

Student Name

Mathematics Stage 5 Core

INDICES A

Book 2 Simplify algebraic products and quotients
using index laws

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OVERVIEW

SYLLABUS CONTENT

MA5-IND-C-01 simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases

Simplify algebraic products and quotients using index laws

- Simplify algebraic expressions that involve powers, products and quotients of simple algebraic terms containing positive-integer indices

REVIEW OF PRIOR KNOWLEDGE

1 Simplify the following expressions.

a $4x^2 + 5x + 3x^2$

b $7y - 2y^3 + 3y$

c $a^4 \times a^5$

d $2^3 \times 2^{17}$

e $d \times d^{15} \times d^{100}$

f $h^6 \div h^3$

g $a^2 \times b^3$

h $q^{16} \div q^{16}$

i $w^{99} \div w^9$

j $\frac{h^5 \times h^7}{h^4}$

k $\frac{y^{100}}{y^{14} \times y^6}$

l $3a^5 \times 2a^7$

m $\frac{28y^6}{4y^3}$

n $(x^4)^5$

o $\frac{8p^{20} \times 10p^5}{16p^7 \div 5p^2}$

p $\frac{(t^3)^5}{t^7}$

q $(h^9)^{10} \times h^8$

r $2k^5 \times 2k^5$

2 Expand these brackets using the distributive law.

a $3(x + y)$

b $8(2x + 3y)$

c $11(2x - y)$

d $-4(x + 7)$

e $\frac{1}{3}(5z + 6)$

f $\frac{4x+12}{2}$

POWER OF A PRODUCT

Power of a Product

$$(ab)^m = a^m b^m$$

Example Simplify algebraic expressions using power of a product.

$(5b)^3$ $= (5)^3(b)^3$ $= 125b^3$	Show why $(5b)^3 = (5)^3(b)^3$ by writing in expanded form. $(5b)^3 = \dots \times \dots \times \dots$ $= \dots$
$(-2x^3y)^2$ $= (-2)^2(x^3)^2(y)^2$ $= 4x^6y^2$	Why is $(-2)^2 = 4$? $(-2)^2 = \dots \times \dots$ $= \dots$
$4(c^2d^3)^5$ $= 4(c^2)^5(d^3)^5$ $= 4c^{10}d^{15}$	In this example, why is 4 not raised to the power of 5? <i>4 is not inside the, so it is not raised to the power of 5.</i>

Simplify these expressions.

a $(2x)^3$	b $(-5y)^2$	c $-(3b)^4$
d $-(7r)^3$	e $9(p^2q^4)^3$	f $(-2h^2)^4$

$8x^3$

$25y^2$

$-81b^4$

$-343r^3$

$9p^6q^{12}$

$16h^8$

Key ideas

1. Power of a product: $(ab)^m = \dots$

1 Complete the following

a

$$\begin{aligned}(5a)^3 &= 5a \times \square \times \square \\ &= 5 \times 5 \times 5 \times a \times \square \times \square \\ &= 5^3 \times \square\end{aligned}$$

$$5^3 \times a^3$$

b

$$\begin{aligned}(ab)^4 &= ab \times \square \times \square \times \square \\ &= a \times \square \times \square \times \square \times b \times \square \times \square \times \square \\ &= a^4 \times \square\end{aligned}$$

$$a^4 \times b^4$$

2 Simplify these expressions by expanding first, then multiplying.

a

$$\begin{aligned}(2x^4)^3 &= 2x^4 \times 2x^4 \times 2x^4 = 2^3 x^{12} \\ &= 8x^{12}\end{aligned}$$

b

$$\begin{aligned}(3h^5)^2 &= \square \times \square = 3^{\square} h^{\square} \\ &= \square\end{aligned}$$

$$9h^{10}$$

c

$$\begin{aligned}(5a^7)^3 &= \square \times \square \times \square = 5^{\square} a^{\square} \\ &= \square\end{aligned}$$

$$125a^{21}$$

d

$$\begin{aligned}(2n^6)^4 &= \square \times \square \times \square \times \square = 2^{\square} n^{\square} \\ &= \square\end{aligned}$$

$$16n^{24}$$

3

a Complete the following

$$(x^3y^2)^3 = x^3y^2 \times \square \times \square = \square$$

$$x^9y^6$$

b Simplify these expressions

i $(a^4b^3)^2$

$$a^8b^6$$

ii $(f^3g^6)^4$

$$f^{12}g^{24}$$

iii $(tm^6)^3$

$$t^3m^{18}$$

iv $(2p^5q^2)^4$

$$16p^{20}q^8$$

v $(3m^{10}n^6)^3$

$$27m^{30}n^{18}$$

vi $(a^3b^5c^7)^5$

$$a^{15}b^{25}c^{35}$$

4 Simplify these expressions using the rule.

a

$$(2x^4)^3 = 2^3 x^{12} = 8x^{12}$$

b

$$(3e^2)^3 = 3^{\square} e^{\square} = \square$$

$$27e^6$$

c

$$(5n^4)^2 = 5^{\square} n^{\square} = \square$$

$$25n^8$$

d

$$(2c^5)^4 = 2^{\square} c^{\square} = \square$$

$$16c^{20}$$

e

$$(3x^3)^4 = 3^{\square} x^{\square} = \square$$

$$81x^{12}$$

f

$$(3k^4)^3 =$$

$$27k^{12}$$

g

$$(4c^5)^4 =$$

$$256c^{20}$$

h

$$(6v^5)^3 =$$

$$216v^{15}$$

i

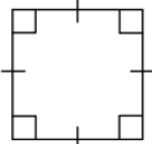
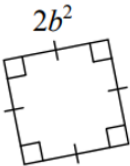
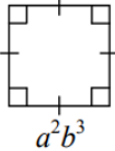
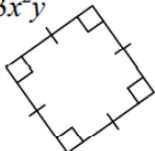
$$(10a^8)^5 =$$

$$100\,000a^{40}$$

5 Simplify

a $(2x)^3$	b $(5y)^2$	c $(4a^2)^3$	d $(-3r)^2$
$8x^3$	$25y^2$	$64a^6$	$9r^2$
e $(a^3b)^5$	f $(x^3y^2)^4$	g $(c^{10}e^2)^8$	h $(a^4b^2c)^5$
$a^{15}b^5$	$x^{12}y^8$	$c^{80}e^{16}$	$a^{20}b^{10}c^5$
i $(3x^5)^2$	j $(2x^3)^4$	k $(5p^3)^3$	l $\left(\frac{1}{2}g^4\right)^3$
$9x^{10}$	$16x^{12}$	$125p^9$	$\frac{g^{12}}{8}$
m $-(3r)^2$	n $(-2h^2)^4$	o $(5c^2d^3)^4$	p $(2x^3y^2)^5$
$-9r^2$	$16h^8$	$625c^8d^{12}$	$32x^{15}y^{10}$

6 Find the area of these squares.

a  $(3a)^2 = \dots 3a \dots$ $9a^2$	b  $4b^4$	c  a^4b^6	d  $9x^4y^2$
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7 Read the example carefully and answer the questions.

$(ab^5)^2$

$(ab^5)^2$

ab^{10}



By expanding and writing out $(ab^5)^2$, show why this is wrong.

$$ab^5 \times ab^5 = a^2b^{10}$$

Simplify $(cj^3)^5$

$$c^5j^{15}$$

$(-10a^6)^2 \cdot a^2$

$(-10a^6)^2 \cdot a^2$

$(-10)^2 \cdot (a^6)^2 \cdot a^2$

$100 \cdot a^{12} \cdot a^2$

$100 \cdot a^{14}$

$100a^{14}$



Explain why we need brackets for $(-10)^2$.

Without brackets, $-10^2 = -100$

Simplify

$$(-2b^3)^2 \times b^4$$

$$4b^{10}$$

8 Expand then simplify these expressions.

a $(x^2b)^3 \times (x^4b^2)^2$ $= \underline{x^6b^3 \times x^8b^4}$ Expand $= \underline{x^{14}b^7}$ Simplify	b $(x^3y^2)^4 \times (xy^5)^2$	c $(b^2k)^3 \times (b^2k^2)^4$
d $(u^6v^3)^2 \times (uv^2)^3$	e $(a^2h^3)^5 \times (2h^2)^2$	

$x^{14}y^{18}$ $b^{14}k^{11}$
 $u^{15}v^{12}$ $4a^{10}h^{19}$

9 Use two methods to simplify these expressions. The first is done for you.

a $(3x^4)^2$ $\downarrow$ $3x^4 \times 3x^4$ $\downarrow$ $3 \times 3 \times x^4 \times x^4$ $(3x^4)^2 = 9x^8$	b $(5y^3)^2$ $\downarrow$ $\square$ $\downarrow$ $\square$ $(5y^3)^2 = \underline{\hspace{2cm}}$
c $\square$ $\downarrow$ $2a^3 \times 2a^3 \times 2a^3$ $\downarrow$ $\square$ $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ $(2a^3)^3, \quad 2^3 \times (a^3)^3$ $2 \times 2 \times 2 \times a^3 \times a^3 \times a^3, \quad 8a^9$ $(2a^3)^3 = 8a^9$	d $\square$ $\downarrow$ $\square$ $\downarrow$ $\square$ $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ $(2x^5)^4$ $2x^5 \times 2x^5 \times 2x^5 \times 2x^5$ $2 \times 2 \times 2 \times 2 \times x^5 \times x^5 \times x^5 \times x^5, \quad 16x^{20}$ $(2x^5)^4 = 16x^{20}$

Top to bottom:
 $5^2 \times (y^3)^2$
 $5y^3 \times 5y^3$
 $5 \times 5 \times y^3 \times y^3, \quad 25y^6$

10 Find the missing numbers.

$(10y^{\square})^{\square} = 100\,000y^{30}$

$(10y^6)^5$

POWER OF A QUOTIENT

Power of a Quotient

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example Simplify algebraic expressions using power of a quotient.

$\left(\frac{6}{b}\right)^3 = \frac{6^3}{b^3}$ $= \frac{216}{b^3}$	Show why $\left(\frac{6}{b}\right)^3 = \frac{6^3}{b^3}$ by writing in expanded form. $\left(\frac{6}{b}\right)^3 = \dots \times \dots \times \dots$ $= \dots$
$\left(\frac{-2}{3b}\right)^4 = \frac{(-2)^4}{(3b)^4}$ $= \frac{16}{3^4 b^4} = \frac{16}{81b^4}$	Why is $(-2)^4 = 16$? $(-2)^4 = \dots \times \dots \times \dots \times \dots$ Describe what happened in the denominator. $(3b)^4 = \dots \times \dots \times \dots \times \dots$ $= \dots$
$\left(\frac{6s^2}{n^3}\right)^2 = \frac{6^2(s^2)^2}{(n^3)^2}$ $= \frac{36s^4}{n^6}$	Describe what happened in the numerator. $(6s^2)^2 = \dots \times \dots$ $= \dots$

Simplify these expressions.

a $\left(\frac{p}{q}\right)^2$ $\frac{p^2}{q^2}$	b $\left(\frac{4}{y}\right)^3$ $\frac{64}{y^3}$	c $\left(\frac{-5}{p^2}\right)^4$ $\frac{625}{p^8}$
d $\left(\frac{2x}{y}\right)^5$ $\frac{32x^5}{y^5}$	e $\left(\frac{3n^3}{2m}\right)^2$ $\frac{9n^6}{4m^2}$	f $\left(\frac{-3f}{2g^2}\right)^3$ $-\frac{27f^3}{8g^6}$

Key ideas

1. Power of a quotient: $\left(\frac{a}{b}\right)^m = \dots$

1 Complete the following

a $\left(\frac{x}{6}\right)^3 = \frac{x}{6} \times \square \times \square$
 $= \frac{x \times \square \times \square}{6 \times \square \times \square}$
 $= \frac{x^3}{\square}$

$$\frac{x^3}{6^3}$$

b $\left(\frac{a}{b}\right)^5 = \frac{a}{b} \times \square \times \square \times \square \times \square$
 $= \frac{a \times \square \times \square \times \square \times \square}{b \times \square \times \square \times \square \times \square}$
 $= \frac{a^5}{\square}$

$$\frac{a^5}{b^5}$$

2 Simplify

a $\left(\frac{p}{q}\right)^3$

$$\frac{p^3}{q^3}$$

b $\left(\frac{x}{y}\right)^4$

$$\frac{x^4}{y^4}$$

c $\left(\frac{4}{y}\right)^3$

$$\frac{64}{y^3}$$

d $\left(\frac{5}{p^2}\right)^4$

$$\frac{625}{p^8}$$

e $\left(\frac{2}{r^3}\right)^2$

$$\frac{4}{r^6}$$

f $\left(\frac{s^3}{7}\right)^2$

$$\frac{s^6}{49}$$

g $\left(\frac{2m}{n}\right)^5$

$$\frac{32m^5}{n^5}$$

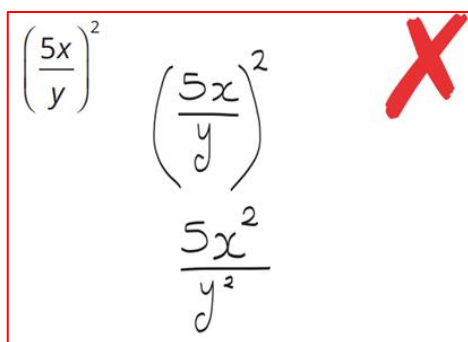
h $\left(\frac{2a^2}{3}\right)^3$

$$\frac{8a^6}{27}$$

i $\left(\frac{3n^3}{2m^4}\right)^3$

$$\frac{27n^9}{8m^{12}}$$

3 Read the example carefully and answer the questions.



There is something wrong with the 5 in the answer. What should there be instead?

$$25$$

Write as an expression without brackets:

$$\left(\frac{3a}{b}\right)^4$$

$$\frac{81a^4}{b^4}$$

4 Expand then simplify these expressions.

a $\frac{(v^3x)^3}{(v^2x^2)^4} = \frac{v^9x^3}{v^8x^8} = \frac{v}{x^5}$ Expand Simplify	b $\frac{(a^5b^2)^4}{(a^2b^3)^2} = \frac{a^{\boxed{}}b^{\boxed{}}}{a^{\boxed{}}b^{\boxed{}}} = \boxed{}$ $a^{16}b^2$	c $\frac{(n^2a^3)^2}{(n^3a^2)^3} = \frac{n^{\boxed{}}a^{\boxed{}}}{n^{\boxed{}}a^{\boxed{}}} = \frac{\boxed{}}{\boxed{}}$ $\frac{1}{n^5}$
d $\frac{(yx^4)^3}{(y^2x^2)^2}$ $\frac{x^8}{y}$	e $\frac{(v^5c^4)^2}{(v^3c^2)^3}$ vc^2	

5 Simplify each of the following.

a $2(x^3y)^2$ $2x^6y^2$	b $(8t^2u^9v^4)^0$ 1	c $(-3w^3y)^3$ $-27w^9y^3$	d $5(v^4x^2)^3$ $5v^{12}x^6$
e $-4(p^4qr)^2$ $-4p^8q^2r^2$	f $(-5s^7t)^2$ $25s^{14}t^2$	g $\left(\frac{5w^4y}{2x^3}\right)^2$ $\frac{25w^8y^2}{4x^6}$	h $\left(\frac{-3x}{2y^3g^5}\right)^2$ $\frac{9x^2}{4y^6g^{10}}$
i $(3q^6e^4)^4$ $81q^{24}e^{16}$	j $\left(\frac{3km^3}{4n^7}\right)^3$ $\frac{27k^3m^9}{64n^{21}}$	k $10(3me^5)^2$ $90m^2e^{10}$	l $\left(\frac{-2r}{n}\right)^4$ $\frac{16r^4}{n^4}$
m $-(-2x^4yz^3)^3$ $8x^{12}y^3z^9$	n $\left(\frac{-3f}{2^3g^5}\right)^2$ $\frac{9f^2}{64g^{10}}$	o $-\left(\frac{-5w^4y}{2zx^3}\right)^2$ $-\frac{25w^8y^2}{4x^6z^2}$	p $2(r^2d^2)^8$ $2r^{16}d^{16}$
q $a(3b)^2$ $9ab^2$	r $9(p^2q^4)^3$ $9p^6q^{12}$	s $a(3b^2)^3$ $27ab^6$	t $-2y(10x^2ny^3)^3$ $-2000x^6n^3y^{10}$
u $-4w(q^5xk^4)^5$ $-4wq^{25}x^5k^{20}$	v $2x(7x^5b^3y)^2$ $98x^{11}b^6y^2$	w $-3(2a^3b^4)^2a^2$ $-12a^8b^8$	x $2(3x^2y^3)^3$ $54x^6y^9$
y $(-4b^2c^5d)^3$ $-64b^6c^{15}d^3$	z $a(2a)^3$ $8a^4$	aa $5a^3(-2a^4b)^3$ $-40a^{15}b^3$	bb $-2(-2^3x^4yz^3)^3$ $1024x^{12}y^3z^9$

- 6 The rule for the number of seeds germinating in a glass house over a two-week period is given by $N = \left(\frac{t}{2}\right)^3$, where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:

i 4 days

8

ii 10 days

125

b Rewrite the rule without brackets using index laws

$$N = \frac{t^3}{8}$$

c Use your rule in part b to find the number of days required to germinate:

i 64 seeds

$$t = \sqrt[3]{512} = 8$$

ii 1 seed

$$t = \sqrt[3]{2} = 2$$

- 7 Simplify each of the following.

a $\frac{(2m^3n)^3}{m^4}$

$$8m^5n^3$$

b $\frac{3(2^2c^4d^5)^3}{(2cd^2)^4}$

$$12c^8d^7$$

c $\frac{-5(3^5m^3n^2)^2}{(-3^3m^2n)^3}$

$$15n$$

- 8 Solve these equations.

a $\left(\frac{a}{3}\right)^2 = \frac{25}{9}$

$$a = \pm 5$$

b $\left(\frac{a}{2}\right)^4 = 16$

$$a = \pm 4$$

c $(5a)^3 = 1000$

$$a = 2$$

d $(2a)^4 = 256$

$$a = \pm 2$$

e $\left(\frac{2a}{5}\right)^2 = \frac{36}{25}$

$$a = \pm 3$$

f $\left(\frac{6a}{7}\right)^3 = 1728$

$$a = \sqrt[3]{2744} = 14$$

- 9 Simplify these fractions without a calculator. The first one is done for you.

a $\frac{2^4}{4^4} = \left(\frac{2}{4}\right)^4$
 $= \left(\frac{1}{2}\right)^4$
 $= \frac{1}{16}$

b $\frac{6^3}{3^3}$

$$8$$

c $\frac{10^4}{5^4}$

$$16$$

d $\frac{3^3}{30^3}$

$$\frac{1}{1000}$$

10 Simplify the following expressions.

a $\frac{3a \times 3a \times 3a}{3a + 3a + 3a}$

$$3a^2$$

b $\frac{3c \times 4c^2 \times 5c^3}{3c + 4c^2 + 5c^3}$

$$\frac{60c^4}{3 + 4c + 5c^2}$$

c $\frac{ab^2 \times 2b^2c^3 \times 3c^3a^4}{a^3b^3 + 2a^3b^3 + 3a^3b^3}$

$$a^2bc^6$$

11 Simplify the following expressions.

a $\frac{(-2x^2)^3}{-4x}$

$$2x^5$$

b $\frac{(3xy^3)^3}{3x^2y^4}$

$$9xy^5$$

c $\frac{(-ab)^3 \times (-ab^2)^2}{-a^5b^3}$

$$b^4$$

d $\frac{(-2a^3b^2)^2 \times 16a^7b}{(2a^2b)^5}$

$$2a^3$$

e $\frac{4m^2n \times 3(m^2n)^3}{6m^2n}$

$$2m^6n^3$$

f $\frac{(7y^2z)^2 \times 3yz^2}{7(yz)^2}$

$$21y^3z^2$$

g $\frac{2(ab)^2 \times (2a^2b)^3}{4ab^2 \times 4a^7b^3}$

$$1$$

h $\frac{w^8}{18} \div \left(\frac{w}{6}\right)^2$

$$2w^6$$

12 Simplify the following expressions.

a $\left(\frac{x^2}{y}\right)^3 \times \frac{2x}{y^4}$

$$\frac{2x^7}{y^7}$$

b $\left(\frac{a^2b}{c}\right)^4 \times \left(\frac{b^2}{c}\right)^3$

$$\frac{a^8b^{10}}{c^7}$$

c $\left(\frac{pq^3}{r^2}\right)^2 \times \left(\frac{p^0q^2}{r}\right)^4$

$$\frac{p^2q^{14}}{r^8}$$

d $\left(\frac{a^3b}{c}\right)^3 \times \left(\frac{ac^4}{b}\right)^2$

$$a^{11}bc^5$$

e $\left(\frac{x^2z}{y}\right)^4 \times \left(\frac{xy^2}{z}\right)^3$

$$x^{11}y^2z$$

f $\left(\frac{r^3s}{t}\right)^2 \div \left(\frac{s}{rt^4}\right)^3$

$$\frac{r^9t^{10}}{s}$$

CHECK YOUR UNDERSTANDING

Power of a product and quotient

1	Expand these brackets $(2a^2)^3$						
a	$6a^5$	b	$8a^5$	c	$2a^6$	d	$8a^6$
2	Expand these brackets $(5a^3)^2$						
a	$10a^6$	b	$25a^6$	c	$5a^6$	d	$25a^5$
3	Expand these brackets $(2p^3q)^3$						
a	$8p^6q^3$	b	$6p^9q^3$	c	$6p^6q^3$	d	$8p^9q^3$
4	Expand these brackets $\left(\frac{a^4}{b^2}\right)^3$						
a	$\frac{a^{12}}{b^6}$	b	$\frac{a^7}{b^5}$	c	$\frac{a^{12}}{b^5}$	d	$\frac{a^7}{b^6}$
5	Expand these brackets $\left(\frac{x^3}{y^5}\right)^2$						
a	$\frac{x^6}{y^7}$	b	$\frac{x^5}{y^7}$	c	$\frac{x^6}{y^{10}}$	d	$\frac{x^5}{y^{10}}$