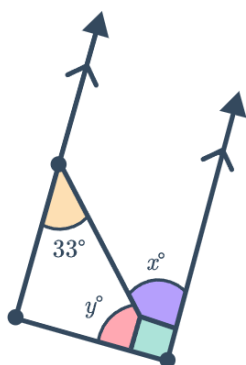
- 5 Calculate $\angle EPQ$ and $\angle EPF$ giving full reasoning.



$\angle EPQ = 83^\circ$ (co-interior angles between parallel lines)

$\angle FPE = 97^\circ$ (corresponding angles on parallel lines)

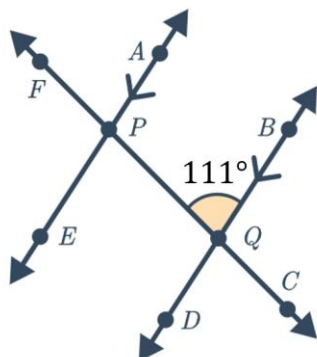
- 6 Calculate x and y giving full reasoning.



$x = 33^\circ$ (alternate angles on parallel lines)

$y = 57^\circ$ (angles in a right-angle)

- 7 Calculate $\angle FPE$ giving full reasoning.

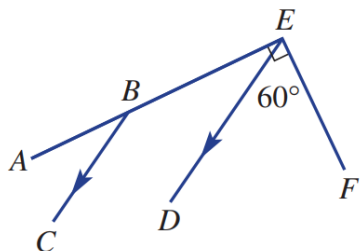


$\angle EPQ = 111^\circ$ (alternate angles on parallel lines)

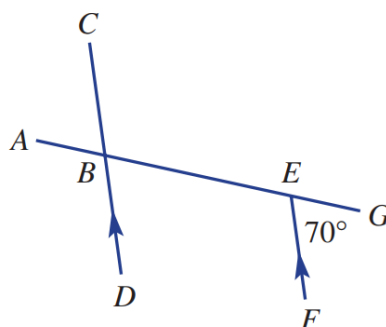
$\angle FPE = 69^\circ$ (angles on a straight line)

- 5 Calculate as many angles as you can in these diagrams. You do not need to show reasoning.

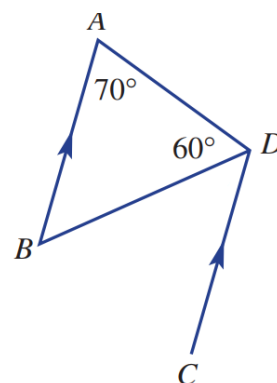
a

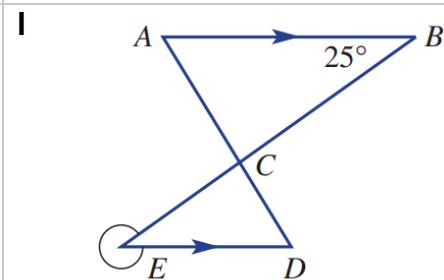
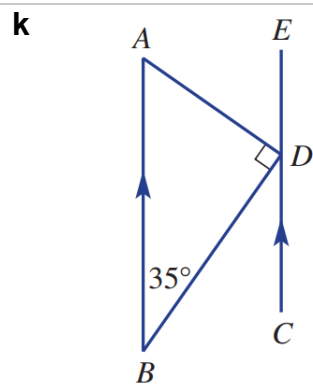
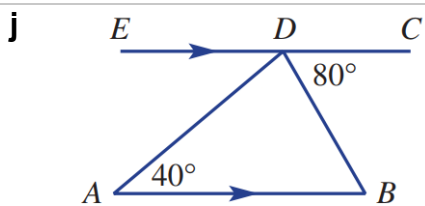
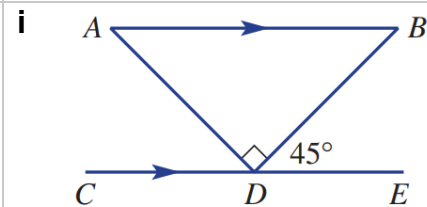
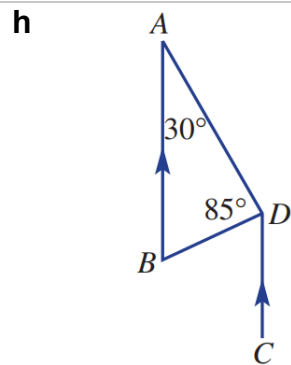
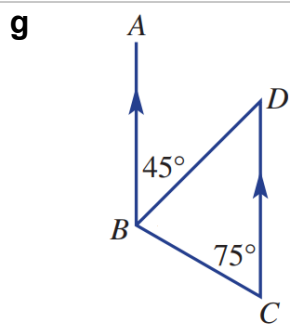
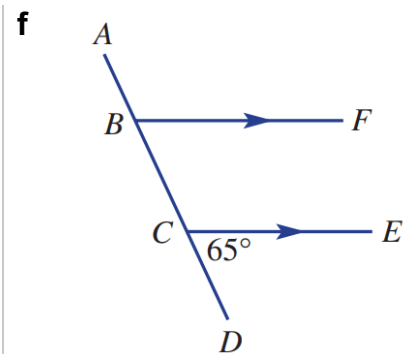
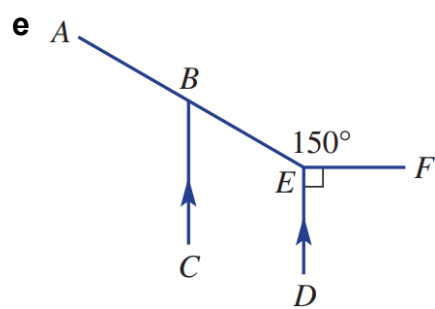
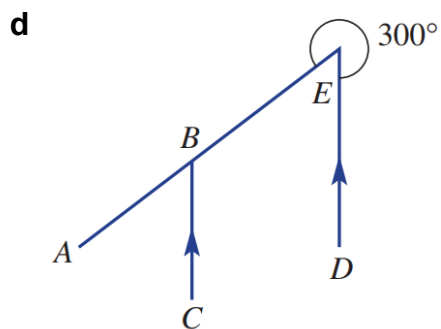


b



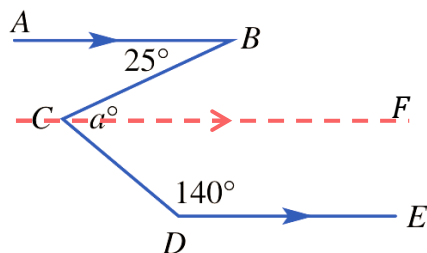
c





8 For these questions, you will need to construct a parallel line. Give full reasoning.

a



Construct CF parallel to AB and DE

$\angle BCF = \dots\dots^\circ$ (alternate angles on $\parallel$ lines)

$\angle FCD + 140 = 180^\circ$ (co-interior angles in $\parallel$ lines)

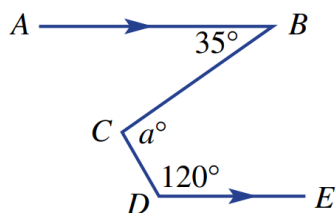
$\angle FCD = \dots\dots\dots$

$\therefore a = \dots\dots + \dots\dots$

$= \dots\dots\dots$

$$25^\circ + 40^\circ = 25^\circ + 40^\circ \mid a = 65^\circ$$

b



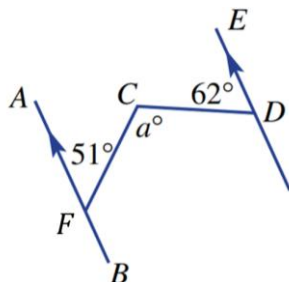
construct CF parallel to AB and DE

$\angle BCF = 35^\circ$ (alternate angles on $\parallel$ lines)

$\angle FCD = 60^\circ$ (co-interior angles in $\parallel$ lines)

$$a = 35 + 60 = 95^\circ$$

c



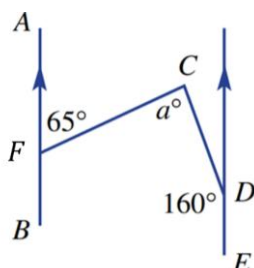
construct CG parallel to AB and DE

$\angle GCF = 51^\circ$ (alternate angles on $\parallel$ lines)

$\angle GCD = 62^\circ$ (alternate angles on $\parallel$ lines)

$$a = 51 + 62 = 113^\circ$$

d



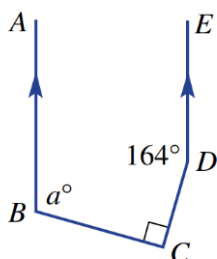
Construct CG parallel to AB and DE

$\angle GCF = 65^\circ$ (alternate angles on $\parallel$ lines)

$\angle GCD = 20^\circ$ (co-interior angles in $\parallel$ lines)

$$a = 65 + 20 = 85^\circ$$

e



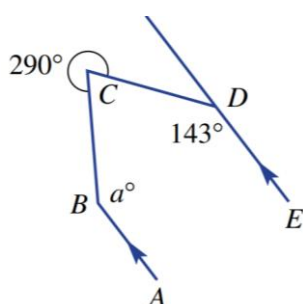
Construct CF parallel to AB and ED

$\angle DCF = 16^\circ$ (co-interior angles in $\parallel$ lines)

$\angle BCF = 74^\circ$ (angles in a right angle)

$$a = 106^\circ \text{ (co-interior angles in } \parallel \text{ lines)}$$

f



Construct CF parallel to AB and DE

$\angle DCF = 37^\circ$ (co-interior angles in $\parallel$ lines)

$\angle BCF = 33^\circ$ (angles at a point)

$$a = 147^\circ \text{ (co-interior angles in } \parallel \text{ lines)}$$

GEOMETRY PROBLEMS INVOLVING EQUATIONS

FOUNDATION

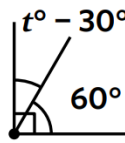
- 1 Use angle facts to form an equation for each diagram.
Then solve your equations to find the value of each letter.

a

$$t - 30 + 60 = 90$$

$$t + \dots = 90$$

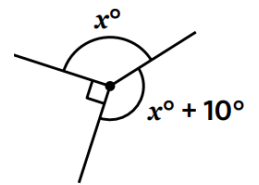
$$t = \dots$$



$$t = 60^\circ$$

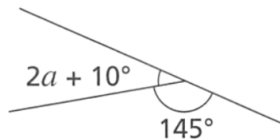
b

$$x + x + 10 + 90 = \dots$$



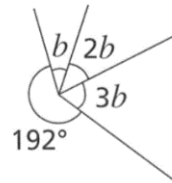
$$x = 130^\circ$$

c



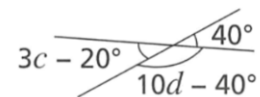
$$a = 12.5^\circ$$

d



$$b = 28^\circ$$

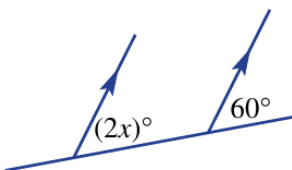
e



$$c = 20^\circ$$

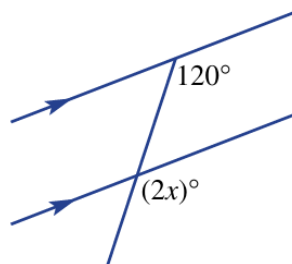
$$d = 18^\circ$$

f



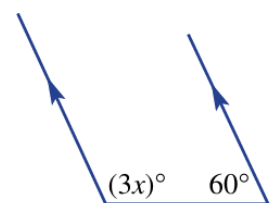
$$x = 30$$

g



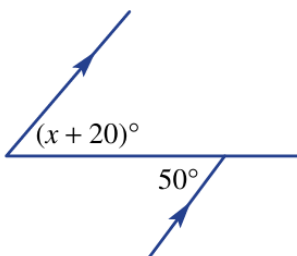
$$x = 60$$

h



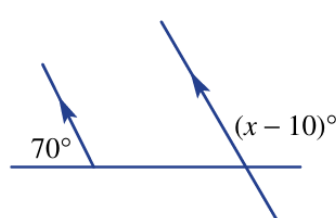
$$x = 40$$

i



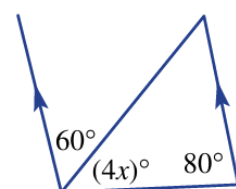
$$x = 30$$

j



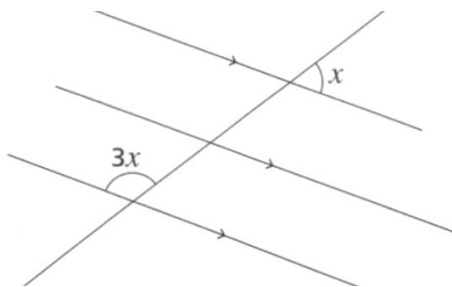
$$x = 120$$

k



$$x = 10$$

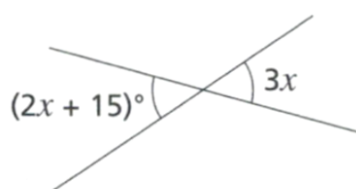
- 2 Work out the value of x .



$$x = 45^\circ$$

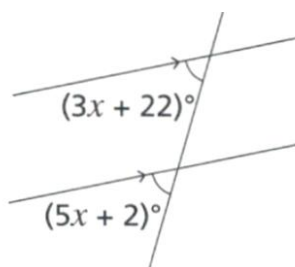
- 3 Work out value of x in these diagrams.

a



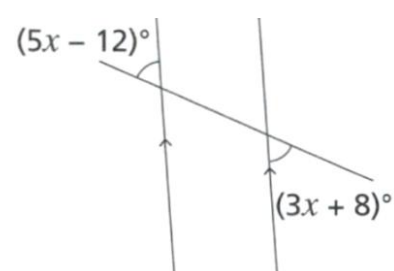
$$x = 15^\circ$$

b



$$x = 10^\circ$$

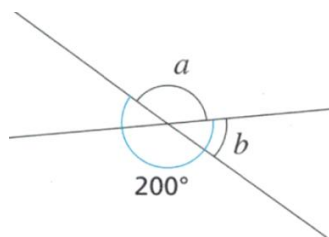
c



$$x = 10^\circ$$

- 4 Determine the value of each pronumeral.

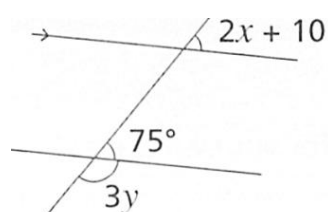
a



$$a = 160^\circ$$

$$b = 20^\circ$$

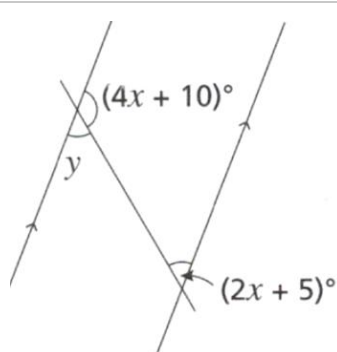
b



$$x = 32.5^\circ$$

$$y = 35^\circ$$

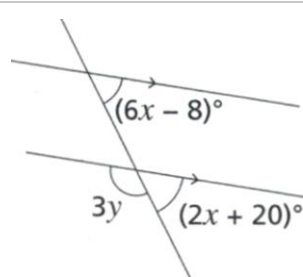
c



$$x = 27.5^\circ$$

$$y = 60^\circ$$

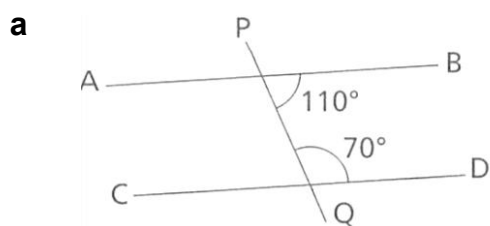
d



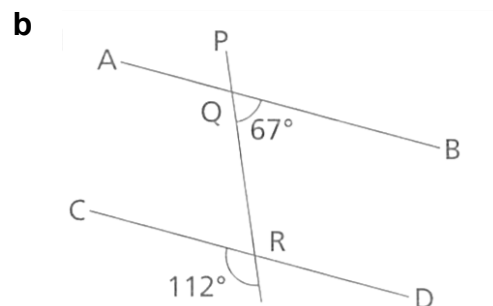
$$x = 7^\circ$$

$$y = 48\frac{2}{3}^\circ$$

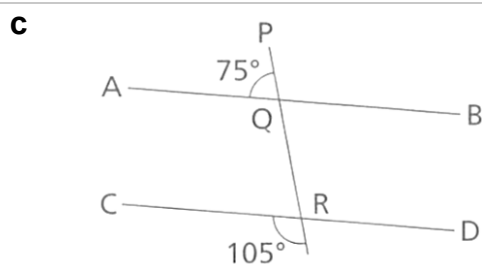
5 In each case, decide if AB is parallel to CD. Give a reason why or why not.



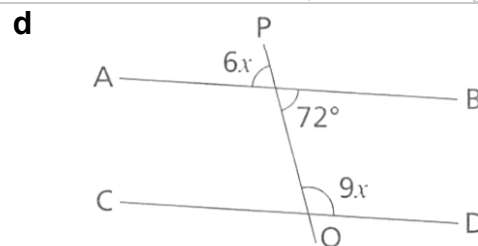
Yes, co-interior angles sum to 180.



$\angle PRD = 112^\circ$ (vertically opposite)
 $\therefore$ no, as co-interior angles don't sum to 180

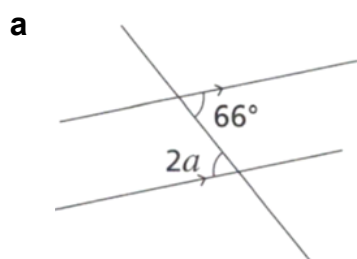


$\angle AQR = 105^\circ$ (angles on a straight line)
 $\therefore$ yes as corresponding angles are equal.

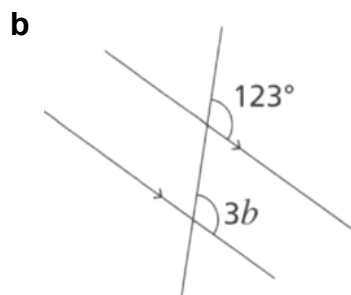


$6x = 72$
 $x = 12$
 $\angle PQD = 9x = 108^\circ$
 $\therefore$ yes as co-interior angles sum to 180

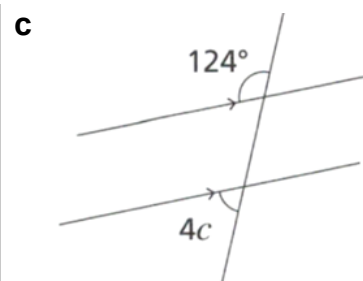
6 Work out the value of the pronumerals in each of these diagrams.



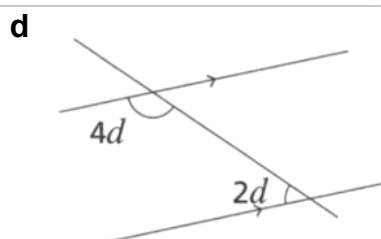
$a = 33^\circ$



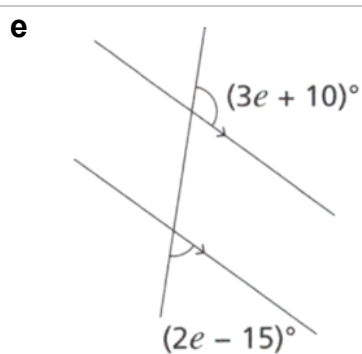
$b = 41^\circ$



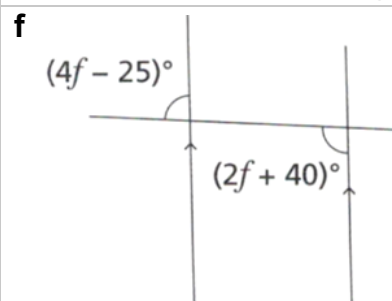
$c = 31^\circ$



$d = 30^\circ$

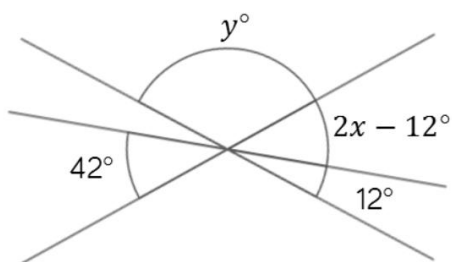


$e = 37$



$f = 27.5^\circ$

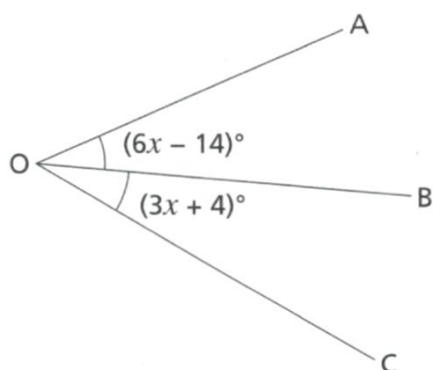
- 7 Calculate the values of x and y .



$$x = 32$$

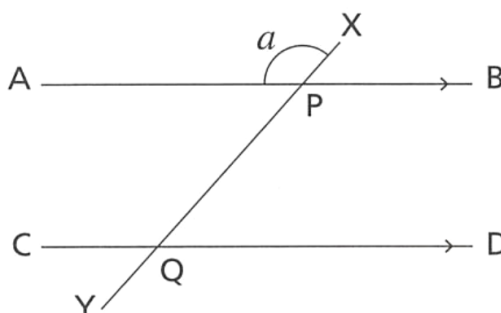
$$y = 116^\circ$$

- 8 OB is the bisector of $\angle AOC$. (in other words, OB splits $\angle AOC$ into equal halves).
Work out the size of $\angle AOC$



$$\angle AOC = 44^\circ$$

- 9 Consider the diagram below.



- a Prove that $\angle CQY = 180^\circ - a$

$$\angle CPX = a \text{ (corresponding angles on } \parallel \text{ lines)}$$

$$\angle CQY = 180^\circ - a \text{ (angles on a straight line)}$$

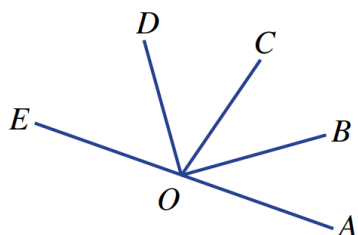
- b Prove that $\angle DQY = a$

$$\angle CQY + \angle DQY = 180^\circ$$

$$180^\circ - a + \angle DQY = 180^\circ$$

$$\therefore \angle DQY = a$$

- 10 In this diagram $\angle AOB = \angle BOC$ and $\angle COD = \angle DOE$. Give reasons why $\angle BOD = 90^\circ$



$$\angle AOB + \angle BOC + \angle COD + \angle DOE = 180^\circ \text{ (angles on a straight line)}$$

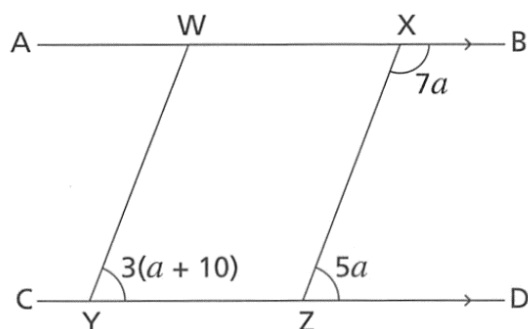
Since $\angle AOB = \angle BOC$ and $\angle COD = \angle DOE$, we can write

$$2\angle BOC + 2\angle COD = 180^\circ$$

$$\angle AOB + \angle COD = 90^\circ$$

$$\therefore \angle BOD = 90^\circ$$

- 11 AB is parallel to CD. Prove that WY is parallel to XZ.



If parallel, then both these equations must be true:

$$3(a + 10) = 5a \text{ (corresponding angles on } \parallel \text{ lines) and}$$

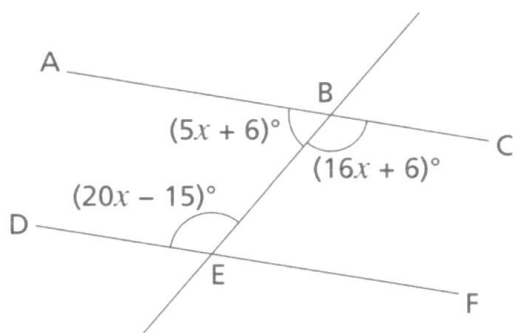
$$5a + 7a = 180 \text{ (co-interior angles in } \parallel \text{ lines)}$$

After we solve for the value of a in the first equation, if this value also works for the second equation, then the lines must be parallel

By solving $3(a + 10) = 5a$ we get $a = 15$

$a = 15$ works for $5a + 7a = 180$, so both lines are parallel

- 12 ABC is a straight line. DEF is a straight line. Is ABC parallel to DEF?
Show working to explain how you know.



If parallel, then both these equations must be true:

$$20x - 15 = 16x + 6 \text{ (alternate angles on } \parallel \text{ lines) and}$$

$$20x - 15 + 5x + 6 = 180 \text{ (co-interior angles in } \parallel \text{ lines)}$$

By solving $20x - 15 = 16x + 6$, we get $a = \frac{21}{4}$

$a = \frac{21}{4}$ does not satisfy $20x - 15 + 5x + 6 = 180$, so not parallel.