Student Name

Mathematics Stage 5 Paths

TRIGONOMETRY C

Book 1 Solve 3-dimensional problems involving right-angled triangles

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-TRG-P-01 applies Pythagoras' theorem and trigonometry to solve 3-dimensional problems and applies the sine, cosine and area rules to solve 2-dimensional problems, including bearings

Solve 3-dimensional problems involving right-angled triangles

- Apply Pythagoras' theorem to solve problems involving the lengths of the edges and diagonals of rectangular prisms and other 3-dimensional objects
- Apply trigonometry to solve problems involving right-angled triangles in 3 dimensions, including using bearings and angles of elevation and depression

PYTHAGORAS' THEOREM IN 3D

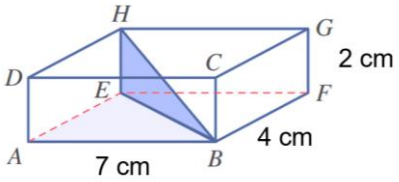
- When working in 3D, simplify the question to 2D.
  1. Break down 3D diagram into 2D triangles.
  2. Use Pythagoras' Theorem to find unknown sides.
     Do not round until your final answer.

Pythagoras in 3D: https://www.geogebra.org/m/mpnd5fck

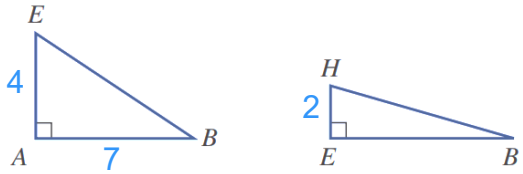
Example Solve problems involving Pythagoras' Theorem in 3D.

Calculate the length of diagonal BH.

Round your answer to 2 decimal places.



Step 1: Consider relevant triangles.



Step 2: Use triangles to find unknown sides

Calculate EB

$EB^2 = 4^2 + 7^2$

$EB = \sqrt{4^2 + 7^2}$

$= \sqrt{65}$

Calculate BH

$BH^2 = 2^2 + \left(\sqrt{65}\right)^2$

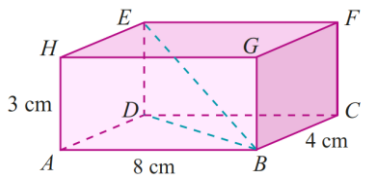
$= 69$

$BH = \sqrt{69} \approx 8.31$ cm

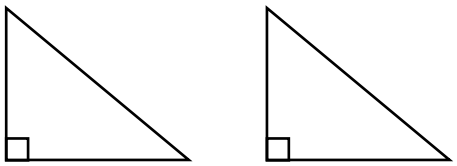
Why did we leave EB in exact form, rather than as a rounded decimal?

Leaving in exact ..... form gives a more ..... answer compared to using the rounded decimal.

a

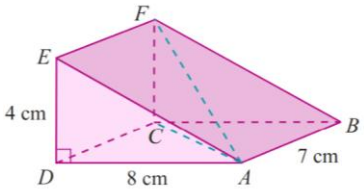


Calculate the length EB.

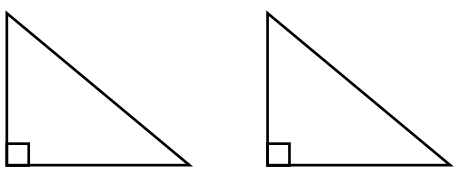


$\sqrt{89}$ cm

b



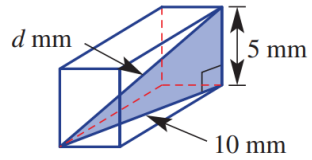
Calculate the length AF.



$\sqrt{129}$ cm

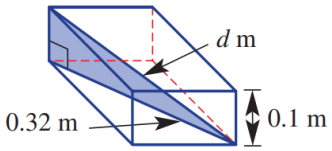
1 In these questions you will only need to consider a single triangle. Find the value of d. Answer to 2 decimal places.

a



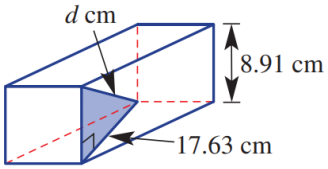
11.18 mm

b



0.34 m

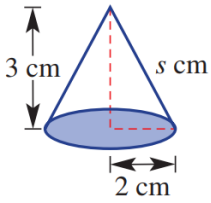
c



19.75 cm

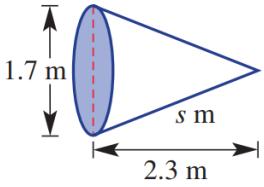
2 Calculate the slant height of these cones. Answer rounded to 1 d.p.

a



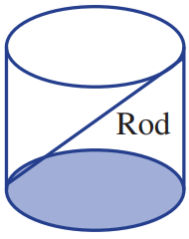
3.6 cm

b



2.5 cm

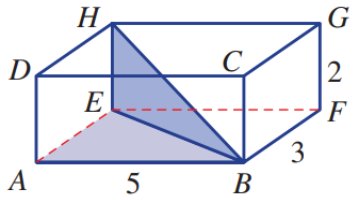
3 Calculate the length of the longest rod that can fit inside this cylinder with a diameter of 10 cm and a height of 15 cm. Round to 1 d.p.

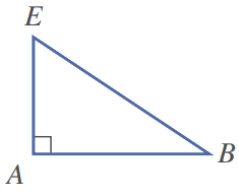


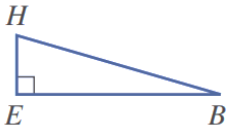
18.0 cm

4 For each of these rectangular prisms, first find BE, leaving your answer in exact form, then find BH, rounded to 2 decimal places. Draw and label the right-angled triangles you are using.

a

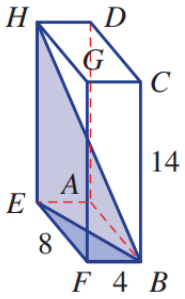


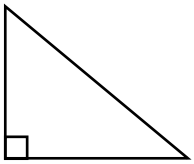


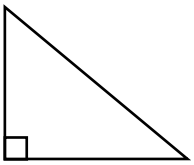


$BE = \sqrt{34}, BH = 6.16$

b

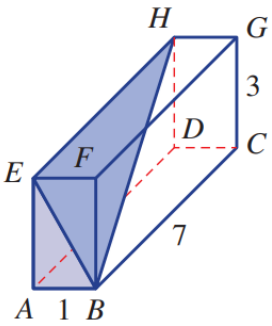






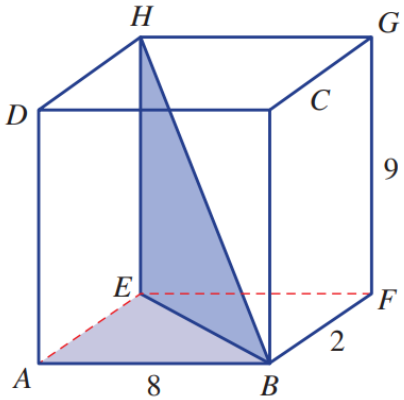
$BE = \sqrt{80}, BH = 16.61$

c



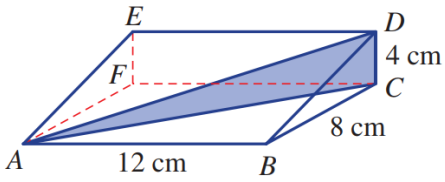
$BE = \sqrt{10}, BH = 7.68$

d



$BE = \sqrt{68}. BH = 12.21$

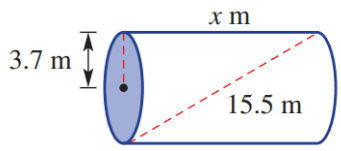
5 Calculate the length of the diagonal AD, correct to 1 d.p.



15.0 cm

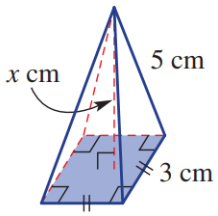
6 Calculate the value of x in each of these diagrams, to 2 decimal places.

a



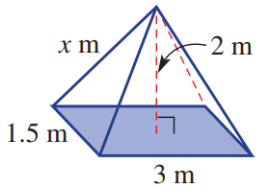
13.62 m

b



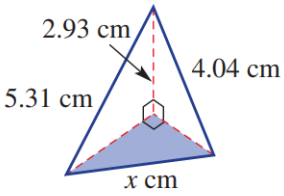
4.53 m

c



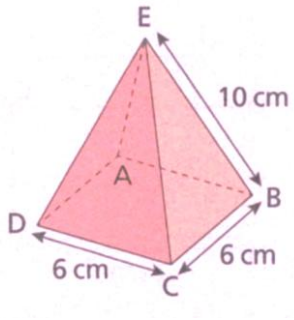
2.61 m

d



5.23 cm

7 Here is a square pyramid.



a Work out the length of BD in exact form.

$\sqrt{72}$ cm

b M is the midpoint of BC

i Write the length of BM.

3 cm

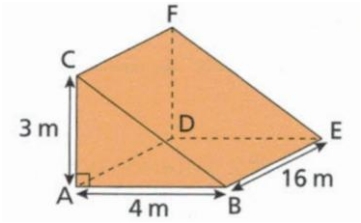
ii Work out the length of EM, in exact form.

$\sqrt{10^2 - 3^2} = \sqrt{91}$ cm

iii Work out the perpendicular height of the pyramid, to 1 decimal place.

9.1 cm

8 Here is a triangular prism.
Calculate these lengths, give your answers to 1 decimal place, where appropriate.



a BC

5 m

b AE

16.5 m

c AF

16.3 m

9 The diagonal of a cube measures 20 m. Calculate the following, rounding your answers to 1 d.p.

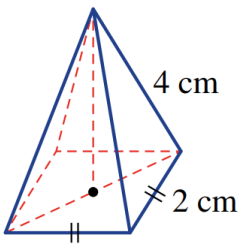
a the volume of the cube

b the surface area of the cube.

1539.6 m^3, 800 m^2

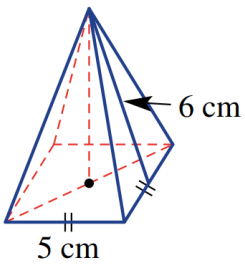
10 There are different ways to approach finding the height of a pyramid depending on what information is given. For each of the following square-based pyramids, find the exact length of the diagonal of the base and the height of the pyramid to 2 decimal places.

a



Diagonal: $\sqrt{8}$ cm
Height: 3.74 cm

b



Diagonal: $\sqrt{50}$ cm
5.45 cm

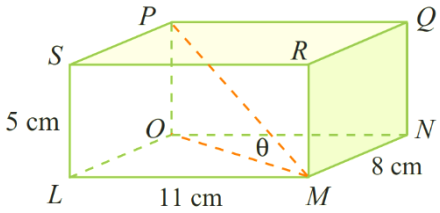
TRIGONOMETRY IN 3D

- When working in 3D, simplify the question to 2D.
  1. Break down 3D diagram into 2D triangles.
  2. Use trigonometry to find unknown sides and angles.

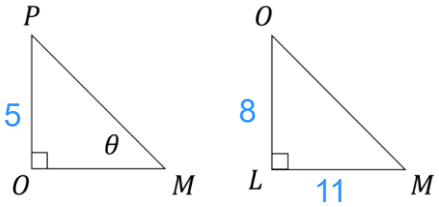
Example Solve problems involving trigonometry in 3D.

Calculate the angle θ.

Round your answer to the nearest minute.



Step 1: Consider relevant triangles.



Step 2: Use triangles to find unknown sides

Calculate OM

$OM^2 = 8^2 + 11^2$

$OM^2 = 185$

$OM = \sqrt{185}$

Calculate θ

$\tan\theta = \frac{5}{\sqrt{185}}$

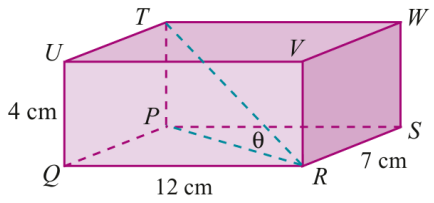
$\theta = \tan^{-1}\frac{5}{\sqrt{185}}$

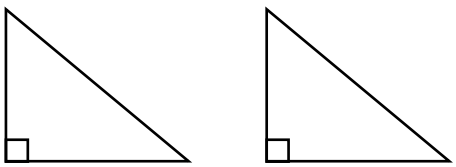
$= 20°11'$

How do we know to use trigonometry in this question?

The question involves finding an ..... in a .....-..... triangle, so it involves trigonometry.

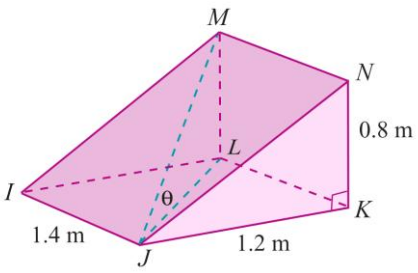
a Find θ to the nearest degree.

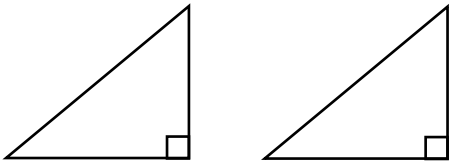




16°

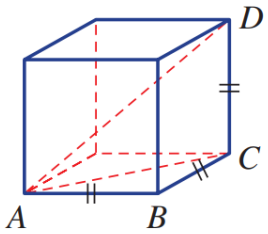
b Find θ to the nearest degree.



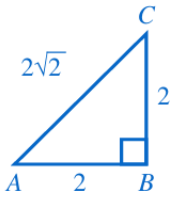


23°

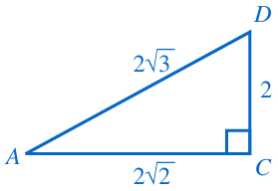
1 The cube shown here has a side length of 2 m.



a Draw the right-angled triangle ΔABC and label all the side lengths.
Answer using exact values.



b Draw the right-angled triangle ΔACD and label all the side lengths.
Answer using exact values.



c Use trigonometry to find ∠DAC, correct to 1 decimal place.

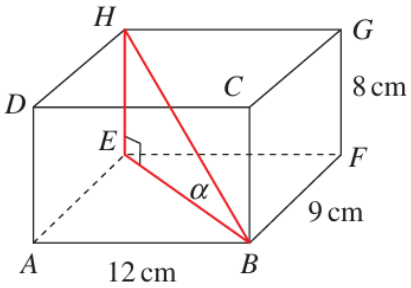
35.3°

2 The diagram shows a rectangular prism.

a Use Pythagoras' theorem to find the length of the base diagonal BE.

b Hence find the length of the prism diagonal BH.

c Find, correct to the nearest degree, the angle α that BH makes with the base of the prism.



a) 15 cm

b) 17 cm

c) 28°

3 The diagram shows a triangular prism.

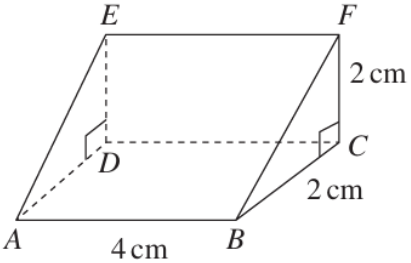
a Find the exact length of:

i AC

ii AF

b What is the size of ∠ACF?

c Find ∠AFC, correct to the nearest degree.

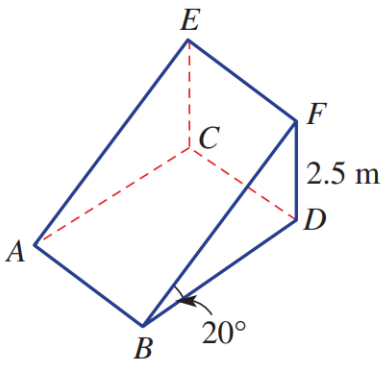


a) i) $2\sqrt{5}$ cm ii) $2\sqrt{6}$ cm

b) 90°

c) 66°

4 A ramp, ABCDEF, rests at an angle of 20° to the horizontal and the highest point on the ramp is 2.5 metres above the ground, as shown.



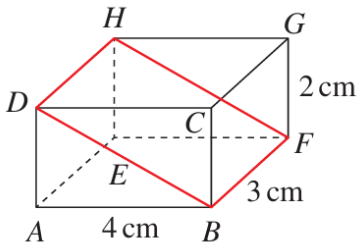
a Find the length BF to 2 d.p.

7.31 m

b Find the length BD to 2 d.p.

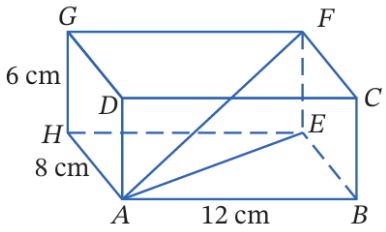
6.87 m

5 The diagram shows a rectangular prism.
Find, correct to the nearest degree, the angle that the diagonal plane DBFH makes with the base of the prism.



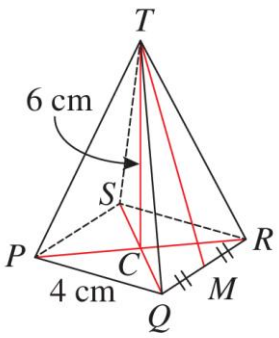
27°

6 Consider the rectangular prism. Calculate the angle $\angle FAE$ to the nearest degree.



23°

7 The diagram shows a square pyramid.
The point C is the centre of the base, and TC is perpendicular to the base.



a Write down the size of:
- **i** $\angle CMQ$
- **ii** $\angle TCM$
- **iii** $\angle TCQ$

b Find the length of:
- **i** CM
- **ii** CQ

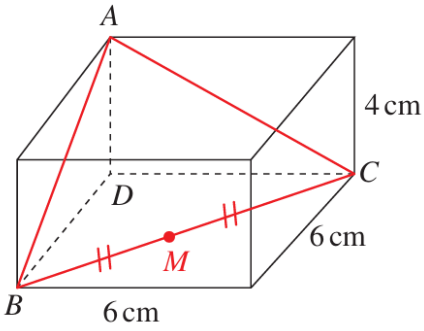
c Find, correct to the nearest degree:
- **i** The angle between a side face and the base.
- **ii** The angle between a slant edge and the base.

a) i) 90° ii) 90° iii) 90°
b) i) 2 cm ii) $2\sqrt{2}$ cm
c) i) 72° ii) 65°

8 The diagram shows a square prism. The plane ABC is inside the prism, and M is the midpoint of the base diagonal BC.

a Find the exact length of MD.

b Hence find, to the nearest degree, the angle that the plane ABC makes with the base of the prism.



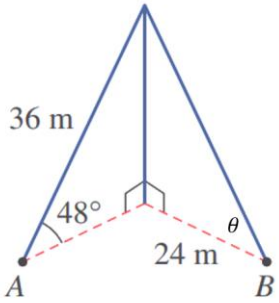
a) $3\sqrt{2}$ cm
b) 43°

PROBLEMS INVOLVING 3D TRIGONOMETRY

3D Trig Visualisation: https://www.geogebra.org/m/zksxcu7h

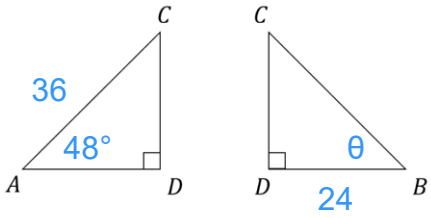
Example Solve problems involving trigonometry in 3D.

A vertical mast is supported at the top by two cables attached to the ground at A and B.
The cable from point A is 36 metres long and is at an angle of 48° to the horizontal.
Point B is 24 metres from the base of the mast.



Calculate the angle made between the cable and the ground at point B.

Step 1: Draw right-angled triangles.



Step 2: Use triangles to find unknown sides.

Calculate CD

$$\sin 48° = \frac{CD}{36}$$

$$36 \sin 48° = CD$$

Calculate θ

$$\tan\theta = \frac{36 \sin 48°}{24}$$

$$\theta = \tan^{-1}\frac{36 \sin 48°}{24}$$

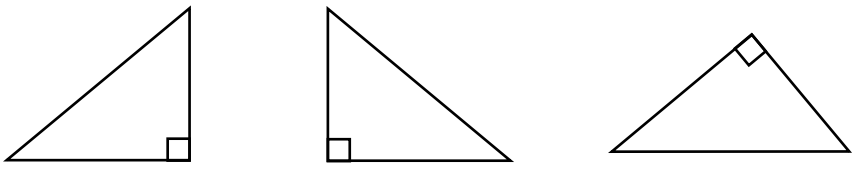
$$= 48°6'$$

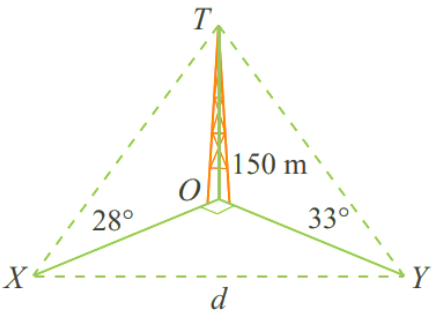
Why did we leave CD as $36 \sin 48°$, rather than a rounded decimal?

Leaving in ..... form gives a more ..... answer compared to using the rounded decimal.

From a point X due south of a tower, the angle of elevation of the top of the 150 m tower is 28°.
From a point Y due east of the tower, the angle of elevation of the top of the tower is 33°.

Find d, the distance between X and Y, to the nearest metre.





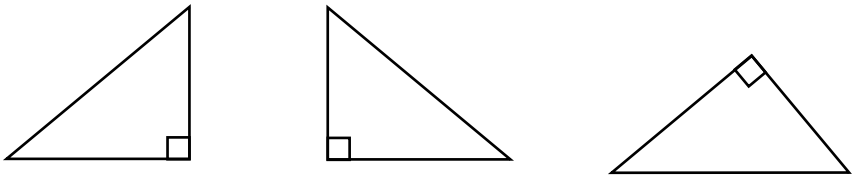
365 m

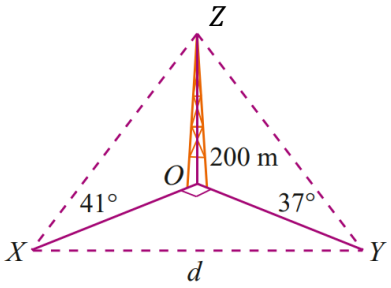
1 From a point X due south of a tower 200 m tall on level ground, the angle of elevation of the top of the tower is 41°. From a point Y due east of the tower, the angle of elevation of the top of the tower is 37°.

a Draw and label ΔZOX

b Draw and label ΔZOY

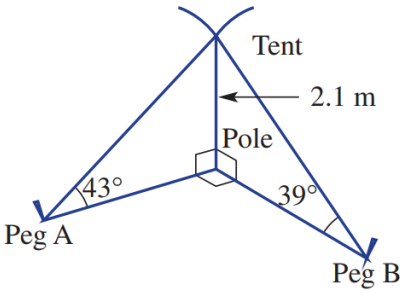
c Draw and label ΔXOY and hence find d.





$d = 351$ m

2 Consider the diagram of a tent pole secured by two wires and pegs.



a Find the distance from the base of the pole to peg A.

2.25 m

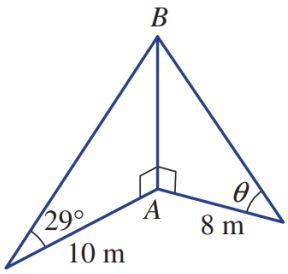
b Find the distance from the base of the pole to peg B.

2.59 m

c Find the distance between peg A and peg B.

3.43 m

3 For this 3D diagram, complete these steps.



a Find AB in exact form.

$10 \tan 29°$

b Find θ to 1 d.p.

$\theta = \tan^{-1}\left(\frac{10 \tan 29°}{8}\right) = 34.7°$

a Why is it important to keep values in exact form and only round the final answer?
Try finding θ by rounding AB to 1 d.p. and using the rounded result.

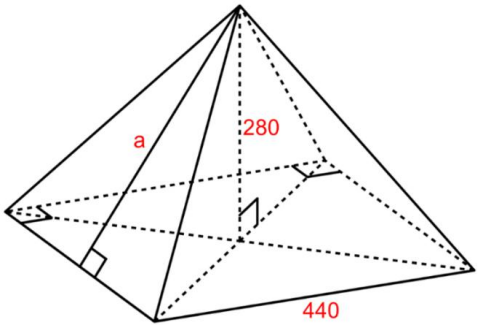
If you round too early your final answer will be inaccurate.

$10 \tan 29° \approx 5.5$

$\theta = \tan^{-1}\left(\frac{5.5}{8}\right) = 34.5°$

This answer is 0.2° different to the answer without rounding.

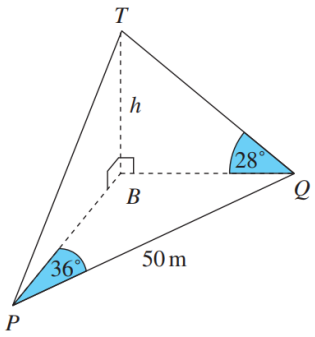
4 The Great Pyramid of Giza is a square pyramid.
It has a perpendicular height of 280 royal cubits and a base length of 440 royal cubits.
Show that the apothem (a) is around 356 royal cubits and it has an angle of inclination of $52°$.



$a = \sqrt{280^2 + 220^2} \approx 356$

$\theta = \tan^{-1}\left(\frac{280}{220}\right) \approx 52°$

5 A tree BT is due north of an observer at P and due west of an observer at Q. The two observers are 50 m apart and the bearing of Q from P is 36°. The angle of elevation of T from Q is 28°.



a Show that BQ = 50sin36°.

$\sin 36° = \frac{BQ}{50}$

$BQ = 50 \sin 36°$

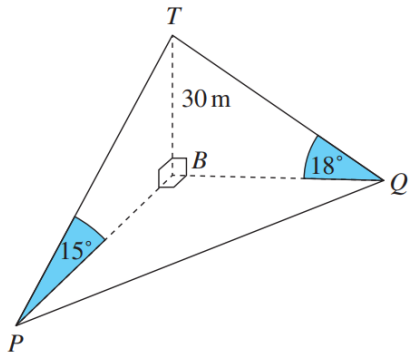
b Hence find the height h of the tree, correct to the nearest metre.

16 m

c Find the angle of elevation of T from P, correct to the nearest degree.

21°

6 Two landmarks P and Q on level ground are observed from the top T of a vertical tower BT of height 30m. Landmark P is due south of the tower, while landmark Q is due east of the tower. The angles of elevation of T from P and Q are 15° and 18° respectively.



a Show that BP = 30 tan75°,

$\angle PTB = 75°$ (angle sum of a triangle)

$\tan 75° = \frac{BP}{30}$

$BP = 30 \tan 75°$

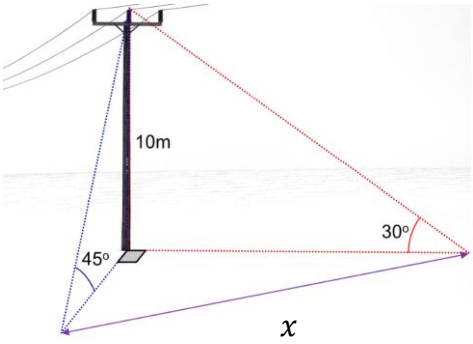
b Find a similar expression for BQ.

Similarly, $BQ = 30 \tan 72°$

c Find, correct to the nearest metre, the distance between the two landmarks.

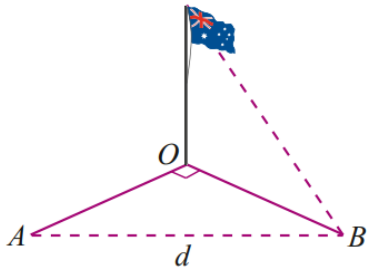
145 m

7 Calculate the ground distance x.



20 m

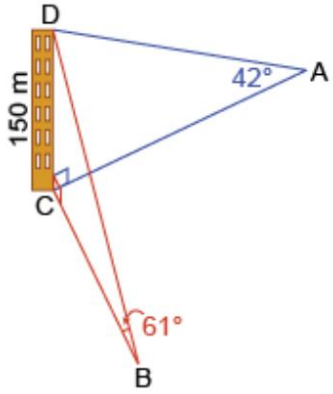
8 From a point A due north of a flagpole 80 m tall on level ground, the angle of elevation of the top of the flagpole is 38°. From a point B due west of the flagpole, the angle of elevation of the top of the flagpole is 47°. Complete the diagram and calculate the distance AB to the nearest metre.



127 m

9 Andrew (A) is on a bearing of 065° from a building which is 150 m tall.
Bret (B) is on a bearing of 155° from the same building.
The angle of elevation from Andrew to the top of the building is 42° while the angle of elevation for Bret is 61°. The land surrounding the building is level.

How far apart are Andrew and Bret, to the nearest metre?



$AC = \frac{150}{\tan 42°}, CB = \frac{150}{\tan 61°}$

$\angle ACB = 155° - 65° = 90°$

$\therefore AB = \sqrt{AC^2 + CB^2}$

$= 186$ m

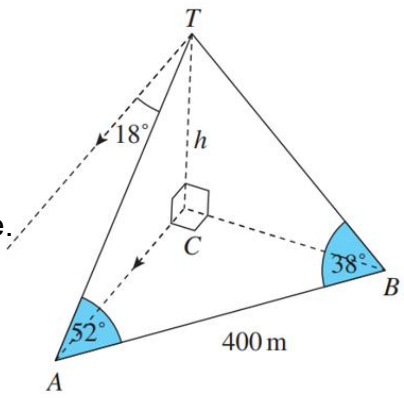
10 A tree is 120 m tall.
From a point C due north on level ground, the angle of elevation of the top of the tree is 51°.
From the point D due east of the tree, the angle of elevation of the top of the tree is 37°.

a Draw a diagram showing this information.

b Calculate the distance CD to the nearest metre.

187 m

11 Two monuments A and B are 400 m apart on the horizontal plane. The angle of depression of A from the top T of a tall building is $18°$. $\angle TAB = 52°$ and $\angle TBA = 38°$



a Show that $TA = 400 \cos 52°$

b Find the height h of the building, correct to the nearest metre.

c Find the angle of depression of B from T, to the nearest degree.

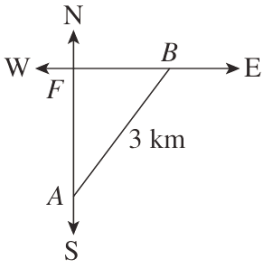
a) $\angle ATB = 180 - 52 - 38 = 90°$

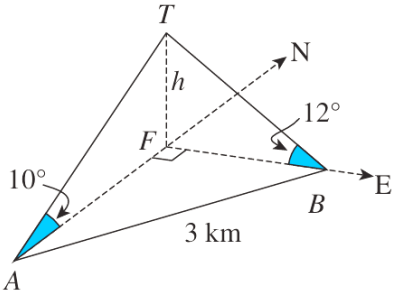
$\cos 52° = \frac{TA}{400}, \therefore TA = 400 \cos 52°$

b) 76 m

c) 14°

12 A motorist on level ground sees, due north of her, a tower whose angle of elevation is 10°. After driving 3 km further in a straight line, the tower is in the direction due west, with angle of elevation 12°.





In this question, no more than two measurements are known in any one of the triangles. Therefore, we must introduce a pronumeral for the height, and work around until we have enough expressions in terms of h such that an equation can be formed and solved.

a Find an expression for AF.

$\frac{h}{\tan 10°}$

b Find and expression for BF

$\frac{h}{\tan 12°}$

c Use Pythagoras' Theorem to form an equation using h, then solve for h to 3 d.p.

$AF^2 + BF^2 = AB^2$

$\therefore h \approx 0.407$ km

d In what direction was the motorist driving?

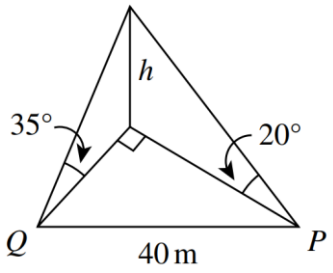
Let $\theta = \angle FAB$, $\sin\theta = \frac{FB}{AB}$

$\theta \approx 40°$

she was travelling at a bearing of 040°

13 From a point P due south of a vertical tower, the angle of elevation of the top of the tower is $20°$. From a point Q 40 metres from P and due east of the tower, the angle of elevation is $35°$. h is the height of the tower.

Show that $h = \frac{40}{\sqrt{\tan^2 70° + \tan^2 55°}}$ and evaluate h to the nearest metre.



13 metres

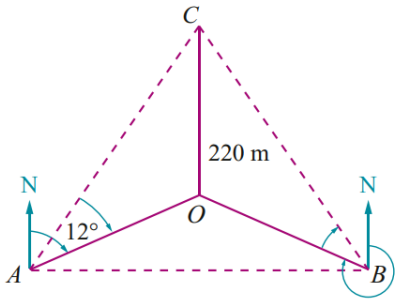
14 Two boats at sea are both observing the same lighthouse, which is at a height of 220 m above sea level. From boat A the bearing of the lighthouse is 040°T and the angle of elevation is 12°. From boat B the bearing of the lighthouse is 310°T and the angle of elevation is 17°.

a Complete this diagram.

b Show that $\angle AOB$ is $90°$
(Hint: draw the top-down bearing diagram of ΔAOB)

c Calculate the distances OA and OB and hence AB

d Calculate $\angle OAB$ and hence the bearing of boat B from A.



b) ∠AOB = (360° − 310°) + 40° = 90°

c) OA = 1035 m, OB = 720 m, AB = 1261 m

d) ∠OAB = 35° , bearing = 075° T

15 Two boats at sea are both observing the same lighthouse, which is at a height of 520 m above sea level. From boat A the bearing of the lighthouse is 067°T and the angle of elevation is 15°. From boat B the bearing of the lighthouse is 337°T and the angle of elevation is 9°.

a Calculate the distance between the boats.

b Calculate the bearing of boat B from boat A.

a) 3814 m

b) 126°T

16 Two boats are observed from a lighthouse 350 m above sea level.
Boat X is on a bearing of 138°T and at an angle of depression of 23°.
Boat Y is on a bearing of 228°T and at an angle of depression of 16°.

a Calculate the distance between boat X and boat Y.

b Calculate the bearing of boat Y from boat X.

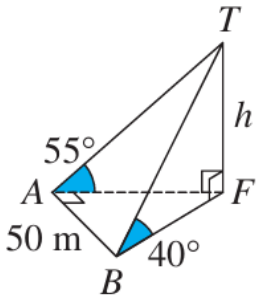
a) XY = 1473 m

b) 262°T

17 The diagram shows a tower of height h metres standing on level ground.
The angles of elevation of the top T of the tower from two points A and B on the ground nearby are 55° and 40° respectively.
The distance AB is 50 metres, and the interval AB is perpendicular to the interval AF, where F is the foot of the tower.

a Find AT and BT in terms of h.

$AT = \frac{h}{\sin 55°}$, $BT = \frac{h}{\sin 40°}$



b What is the size of $\angle BAT$?

90°, as AB is perpendicular to AF

c Hence show that $h = \frac{50 \sin 55° \sin 40°}{\sqrt{\sin^2 55° - \sin^2 40°}}$

$BT^2 = AT^2 + AB^2$

$\frac{h^2}{\sin^2 40°} = \frac{h^2}{\sin^2 55°} + 50^2$

$\frac{h^2}{\sin^2 40°} - \frac{h^2}{\sin^2 55°} = 50^2$

$\frac{h^2 \sin^2 55° - h^2 \sin^2 40°}{\sin^2 40° \sin^2 55°} = 50^2$

$\frac{h^2(\sin^2 55° - \sin^2 40°)}{\sin^2 40° \sin^2 55°} = 50^2$

$h^2(\sin^2 55° - \sin^2 40°) = 50^2(\sin^2 40° \sin^2 55°)$

$h^2 = \frac{50^2(\sin^2 40° \sin^2 55°)}{(\sin^2 55° - \sin^2 40°)}$

$\therefore h = \frac{50 \sin 40° \sin 55°}{\sqrt{\sin^2 55° - \sin^2 40°}}$