

Student Name

Mathematics Stage 5 Core

EQUATIONS A

Book 2 Solve linear equations involving
one algebraic fraction

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OVERVIEW

SYLLABUS CONTENT

MA5-EQU-C-01 solves linear equations of up to 3 steps, limited to one algebraic fraction

Solve linear equations involving one algebraic fraction

- Solve linear equations involving one algebraic fraction using up to 3 steps

REVIEW OF PRIOR KNOWLEDGE

1 Solve these equations.

a $2p + 5 = 6$

b $\frac{p}{2} + 5 = 6$

c $5 + \frac{p}{2} = 6$

d $6 = \frac{p}{2} - 5$

e $6 = -5 + \frac{p}{2}$

f $6 = 5 - \frac{p}{2}$

g $6 = \frac{p-5}{2}$

h $6 = \frac{5-p}{2}$

i $6 = -\frac{5-p}{2}$

2 Solve these equations.

a $3x - 6 = 2x + 5$

b $5x - 3 = -4x + 33$

c $9x + 8 + 2x = 3x - 16$

d $5x + 7 - 3 = x - 10$

e $4(w + 3) = 7w + 15$

f $3(5z + 2) = -2(5z + 8)$

MULTI-STEP EQUATIONS WITH FRACTIONS

- **Invisible brackets** group terms in the **numerator**.

1. Undo addition/subtraction outside brackets.
2. Undo multiplication/division outside brackets.
3. Undo addition/subtraction inside brackets.
4. Undo multiplication/division inside brackets.

Example Solve multi-step equations involving fractions.

$\frac{3x}{5} - 6 = 2$ $+6 \quad +6$ $\frac{3x}{5} = 8$ $\times 5 \quad \times 5$ $3x = 40$ $\div 3 \quad \div 3$ $x = \frac{40}{3}$	$\frac{4x + 3}{6} = -2$ $\times 6 \quad \times 6$ $4x + 3 = -12$ $-3 \quad -3$ $4x = -15$ $\div 4 \quad \div 4$ $x = -\frac{15}{4}$
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Solve these equations.

a

$$\frac{8d + 15}{2} = 8$$

$$d = \frac{1}{8}$$

b

$$\frac{5a}{2} - 8 = 12$$

$$a = 8$$

c

$$\frac{4c + 15}{9} = 3$$

$$c = 3$$

d

$$12 = \frac{2(k - 10)}{5}$$

$$k = 40$$

e

$$2\left(\frac{a}{3} - 5\right) = 16$$

$$a = 39$$

f

$$15 = \frac{3(y + 20)}{10}$$

$$y = 30$$

1 Write the inverse operations done between each line.

a

$$\frac{7j - 12}{9} = 1$$

$$\boxed{} \quad \boxed{} \quad \times 9$$

$$7j - 12 = 9$$

$$\boxed{} \quad \boxed{} \quad +12$$

$$7j = 21$$

$$\boxed{} \quad \boxed{} \quad \div 7$$

$$j = 3$$

b

$$\frac{3r}{5} + 2 = 8$$

$$\boxed{} \quad \boxed{} \quad -2$$

$$\frac{3r}{5} = 6$$

$$\boxed{} \quad \boxed{} \quad \times 5$$

$$3r = 30$$

$$\boxed{} \quad \boxed{} \quad \div 3$$

$$r = 10$$

c

$$2 = \frac{2x + 4}{7}$$

$$\boxed{} \quad \boxed{} \quad \times 7$$

$$14 = 2x + 4$$

$$\boxed{} \quad \boxed{} \quad -4$$

$$10 = 2x$$

$$\boxed{} \quad \boxed{} \quad \div 2$$

$$5 = x$$

d

$$8 = \frac{10y}{4} - 7$$

$$\boxed{} \quad \boxed{} \quad +7$$

$$15 = \frac{10y}{4}$$

$$\boxed{} \quad \boxed{} \quad \times 4$$

$$60 = 10y$$

$$\boxed{} \quad \boxed{} \quad \div 10$$

$$6 = y$$

e

$$\frac{3(c - 8)}{5} = 6$$

$$\boxed{} \quad \boxed{} \quad \times 5$$

$$3(c - 8) = 30$$

$$\boxed{} \quad \boxed{} \quad \div 3$$

$$c - 8 = 10$$

$$\boxed{} \quad \boxed{} \quad +8$$

$$c = 18$$

f

$$\frac{9(x + 1)}{3} = 12$$

$$\boxed{} \quad \boxed{} \quad \times 3$$

$$9(x + 1) = 36$$

$$\boxed{} \quad \boxed{} \quad \div 9$$

$$x + 1 = 4$$

$$\boxed{} \quad \boxed{} \quad -1$$

$$x = 3$$

g

$$10 = \frac{5(x - 4)}{8}$$

$$\boxed{} \quad \boxed{} \quad \times 8$$

$$80 = 5(x - 4)$$

$$\boxed{} \quad \boxed{} \quad \div 5$$

$$16 = x - 4$$

$$\boxed{} \quad \boxed{} \quad +4$$

$$20 = x$$

h

$$1 = \frac{25(x + 6)}{100}$$

$$\boxed{} \quad \boxed{} \quad \times 100$$

$$100 = 25(x + 6)$$

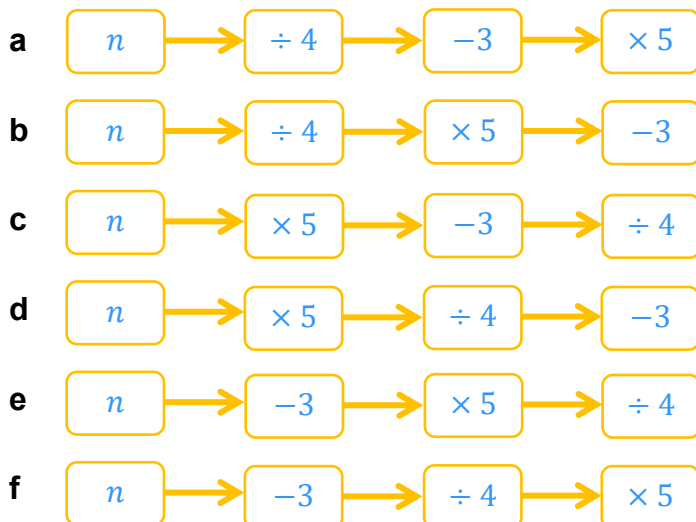
$$\boxed{} \quad \boxed{} \quad \div 25$$

$$4 = x + 6$$

$$\boxed{} \quad \boxed{} \quad -6$$

$$-2 = x$$

2 Match the sequences of operations to each expression.



- A $5\left(\frac{n-3}{4}\right)$
- B $\frac{5(n-3)}{4}$
- C $5\left(\frac{n}{4} - 3\right)$
- D $\frac{5n-3}{4}$
- E $\frac{5n}{4} - 3$

a - C, b - E, c - D, d - E, e - B, f - A

3 Solve these equations.

a

$$\frac{5x}{9} - 9 = -4$$

b

$$-\frac{u}{4} + 15 = 8$$

c

$$-\frac{y}{3} + 10 = 17$$

d

$$\frac{8c}{3} + 5 = -11$$

e

$$-\frac{3x}{4} + 5 = -7$$

f

$$-\frac{5x}{6} - 6 = 14$$

g

$$17 - \frac{2x}{3} = 1$$

h

$$2 - \frac{6x}{5} = 3$$

i

$$-5 - \frac{12x}{7} = 3$$

4 Solve these equations.

a

$$\frac{2x - 12}{3} = 1$$

b

$$\frac{3x + 6}{2} = 15$$

c

$$\frac{-3t - 6}{7} = -3$$

$$\frac{15}{2}$$

$$8$$

$$5$$

d

$$\frac{-13 - 4r}{3} = -15$$

e

$$\frac{-9 + 5x}{2} = -22$$

f

$$\frac{12 - x}{3} = 5$$

$$8$$

$$-7$$

$$-3$$

5 Read the example carefully and answer the questions.

$$\frac{3x - 6}{7} = 3$$

$$\frac{3x}{7} = 9$$

$$3x = 63$$

$$x = 21$$



Substitute $x = 21$ into the original equation and show that it is not the correct solution.

Describe the mistake made by the student.
The of inverse operations is incorrect. They should have first.

Solve the equation correctly.

$$x = 9$$

6 Read the example carefully and answer the questions.

Pablo's work:

$$\begin{array}{l} \frac{6-4x}{3} = 5 \\ 3\left(\frac{6-4x}{3}\right) = (5)3 \\ 6-4x = 15 \\ -6 \quad -6 \\ 4x = 9 \\ \div 4 \quad \div 4 \\ x = \frac{9}{4} \end{array}$$

What did Pablo forget to do in the step marked with an arrow?

Keep the symbol

Solve:

$$\frac{4x - 6}{3} = 6$$

$$x = 6$$

7 Solve these equations.

a

$$\frac{2x}{3} - 1 = 7$$

b

$$\frac{3x}{4} - 2 = 7$$

c

$$1 = 3 + \frac{x}{2}$$

12

12

-4

d

$$\frac{3b - 8}{2} = 5$$

e

$$\frac{2x + 2}{3} = 4$$

f

$$9 = \frac{7m - 8}{3}$$

6

5

5

g

$$\frac{x - 9}{5} + 4 = 7$$

h

$$\frac{x - 3}{5} - 8 = -7$$

i

$$\frac{x + 16}{3} + 5 = 3$$

24

8

-22

j

$$\frac{5 - y}{3} = 2$$

k

$$\frac{4 - 2t}{6} = 3$$

l

$$5 = 1 - \frac{4x}{3}$$

-1

-7

-3

m

$$\frac{2(x - 4)}{3} = 8$$

n

$$\frac{5(x + 1)}{2} = 10$$

o

$$\frac{-3(x - 3)}{4} = 0$$

16

3

3

p

$$\frac{9(x + 10)}{2} = -6$$

q

$$\frac{4(2 - x)}{3} = 3$$

r

$$\frac{7(x - 9)}{4} = 14$$

-34
31
4

17

8 Solve these three equations.

a

$$-\frac{3c}{4} + 5 = 14$$

b

$$\frac{-3c}{4} + 5 = 14$$

c

$$5 - \frac{3c}{4} = 14$$

-12

-12

-12

What do you notice?

The solutions are the same.

9 Solve these equations.

a

$$2\left(\frac{x}{3} + 1\right) = 6$$

b

$$5\left(\frac{x}{2} + 7\right) = 60$$

c

$$4\left(\frac{x}{5} - 3\right) = -12$$

6

10

0

10 Solve these equations.

a

$$\frac{y}{2} + 7 = 19$$

b

$$\frac{3y}{2} + 7 = 19$$

c

$$3\left(\frac{y}{2} + 7\right) = 19$$

d

$$3\left(\frac{y}{2} - 7\right) = 19$$

24

8

$-\frac{4}{3}$

$\frac{80}{3}$

11 Consider the equation $\frac{x+4}{5} + 3 = 6$. Solve this equation by:

Subtracting 3 first

Multiplying all terms by 5 first

11

You should have gotten the same answer for both methods above. Which do you prefer?

.....

FRACTION EQUATIONS WITH TWO UNKNOWN TERMS

- If an equation has fractions and more than one pronumeral:
 - Multiply every term on both sides by the denominator.
 - Move all pronumerals to one side.
 - Solve for the unknown using inverse operations.

Example Solve equations with one algebraic fraction and pronumerals on both sides

$\frac{3x - 5}{2} = x$ 1. Multiply $\times 2 \quad \times 2$ $3x - 5 = 2x$ 2. Move $-2x \quad -2x$ $x - 5 = 0$ 3. Solve $+5 = +5$ $x = 5$	$\frac{2x}{3} - 5 = 6x$ 1. Multiply $\times 3 \quad \times 3$ $2x - 15 = 18x$ 2. Move $-2x \quad -2x$ $-15 = 16x$ 3. Solve $\div 16 \quad \div 16$ $-\frac{15}{16} = x$
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Solve these equations.

a $\frac{7x - 12}{3} = x$ $x = 3$	b $\frac{3x + 4}{5} = x$ $x = 2$	c $\frac{3x - 4}{5} = 3x$ $x = -\frac{1}{3}$
d $\frac{2x}{3} + 4 = x - 1$ $x = 15$	e $\frac{2x + 4}{3} = 3x + 1$ $x = \frac{1}{7}$	f $x + \frac{x - 1}{5} = 3$ $x = \frac{8}{3}$

1 Solve these equations.

a

$$\frac{4x - 40}{8} = 3x$$

$$x = -2$$

b

$$\frac{4x + 84}{8} = 3x$$

$$x = \frac{21}{5}$$

c

$$\frac{-6x + 54}{4} = 3x$$

$$x = 3$$

d

$$\frac{3x + 6}{7} = x + 2$$

$$x = -2$$

e

$$\frac{4x - 51}{7} = 3x$$

$$x = -3$$

f

$$\frac{-4x + 57}{3} = 5x$$

$$x = 3$$

g

$$\frac{5x}{4} - 6 = 2x$$

$$x = -8$$

h

$$\frac{3x}{8} = -5x + 26$$

$$x = \frac{208}{43}$$

2 Read the example carefully and answer the questions.

Worked Solution	Explanation
Solve: $x + \frac{2x + 1}{7} = 4$ $7x + 2x + 1 = 28$ $9x + 1 = 28$ $9x = 27$ $x = 3$	Substitute $x = 3$ into the original equation to show that this is the correct solution. What operation was done to both sides to get the first line of working out? Annotate the example to show the operation done between each line.

Solve these equations.

a

$$x + \frac{x - 2}{3} = 6$$

$$x = 5$$

b

$$\frac{2x + 3}{5} + 2 = 7$$

$$x = 11$$

c

$$\frac{x - 4}{3} - 2x = 11$$

$$x = -\frac{37}{5}$$

d

$$\frac{4x + 6}{5} - 3x = 8$$

$$x = -\frac{34}{11}$$

- 3 Read the example carefully and answer the questions.

Worked Solution	Explanation
$x - \frac{x-6}{5} = 6$ $5x - (x-6) = 30$ $5x - x + 6 = 30$ $4x + 6 = 30$ $4x = 24$ $x = 6$	Substitute $x = 6$ into the original equation and show that this is the correct solution. In the first line of working out, why is the RHS 30? In the first line of working out, why is $x - 6$ in brackets?

Solve these equations.

a

$$x - \frac{x-2}{3} = 4$$

$$x = 5$$

b

$$x - \frac{2x+3}{5} = 3$$

$$x = 6$$

c

$$2x - \frac{x-4}{3} = 11$$

$$x = \frac{29}{5}$$

d

$$-3x - \frac{4x-6}{5} = 8$$

$$x = -\frac{34}{19}$$

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Solve:

$$x + \frac{x-1}{2} = 9$$

$$x = \frac{19}{3}$$

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Solve:

$$\frac{5x+1}{3} - 4 = 5 - 7x$$

$$x = 1$$

6 Solve these equations.

a $\frac{x}{6} = \frac{5}{3}$

10

b $\frac{x}{9} = \frac{2}{3}$

6

c $\frac{x}{3} = x - 8$

12

d $3(2x + 3) = 14$

$\frac{5}{6}$

e $43 = 4 - 5(3x - 2)$

$-\frac{29}{15}$

f $10 - (3y + 2) = 2y$

$\frac{8}{5}$

g $2x - 9 = \frac{x}{4}$

$\frac{36}{7}$

h $\frac{x}{2} = x + 5$

-10

i $6m - 4 = \frac{m}{4}$

$\frac{16}{23}$

j $4 = 4 - \frac{2n}{3}$

0

k $-\frac{3}{8}x + 10 = 19$

-24

l $x + \frac{2x - 4}{4} = 3$

$\frac{8}{3}$

m $x - \frac{x + 5}{6} = 2$

$\frac{17}{5}$

n $\frac{2k}{5} - k = 54$

-90

o $\frac{5(x - 1)}{6} - x = 1$

-11

p $x - \frac{4x + 2}{6} = 5$

16

q $\frac{5x - 6}{7} = -5x + 22$

4

r $-\frac{3(a - 40)}{12} = 0.5$

38

s $\frac{y + 4}{2} = y + 1$

2

t $x - \frac{14(x + 10)}{7} = 7$

-27

7 Solve

$\frac{7 - 3n}{6} + 4 = 5(2n + 1)$