

Student Name

Mathematics Stage 4

RATIOS AND RATES

Book 3 Recognise and simplify rates

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OVERVIEW

SYLLABUS CONTENT

MA4-RAT-C-01 solves problems involving ratios and rates, and analyses distance–time graphs

Recognise and simplify rates

- Explain the differences between ratios and rates
- Represent given information as a simplified rate
- Convert between units for rates

SUMMARY

Recognise and Simplify Rates

Rates

A relationship between two quantities of different units.

E.g. Speed compares distance and time.

$$40 \text{ km/h} \rightarrow \frac{40 \text{ km}}{1 \text{ hr}}$$

Simplifying Rates

Write as a unit rate; the second quantity is 1.

1. Write rate as a fraction.
2. Divide both quantities by the denominator.
You can do this by entering the fraction in a calculator.
3. Rewrite the simplified rate using the /.

\$28 for 4 kilograms 150 heartbeats every 3 minutes.

- | | |
|---|--|
| 1. $\frac{\$28}{4 \text{ kg}}$ | 1. $\frac{150 \text{ beats}}{3 \text{ minutes}}$ |
| 2. $\frac{\$28 \div 4}{4 \text{ kg} \div 4} = \frac{\$7}{1 \text{ kg}}$ | 2. $\frac{150 \text{ beats} \div 3}{3 \text{ minutes} \div 3} = \frac{50 \text{ beats}}{1 \text{ minute}}$ |
| 3. = \$7/kg | 3. = 50 beats/minute |

Converting Rates

1. Write the rate as a fraction with units.
2. Convert the quantities to the required units.
3. Simplify the fraction using a calculator.

Convert 72 L/h to mL/s

1. $\frac{72 \text{ L}}{1 \text{ hr}} = \frac{(\quad) \text{ mL}}{(\quad) \text{ s}}$
2. $= \frac{(72 \times 1000) \text{ mL}}{(1 \times 60 \times 60) \text{ s}}$
3. = 20 mL/s

Convert 35c/min to \$/hr

1. $\frac{35 \text{ c}}{1 \text{ min}} = \frac{\$(\quad)}{(\quad) \text{ hr}}$
2. $= \frac{\$(35 \div 100)}{(1 \div 60) \text{ hr}}$
3. = \$21/hr

REVIEW OF PRIOR KNOWLEDGE

1 Simplify the following:

a $\frac{10}{30}$

b $\frac{15}{24}$

c $\frac{64}{80}$

d 10:30

e 15:24

f 64:80

g $\frac{21}{56}$

h 15:55

i 35:105

2 Describe what is meant by:

a My car's speed is *40 kilometres per hour*.

.....

b My heartrate is *60 beats per minute*.

.....

c Bananas are *\$5.50/kg*

.....

3 Match each rate with the most likely units.

a	Employee Wage	90 people/day
b	Speed of a car	\$2100/m ²
c	Cost of building a new home	68 km/h
d	Population growth	64 beats/min
e	Resting heart rate	\$25/h

INTRODUCTION TO RATES

- **Ratio:** Compares quantities with the same units (e.g., student : teacher).
- **Rate:** Compares quantities with different units (e.g., distance / time).

- Rates show a division of one quantity by another.
- Always include **units** in rates.
- The slash (/) means "per" or "for each".
- Example: 40 km/h = "40 km per hour" or 40 km for each hour.

- Write rates as **fractions** for clarity.
- These fractions show division, not parts of a whole.
- Always write the **units**.

$$40 \text{ km/h} = \frac{40 \text{ km}}{1 \text{ hr}}$$

Key ideas

1. Ratios and rates are both ways of quantities.
2. Ratios compare two similar quantities, while rates compare two quantities with units.
3. Rates need to specify the
4. A rate can be written as a; this represents division, not parts of a whole.

SIMPLIFYING RATES

- **Simplifying** a rate means to write it as a **unit** rate; the second quantity is 1.
 1. Write the rate as a fraction with units.
 2. Divide both quantities by the denominator.
You can do this by entering the fraction in a calculator
 3. Rewrite the simplified rate using the /.

Example Simplify rates.

150 heartbeats every 3 minutes. 1. $\frac{150 \text{ beats}}{3 \text{ minutes}}$ 2. $\frac{150 \text{ beats} \div 3}{3 \text{ minutes} \div 3} = \frac{50 \text{ beats}}{1 \text{ minute}}$ 3. = 50 beats/minute	What is meant by a unit rate? <i>A unit rate means the quantity is 1.</i> How do we simplify to a unit rate? <i>We write the rate as a, then put it into the calculator.</i>
\$28 for 4 kilograms 1. $\frac{\\$28}{4 \text{ kg}}$ 2. $\frac{\\$28 \div 4}{4 \text{ kg} \div 4} = \frac{\\$7}{1 \text{ kg}}$ 3. = \$7/kg	

Simplify these rates

a \$84 for every 4 people $\frac{\\$84}{4 \text{ people} \div 4} = \frac{\\$}{1 \text{ person}}$ = \$ /person \$21/person	b 21 apples for \$3 $\frac{21 \text{ apples}}{\\$3} \div \frac{1}{\\$1} = \frac{\text{apples}}{\\$1}$ = apples /\$1 7 apples/\$1	c 300 words in 5 minutes 60 words/min
d \$10 per 2.5 kg \$4/kg	e \$18 for 5 kg of seed \$3.60/kg	f 3 kg of seed per 2 m² 1.5 kg/m²

Key ideas

1. Simplifying a rate involves finding the rate.
2. A rate is called a unit rate if the second quantity is
3. We do this by dividing the by the
4. This can be done on a calculator by entering the rate as a

1 Simplify these to unit rates.

a \$12 in 4 hours

$$\frac{\$12}{4 \text{ hours}} = \frac{\$}{1 \text{ hour}}$$

= \$..... /hour

\$3 / h

b 15 goals per 3 games

5 goals / game

c \$160 for 6 hours

\$26.67 / h

d \$17.50 for every 5 kg

\$3.50 / kg

e 36000 cans in 8 hours

4500 cans/ h

f 80 mm rain in 5 days

16 mm / day

g 15 km in 60 minutes

0.25 km / min

h 78 runs for 12 overs

6.5 runs / over

i 91 people in 3 buses

30.3 people / bus

j 448 km in 8 hours

56 km / h

k \$16.50 for 6 kg

\$2.75 / kg

l 243 km using 30 litres

8.1 km / L

2 A lift should carry no more than 1600 kg or 20 people.
What is this weight allowance, in kg/person?

80 kg / person

3 The cost of 53 litres of petrol is \$73.67. Express this cost in \$/L

\$1.39 / L

4 30 salad rolls were bought to feed 20 people at a picnic. The total cost was \$120.
Find these unit rates:

a Salad rolls/person

1.5 rolls / person

b Cost/person

\$6 / person

c Cost/roll

\$4 / roll

- 5** On a particular day, Dr Rosanna starts seeing patients at 8:30 am and finishes by 5:30 pm. If she saw 40 patients that day, then what is the rate at which Dr Rosanna saw her patients, in patients/hour?

4.44 patients / hr

- 6** Harvey finished a 10 km race in 37 min 30 sec.
Jack finished a 16 km race in 53 min 15 sec.

a Calculate the running rate of Harvey.

0.26 km / min

b Calculate the running rate of Jack.

0.3 km / min

c Who ran faster?

Jack

- 7** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:

a Kilometres per litre

9 L / h

b Litres per kilometre

 $\frac{1}{9}$ h

c How can you convert from km/L to L/km?

Flip the fraction (take the reciprocal)

- 8** Harvey gets paid \$296 per day and works 8 hours per day.
Jackie gets paid \$1500 per week and works 46 hours per week.
Who has the higher hourly rate, and by how much?

Harvey by \$4.39 / hr

- 9** Tom was 120 cm tall when he turned 10 years old.
He was 185 cm tall when he turned 20 years old.
Find Tom's average rate of growth per year between 10 and 20 years of age.

$$\text{Growth} = 185 - 120 = 65 \text{ cm}$$

$$\text{Time} = 20 - 10 = 10 \text{ years}$$

$$\text{Average rate} = 65 \div 10 = 6.5 \text{ cm / year}$$

- 10** The temperature was 20°C at 9 am. At noon the temperature had risen to 32°C.
What was the average increase in temperature per hour?

$$\text{Temperature increase} = 32 - 20 = 12^\circ\text{C}$$

$$\text{Time} = 12 - 9 = 3 \text{ hours}$$

$$\text{Average increase} = 12/3 = 4^\circ\text{C / hour}$$

- 11** The Burwood Football Club had 12 000 members.
After five successful years, they now have 18 000 members.
What has been the average rate of membership growth per year for the past 5 years?

$$\text{Membership increase} = 18000 - 12000 = 6000 \text{ members}$$

$$\text{Time} = 5 \text{ years}$$

$$\text{Average rate} = 6000/5 = 1200 \text{ members / year}$$

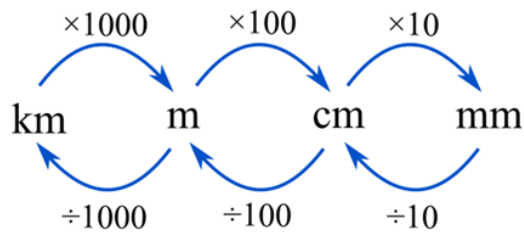
- 12** A car had a full tank of 56 L at the start of a 75 km journey.
At the end of the journey, the fuel tank was three quarters full.
What is the fuel consumption rate, in kilometres per litre?

$$\text{Fuel consumed} = 56 - (3/4)(56) = 56 - 42 = 14 \text{ L}$$

$$\text{Fuel consumption rate} = 75/14 = 75/14 \approx 5.36 \text{ km / litre}$$

REVIEW OF CONVERTING UNITS

CONVERTING LENGTH



- When converting to a larger unit, we divide, since we will need fewer units.
- When converting to a smaller unit, we multiply, since we will need more units.

1 When converting to a larger unit, we, since we will need fewer units.

2 When converting to a unit, we multiply, since we will need more units.

3 Convert these units of length. Only show the operation.

You do not need to find the answer.

a 3.6 km
= 3.6×1000 m

b 8.4 mm
= cm

c 2.9 m
= cm

d 7000 m
= km

e 594 cm
= m

f 40 mm
= cm

g 0.964 m
= mm

h 0.658 m
= cm

i 620 mm
= m

j 14300 m
= km

k 86 cm
= m

l 3.69 cm
= mm

m 70 mm
= m

n 630 m
= km

o 3600 cm
= m

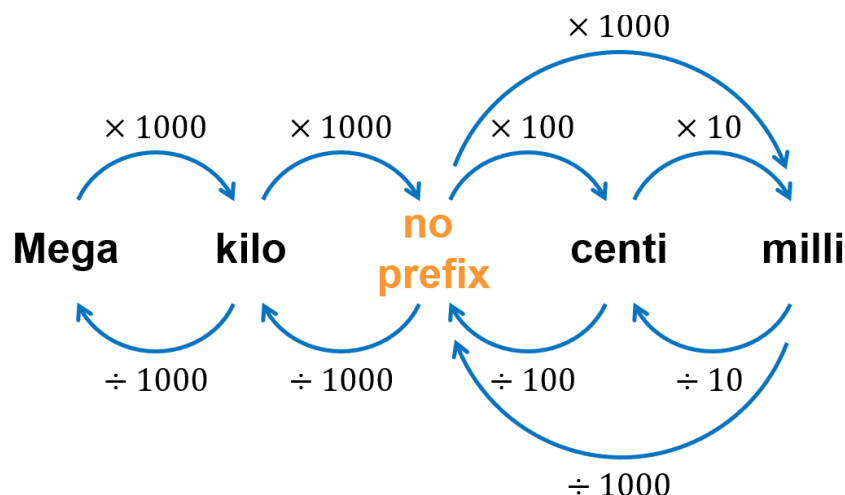
p 4 km
= cm.

q 0.78 km
= mm

r 72945 mm
= m

METRIC PREFIXES

- Metric prefixes are based on powers of 10.



4 Convert these units of capacity. The base unit is **litres**.

Only show the operation. You do not need to find the answer.

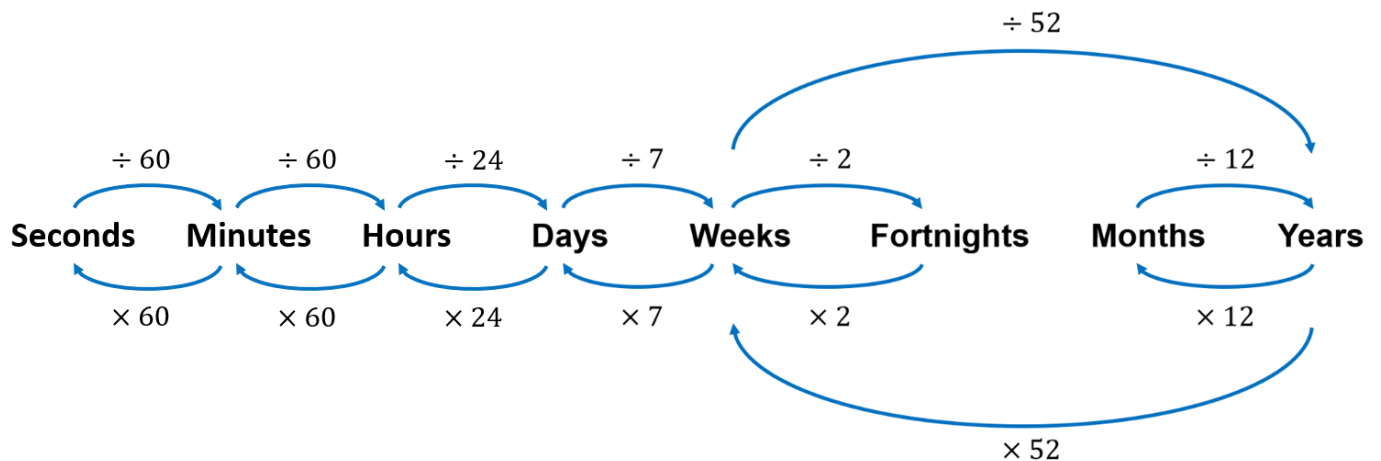
- | | | |
|--------------------------------------|-------------------------------|-------------------------------|
| a 35 mL
= $35 \div 1000$ L | b 15.9 L
= mL | c 1.65 L
= mL |
| d 15 000 mL
= L | e 0.06 L
= mL | f 8000 L
= kL |
| g 0.015 L
= mL | h 4725 L
= kL | i 95 mL
= L |

5 Convert these units of mass. The base unit is **grams**.

A tonne is 1 Megagram (1000 kg)

- | | | |
|--|-------------------------------|--------------------------------|
| a 2.63 kg
= 3.63×1000 g | b 7.1 g
= mg | c 17.84 kg
= g |
| d 8000 kg
= t | e 4300 g
= kg | f 2740 mg
= g |
| g 0.63 t
= kg | h 4.8 kg
= g | i 690 g
= kg |
| j 21.59 t
= kg | k 700 kg
= t | l 80 g
= kg |
| m 7 mg
= g | n 6.06 kg
= g | o 1.07 t
= kg |

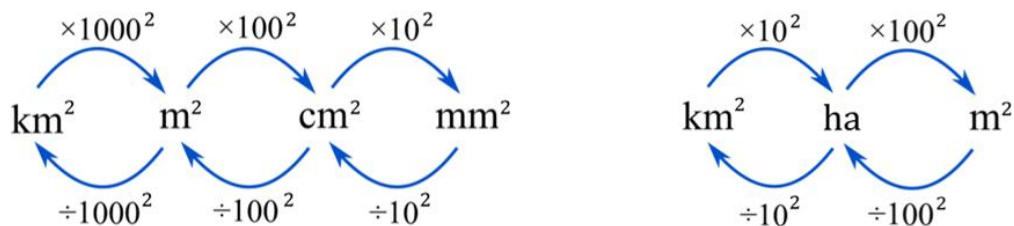
UNITS OF TIME



6 Convert these units of time. Only show the operation. You do not need to find the answer.

- | | | |
|-------------------------------------|-----------------------------------|-----------------------------------|
| a 15 min
= $15 \div 60$ h | b 200 min
= h | c 300 s
= min |
| d 193 min
= s | e 4.5 h
= min | f 635 min
= h |
| g 4 h 37 min
= min | h 47 min 12 s
= s | i 5 days 17 h
= h |
| j 100 h
= days | k 1 week
= h | l 10 000 h
= days |

UNITS OF AREA



7 Convert these units of area.

- | | | |
|---|--|--|
| a 2.6 ha
= 2.6×100^2 m ² | b 4.9 m ²
= cm ² | c 14 cm ²
= mm ² |
| d 63 000 m ²
= ha | e 127 000 cm ²
= m ² | f 24.8 cm ²
= mm ² |
| g 5.671 km ²
= cm ² | h 298 000 m ²
= km ² | i 2.4 km ²
= ha |

CONVERTING RATES (ONE QUANTITY)

- To convert rates:
 - Write the rate as a fraction with units.
Numerator is the first quantity.
Denominator is 1.
 - Convert the quantities to the required units.
 - Simplify the fraction using a calculator.

Example Convert rates (one quantity).

Convert 120 c/L to \$/L 1. $\frac{120 \text{ c}}{1 \text{ L}} = \frac{\\$(\quad)}{1 \text{ L}}$ 2. $= \frac{\\$(120 \div 100)}{1 \text{ L}}$ 3. $= \\$1.20/\text{L}$	Usually when working with fractions, we multiply/divide the numerator and denominator. Why don't we do that when converting rates? <i>When converting, we are not changing the of the denominator or numerator, so we do not need to do the operation to both.</i>
Convert \$35/kg to \$/g 1. $\frac{\\$35}{1 \text{ kg}} = \frac{\\$35}{(\quad) \text{ g}}$ 2. $= \frac{\\$35}{(1 \times 1000) \text{ g}}$ 3. $= \\$0.035/\text{g}$	

Convert these rates

a \$15/kg to \$/g $\frac{\$15}{1 \text{ kg}} = \frac{\$15}{\quad \text{g}}$ $=$ \$0.015/g	b 13 L/h to mL/h $\frac{13 \text{ L}}{1 \text{ h}} = \frac{\quad \text{mL}}{1 \text{ h}}$ $=$ 13000 mL/h	c 900 km/h to km/min 15 km/min
d 34.5 m/s to m/h 124200 m/h	e 0.6 c/min to c/h 36 c/h	f 0.47 g/mL to g/L 470 g/L

Key ideas

- When converting rates, write the rate as a
- Convert the numerator or denominator to the required units by
or
- Then use a to find the simplified rate.

1 Convert these units

a 5 t/ha to kg/ha

$$\frac{5 \text{ t}}{1 \text{ ha}} = \frac{\text{kg}}{1 \text{ ha}}$$

$$= \dots\dots\dots \text{ kg/ ha}$$

5000

b \$150/year to \$/month

$$\frac{\$150}{1 \text{ year}} = \frac{\$150}{\text{month}}$$

$$= \$\dots\dots\dots / \text{ mth}$$

12.5

c 300 cm/year to cm/week

5.77 cm/wk

d 20 m/s to m/h

72 000 m/h

e 500 g/L to g/mL

0.5 g/mL

f 560 c/kg to \$/kg

\$5.60/kg

g 750 kg/m² to kg/ha

7 500 000 kg/ha

h 0.8 c/m to c/km

800 c/km

i 400 mL/h to mL/s

0.1 mL/s

2 The local council gets 2190 complaints per 3 years. How many is this per month?

60.83 complaints/mth

3 The cost of 50 litres of petrol is \$94.68. Express this cost in c/L to 2 d.p.

189.36 c/L

CONVERTING RATES (BOTH QUANTITIES)

- To convert rates:
 - Write the rate as a fraction with units.
Numerator is the first quantity.
Denominator is 1.
 - Convert the quantities to the required units.
 - Simplify the fraction using a calculator.

Example Convert rates (both quantities).

Convert 72 L/h to mL/s 1. $\frac{72 \text{ L}}{1 \text{ hr}} = \frac{(\quad) \text{ mL}}{(\quad) \text{ s}}$ 2. $= \frac{(72 \times 1000) \text{ mL}}{(1 \times 60 \times 60) \text{ s}}$ 3. $= 20 \text{ mL/s}$	When two units are involved, many students don't know whether to multiply or divide. How does writing as a fraction help? <i>By writing as a fraction and working on each quantity, we can more easily identify whether to multiply or divide.</i>
Convert 35c/min to \$/hr 1. $\frac{35 \text{ c}}{1 \text{ min}} = \frac{\\$(\quad)}{(\quad) \text{ hr}}$ 2. $= \frac{\\$(35 \div 100)}{(1 \div 60) \text{ hr}}$ 3. $= \\$21/\text{hr}$	

Convert these rates.

a 45 L/h to mL/s $\frac{45 \text{ L}}{1 \text{ hr}} = \frac{\text{mL}}{\text{s}}$ = 12.5 mL/s	b \$11/kg to cents/g 1.1 c/g	c 29 L/min to mL/s 483.3 mL/s
d 125 kg/s to t/min 7.5 t/min	e 57 km/h to m/s 15.83 m/s	f 8.2 kg/day to g/min 5.69 g/min

1 Complete the following to convert these rates.

a \$3.75/h to cents/min

$$\frac{\$3.75}{1 \text{ h}} = \frac{\text{cents}}{\text{min}}$$

$$= \dots\dots\dots \text{c/ min}$$

6.25 c/min

b 16 t/ha to kg/m²

$$\frac{16 \text{ t}}{1 \text{ ha}} = \frac{\text{kg}}{\text{m}^2}$$

$$= \dots\dots\dots \text{kg/ m}^2$$

1.6 kg/m²

c \$1.80/m to cents/mm

$$\frac{\$1.80}{1 \text{ m}} = \frac{\text{cents}}{\text{mm}}$$

$$= \dots\dots\dots \text{c/ mm}$$

0.18 c/mm

d 2.4 kg/L to g/mL

$$\frac{2.4 \text{ kg}}{1 \text{ L}} = \frac{\text{g}}{\text{mL}}$$

$$= \dots\dots\dots \text{g/ mL}$$

2.4 g/mL

e 6 m/s to km/h

$$\frac{6 \text{ m}}{1 \text{ s}} = \frac{\text{km}}{\text{h}}$$

$$= \dots\dots\dots \text{km/ h}$$

21.6 km/h

f 3 cents/min to \$/day

$$\frac{3 \text{ cents}}{1 \text{ min}} = \frac{\$}{\text{day}}$$

$$= \$\dots\dots\dots / \text{day}$$

\$43.20 /day

j \$12/ kg to cents/g

1.2 c/g

k 18 km/h to m/s

5 m/s

l 3.8 cm/s to m/h

140 m/h

m 27 L/h to mL/s

7.5 mL/s

n 7.2 kg/day to g/min

5g /min

o 17 g/mL to kg/L

17 kg/L

p \$8/m to c/cm

8 c/cm

q 0.4 L/min to mL/sec

6.6 mL/s

r 0.3 kg/ha to kg/m²

0.000 03 kg/m²

CHECK YOUR UNDERSTANDING

Simplifying rates

1	300 L/s is the same as L/min						
a	18000	b	5	c	1800	d	50
2	60 kg/s is the same as kg/min						
a	6	b	3600	c	1	d	3600
3	30 m/s is the same as m/min						
a	0.5	b	1800	c	3000	d	180
4	600 L/s is the same as L/min						
a	3600	b	60	c	36000	d	10
5	60 kg/min is the same as kg/s						
a	6	b	36000	c	1	d	300