Student Name

Mathematics Stage 4

LINEAR RELATIONSHIPS

Book 3 Solve linear equations using graphical techniques

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA4-LIN-C-01 creates and displays number patterns and finds graphical solutions to problems involving linear relationships

Solve linear equations using graphical techniques

- Recognise that each point on the graph of a linear relationship satisfies the equation of a line
- Apply graphs of linear relationships to solve a corresponding linear equation using graphing applications
- Graph 2 intersecting lines on the same set of axes and identify the point of intersection using either graphing applications or a table of values
- Verify that the point of intersection satisfies the equations of both lines

SUMMARY

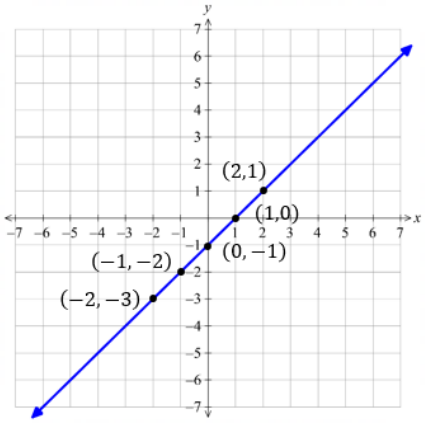
Solve Equations Graphically

Every Point on a Line is a Solution to the Equation

Equation

$y = x - 1$

Line



Solutions

Values of the unknowns that make the equation true

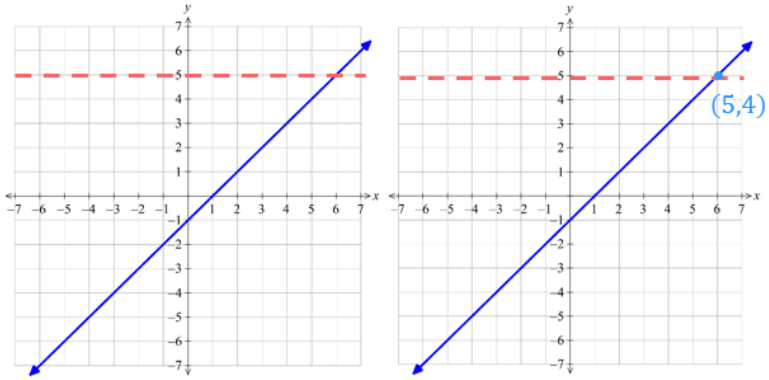
$x = -2,$	$y = -3$	$(-2, -3)$
$x = -1,$	$y = -2$	$(-1, -2)$
$x = 0,$	$y = -1$	$(0, -1)$
$x = 1,$	$y = 0$	$(1,0)$
$x = 2,$	$y = 1$	$(2,1)$

There are an infinite number of solutions; the line extends forever.

Solving Equations Graphically

1. Draw a horizontal line at the y-value.
2. Identify the point of intersection of the horizontal line and the graph.
3. The x-value of the point of intersection is the solution.

Solve $x - 1 = 4$ using the graph



The solution is $x = 5$

Solving Equations Simultaneously

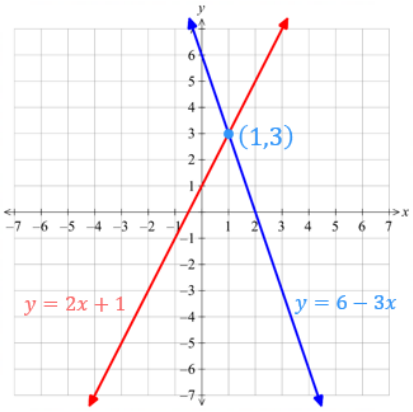
The point of intersection of two lines is the solution to both equations.

1. Identify the point of intersection.
2. State the x & y values at the point of intersection.

Solve these equations simultaneously.

$y = 6 - 3x$

$y = 2x + 1$



The solution is $x = 1, y = 3$

SOLVING EQUATIONS GRAPHICALLY

Investigation Solving equations using graphs.

Find pairs of possible x and y values that satisfy this equation:

$$y = x + 1$$

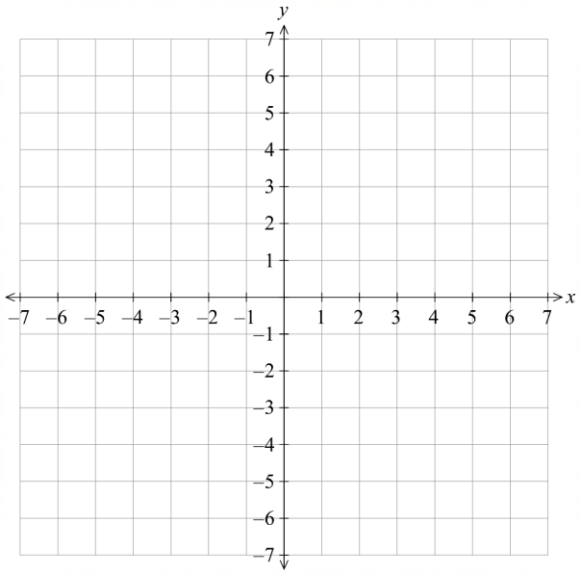
.....

.....

.....

List out the (x, y) pairs systematically

x							
y							



Plot this table of values on the Cartesian plane.
Draw a straight line through all the points and extend the line.
You have drawn the graph of $y = x + 1$.

Using the graph:

Find x when $y = -3$ $x =$ .....

Find x when $y = -6$ $x =$ .....

Find x when $y = -6$ $x =$ .....

Does every single point on the graph satisfy the equation?

.....

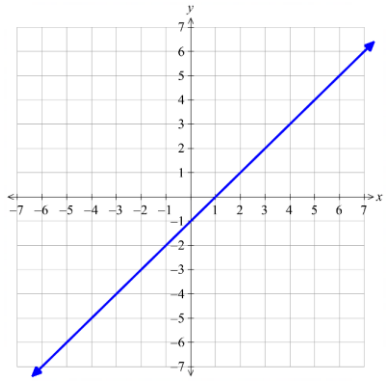
SOLUTIONS

- The **solution** to an equation is the value of the variables that make the equation true.
- Every point (x, y) on the line is a solution to the equation $y = mx + c$.
- To solve equations graphically:
  1. Draw a horizontal line at the given y-value.
  2. Find where this line meets the graph.
  3. The x-value at this point is the solution.

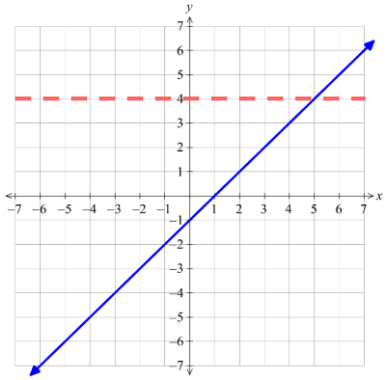
Example Solve equations graphically.

The graph of $y = x - 1$ is shown.

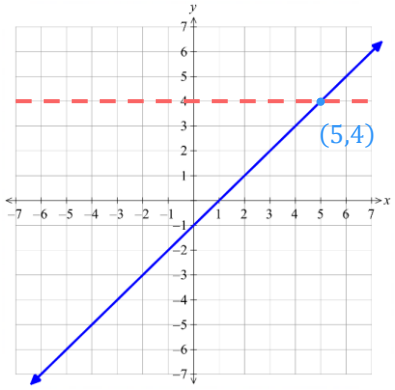
Solve $x - 1 = 4$ using the graph.



1. Draw horizontal line at the y-value.

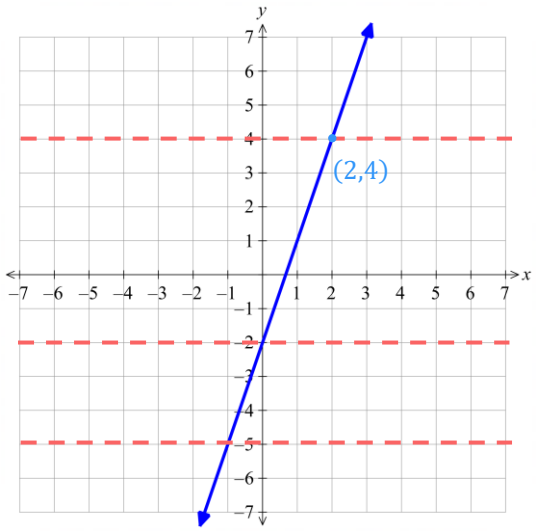


2. Identify point of intersection.



3. The solution to $x - 1 = 4$ is $x = 5$

a The graph of $y = 3x - 2$ is given.



Solve using the graph:

$3x - 2 = 4$ $x =$ .....

$x = 2$

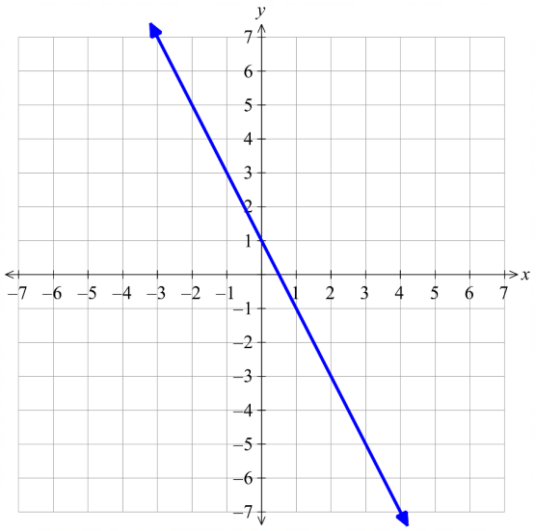
$3x - 2 = -5$ $x =$ .....

$x = -1$

$3x - 2 = -2$ $x =$ .....

$x = 0$

b The graph of $y = -2x + 1$ is given.



Solve using the graph:

$-2x + 1 = 3$ $x =$ .....

$x = -1$

$-2x + 1 = -5$ $x =$ .....

$x = 3$

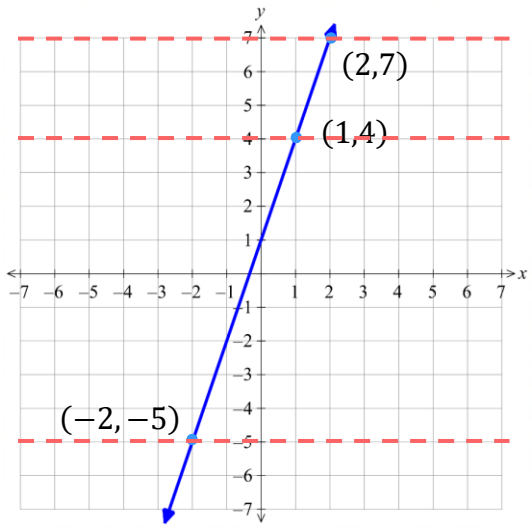
$-2x + 1 = 4$ $x =$ .....

$x = -1.5$

Key ideas

1. Every point (x, y) on a line $y = 2x + 1$ is a pair of x-values and y-values that ..... the equation.
2. Solving $2x + 1 = 5$ means to find the value of x when $y =$ ..... is substituted into the equation $y = 2x + 1$
3. To solve $2x + 1 = 5$, look at the graph of $y = 2x - 3$ and see where the y-coordinate is at 5, then read off the ..... coordinate.

1 The graph of $y = 3x + 1$ is given. Use the coordinates to identify the x value.



$y = 3x + 1$

a $7 = 3x + 1$ $x =$……

$x = 2$

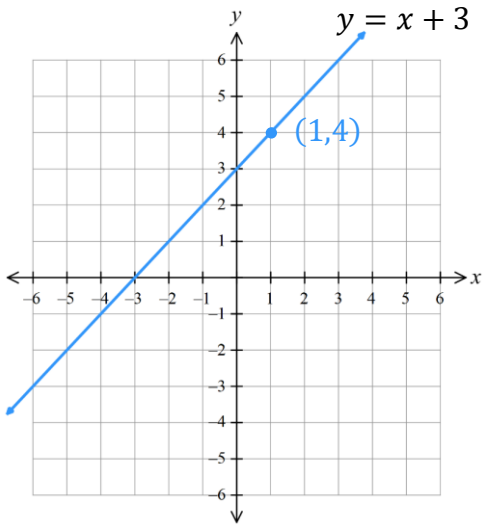
b $4 = 3x + 1$ $x =$……

$x = 1$

c $-5 = 3x + 1$ $x =$……

$x = -2$

2 Consider the diagram and complete the sentences.



a $(1,4)$ lies on the line, so $x =$ …… $y =$ …… is a solution to the equation $y = x + 3$

$x = 1, y = 4$

b Other solutions to $y = x + 3$ include:

i $x = 0$ $y =$ …… Show this coordinate (……, ……) on the graph.

$y = 3, (0,3)$

ii $x = 2$ $y =$ …… Show this coordinate (……, ……) on the graph.

$y = 5, (2,5)$

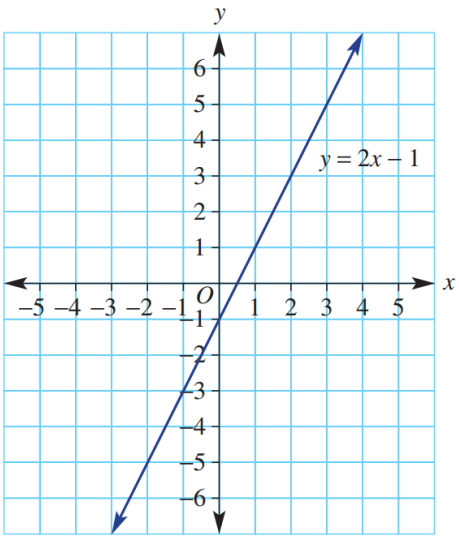
iii $x = -5$ $y =$ …… Show this coordinate (……, ……) on the graph.

$y = -2, (-5,-2)$

c Every point on the line $y = x + 3$ is a pair of ……-values and ……-values that …… the equation.

x-values, y-values, satisfy

3 Use the graph of $y = 2x - 1$ to find the solution to each of these equations.



a $2x - 1 = 3$ $x =$.....

$x = 2$

b $2x - 1 = 0$ $x =$.....

$x = \frac{1}{2}$

c $2x - 1 = 5$ $x =$.....

$x = 3$

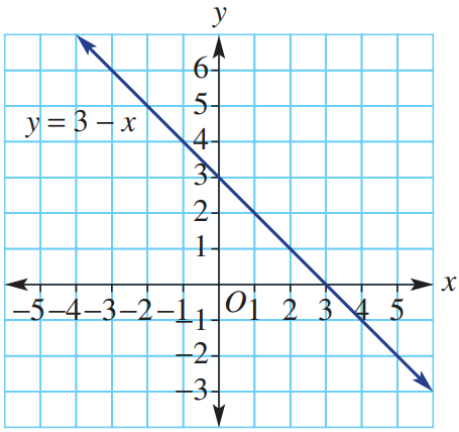
d $2x - 1 = -6$ $x =$.....

$x = -\frac{5}{2}$

e $2x - 1 = -4$ $x =$.....

$x = -\frac{3}{2}$

4 Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.



a $3 - x = 6$ $x =$.....

$x = -3$

b $3 - x = 0$ $x =$.....

$x = 3$

c $3 - x = 3.5$ $x =$.....

$x = -0.5$

d $3 - x = -1$ $x =$.....

$x = 4$

e $3 - x = -2$ $x =$.....

$x = 5$

f Write down some solutions to the equation $y = 3 - x$

Answers will vary

g Is every (x, y) pair on the line in **Q4** a solution to the equation $y = 3 - x$?

yes

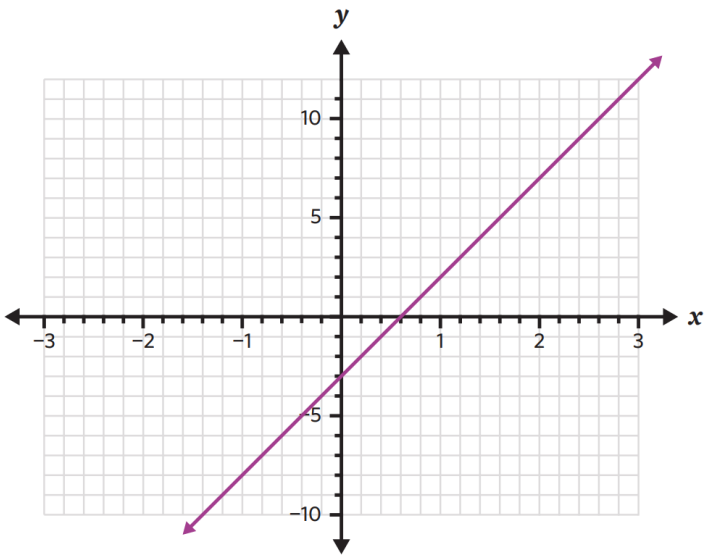
h Is there a limit to the number of solutions for $y = 3 - x$?

no

i What does that tell you about the length of the line $y = 3 - x$?

The line extends forever

5 Use the given graph of $y = 5x - 3$ to solve these equations. Pay attention to the axis scale.



a $5x - 3 = 2$ ..... $x = 1$

b $5x - 3 = 7$ ..... $x = 2$

c $5x - 3 = 12$ ..... $x = 3$

d $5x - 3 = -3$ ..... $x = 0$

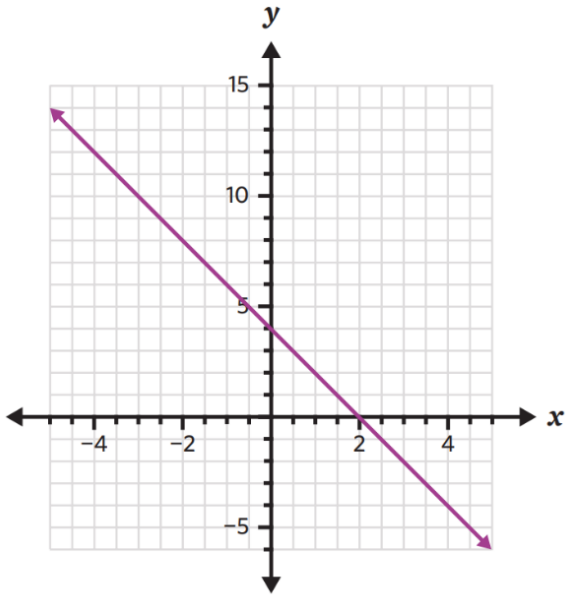
e $5x - 3 = 4$ ..... $x = 1.4$

f $5x - 3 = -3$ ..... $x = 0$

g $5x - 3 = -7$ ..... $x = -0.8$

h $5x - 3 = 0$ ..... $x = 0.6$

6 Use the given graph of $y = -2x + 4$ to solve these equations. Pay attention to the axis scale.



a $-2x + 4 = 2$ ..... $x = 1$

b $-2x + 4 = 0$ ..... $x = 2$

c $-2x + 4 = 3$ ..... $x = 0.5$

d $-2x + 4 = -2$ ..... $x = 3$

e $-2x + 4 = 6$ ..... $x = -1$

f $-2x + 4 = 4$ ..... $x = 0$

g $-2x + 4 = 12$ ..... $x = -4$

h $-2x + 4 = 13$ ..... $x = -4.5$

SIMULTANEOUS EQUATIONS

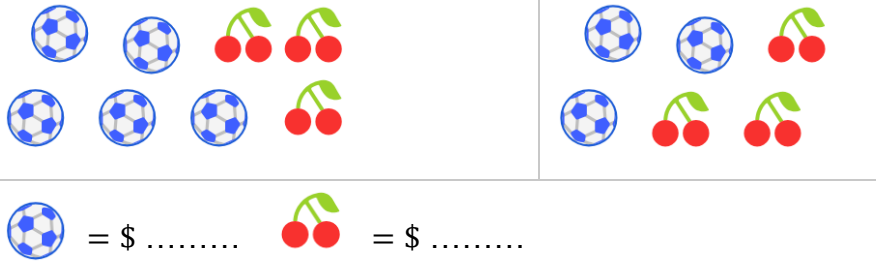
Review of prior knowledge

The total cost of a basket of goods is given. Work out the price of each item.

a

= $ ..... = $ .....

b



= $ ..... = $ .....

c

$m =$ = $ ..... $k =$ = $ .....

d

$l =$ $ ..... $d =$ $ .....

e

$2x + 2y = 22$ $2x + 3y = 29$

$x =$ ..... $y =$ .....

IDENTIFY SIMULTANEOUS SOLUTION

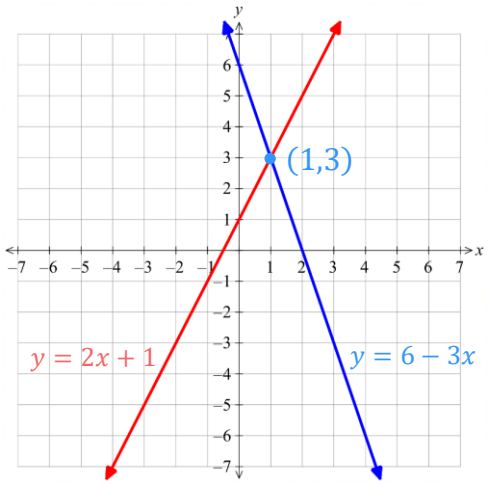
- Solving equations simultaneously means finding the x and y values that make two equations true at the same time.
- The x, y coordinates where two lines meet solves both equations.
- To find the **simultaneous solution**:
  1. Identify point of intersection.
  2. State the x and y values at the point of intersection.

Example Identify the simultaneous solution of two equations.

Solve these equations simultaneously.

$y = 6 - 3x$

$y = 2x + 1$



The point of intersection is: $(1,3)$

The solution to both equations is:

$x = 1, y = 3$

What does it mean to solve two equations simultaneously?

Solving two equations simultaneously means finding the value of …… and …… that make both equations …… at the same time.

Verify that $x = 1$ and $y = 3$ solve both equations.

$$y = 6 - 3x$$

$$y = 2x + 1$$

Why does the point of intersection solve both equations?

Every point on a line is a …… to the equation. When two lines meet, the point of intersection lies on …… lines, so the coordinates …… both equations.

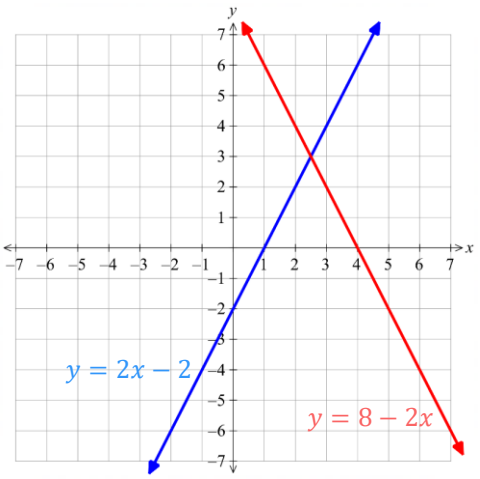
Solve these equations simultaneously.

$y = 2x - 2$

$y = 8 - 2x$

The point of intersection is: (……, ……)

(2.5, 3)



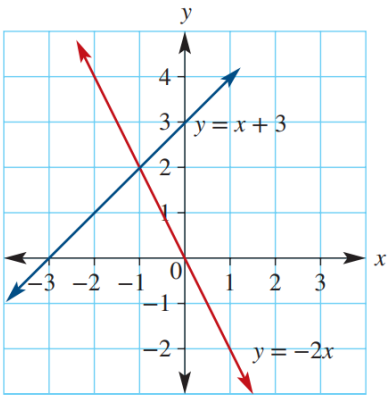
Verify the point of intersection solves both equations.

$y = 2x - 2$

$y = 8 - 2x$

1 Determine the point of intersection and hence solve these equations simultaneously.

a



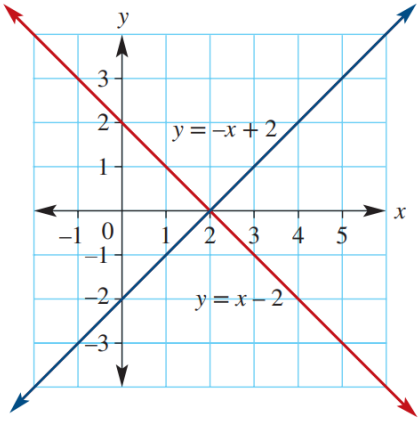
The point of intersection is: .....

The solution to both equations is:

$x =$ ..... $y =$ .....

$x = -1, y = 2$

b

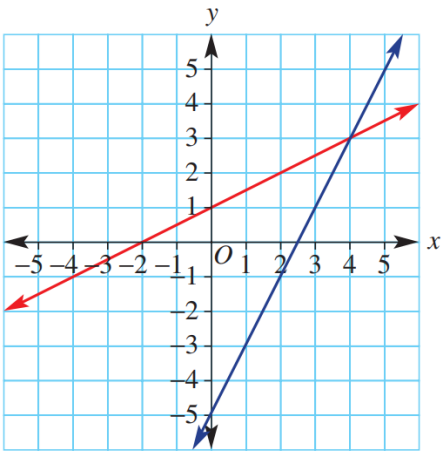


The point of intersection is: .....

The solution to both equations is:

$x = 2, y = 0$

c

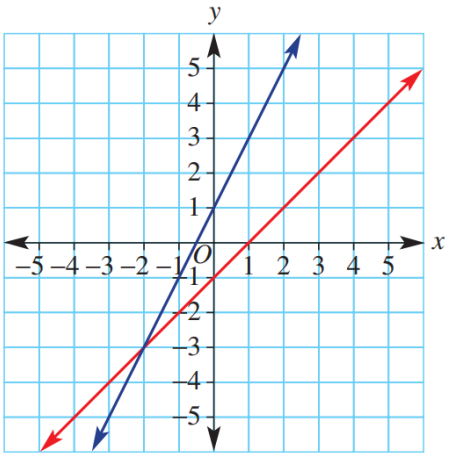


The point of intersection is: .....

The solution to both equations is:

$x = 4, y = 3$

d

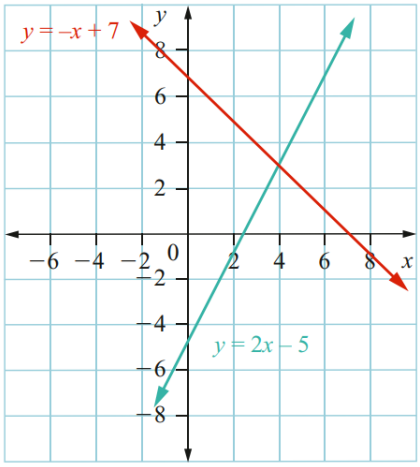


The point of intersection is: .....

The solution to both equations is:

$x = -2, y = -3$

e

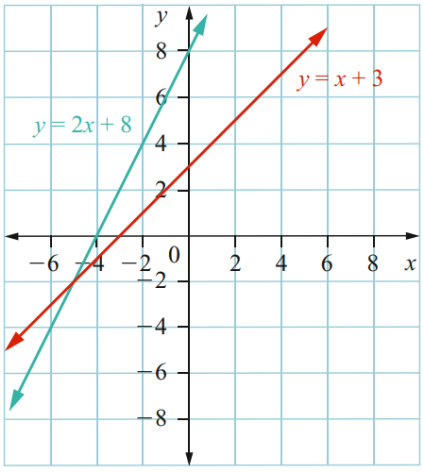


The point of intersection is: .....

The solution to both equations is:

$x = 4, y = 3$

f



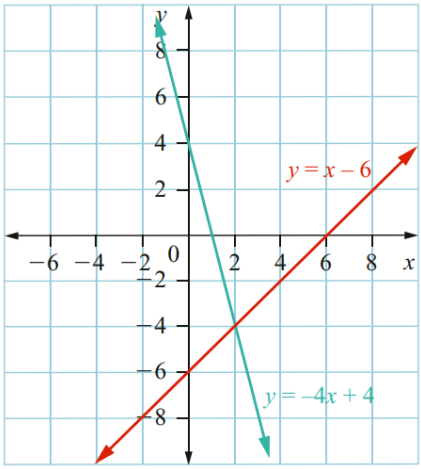
The point of intersection is: .....

The solution to both equations is:

$x = -5, y = -2$

2 Verify that the point of intersection solves both equations simultaneously.

a



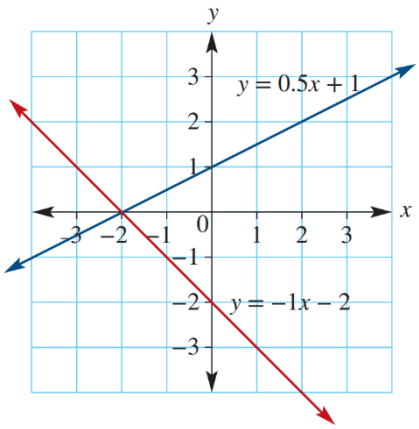
Point of intersection is: ……

(2, −4)

Verify:

$y = x - 6$ $\qquad$ $y = -4x + 4$

b



Point of intersection is: ……

(−2,0)

Verify:

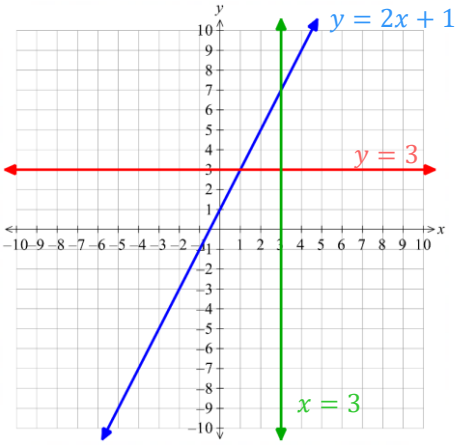
$y = 0.5x + 1$ $\qquad$ $y = -x - 2$

3 Here are the graphs of

$y = 2x + 1$

$x = 3$

$y = 3$



a Identify the coordinates of the point of intersection of $y = 2x + 1$ and $x = 3$

(3, 7)

b Substitute $x = 3$ into the equation $y = 2x + 1$ and evaluate y.

$y = 7$

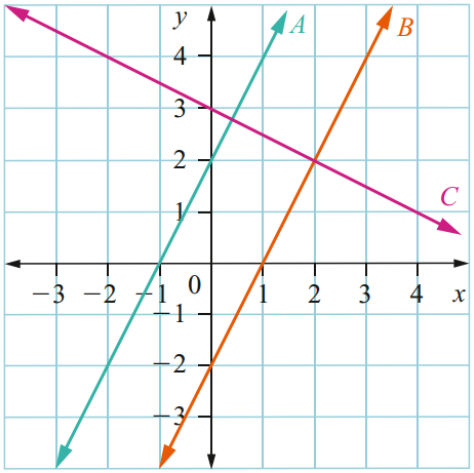
c Identify the coordinates of the point of intersection of $y = 2x + 1$ and $y = 3$

(1,3)

d Substitute $y = 3$ into the equation $y = 2x + 1$ and solve for x.

$x = 1$

4 Consider the straight lines A, B and C shown on this graph.



a Write the coordinates of the point of intersection of lines B and C and hence give the solution to the simultaneous equations for these lines.

(2,2)

b Use the graph to solve the equations for lines A and C simultaneously (give an estimate).

$\approx (0.4, 2.8)$

c Can the equations for lines A and B be solved simultaneously? Explain why or why not.

No, they are parallel so will never meet. Theres no pair of x and y values that will solve both equations.

5 When two different lines have the same gradient, there will be no intersection point. Use this idea to decide if these pairs of simultaneous equations will have a solution.

a $y = 3x - 1$, $y = 3x + 1$

No solution

b $y = 7x + 2$, $y = 7x + 6$

No solution

c $y = 2x - 6$, $y = -2x + 1$

Solution possible

d $y = -x + 7$, $y = 2x - 1$

Solution possible

e $2x - y = 4$, $y = 2x + 1$

No solution

f $7x - y = 1$, $y = -7x + 2$

Solution possible

g $2y - 3x = 0$, $y = -\frac{3x}{2} - 1$

Solution possible

h $y = \frac{x}{2} + 3$, $\frac{x - 2y}{4} = 0$

The two equations are the same line.

6 For what value of k will these pairs of simultaneous equations have no solution?

a $y = -4x - 7$, $y = kx + 2$

$k = -4$

b $y = kx + 4$, $3x - 2y = 5$

$k = \frac{3}{2}$

c $kx - 3y = k$, $y = 4x + 1$

$k = 12$

SOLVING EQUATIONS SIMULTANEOUSLY

- To solve two equations simultaneously:
  1. Construct table of values from each equation.
  2. Graph each line from the table of values.
  3. Identify point of intersection.

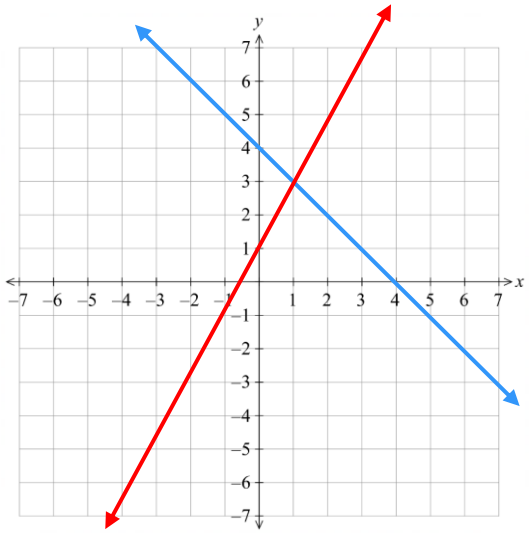
Example Solve simultaneous linear equations graphically.

Worked Solution	Your Turn
Solve these equations simultaneously. $y = 2x + 1$ $y = 4 - x$	Solve these equations simultaneously. $y = x + 2$ $y = 2x - 1$

Worked Solution

x	0	1	2	3
y	1	3	5	7

x	0	1	2	3
y	4	3	2	1



The point of intersection is: $(1,3)$

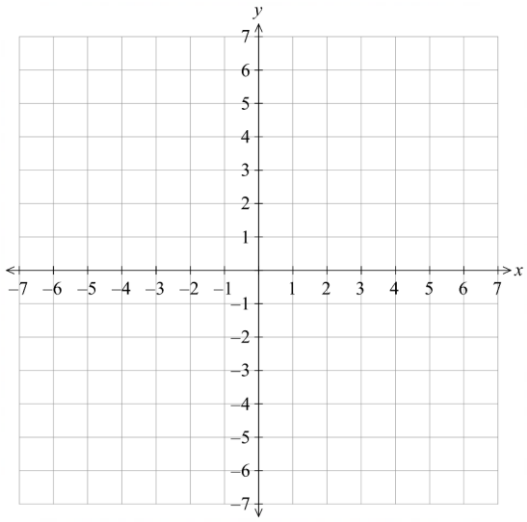
The solution to both equations is:

$x = 1, y = 3$

Your Turn

x	0	1	2	3
y				

x	0	1	2	3
y				



The point of intersection is: ……

The solution to both equations is:

……

$x = 3, y = 5$

Key ideas

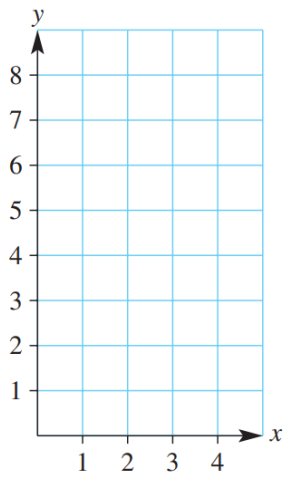
1. Every point on a line is a …… to the equation.
2. To solve two equations simultaneously means to find the values of x and y that satisfy …… equations.
3. The point of …… of two lines is the simultaneous solution, as it lies on …… the lines

1 Plot the points on the Cartesian Plane and join them to form two straight lines.
What is the point of intersection?

a

x	0	1	2	3	4
y	0	2	4	6	8

x	0	1	2	3	4
y	6	5	4	3	2



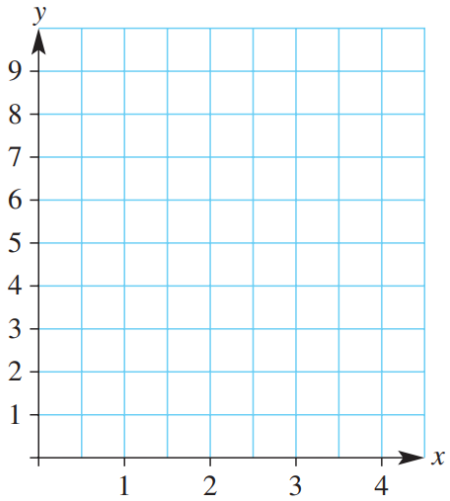
Point of intersection: ……

(2,4)

b

x	0	1	2	3	4
y	1	3	5	7	9

x	0	1	2	3	4
y	4	3	2	1	0



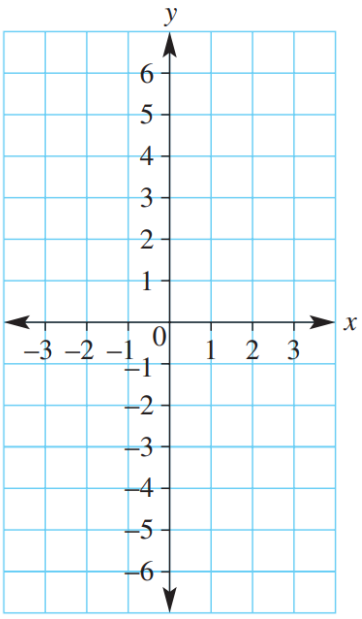
Point of intersection: ……

(1,3)

c

x	−2	−1	0	1	2
y	−6	−5	−4	−3	−2

x	−2	−1	0	1	2
y	6	3	0	−3	−6



Point of intersection: ……

(1, −3)

2 Looking at your answers to **Q1**, does the point of intersection appear on both tables of values?

yes

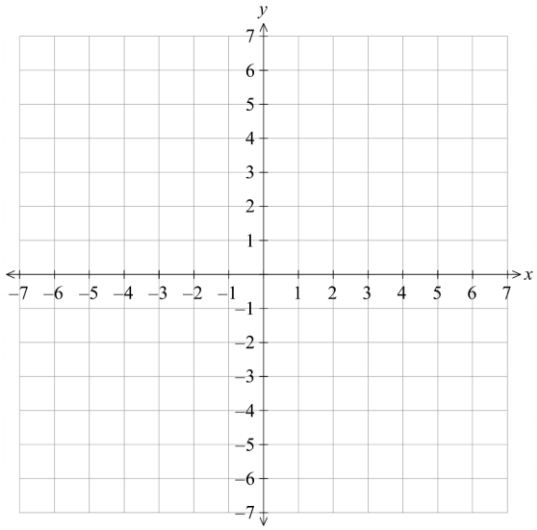
3 Complete the following table of values and plot the points on a number plane.
What is the solution to each pair of simultaneous equations?

a $y = 2x - 1$

x	-1	0	1
y			

$y = x + 1$

x	-1	0	1
y			



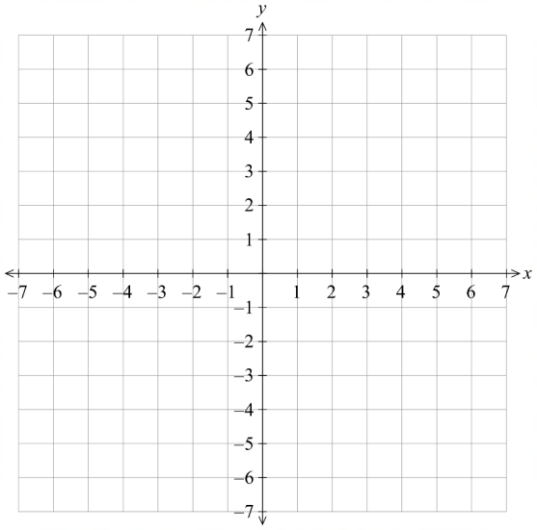
(2,3)

b $y = -x$

x	-1	0	1
y			

$y = x + 2$

x	-1	0	1
y			



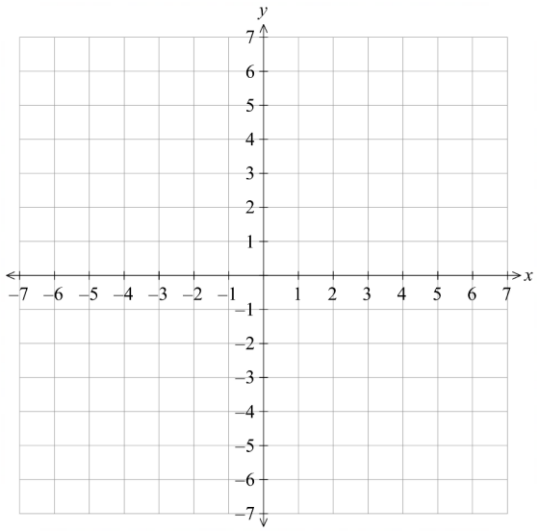
(−1,1)

c $y = 3x + 1$

x	-1	0	1
y			

$y = 5x - 3$

x	-1	0	1
y			



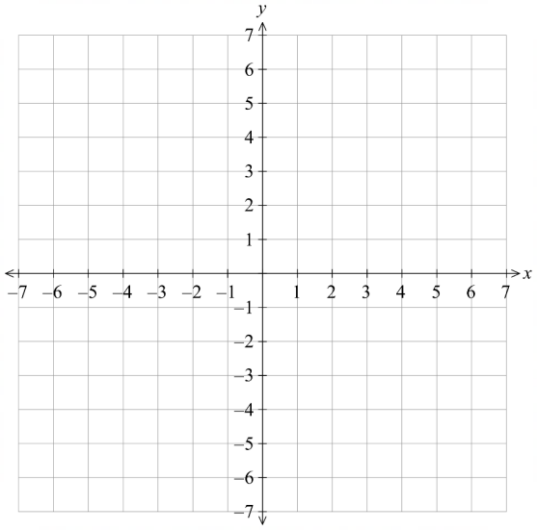
(2,7)

d $y = 3x - 3$

x	-1	0	1
y			

$y = x + 1$

x	-1	0	1
y			



(2,3)

4 Consider the tables given below to find the x-value and the y-value that is the solution to each pair of simultaneous linear equations.

a $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

$x =$ .....

$y = 6 - x$

x	-3	-2	-1	0	1	2	3
y	9	8	7	6	5	4	3

$y =$ .....

(2,4)

b $y = 2x - 1$

x	0	1	2	3	4
y	-1	1	3	5	7

$y = 8 - x$

x	0	1	2	3	4
y	8	7	6	5	4

(3,5)

c $y = 3x - 5$

x	5	6	7	8	9
y	10	13	16	19	22

$y = 23 - x$

x	5	6	7	8	9
y	18	17	16	15	14

(7,16)

5 Graph these pairs of lines on DESMOS to solve these pairs of simultaneous equations.
https://www.desmos.com/calculator

a $x + y = 5$ and $2x + y = 6$ .....

(1,4)

b $3y + x = 9$ and $2x - y = 4$ .....

(3,2)

c $x + y = -4$ and $x - y = 2$ .....

$(-1, -3)$

d $y = x^2 + 3$ and $y = 2x + 2$ .....

(1,4)

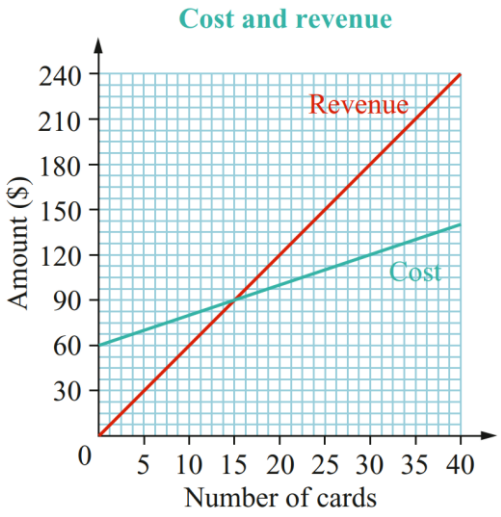
6 Suzy makes hand-painted cards. The production cost and the revenue generated by the sale of the cards can be modelled with two linear equations and shown on a graph.

a In business, "breaking even" means that revenue from selling items is equal to the cost of producing the items. How many cards does Suzy need to sell to break even?

15

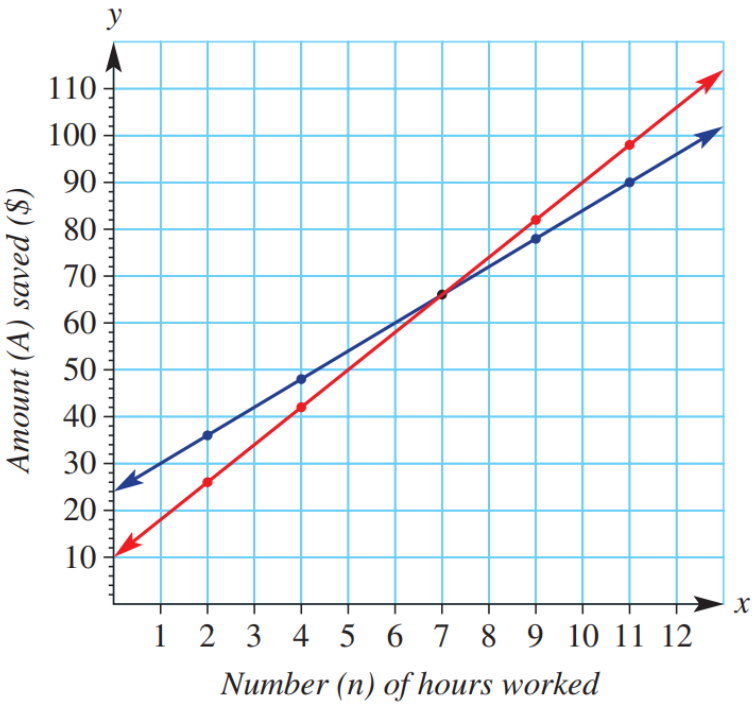
b What is the revenue at this break-even point?

$90



7 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved \$24 and earns \$6 per hour mowing lawns.
- Ruby has saved \$10 and earns \$8 per hour babysitting.



a The two graphs represent Jayden and Ruby's savings over time.
Which line represents Jayden, and which line represents Ruby? Why?

Jayden is blue, his line has gradient 6 (\$6/hour) and y-int 24 (\$24 initially)

Ruby is red, his line has gradient 8 (\$8/hour) and y-int 10 (\$10 initially)

b Use the appropriate line on the graph above to find the solution to the following equations.
Do not solve using the inverse operations method. Solve graphically.

$10 + 8n = 42$ $n = 4$	$24 + 6n = 48$ $n = 4$	$10 + 8n = 66$ $n = 7$
$24 + 6n = 66$ $n = 7$	$10 + 8n = 98$ $n = 11$	$24 + 6n = 90$ $n = 11$

c Write the coordinates of the point of intersection.
Show that these values satisfy both Ruby and Jayden's equations.

(7,66)

d Identify how many hours have been worked and what their savings are at the point of intersection of the two lines.

7 hours of work, \$66 in savings

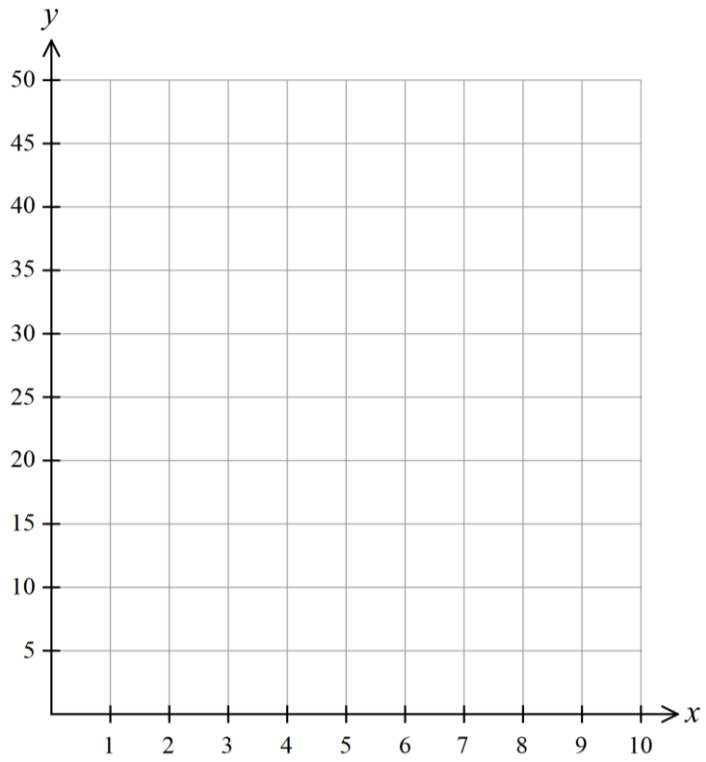
8 Jessica and Max have a 10 second running race.

- Max runs at 6 m/s.
- Jessica is given a 10 m head-start and runs at 4 m/s.

a Complete the table of values.

Time (t), in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d), in metres	0										
Jessica's distance (d), in metres	10										

b Plot these points on a distance–time graph and join them to form two straight lines, labelling them 'Jessica' and 'Max'.



c Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

(5,30)

d Explain what is happening in the race at the point of intersection.

After 5 seconds, both athletes are 30 m from the starting point, i.e., they pass each other.

9 2017 HSC Standard 2 Band 4

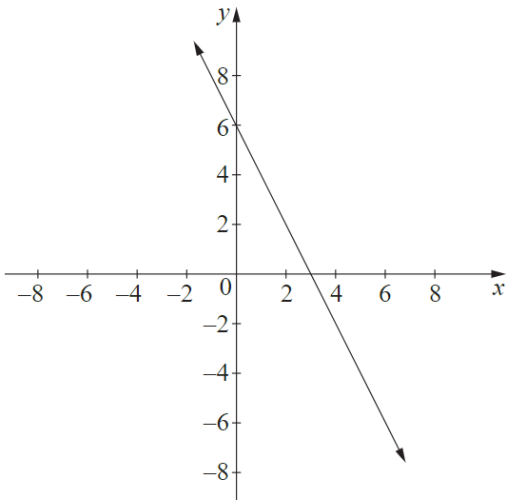
The graph of the line with equation $y = 6 - 2x$ is shown. When the graph of the line with equation $y = x + 3$ is also drawn on the number plane, what will be the point of intersection?

A. $(0, 6)$

B. $(1, 4)$

C. $(2, 2)$

D. $(3, 0)$



B

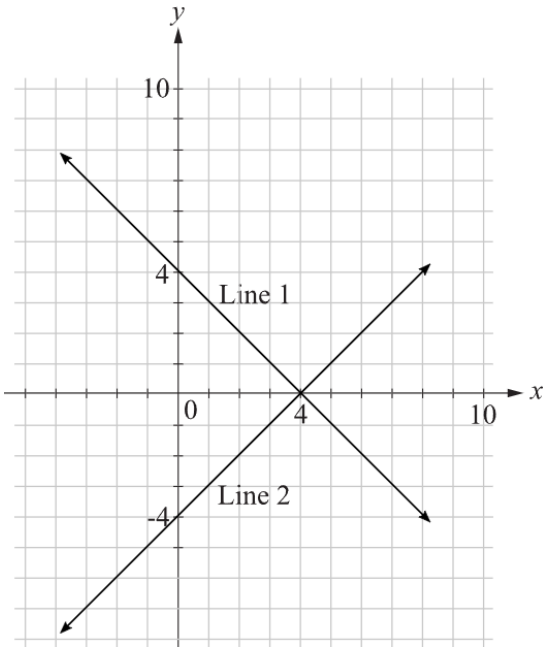
10 HSC Standard 2 Sample Question Band 4

A computer application was used to draw the graphs of the equations:

$$x + y = 4 \text{ and } x - y = 4$$

Which row of the table correctly matches the equations with the lines drawn and identifies the solution when the equations are solved simultaneously?

	$x + y = 4$	$x - y = 4$	Solution
A.	Line 1	Line 2	$x = 4, y = 0$
B.	Line 1	Line 2	$x = 4, y = 4$
C.	Line 2	Line 1	$x = 4, y = 0$
D.	Line 2	Line 1	$x = 4, y = 4$

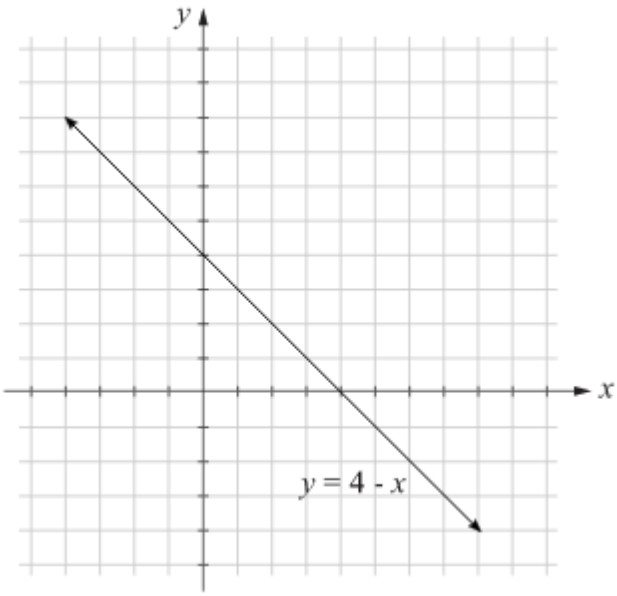


A

11 HSC Standard 2 Sample Question Band 4

The graph of the line $y = 4 - x$ is shown.

By graphing $y = 2x + 1$ on the grid provided, find the simultaneous solution.



(1,3)

12 HSC Sample Question Band 4

A student was asked to solve the following simultaneous equations.

$$y = 2x - 8$$

$$x - 4y + 3 = 0$$

After graphing the equations, the student found the point of intersection to be (5,2).

Is the student correct? Support your answer with mathematical reasoning.

Yes, (5,2) satisfies both equations.