

Student Name

Mathematics Stage 5 Core

EQUATIONS A

Book 3 Solve linear equations arising from
word problems and substitution into formulas

Version: 260207
Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview2

Solving for an Unknown after Substitution5

Using Formulas with Quadratic Equations**Error! Bookmark not defined.**

Problem-Solving using Equations13

Check Your Understanding22

OVERVIEW

SYLLABUS CONTENT

MA5-EQU-C-01 solves linear equations of up to 3 steps, limited to one algebraic fraction

Solve linear equations arising from word problems and substitution into formulas

- Solve linear equations arising from substitution into formulas
- Represent word problems as linear equations, solve the equations and interpret the solutions in the context of the problem

REVIEW OF PRIOR KNOWLEDGE

1 Solve these equations

a $4x = 12$

b $32 = 8 \times x$

c $49 = 7 \times 2 \times x$

d $3y + 7 = 40$

e $3y - 7 = 41$

f $40 + 3y = 16$

g $40 - 3y = 16$

h $4x - 3 = 17$

i $17 = 4x - 3$

j $3g + 12 = 33$

k $40 = 2p - 7$

l $10(q - 3) = 80$

m $\frac{m}{4} - 3 = 12$

n $\frac{n+3}{4} = 12$

o $8 = 24 - 2a$

p $x^2 = 16$

q $x^2 + 15 = 24$

r $\sqrt{d} = 9.5 + 1.5$

2 Substitute $P = 10$ and $x = 2$ into the equation $P = 3m + x$.

3 Substitute $k = 10$ and $L = 12$ into the equation $L = 4k + Q$.

4 Write expressions to represent the following: a Josh is n years old this year. How old will be he in 2 years' time?	b Erica is n years old this year. Her brother is 3 years younger than her. What is the age of her brother?
c Find the total cost of 3 movie tickets at m dollars each.	d The total cost of an excursion of 15 students is C. What is the cost per student?

- 5** Elijah is x years old. His sister is 2 years younger than him.
Write an expression for the sum of their ages.

- 6** Mike saved x dollars. Georgia saved double Mike's amount.
Write an expression to represent the total amount they have saved.

- 7** A rectangle has a width of x cm. The length is 1 cm longer than the width.
Write an expression for the perimeter of the rectangle.

- 8** A taxi costs \$10 to start and \$8 per kilometre.
Write an expression to represent the cost of a trip of distance n kilometres.

SOLVING FOR AN UNKNOWN AFTER SUBSTITUTION

- We can use formulas to find any unknown value when we know all other quantities.
 - Write the formula.
 - Substitute given values.
 - Solve for the unknown value using inverse operations.

Example Substitute values into a formula then solve for the unknown.

$y = 2x + 7$ Find x when $y = 29$ 1. Write formula $y = 2x + 7$ 2. Substitute $29 = 2x + 7$ 3. Solve $22 = 2x$ $11 = x$	How do we use formulas? 1. w t f 2. given values. 3. to find the unknown. Annotate every step of the worked example, showing the operation applied to both sides
--	--

A rectangle has length l and width w . The formula for the area is $A = lw$.

- a** Find the area when
 $l = 40$ cm and $w = 17$ cm.
 $A = lw$

$$A = 680$$

- b** The area of a rectangle is 720 cm^2 .
 The length is 45 cm. Find the width.
 $A = lw$

$$w = 16$$

The rule for a relationship is given by $y = 2x + 5$

- a** Find y when $x = 17$
 $y = 2x + 5$

$$y = 39$$

- b** Find x when $y = 20$
 $y = 2x + 5$

$$x = 7.5$$

Key ideas

- We can find any unknown value in a formula if know the value of the other
 - Write the
 - given values.
 - the equation using inverse operations.

1 Consider the formula $U = 8a + 4$

a Find U if $a = -1$

b Find a if $U = 44$. Set up an equation.

c Find a if $U = 92$

d If $U = -12$, find the value of a

2 Consider the formula $y = 2x + 4$

a Find the value of x that makes $y = 16$

b Find the value of x if $y = 0$

3 Consider the formula $v = u + at$

a Find t when $v = 102$, $u = 18$ and $a = 7$

b Find a when $v = 54$, $u = 12$ and $t = 14$

4 Consider the formula $d = \frac{1}{2}ct$

a Find c when $d = 100$, $t = 8$

b Find t when $d = 320$, $c = 18$

5 Consider the formula $I = Prn$

a Find P when $I = 19500$, $r = 0.03$, $n = 25$

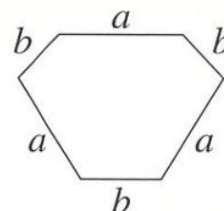
b Find n when $I = 2100$, $P = 5000$, $r = \frac{6}{100}$

- 6** A formula for the area of a rectangle is $A = bh$, where b is the base and h is the height.
- | | |
|--|---|
| <p>a Calculate the area of a rectangle with base length of 8 cm and height of 9 cm.</p> | <p>b Calculate the base length of a rectangle with area of 20 cm² and height of 4 cm.</p> |
|--|---|
- c** Calculate the height of a rectangle with area of 25 cm² and base length of 5 cm.

- 7** The perimeter of a rectangle of length l and width w is given by the formula $P = 2(l + w)$. Use the formula to:
- | | |
|---|---|
| <p>a find P when $l = 6$ and $w = 20$</p> | <p>b find l when $w = 6$ and $P = 20$</p> |
|---|---|
-
- | | |
|---|---|
| <p>c find w when $P = 10$ and $l = 2$</p> | <p>d find area of a rectangle if its perimeter is 20 and width is 4.</p> |
|---|---|

- 8** The area of a triangle is given by the formula $A = \frac{1}{2}bh$. Use the formula to:
- | | |
|---|--|
| <p>a find A when $b = 20$ and $h = 9$</p> | <p>b to find b when $A = 20$ and $h = 5$</p> |
|---|--|

- 9** Consider the diagram to the right.
- a** Find a formula for the perimeter, P , of the hexagon in terms of a and b
-



- | | |
|--|---|
| <p>b Work out P when $a = 7$ and $b = 3$</p> | <p>c Work out a when $b = 2$ and $P = 21$</p> |
|--|---|

- 10** The side length of a square is l
- a** Write a formula to find the perimeter, P , of the square.
- b** Use the formula to find P when $l = 23$
- c** Use the formula to form and solve an equation to find l when $P = 23$
-
-
- 11** A rectangle has length l and width w . The formula for the area is $A = lw$
- a** Use the formula to find A when $l = 40$ and $w = 17$
- b** The area of a different rectangle is 720 cm^2 . The length of this rectangle is 45 cm .
Use $A = lw$ to form an equation in w
-
-
- c** Solve your equation to find the width of this rectangle.
-
-
- 12** The cost $\$C$ to hire a taxi for a trip of length $d \text{ km}$ is $C = 3 + 2d$
- a** Find the cost of a 10 km trip. ($d = 10$)
-
- b** A trip cost a total of $\$161$. Determine the distance the taxi travelled.
-
-
- 13** The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$,
where t is the time in seconds after a tap is turned on.
- a** Find the volume after 2 minutes.
- b** How long, to the nearest minute, does it take to empty the tank?
-
-

14 A car salesman has a basic salary of \$800 a month.

For every car he sells, he earns a commission of \$150.

a Write a formula to find his wage, W , in terms of the number, n , of cars he sells.

.....

b Use the formula to find his wage in a month when he sells 12 cars.

.....

c Use the formula to form and solve an equation to find the number of cars he sells in a month when he earns \$1850.

.....

15 Solve the equations to determine the value of the unknown pronumeral.

Round to 2 d.p. where appropriate.

a $F = ma$ $F = 12, m = 3$	b $m = \frac{F}{a}$ $m = 12, a = 3$	c $A = lb$ $A = 30, l = 5$
d $M = \frac{5k}{18},$ $M = 15$	e $C = \frac{890-2r}{n}$ $C = 50, n = 10$	f $v = u + at$ $v = 6, u = 2, t = 4$
g $S = \frac{n}{2}(a + l)$ $S = 560, n = 20, l = 53$	h $V^2 = gR,$ $V = 12, g = 10$	i $A = P\left(1 + \frac{r}{100}\right)$ $A = 1000, P = 800$
j $S = 2\pi r^2$ $S = 72, \pi = 3.14$	k $v^2 = u^2 + 2as$ $v = 22, u = 6, a = 12$	l $m = \sqrt{\frac{x}{y}}$ $m = 8, y = 4$

- 16 The formula for the area of a trapezium is $A = \frac{1}{2}(a + b)h$
- a Find the value of A given that $a = 7$, $b = 8$ and $h = 10$

.....

.....

- b Find the value of h given that $A = 72$, $a = 6$ and $b = 10$

.....

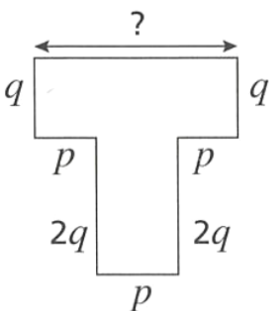
.....

- c Find the value of b given that $A = 78$, $a = 7$ and $h = 12$

.....

.....

- 17 Consider the diagram to the right.
- a Explain why the length of the line with a question mark is $3p$.



.....

- b Find a formula for the perimeter, P , of the shape,

.....

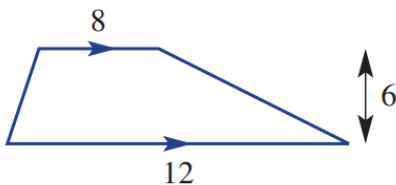
- 18 Consider the formula $c^2 = a^2 + b^2$. Determine the value of b if $c = 35$ and $a = 20$.

.....

.....

- 19 The formula for the area of the trapezium is $A = \frac{1}{2}h(a + b)$, where a, b are the parallel sides and h is the perpendicular height.

- a Find the area of the trapezium on the right.



.....

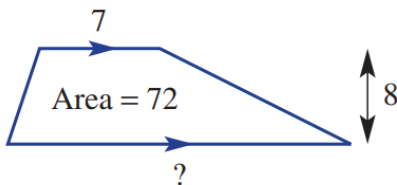
.....

- b Find the value of h if $A = 20$, $a = 3$ and $b = 7$

.....

.....

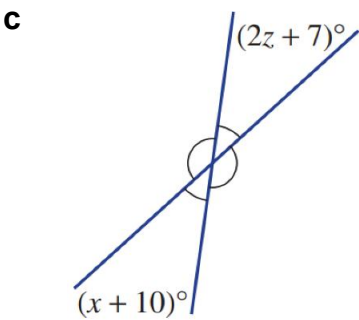
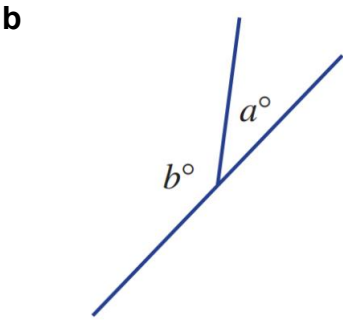
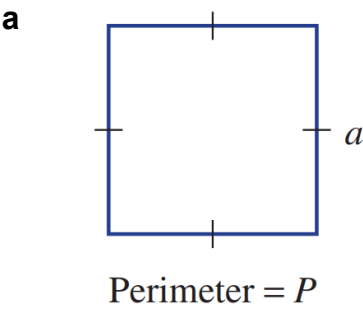
- c Find the missing value in the trapezium shown on the right.



.....

.....

20 Write a formula describing these relationships.



21 $s = \frac{d}{t}$ gives the speed in km/h of a car that has travelled a distance of d km in t hours.
Calculate the time taken, to the nearest minute, to travel 380 km if the car averages 75 km/h.

.....

.....

.....

22 To convert between temperatures in Celsius and Fahrenheit, use this formula:

$$F = \frac{9C}{5} + 32$$

a Find F if $C = 20$

b Find the value of C if $F = 50$.
.....
.....

c When watching American TV, you hear “it’s so hot, it’s 100 degrees outside!”
Fahrenheit is used in the USA. What is this in Celsius?
.....
.....

d Books burn at 451°F . What is this in Celsius?
.....
.....

23 Write a formula to show these mathematical relationships.
Make the first pronumeral the subject.

a How many \$ D you have, if you have c cents.

b How long something is in d cm, if it is m metres.

c The discounted price \$ D that is 30% off the original price \$ P

d The cost \$ C of hiring a car at \$50 upfront plus \$18 per hour for t hours.

- 24** The velocity of an object is described by the formula $v = u + at$ where u is the initial velocity in m/s, a is acceleration in m/s^2 , and t is the time in seconds.

Find the acceleration of a car that can go from 0 to 100 km/h in 7 seconds.

You can convert between speed in m/s and km/h using the formula $s_{\text{kmh}} = \frac{s_{\text{ms}}}{3.6}$.

.....
.....
.....

25 2004 HSC Standard 2 Band 4

If $d = 6t^2$, what are two possible values of t when $d = 2400$?

26 HSC Standard 2 Sample Question Band 4

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere.

If the volume is 220 cm^3 , find the radius, to 1 decimal place.

27 2015 HSC Standard 2 Band 4

The formula $C = \frac{5}{9}(F - 32)$ is used to convert temperatures between degree Fahrenheit (F) to degrees Celsius (C). Covert 3 degrees Celsius to the equivalent temperature in Fahrenheit.

28 2019 HSC Standard 2 Band 4

Given the formula $C = \frac{A(y+1)}{24}$, calculate the value of y when $C = 120$ and $A = 500$

29 2020 HSC Standard 2 Band 4

The distance, d metres, travelled by a car slowing down from u km/h to v km/h can be obtained using the formula $v^2 = u^2 - 100d$.

What distance does as car travel while slowing down from 70 km/h to 40 km/h?

PROBLEM-SOLVING USING EQUATIONS

- Model scenarios with unknown numbers as **equations**.
 - Let a pronumeral represent an unknown quantity.
 - Write an equation describing the problem.
 - Solve the equation.
 - Check your answer makes sense and has correct units.

Example Model and solve word problems using equations.

When Adam adds his current age and his age next year, the result is 27. How old is he now? - Let a be Adam's current age. $a + (a + 1) = 27$ $2a + 1 = 27$ $2a = 26$ $a = 13$ - Adam is 13 years old this year.	Why did we make a represent Adam's current age? <i>The pronumeral should represent</i> How do we represent Adam's age next year? <i>If Adam's current age is a, then his age next year is</i>
Asher works for 12 hours this week. He earned a wage and a bonus of \$50. If he earned \$410 in total, how much does he make an hour? - Let n be how much he makes an hour. $12n + 50 = 410$ $12n = 360$ $n = 30$ - Asher makes \$30 an hour.	How did we know $12n + 50$ represents Asher's total pay?

a When Mila adds her current age and her age last year, the result is 23.
How old is she now?

- Let be
-
-
-

12 years old

b Charlotte earned \$180 total this week.
If this includes a \$60 bonus and she earns \$15 an hour, how many hours did she work this week?

- Let be
-
-
-

8 hours

- 1 The sum of the ages of Sam and his brother Bernard is 34.
If Sam is 4 years older than Bernard, find their ages.

Let x be the of Bernard.

The age of Sam is .

The sum of their ages is 34.

$$\therefore x + \text{} = 34$$

$$\text{} + 4 = 34$$

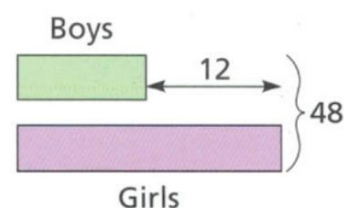
$$2x = \text{}$$

$$x = \text{}$$

$\therefore$ Bernard is _____ years old and Sam is _____ years old.

- 2 There are 12 less boys than girls in a choir.
There are 48 students altogether in the choir.
Find the number of girls in the choir.

Let x be the number of girls.



- 3 The total price of three pens and one pencil is 99 cents.
The pencil costs 18 cents.
Let the price of one pen be x cents.

Which of these equations represents the information given?

A $\frac{x}{3} + 18 = 99$

B $3(x + 18) = 99$

C $3x + 18 = 99$

D $x + 3 \times 18 = 99$

Solve the correct equation to find the price of one pen.

- 4 Jonas is paid \$19.50 per hour and gets paid a bonus of \$65 each week.
One week he earned \$502. Calculate how many hours Jonas worked that week.

- a Let n be the number of hours Jonas worked.
Write an equation relating the information given.

.....

- b Solve the equation for n .

.....

.....

- 5 The sum of Kate's age now, and her age next year, is 47.
How old is Kate?

Let x be Kate's current age.

$$x + (x + 1) = 47$$

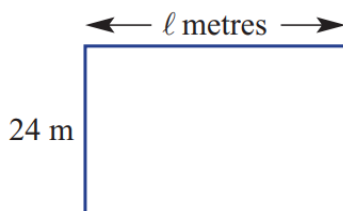
- 6 The sum of the ages of two brothers Keynai and Kamire is 28.
If Kamire is 4 years older than Keynai, find their ages.

Let x be the age of Keynai.

- 7 A prize of \$1000 is divided between Asher and Leo.
Asher receives \$287 more than Leo.
How much did Leo receive?

Let x be

- 8 This rectangular paddock has an area of 720 m. Find the length.



- 9 A prize of \$1000 is divided between Adele and Benita so that Adele receives \$285 more than Benita. How much did each person receive?

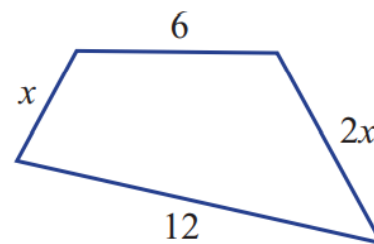
- 10 A taxi ride is \$10 flag fall and \$2.50 per kilometre.
How many kilometres did you travel if the final cost was \$220?

- 11** Jane and Luke have a combined age of 60.
Given that Jane is twice as old as Luke, find the ages of Luke and Jane.
- 12** Zach buys four copies of a game.
He also buys a gaming accessory for \$160.
He spent a total of \$388.
How much was the game?
- 13** Two people are eating at a restaurant.
A fancy dish costs three times as much as a regular dish.
If they order one of each and the total bill is \$68, how much is the fancy dish?
- 14** A theme park charges \$18 for a child ticket.
The total cost of two adult tickets and three child tickets is \$110.
a Find the value of an adult ticket.
- b** Find the cost for a party of 6 adults and 13 children to visit the theme park.
- 15** Molly is paid \$17 an hour and gets paid a bonus of \$65 each week.
One week she earned \$643.
How many hours did she work?

DEVELOPMENT

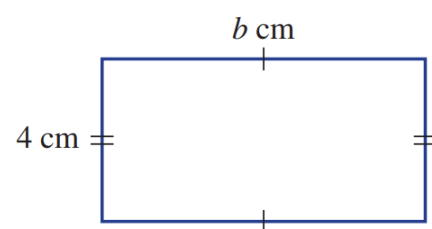
16 The perimeter of the shape shown is 30.

a Find the value of x .

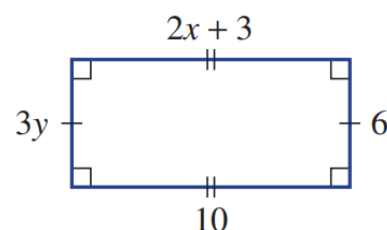


b Explain why it is impossible for the perimeter to be 17.

17 Determine the breadth of this rectangle if the perimeter is 72.7 cm.

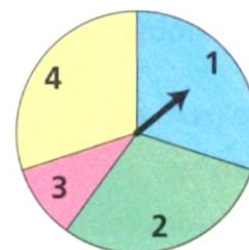


18 Find the values of x and y in the rectangle shown.



19 The spinner is spun once. The table shows the probabilities that it lands on 1, 2, 3 or 4.

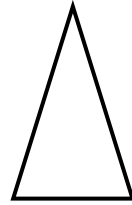
Score	1	2	3	4
Probability	0.3	0.3	x	$3x$



a Find the value of x . (hint: the sum of all probabilities is 1)

b Find the probability that the spinner lands on an even number.

- 20** The equal sides of an isosceles triangle are 12 cm long.
The perimeter is 35 cm. How long is the third side?



- 21** In a rectangle, one side is 4 times longer than the other.
The perimeter is 60 cm. How long is the shorter side?



- 22** In a rectangle, one side is 4 cm longer than the other.
The perimeter is 60 cm. How long is the shorter side?



- 23** In an isosceles triangle, one side is 3 cm longer than the equal sides.
The perimeter is 60 cm. How long are the sides?

- 24** I can buy 3 rulers and 4 pencils for \$13.50.
The pencils cost \$1.50 each. What is the price for each ruler?

- 25** A 50 m long rope is cut into four pieces.
Three of them are equal but the other is 6 m shorter. How long are the pieces?

- 26** A rectangular paddock has an area of 720 m.
One of the sides is 22.5 m. Calculate its perimeter.
- 27** Sarah is x years old. Thomas is 3 years older than Sarah. David is twice as old as Sarah.
The total of their ages is 51. How old is Sarah?
- 28** Rayyan buys 3 identical pens from a shop. For each pen he was given a 12-cent discount.
He paid \$25 and got \$1.36 change. How much was each pen?
- 29** Maya, Charlotte, and Emma are sharing \$1850. They don't share it evenly.
Let Emma's share be x .
Charlotte gets \$400 more than Emma.
Maya gets \$350 less than double Emma's share.
How much does each person receive?
- 30** Shay recorded three songs, A, B and C.
Song B is 40 seconds longer than song A.
Song C is twice as long as song A.
The total length of the three songs is 680 seconds.
Work out the length of song B. Give your answer in minutes and seconds.

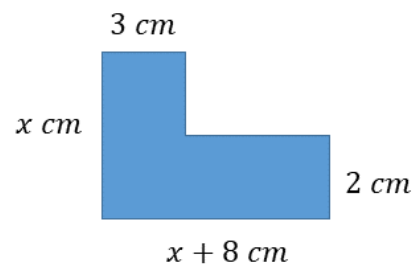
31 Consider this shape.

a Find the perimeter if $x = 5$

b Find the area if $x = 7$

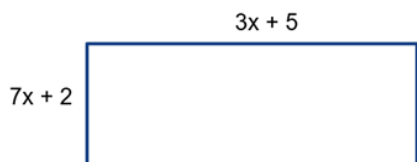
c The perimeter is 46 cm. Find the value of x .

d The area of the shape is 65 cm^2 . Find the value of x

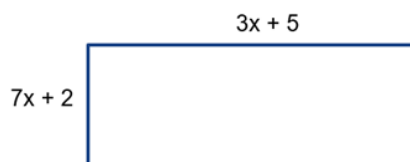


32

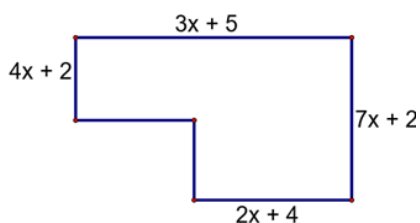
a Write an expression to find the perimeter of the rectangle. Simplify the expression.



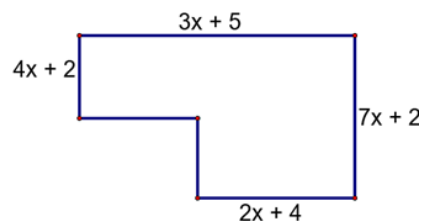
b If $x = 2.5$, find the perimeter of the rectangle.



c Write an expression to find the perimeter of the shape. Simplify the expression.



d Determine the value of x if the perimeter is 64 cm.



33 Consecutive integers are numbers that occur one after the other, such as 5, 6, 7.

Any number can be represented as n

Consecutive numbers would $n, (n + 1), (n + 2) \dots$

- | | |
|--|--|
| <p>a The sum of three consecutive integers is 261. Find the integers.</p> | <p>b The sum of three consecutive integers is 342. Find the integers.</p> |
|--|--|

Consecutive even numbers are even numbers that occur one after the other, such as 14, 16, 18

Any even number can be represented as $2n$.

Consecutive even numbers would be $2n, (2n + 2), (2n + 4), (2n + 6) \dots$

- | | |
|---|---|
| <p>c Find three consecutive even numbers that add up to 42</p> | <p>d Find four consecutive even numbers that add up to 60.</p> |
|---|---|

Consecutive odd numbers are odd numbers that occur one after the other, such as 11, 13, 15

Any odd number can be represented as $2n + 1$.

Consecutive even numbers would be $2n + 1, (2n + 3), (2n + 5), (2n + 7) \dots$

- | | |
|---|--|
| <p>e Find three consecutive odd numbers that add up to 57.</p> | <p>f Find four consecutive odd numbers that add up to 48.</p> |
|---|--|

34 Four consecutive numbers add up to 27.

a Find the integers.

b What do you notice? What does this mean?

CHECK YOUR UNDERSTANDING

Solving for an unknown after substitution

1 $y = 6x$

If $y = 2$, find the value of x .

a $\frac{1}{2}$

b 12

c $\frac{1}{3}$

d 3

2 $a = 2 - c$

If $c = 0.25$, find the value of a .

a $\frac{9}{4}$

b $\frac{7}{4}$

c $\frac{1}{4}$

d $\frac{3}{4}$

3 $d = st$

If $d = 4$ and $t = 10$, what is the value of s .

a 0.4

b 40

c 2.5

d 4

4 $z = 4x - 2y$

If $x = 2.5$ and $z = 2$, what is the value of y ?

a 8

b -8

c 6

d 4

5 $p = 6\sqrt{q}$

What is the value of q when $p = 9$?

a 18

b $\sqrt{\frac{3}{2}}$

c 3

d 2.25

6 $\sqrt{p} = 6q$

What is the value of q when $p = -9$?

a -0.5

b 2916

c 0.5

d Impossible to solve.