Student Name

Mathematics Stage 5 Paths

EQUATIONS B

Book 1 Solve monic quadratic equations

Version: 260207

Report mistakes & suggest improvements:
https://MrDingMaths.com

CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-EQU-P-01 solves monic quadratic equations, linear inequalities and cubic equations of the form $ax^3 = k$

Solve monic quadratic equations

- Solve quadratic equations of the form $ax^2 + bx + c = 0$, limited to $a = 1$, using factors

REVIEW OF PRIOR KNOWLEDGE

1 Factorise these expressions.

a	$y^2 - 7y$	**b**	$x^2 + 36x$	**c**	$-x^2 - 36x$	**d**	$-x^2 + 12x$
e	$4x^2 + 4x$	**f**	$-8x^2 + 4x$	**g**	$9x - 3x^2$	**h**	$12x - 18x^2$

2 Factorise these expressions.

a	$x^2 + 4x + 3$	**b**	$y^2 - 7y + 10$	**c**	$x^2 + 6x + 9$
d	$x^2 - x - 6$	**e**	$a^2 + 9x + 20$	**f**	$4x^2 - 20x + 24$

SOLVING QUADRATIC EQUATIONS USING NULL FACTOR LAW

Null Factor Law	
If $a \times b = 0$	If $(x + a)(x + b) = 0$
Then $a = 0$ or $b = 0$	Then $x + a = 0$
	or $x - b = 0$

Example Identify the solution of a quadratic equation using the null factor law.

a $x(x + 3) = 0$ $x = 0$ or $x + 3 = 0$ $x = \ldots\ldots$ $x = 0$ or -3	**g** $(x - 6)(x - 3) = 0$ $x = 6$ or 3
b $x(x + 6) = 0$ $x = 0$ or -6	**h** $(x - 6)(x - 2) = 0$ $x = 6$ or 2
c $(x + 6)(x + 3) = 0$ $x = -6$ or -3	**i** $(x + 3)^2 = 0$ $x = -3$
d $(x + 3)(x + 6) = 0$ $x = -3$ or -6	**j** $(x + 3)(x + 3) = 0$ $x = -3$
e $(x - 3)(x + 6) = 0$ $x = 3$ or -6	**k** $2x(x - 5) = 0$ $x = 0$ or 5
f $(x - 6)(x + 6) = 0$ $x = 6$ or -6	**l** $2(x + 3) = 0$ $x = -3$

SOLVING QUADRATIC BINOMIAL EQUATIONS

- To solve binomial equations:
  1. Write equation in the form $ax^2 + bx = 0$, if it isn't already.
  2. Factorise the expression.
  3. Use the **null factor law** to solve for x.

Example Solve quadratic equations of the form $ax^2 + bx = 0$.

$y^2 - 7y = 0$ 2. $y(y-7) = 0$ 3. $y = 0$ or $y - 7 = 0$ $y = 0$ $\quad$ $y = 7$	What is the null factor law? *The null factor law states that if two factors multiply to make ……., then one or both factors must be …….*
$x^2 = 36x$ 1. $x^2 - 36x = 0$ 2. $x(x-36) = 0$ 3. $x = 0$ or $x - 36 = 0$ $x = 0$ $\quad$ $x = 36$	What did we have to do to first to solve this equation? Why? *We had to make the RHS = ……. to be able to use the ……. ……. law.*

Solve these quadratic equations.

a $x^2 - 5x = 0$ $x(x-5) = 0$ $x = 0$ or 5	**b** $x^2 + 4x = 0$ $x = 0$ or -4	**c** $2x^2 = 8x$ $x = 0$ or 4
d $x^2 = 16$ $x = -4$ or 4	**e** $x^2 + 16 = 0$ no real solution	**f** $3x^2 = 18x$ $x = 0$ or 6

Key ideas

1. The null factor law states that if the ……. of two or more factors is zero, then one or both of the factors must be …….
2. To solve a quadratic equation that in the form of $ax^2 + bx = 0$, we ……. the LHS then use the null factor law.

1 Use the null factor law to write down the solutions to these equations.

a $x(x-3)=0$

$x =$ ..... or .....

$x=0$ or 3

b $4x(x+1)=0$

$x =$ ..... or .....

$x=0$ or -1

c $-7x(x+2)=0$

$x =$ ..... or .....

$x=0$ or -2

d $(x+1)(x-1)=0$

$x =$ ..... or .....

$x=-1$ or 1

e $(x+5)(x-5)=0$

$x =$ ..... or .....

$x=-5$ or 5

f $(2x+1)(2x-1)=0$

$x =$ ..... or .....

$x=-\frac{1}{2}$ or $\frac{1}{2}$

2 Solve each of these equations.

a $x^2+3x=0$

$x(x+3)=0$

$x =$ ..... or .....

$x=0$ or -3

b $x^2+7x=0$

$x=0$ or -7

c $x^2+4x=0$

$x=0$ or -4

d $x^2-5x=0$

$x=0$ or 5

e $x^2-2x=0$

$x=0$ or 2

f $x^2+\frac{1}{3}x=0$

$x=0$ or $-\frac{1}{3}$

3 Solve these equations by first taking out the highest common factor.

a $2x^2-6x=0$

$2x(x-3)=0$

$x =$ ..... or .....

$x=0$ or 3

b $3x^2-12x=0$

$x=0$ or 4

c $4x^2+20x=0$

$x=0$ or -5

d $6x^2-18x=0$

$x=0$ or 3

e $-5x^2+15x=0$

$x=0$ or 3

f $-2x^2-8x=0$

$x=0$ or -4

g $2x^2-2x=0$

$x=0$ or 1

h $5x-x^2=0$

$x=0$ or 5

i $3x^2+12x=0$

$x=0$ or -4

j $-x^2+2x=0$

$x=0$ or 2

k $-x^2-4x=0$

$x=0$ or -4

l $-8x^2-16x=0$

$x=0$ or -2

4 Solve each of the following equations.

a $x^2 = 3x$

$x = 0$ or 3

b $5x^2 = 10x$

$x = 0$ or 2

c $4x^2 = 16x$

$x = 0$ or 4

d $3x^2 = -9x$

$x = 0$ or -3

e $2x^2 = -8x$

$x = 0$ or -4

f $7x^2 = -21x$

$x = 0$ or -3

5 Consider two methods of solving $x^2 - 4 = 0$

Method 1: Isolate x^2 and square root

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm\sqrt{4} = \pm 2$$

Method 2: Factorise and use null factor law

$$(x + 2)(x - 2) = 0$$
$$x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -2 \qquad x = 2$$

Which method was more efficient for this question?

First method; if the equation has only an x^2 term and no x term, its probably easier to isolate and square root.

6 Solve these equations.

a $4 = x^2$

$x = \pm 2$

b $-x^2 + 25 = 0$

$x = \pm 5$

c $-x^2 = -100$

$x = \pm 10$

d $-3x^2 = 21x$

$x = 0$ or -7

e $-5x^2 + 35x = 0$

$x = 0$ or 7

f $1 - x^2 = 0$

$x = \pm 1$

g $12x = 18x^2$

$x = 0$ or $\frac{2}{3}$

h $2x^2 + x = 7x^2 - 2x$

$x = 0$ or $\frac{3}{5}$

i $9 - 2x^2 = x^2$

$x = \pm\sqrt{3}$

7 Write an equation and solve it to find the number.

a The square of the number is 7 times the same number

$x^2 = 7x$
$x = 0$ or 7

b The difference between the square of a number and 64 is zero.

$x^2 - 64 = 0$
$x = \pm 8$

c 3 times the square of a number is equal to −12 times the number.

$3x^2 = -12x$
$x = 0$ or -4

8 An equation of the form $ax^2 + bx = 0$ will always have two solutions if a and b are not zero.

a By factorising the LHS, explain why one of the solutions will always be $x = 0$.

$x(ax + b) = 0$, so x is always a solution

b What would the other solution be, in terms of a and b?

$ax + b = 0; x = -\frac{b}{a}$

9 Solve the following equations.

a $x - \frac{4}{x} = 0$

$x = \pm 2$

b $\frac{36}{x} - x = 0$

$x = \pm 6$

c $\frac{3}{x^2} = 3$

$x = \pm 1$

d $5(x^2 + 1) = 3x^2 + 7$

$x = \pm 1$

e $x(x - 3) = 2x^2 + 4x$

$x = 0$ or -7

f $3x(5 - x) = x(7 - x)$

$x = 0$ or 4

SOLVING MONIC QUADRATIC TRINOMIAL EQUATIONS

- To solve quadratic equations that are monic quadratic trinomials:
  1. Write equation in the form $x^2 + bx + c = 0$, if it isn't already.
  2. Factorise the expression using the PSF method.
  3. Use null factor law to solve for x.

Example Solve quadratic equations of the form $x^2 + bx + c = 0$.

$x^2 + x - 6 = 0$ 2. $(x + 3)(x - 2) = 0$ 3. $x = -3$ or 2	How do we factorise quadratic trinomials? *Think of two numbers that ..... to make the constant term and ..... to make the coefficient of x* Why is the solution -3 or 2?
$x^2 - 13x = -36$ 1. $x^2 - 13x + 36 = 0$ 2. $(x - 4)(x - 9) = 0$ 3. $x = 4$ or 9	*Using the null factor law,* $x + 3 =$ ..... *or* $x - 2 =$ ..... $x =$ ..... $x =$ ..... Why is it important that RHS = 0? *We need RHS = 0 to use the ....*

Solve for x.

a $x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x =$ ... or ...

$x = -3$ or -1

b $x^2 - 7x + 10 = 0$

$x = 5$ or 2

c $x^2 = x + 6$

$x = 3$ or -2

d $x^2 - 8x + 16 = 0$

$x = 4$

e $x^2 + 4x = 5$

$x = -5$ or 1

f $x^2 + x = 30$

$x = 5$ or -6

Key ideas

1. The null factor law states that if the ..... of two or more expressions is zero, then one of the expressions must be zero.
2. To solve a quadratic equation that in the form of $x^2 + bx + c = 0$, we ..... the LHS using the ..... method, then use the null factor law.
3. For the null factor law to work, the RHS must be equal to .....

1 Fill in the table.

General Form	Product	Sum	Factors	Factorised Form	Solutions
$x^2+3x+2=0$	2	3	1 & 2	$(x+1)(x+2)=0$	$x=-1,-2$
$x^2+4x+4=0$	4	4	2, 2	$(x+2)(x+2)=0$	$x=-2$ only
$x^2+5x+6=0$	6	5	2, 3	$(x+2)(x+3)=0$	$x=-2,-3$
$x^2+10x+24=0$	24	10	4, 6	$(x+4)(x+6)=0$	$x=-4,-6$
$x^2+6x+9=0$	9	6	3, 3	$(x+3)^2=0$	$x=-3$ only
$x^2+12x+36=0$	36	12	6, 6	$(x+6)^2$	$x=-6$ only
$x^2+2x-3=0$	–3	2	–1, 3	$(x-1)(x+3)=0$	$x=1,-3$
$x^2-2x-3=0$	–3	–2	–3, 1	$(x-3)(x+1)=0$	$x=3,-1$
$x^2-4x+3=0$	3	–4	–1, –3	$(x-1)(x-3)=0$	$x=1,3$
$x^2-5x+6=0$	6	–5	–2, –3	$(x-2)(x-3)=0$	$x=2,3$
$x^2-x-6=0$	–6	–1	–3, 2	$(x-3)(x+2)=0$	$x=3,-2$

2 Solve these quadratic equations.

a $x^2+8x+12=0$

$(x+6)(x+2)=0$

$x=$ ..... or .....

$x=-6,-2$

b $x^2+7x+10=0$

$x=-2,-5$

c $x^2+5x-14=0$

$x=-7,2$

d $x^2+7x-30=0$

$x=-10,3$

e $x^2-12x+32=0$

$x=4,8$

f $x^2-10x+21=0$

$x=3,7$

g $x^2-2x-15=0$

$x=5,-3$

h $x^2-4x-45=0$

$x=9,-5$

i $x^2-10x+24=0$

$x=4,6$

j $x^2+5x-84=0$

$x=-12,7$

k $x^2+4x+3=0$

$x=-3,-1$

l $x^2-12x+20=0$

$x=10,2$

3 Solve these equations.

a $x^2 + 9x + 20 = 0$

$x = -5, -4$

b $x^2 + 12x + 20 = 0$

$x = -10, -2$

c $x^2 - 9x + 20 = 0$

$x = 4, 5$

d $x^2 - 12x + 20 = 0$

$x = 10, 2$

e $21x + 20 + x^2 = 0$

$x = -20, -1$

f $x^2 + 9x = -20$

$x = -4, -5$

g $x^2 + 21x = -20$

$x = -20, -1$

h $x^2 + 20 = 9x$

$x = 4, 5$

i $x^2 = 12x - 20$

$x = 10, 2$

j $20 = 9x - x^2$

$x = 4, 5$

4 Solve $x^2 + 16x + 64 = 0$ and explain why it has only one solution.

This is a perfect square; $(x + 8)^2 = 0$, so it only has one solution $x = -8$

5 Solve these quadratic equations.

a $x^2 + 6x + 9 = 0$

$x = -3$

b $x^2 + 4x + 4 = 0$

$x = -2$

c $x^2 + 14x + 49 = 0$

$x = -7$

d $x^2 + 24x + 144 = 0$

$x = -12$

e $x^2 - 10x + 25 = 0$

$x = 5$

f $x^2 - 16x + 64 = 0$

$x = 8$

g $x^2 - 12x + 36 = 0$

$x = 6$

h $x^2 - 18x + 81 = 0$

$x = 9$

i $x^2 + 18x + 81 = 0$

$x = -9$

6 Fill in the table.

General Form	Sum	Product	Factors	Factorised Form	Solutions
$x^2 + x - 6 = 0$	1	-6	3, –2	$(x+3)(x-2)=0$	$x = -3, 2$
$x^2 - 4x = 0$	–4	0	0, –4	$x(x-4)=0$	$x = 0, x = 4$
$x^2 - 8x + 16 = 0$	–8	16	–4, –4	$(x-4)^2 = 0$	$x = 4$ only
$x^2 - 16x + 64 = 0$	–16	64	8, 8	$(x-8)^2 = 0$	$x = 8$ only
$x^2 - 4 = 0$	0	–4	–2, 2	$(x-2)(x+2)=0$	$x = \pm 2$
$x^2 - 6x + 9 = 0$	–6	9	–3, –3	$(x-3)^2 = 0$	$x = 3$ only
$x^2 - 12x + 36 = 0$	–12	36	–6, –6	$(x-6)^2 = 0$	$x = 6$ only
$x^2 - 9 = 0$	0	–9	–3, 3	$(x-3)(x+3)=0$	$x = 3, x = -3$
$x^2 - 2x + 1 = 0$	–2	1	–1, –1	$(x-1)^2 = 0$	$x = 1$ only
$x^2 - 1 = 0$	0	–1	1, –1	$(x-1)(x+1)=0$	$x = \pm 1$

7 The following working out is **incorrect**.

Solve:

$$x^2 - 2x - 8 = 7$$
$$(x-4)(x+2) = 7$$
$$x - 4 = 7 \text{ or } x + 2 = 7$$
$$x = 11 \text{ or } x = 5$$

The factorisation is correct.

Writing $x - 4 = 7$ and $x + 2 = 7$ is incorrect. Why?

We can only use the null factor law if RHS=0

What do you need to do before you factorise?

Rearrange so all terms are on one side

Hence solve $x^2 - 2x - 8 = 7$ correctly.

$x = 5, -3$

8 Solve these quadratic equations by first rearranging to $ax^2 + bx + c = 0$

a $x^2 = 3x + 10$

$x = 5, -2$

b $x^2 = 7x - 10$

$x = 5, 2$

c $x^2 = 6x - 9$

$x = 3$

d $x^2 = 4 - 3x$

$x = -4.1$

e $14 - 5x = x^2$

$x = -7, 2$

f $x^2 + 16 = 8x$

$x = 4$

g $x^2 - 12 = -4x$

$x = -6, 2$

h $6 - x^2 = 5x$

$x = -6, 1$

i $15 = 8x - x^2$

$x = 3, 5$

9 The temperature in °C inside a room at a base in Antarctica is given by the expression $t^2 - 9t + 8$, where t is hours after 10 am.
Find the times in the day when the temperature is 0°C.

$t^2 - 9t + 8 = 0$

$(t - 8)(t - 1) = 0$

$t = 1, 8$

Temperatures are at 0 at 11 am and 6 pm

10 Solve these equations by factorising out a common factor.

a $2x^2 - 2x - 12 = 0$

$x = 3, -2$

b $4x^2 - 24x - 64 = 0$

$x = 8, -2$

c $4x^2 - 20x + 24 = 0$

$x = 3, 2$

d $3x^2 + 6x + 3 = 0$

$x = -1$

e $7x^2 - 70x + 175 = 0$

$x = 5$

f $5x^2 = 35 - 30x$

$x = 7, 1$

11 Solve these equations by first writing in the form $x^2 + bx + c = 0$.

a $x^2 = 5(x - 1.2)$

$x = 2, 3$

b $2x(x - 3) = x^2 - 9$

$x = 3$

c $x(x + 6) = 4(x + 1) - x$

$x = -4, 1$

d $\frac{5x+84}{x} = x$

$x = 12, -7$

e $\frac{9x+70}{x} = x$

$x = 14, -5$

f $x - 2 = \frac{35}{x}$

$x = 7, -5$

g $x + 3 = -\frac{2}{x}$

$x = -1, -2$

h $-\frac{1}{x} = x - 2$

$x = 1$

i $-\frac{8}{x} = 2x - 10$

$x = 1, 4$

j $\frac{x^2-48}{x} = 2$

$x = 8, -6$

k $\frac{x^2+12}{x} = -8$

$x = -2, -6$

l $\frac{x-5}{4} = \frac{6}{x}$

$x = 8, -3$

m $\frac{x-4}{2} = \frac{5}{2x}$

$x = 5, -1$

n $\frac{x+4}{2} - \frac{3}{x-3} = 1$

$x = 4, -3$

o $\frac{x}{x-2} - \frac{x+1}{x+4} = 1$

$x = 5, -2$

CHECK YOUR UNDERSTANDING

Solving monic quadratic binomials

1 What should replace the star?

$$(h+4)(h-3)=0$$

$$h = \bigstar \quad \text{or} \quad h = 3$$

a 4 **b** -4 **c** 3 **d** 0

2 $x^2 + 4x = 0$

Tom says the solution is $x = 2$

Katie says the solution is $x = 0$

Who is correct?

a Only Tom **b** Only Katie **c** Both **d** Neither

3 $k^2 - 8k = 0$

a $k = -8$ or 8 **b** $k = \sqrt{8}$ **c** $k = 8$ **d** $k = 0$ or 8

Solving monic quadratic trinomials

1 $y^2 + 9y = 10$

Tom says the first line should be $y + 3y = \sqrt{10}$

Katie says the first line should be $y^2 + 9y - 10 = 0$

a Only Tom **b** Only Katie **c** Both **d** Neither

2 Isaac is trying to solve this equation: $\boldsymbol{p^2 + 3p - 18 = 0}$

Here are his steps.

Which steps, if any, are correct?

Step 1 $(p + 6)(p - 3) = 0$

Step 2 $p = 6$ or $p = -3$

a Only step 1 **b** Only step 2 **c** Both **d** Neither

3 Olivia is trying to solve this equation: $\boldsymbol{p^2 - p - 12 = 0}$

Here are her steps.

Which steps, if any, are correct?

Step 1 $(p + 4)(p - 3) = 0$

Step 2 $p = -4$ or $p = 3$

a Only step 1 **b** Only step 2 **c** Both **d** Neither

4 Olivia is trying to solve this equation: $\boldsymbol{p^2 - p - 12 = 0}$

Here are her steps.

Which steps, if any, are correct?

Step 1 $(p + 4)(p - 3) = 0$

Step 2 $p = -4$ or $p = 3$

a Only step 1 **b** Only step 2 **c** Both **d** Neither

5 Emily is trying to solve the equation by factorising. $\boldsymbol{p^2 + 7p + 10 = 4}$

What mistake has she made?

Step 1 $(p + 5)(p + 2) = 4$

a She should have square rooted everything

b She should have subtracted 10 from both sides before factorising

c Incorrect factorisation

d She should have subtracted by 4 from both sides before factorising