

Student Name

Mathematics Stage 4

EQUATIONS

Book 2 Solve and verify equations by substitution

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA4-EQU-C-01 solves linear equations of up to 2 steps and quadratic equations of the form $ax^2 = c$

Solve and verify linear equations by substitution

- Solve problems involving linear equations, including those arising from substituting given values into formulas

SUMMARY

Formulas

A formula is an equation showing a relationship between two or more variables.

$$\text{Area} = \text{length} \times \text{breadth}$$

Subject

The isolated variable on the LHS

Using Formulas

- Write the formula.
- Substitute given values.
- Solve to find the unknown.

Finding the subject

$$v = u + at$$

Find v when $u = 50$, $a = -10$ and $t = 6$

- | | |
|------------------|-------------------|
| 1. Write formula | $v = u + at$ |
| 2. Substitute | $= 50 + (-10)(6)$ |
| 3. Solve | $= 50 - 60$ |
| | $= -10$ |

Finding any unknown value

$$y = 2x + 7$$

Find x when $y = 29$

- | | |
|------------------|---------------|
| 1. Write formula | $y = 2x + 7$ |
| 2. Substitute | $29 = 2x + 7$ |
| 3. Solve | $22 = 2x$ |
| | $11 = x$ |

REVIEW OF PRIOR KNOWLEDGE

1 Substitute the given value into the expression and evaluate the result.

a $7x$ $x = 5$	b $2p - 3$ $p = 4$	c $\frac{3x}{2}$ $x = 8$
d $r - q$ $r = 7.5,$ $q = -1$	e $pq - q$ $p = 4, q = -3$	f $\frac{x}{2+y}$ $x = 3, y = 0$
g $\frac{x}{2-y}$ $x = 15,$ $y = -3$	h $\sqrt{c} + 5$ $c = 25$	i $x + y^2$ $x = 3, y = 0.5$
j $2f - h$ $f = -3,$ $h = 9$	k $\sqrt{6m + n}$ $m = 3$ $n = 7$	l $\sqrt{g^2 + h^2}$ $g = 6, h = 8$

2 Solve these equations

a $4x = 12$	b $32 = 8 \times x$	c $49 = 7 \times 2 \times x$
d $3y + 7 = 40$	e $3y - 7 = 41$	f $40 + 3y = 16$
g $40 - 3y = 16$	h $4x - 3 = 17$	i $17 = 4x - 3$
j $3g + 12 = 33$	k $40 = 2p - 7$	l $10(q - 3) = 80$
m $\frac{m}{4} - 3 = 12$	n $\frac{n+3}{4} = 12$	o $8 = 24 - 2a$

SUBSTITUTION INTO FORMULAS

- **Formulas** are equations that show how **variables** relate to each other.

$$\text{Area} = \text{length} \times \text{breadth}$$

$$A = lb$$

- The **subject** is the **isolated variable** on the left side.
- We use formulas to find the subject when we know the other values.
 1. Write the formula.
 2. Substitute given values.
 3. Solve to find the unknown.

Example Substitute values into a formula and evaluate the subject.

$v = u + at$ Find v when $u = 50$, $a = -10$ and $t = 6$ 1. Write formula $v = u + at$ 2. Substitute $= 50 + (-10)(6)$ 3. Solve $= 50 - 60$ $= -10$	What is the subject of the formula? What are the steps to using formulas? 1. w..... t..... f..... 2. given values. 3. to find the unknown.
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Find the unknown using the formula.

a $t = a + (n - 1)d$
 Find t when $a = -3$, $n = 20$, $d = 2$
 $t = a + (n - 1)d$
 $t = (\quad) + (\quad - 1)(\quad)$
 $= \dots\dots\dots$

$$t = 35$$

b $V = IR$
 Find V when $I = 10$, $R = 250$

$$V = 2500$$

c $a = \frac{F}{m}$
 Find a when $F = -80$, $m = 5$

$$a = -16$$

d $C = I + 20x$
 Find C when $I = -13$, $x = 15$

$$C = 287$$

e $c = \sqrt{a^2 + b^2}$
 Find c when $a = 3$, $b = 4$

$$c = 5$$

f $z = \frac{n+1}{4r}$
 Find z when $n = -13$, $r = 2$

$$z = -\frac{3}{2}$$

Key ideas

1. A formula describes a between two or more variables.
2. The isolated variable is called the of the formula.
3. We can find the subject by known values and the result.

1 Consider the formula $A = 4p + 7$

a Find A if $p = 3$

b Find A if $p = 11$

c Find A if $p = -2$

d Find A if $p = 13.5$

2 Given these formulae, substitute known values and evaluate the subject.

a $V = IR$

find V when $I = 50$ and $R = 20$.

b $W = Fs$

find W when $F = 100$ and $s = 12$

c $s = \frac{d}{t}$

find s when $d = 50$ and $t = 10$

d $v = u + at$

find v when $u = 5$, $a = 10$ and $t = 7$

e $s = ut + \frac{1}{2}at^2$

find s when $u = -3$, $a = 10$ and $t = 5$

f $d = 180 - \frac{360}{n}$

Find d when $n = 12$

g $a = \frac{2Rn}{n+1}$

find a when $R = 8$, and $n = 24$

h $V = \frac{4}{3}\pi r^3$

Find V when $r = 8.5$

i $A = P(1 + r)^n$

find A when $P = 1000$, $r = 0.1$, $n = 5$

j $A = P(1 - r)^n$

find A when $P = 1000$, $r = 0.1$, $n = 5$

k $c = \sqrt{a^2 + b^2}$

find c when $a = 5$, $b = 12$

l $a = \sqrt{c^2 - b^2}$

find a when $c = 20$, $b = 16$

3 Given these formulae, substitute known values and evaluate the subject.

a $S = V - Dn$

find S when $V = 17.5$, $D = 5$ and $n = 3$.

b $z = \frac{x-y}{s}$

find z when $x = 24$, $y = 4.5$ and $s = 4$

4 The formula used to convert between degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$.

A recipe says to bake a roast at $400^\circ F$. How much is this in degrees Celsius?

Round your answer to the nearest degree.

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5 The area of an annulus (or donut) can be calculated using $A = \pi(R^2 - r^2)$, where R is the radius of the large circle and r is the radius of the smaller circle.

Find the area of the annulus if $R = 8$ cm and $r = 4$ cm. Round your answer to 2 d.p.

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6 Body mass index (BMI) is used to determine if an adult's mass is within a recommended range.

The formula for BMI is $b = \frac{m}{h^2}$ where m is mass in kilograms and h is height in metres.

a Mr Ding has a mass of 68 kg and height of 181 cm. Calculate his BMI.

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b Someone found Mr Ding's BMI to be 0.0020756... What mistake have they made?

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c The recommended range for BMI is $21 < b < 25$. Based on Mr Ding's BMI, what would you recommend he do to get into the recommended range?

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7 2008 HSC Standard 2 Band 4

What is the value of $\sqrt{\frac{x+2y}{8y}}$ if $x = 5.6$ and $y = 3.1$, correct to 2 decimal places?

- (A) 0.69
- (B) 2.62
- (C) 2.83
- (D) 4.77

8 HSC Standard 2 Sample Question Band 4

What is the value of $\frac{a+b}{ab}$ if $a = -2.1$ and $b = -3.6$, correct to 1 decimal place?

9 HSC Standard 2 Sample Question Band 4

For adults (18 years and older), the Body Mass Index is given by $B = \frac{m}{h^2}$ where m is mass in kg and h is height in metres.

The medically accepted healthy range is $21 \leq B \leq 25$

What is the minimum weight for a 163 cm adult to be considered healthy?

SOLVING FOR AN UNKNOWN AFTER SUBSTITUTION

- We can use formulas to find any unknown value when we know all other quantities.
 - Write the formula.
 - Substitute given values.
 - Solve for the unknown value using inverse operations.

Example Substitute values into a formula then solve for the unknown.

$y = 2x + 7$ Find x when $y = 29$ 1. Write formula $y = 2x + 7$ 2. Substitute $29 = 2x + 7$ 3. Solve $22 = 2x$ $11 = x$	How do we use formulas? 1. w t f 2. given values. 3. to find the unknown. Annotate every step of the worked example, showing the operation applied to both sides
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A rectangle has length l and width w . The formula for the area is $A = lw$.

- a** Find the area when
 $l = 40$ cm and $w = 17$ cm.
 $A = lw$

$$A = 680$$

- b** The area of a rectangle is 720 cm^2 .
 The length is 45 cm. Find the width.
 $A = lw$

$$w = 16$$

The rule for a relationship is given by $y = 2x + 5$

- a** Find y when $x = 17$
 $y = 2x + 5$

$$y = 39$$

- b** Find x when $y = 20$
 $y = 2x + 5$

$$x = 7.5$$

Key ideas

- We can find any unknown value in a formula if know the value of the other
 - Write the
 - given values.
 - the equation using inverse operations.

1 Consider the formula $U = 8a + 4$

a Find U if $a = -1$

b Find a if $U = 44$. Set up an equation.

c Find a if $U = 92$

d If $U = -12$, find the value of a

2 Consider the formula $y = 2x + 4$

a Find the value of x that makes $y = 16$

b Find the value of x if $y = 0$

3 Consider the formula $v = u + at$

a Find t when $v = 102$, $u = 18$ and $a = 7$

b Find a when $v = 54$, $u = 12$ and $t = 14$

4 Consider the formula $d = \frac{1}{2}ct$

a Find c when $d = 100$, $t = 8$

b Find t when $d = 320$, $c = 18$

5 Consider the formula $I = Prn$

a Find P when $I = 19500$, $r = 0.03$, $n = 25$

b Find n when $I = 2100$, $P = 5000$, $r = \frac{6}{100}$

- 6** A formula for the area of a rectangle is $A = bh$, where b is the base and h is the height.
- | | |
|--|---|
| <p>a Calculate the area of a rectangle with base length of 8 cm and height of 9 cm.</p> | <p>b Calculate the base length of a rectangle with area of 20 cm² and height of 4 cm.</p> |
|--|---|
- c** Calculate the height of a rectangle with area of 25 cm² and base length of 5 cm.

- 7** The perimeter of a rectangle of length l and width w is given by the formula $P = 2(l + w)$. Use the formula to:

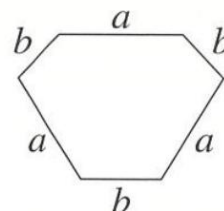
- | | |
|---|---|
| <p>a find P when $l = 6$ and $w = 20$</p> | <p>b find l when $w = 6$ and $P = 20$</p> |
|---|---|
-
- | | |
|---|---|
| <p>c find w when $P = 10$ and $l = 2$</p> | <p>d find area of a rectangle if its perimeter is 20 and width is 4.</p> |
|---|---|

- 8** The area of a triangle is given by the formula $A = \frac{1}{2}bh$. Use the formula to:
- | | |
|---|--|
| <p>a find A when $b = 20$ and $h = 9$</p> | <p>b to find b when $A = 20$ and $h = 5$</p> |
|---|--|

- 9** Consider the diagram to the right.

- a** Find a formula for the perimeter, P , of the hexagon in terms of a and b

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- | | |
|--|---|
| <p>b Work out P when $a = 7$ and $b = 3$</p> | <p>c Work out a when $b = 2$ and $P = 21$</p> |
|--|---|

- 10** The side length of a square is l
- a** Write a formula to find the perimeter, P , of the square.
- b** Use the formula to find P when $l = 23$
- c** Use the formula to form and solve an equation to find l when $P = 23$
-
-
- 11** A rectangle has length l and width w . The formula for the area is $A = lw$
- a** Use the formula to find A when $l = 40$ and $w = 17$
- b** The area of a different rectangle is 720 cm^2 . The length of this rectangle is 45 cm .
Use $A = lw$ to form an equation in w
-
-
- c** Solve your equation to find the width of this rectangle.
-
-
- 12** The cost $\$C$ to hire a taxi for a trip of length $d \text{ km}$ is $C = 3 + 2d$
- a** Find the cost of a 10 km trip. ($d = 10$)
-
- b** A trip cost a total of $\$161$. Determine the distance the taxi travelled.
-
-
- 13** The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$,
where t is the time in seconds after a tap is turned on.
- a** Find the volume after 2 minutes.
- b** How long, to the nearest minute, does it take to empty the tank?
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14 A car salesman has a basic salary of \$800 a month.

For every car he sells, he earns a commission of \$150.

a Write a formula to find his wage, W , in terms of the number, n , of cars he sells.

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b Use the formula to find his wage in a month when he sells 12 cars.

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c Use the formula to form and solve an equation to find the number of cars he sells in a month when he earns \$1850.

.....

15 Solve the equations to determine the value of the unknown pronumeral.

Round to 2 d.p. where appropriate.

a $F = ma$ $F = 12, m = 3$	b $m = \frac{F}{a}$ $m = 12, a = 3$	c $A = lb$ $A = 30, l = 5$
d $M = \frac{5k}{18},$ $M = 15$	e $C = \frac{890-2r}{n}$ $C = 50, n = 10$	f $v = u + at$ $v = 6, u = 2, t = 4$
g $S = \frac{n}{2}(a + l)$ $S = 560, n = 20, l = 53$	h $V^2 = gR,$ $V = 12, g = 10$	i $A = P\left(1 + \frac{r}{100}\right)$ $A = 1000, P = 800$
j $S = 2\pi r^2$ $S = 72, \pi = 3.14$	k $v^2 = u^2 + 2as$ $v = 22, u = 6, a = 12$	l $m = \sqrt{\frac{x}{y}}$ $m = 8, y = 4$

- 16 The formula for the area of a trapezium is $A = \frac{1}{2}(a + b)h$
- a Find the value of A given that $a = 7$, $b = 8$ and $h = 10$

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- b Find the value of h given that $A = 72$, $a = 6$ and $b = 10$

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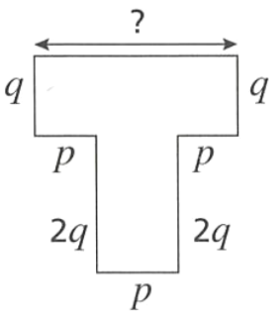
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- c Find the value of b given that $A = 78$, $a = 7$ and $h = 12$

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- 17 Consider the diagram to the right.
- a Explain why the length of the line with a question mark is $3p$.



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- b Find a formula for the perimeter, P , of the shape,

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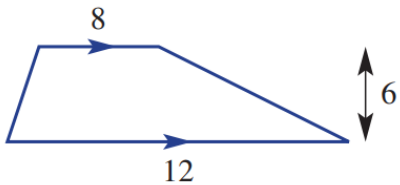
- 18 Consider the formula $c^2 = a^2 + b^2$. Determine the value of b if $c = 35$ and $a = 20$.

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- 19 The formula for the area of the trapezium is $A = \frac{1}{2}h(a + b)$, where a, b are the parallel sides and h is the perpendicular height.

- a Find the area of the trapezium on the right.



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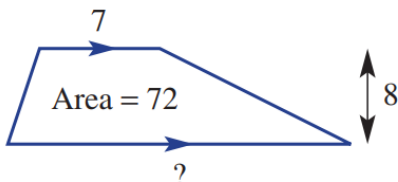
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- b Find the value of h if $A = 20$, $a = 3$ and $b = 7$

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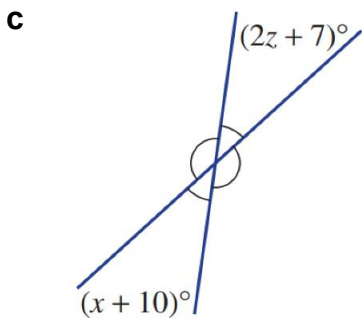
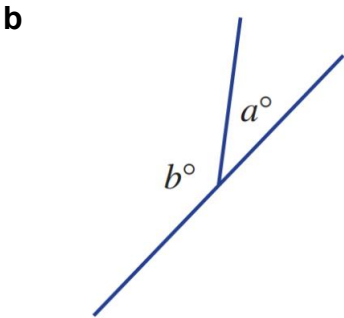
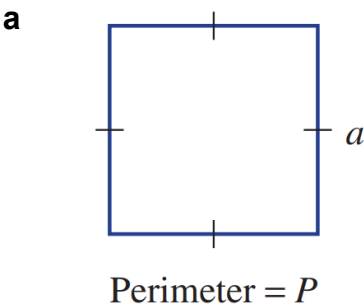
- c Find the missing value in the trapezium shown on the right.



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20 Write a formula describing these relationships.



21 $s = \frac{d}{t}$ gives the speed in km/h of a car that has travelled a distance of d km in t hours.
Calculate the time taken, to the nearest minute, to travel 380 km if the car averages 75 km/h.

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22 To convert between temperatures in Celsius and Fahrenheit, use this formula:

$$F = \frac{9C}{5} + 32$$

a Find F if $C = 20$

b Find the value of C if $F = 50$.
.....
.....

c When watching American TV, you hear “it’s so hot, it’s 100 degrees outside!”
Fahrenheit is used in the USA. What is this in Celsius?
.....
.....

d Books burn at 451°F. What is this in Celsius?
.....
.....

23 Write a formula to show these mathematical relationships.
Make the first pronumeral the subject.

a How many \$ D you have, if you have c cents.

b How long something is in d cm, if it is m metres.

c The discounted price \$ D that is 30% off the original price \$ P

d The cost \$ C of hiring a car at \$50 upfront plus \$18 per hour for t hours.

- 24** The velocity of an object is described by the formula $v = u + at$ where u is the initial velocity in m/s, a is acceleration in m/s^2 , and t is the time in seconds.

Find the acceleration of a car that can go from 0 to 100 km/h in 7 seconds.

You can convert between speed in m/s and km/h using the formula $s_{\text{kmh}} = \frac{s_{\text{ms}}}{3.6}$.

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25 2004 HSC Standard 2 Band 4

If $d = 6t^2$, what are two possible values of t when $d = 2400$?

26 HSC Standard 2 Sample Question Band 4

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere.

If the volume is 220 cm^3 , find the radius, to 1 decimal place.

27 2015 HSC Standard 2 Band 4

The formula $C = \frac{5}{9}(F - 32)$ is used to convert temperatures between degree Fahrenheit (F) to degrees Celsius (C). Covert 3 degrees Celsius to the equivalent temperature in Fahrenheit.

28 2019 HSC Standard 2 Band 4

Given the formula $C = \frac{A(y+1)}{24}$, calculate the value of y when $C = 120$ and $A = 500$

29 2020 HSC Standard 2 Band 4

The distance, d metres, travelled by a car slowing down from u km/h to v km/h can be obtained using the formula $v^2 = u^2 - 100d$.

What distance does as car travel while slowing down from 70 km/h to 40 km/h?

CHECK YOUR UNDERSTANDING

Substitution into formulas

1	If $g = 4$, what is the value of $5 + g$?			
a	1	b	54	c 9
d	Can't tell as g is a variable.			
2	$y = 10x - 5$ If $x = 4$, what is the value of y ?			
a	$5x$	b	35	c 99
d	20			
3	If $t = 2$, what is the value of $13 - 2t$?			
a	$11t$	b	11	c -9
d	9			
4	$A = 10 + \frac{xy}{2}$ Which of these is the value of A when $x = 6$ and $y = 0$?			
a	13	b	$10xy$	c 10
d	40			
5	$C = 15 - 3d$ If $d = -3$, what is the value of C ?			
a	-18	b	6	c 9
d	24			
6	$g = 3k^2 - 4k$ If $k = 4$, what is the value of g ?			
a	4	b	32	c 8
d	128			
7	$z = 3r^2 - 4r$ If $r = -2$, what is the value of z ?			
a	-34	b	-16	c 16
d	0			

Solving for an unknown after substitution

1 $y = 6x$

If $y = 2$, find the value of x .

a $\frac{1}{2}$

b 12

c $\frac{1}{3}$

d 3

2 $a = 2 - c$

If $c = 0.25$, find the value of a .

a $\frac{9}{4}$

b $\frac{7}{4}$

c $\frac{1}{4}$

d $\frac{3}{4}$

3 $d = st$

If $d = 4$ and $t = 10$, what is the value of s .

a 0.4

b 40

c 2.5

d 4

4 $z = 4x - 2y$

If $x = 2.5$ and $z = 2$, what is the value of y ?

a 8

b -8

c 6

d 4

5 $p = 6\sqrt{q}$

What is the value of q when $p = 9$?

a 18

b $\sqrt{\frac{3}{2}}$

c 3

d 2.25

6 $\sqrt{p} = 6q$

What is the value of q when $p = -9$?

a -0.5

b 2916

c 0.5

d Impossible to solve.