Student Name

Mathematics Stage 5 Core

LINEAR RELATIONSHIPS A

Book 3 Recognise and graph equations

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA4-LIN-C-01 determines the midpoint, gradient and length of an interval, and graphs linear relationships, with and without digital tools

Recognise and graph equations

- Recognise that equations of the form $y = mx + c$ represent linear relationships or straight lines
- Construct tables of values and use coordinates to graph a variety of linear relationships on the Cartesian plane, with and without digital tools
- Identify the x- and y- intercepts of lines
- Determine whether a point lies on a line using substitution

REVIEW OF PRIOR KNOWLEDGE

1 Complete these tables of values.

a $y = x + 2$

x	0	1	2	3	4
y					
	$0+2$	$1+2$	$2+2$	$3+2$	$4+2$

b $y = 4x$

x	0	1	2	3	4
y					
	4×0	4×1	4×2	4×3	4×4

c $y = 3x + 6$

x	0	1	2	3	4
y					
	$3\times0+6$	$3\times1+6$	$3\times2+6$	$3\times3+6$	$3\times4+6$

d $y = 8 - 2x$

x	0	1	2	3	4
y					
	$8-2\times0$	$8-2\times1$	$8-2\times2$	$8-2\times3$	$8-2\times4$

e $y = \frac{1}{2}x + 7$

x	0	1	2	3	4
y					
	$\frac{1}{2}\times0+7$	$\frac{1}{2}\times1+7$	$\frac{1}{2}\times2+7$	$\frac{1}{2}\times3+7$	$\frac{1}{2}\times4+7$

2 Complete the tables of values. There is no need to write the substitution.

a $y = x + 2$

x	0	1	2	3
y				

b $y = 2x$

x	0	1	2	3
y				

c $y = x - 2$

x	0	1	2	3
y				

d $y = 2x + 4$

x	0	1	2	3
y				

e $y = \frac{x}{2}$

x	0	1	2	3
y				

f $y = \frac{x}{2} - 1$

x	0	1	2	3
y				

g $y = 3x$

x	-2	-1	0	1	2
y					

h $y = 3x - 2$

x	-2	-1	0	1	2
y					

i $y = -x$

x	-2	-1	0	1	2
y					

j $y = -x + 1$

x	-2	-1	0	1	2
y					

k $y = -2x$

x	-2	-1	0	1	2
y					

l $y = 5 - 2x$

x	-2	-1	0	1	2
y					

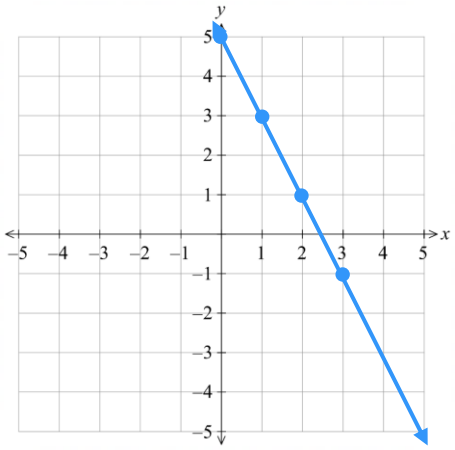
EQUATION OF A LINE

- A **linear** equation is in the form $y = mx + c$.
- We can show each x y **pair** from a table of values as **coordinates**.
  1. Construct a table of values by substituting x values into the equation to find y.
  2. Plot the (x, y) points on a Cartesian plane.
  3. Draw a straight line through the points and extend it.

Example Graph a linear relationship using a table of values.

Draw the graph of the equation $y = 5 - 2x$

x	0	1	2	3
y	5	3	1	-1



Coordinates:

(0, 5)

(1, 3)

(2, 1)

(3, -1)

How do we get coordinates from a table of values?

We look at the …… of the table of values to get the x and y values.

How many coordinates do we need to draw a line?

We need at least ……., but 3 or more is often useful to see if we've made a mistake.

Draw the graph of:

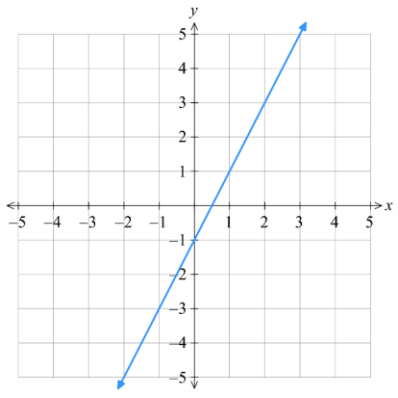
a $y = 2x - 1$

x	0	1	2	3
y				

Coordinates:

……

……

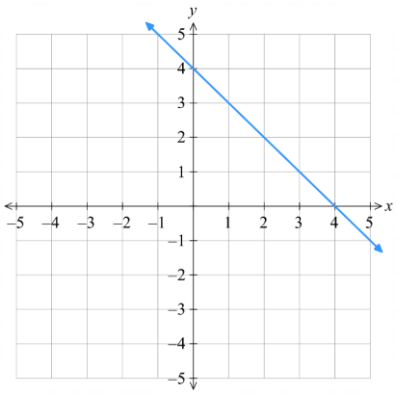


b $y = 4 - x$

x	0	1	2	3
y				

……

……



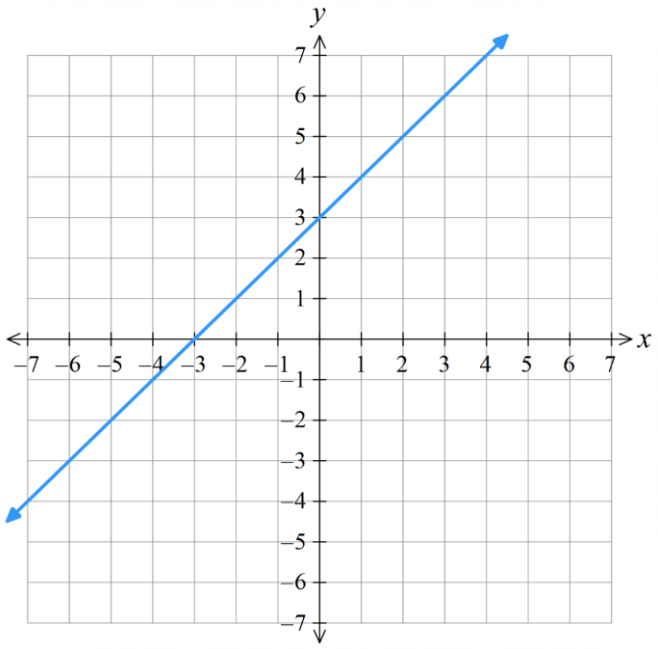
Key ideas

1. If we have a table of values, then we can represent each x y pair as a ……

1 Complete the table of values and graph the result.

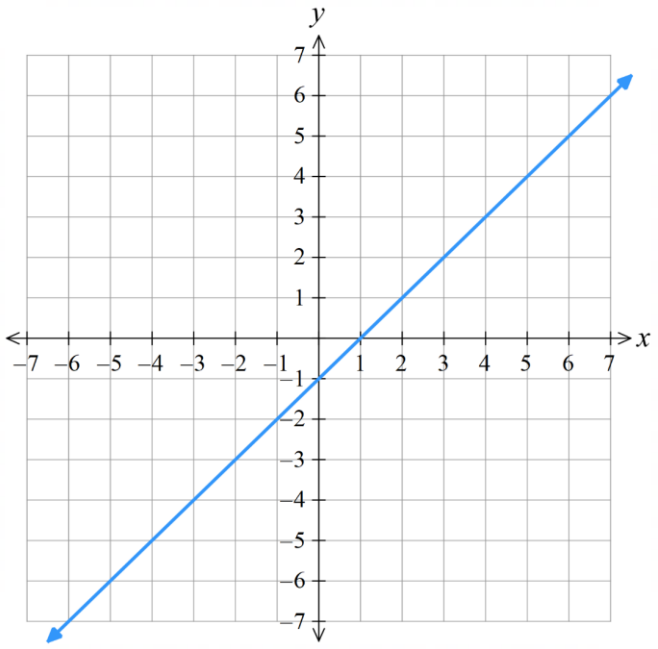
a $y = x + 3$

x	-2	-1	0	1	2
y					



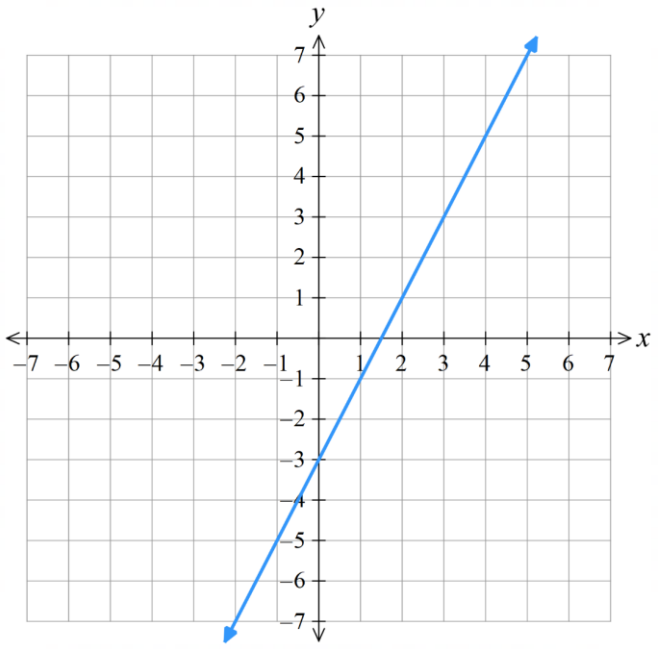
b $y = x - 1$

x	-2	-1	0	1	2
y					



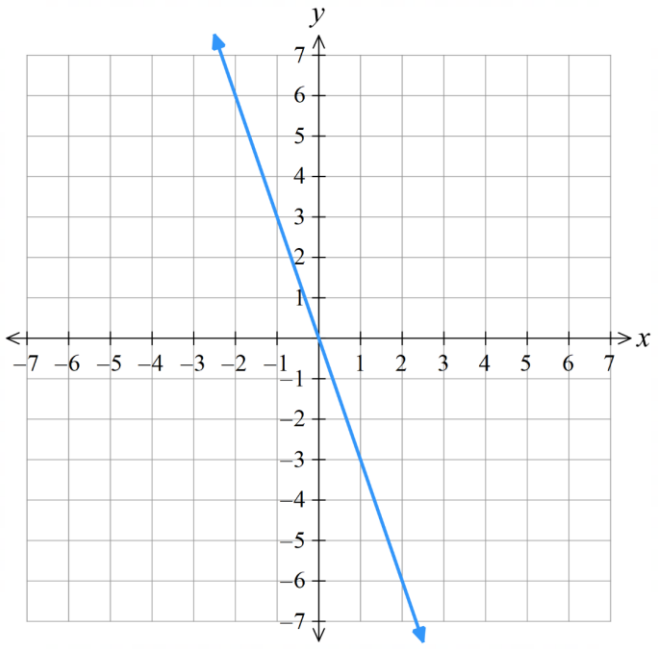
c $y = 2x - 3$

x	-2	-1	0	1	2
y					



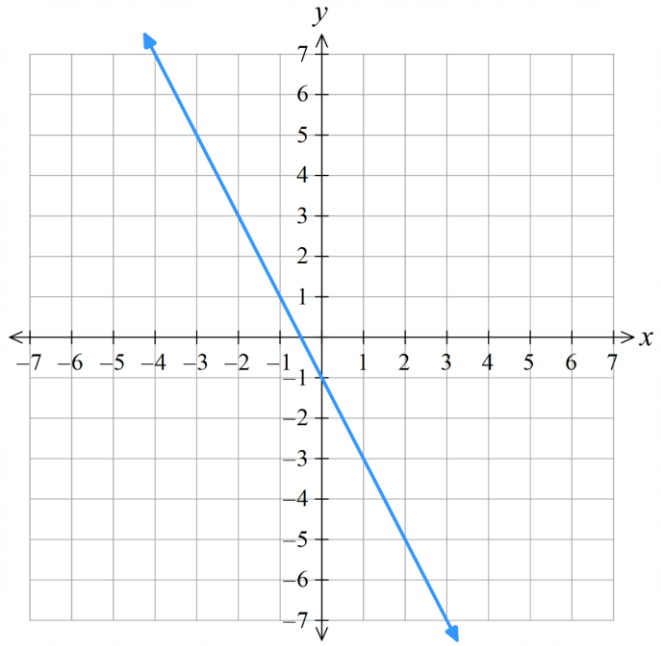
d $y = -3x$

x	-2	-1	0	1	2
y					



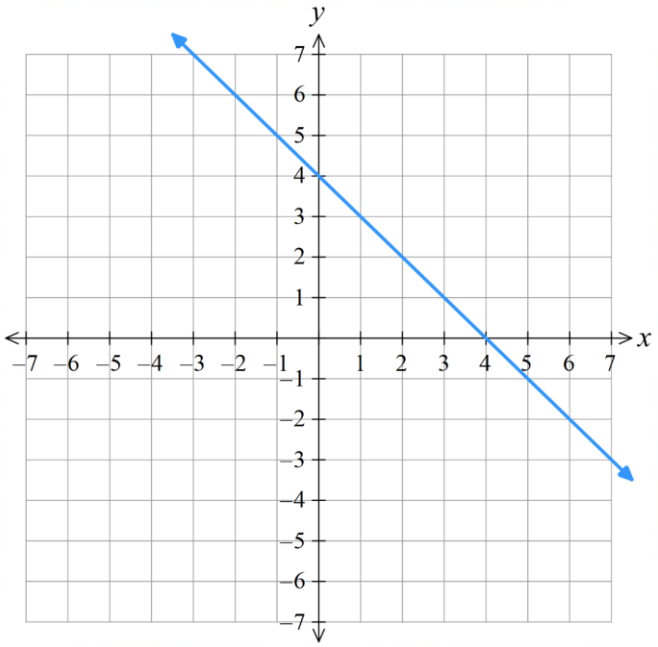
e $y = -2x - 1$

x	-2	-1	0	1	2
y					



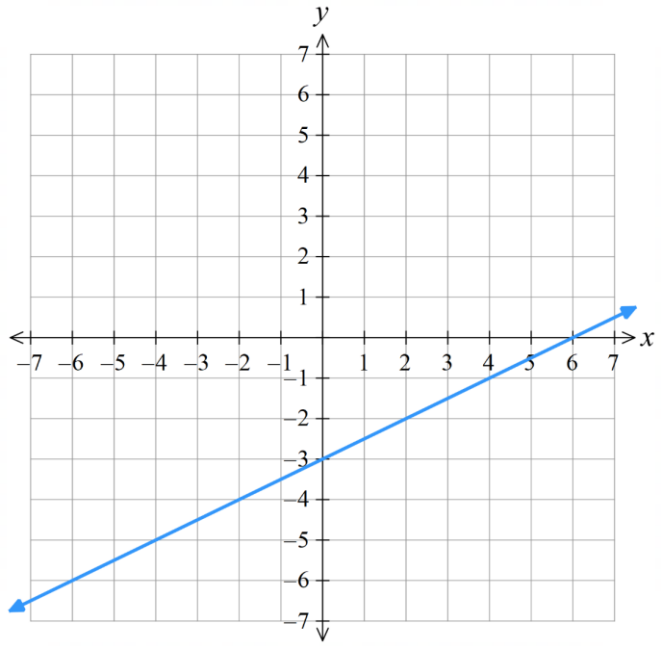
f $y = -x + 4$

x	-2	-1	0	1	2
y					



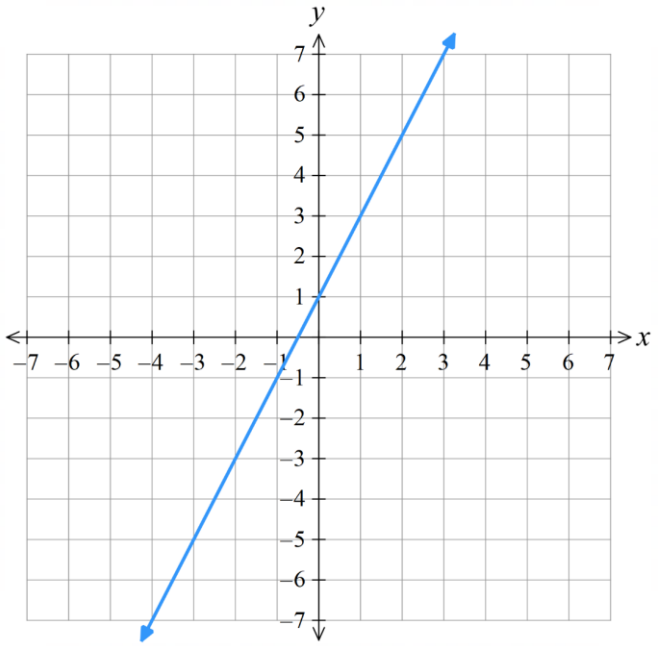
g $y = 0.5x - 3$

x	-2	-1	0	1	2
y					



h $y = 1 + 2x$

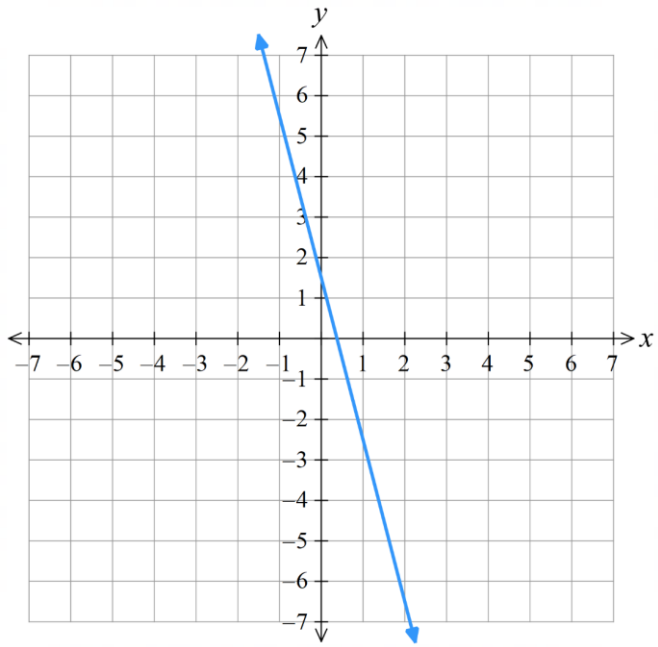
x	-2	-1	0	1	2
y					



2 Complete the table of values and graph the result.

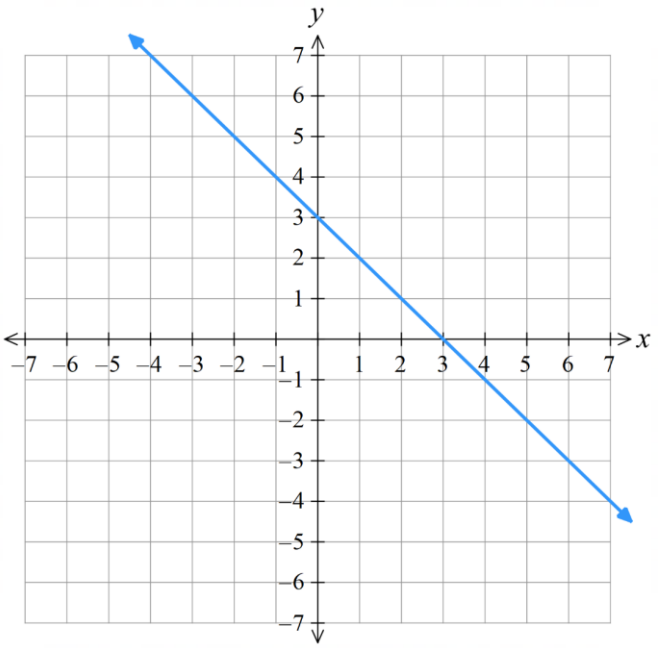
a $y = 1.5 - 4x$

x	-2	-1	0	1	2
y					



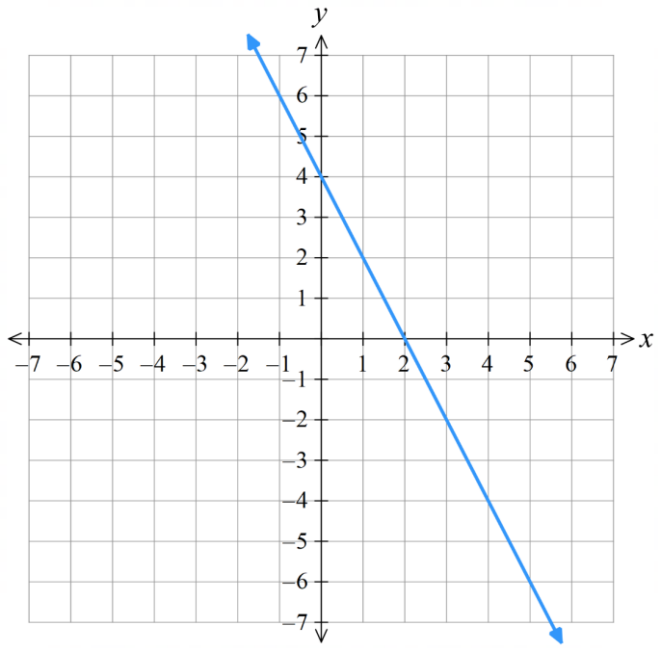
b $y = 3 - x$

x	-2	-1	0	1	2
y					



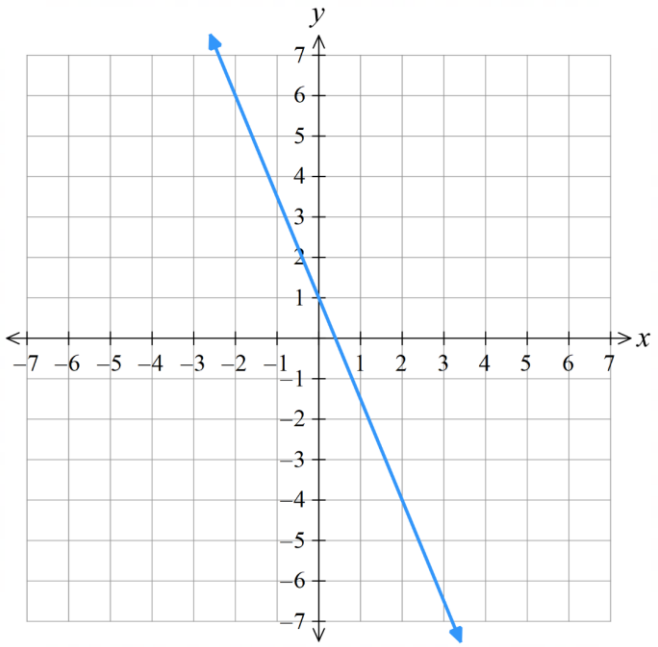
c $y = 4 - 2x$

x	-2	-1	0	1	2
y					



d $y = 1 - \frac{5}{2}x$

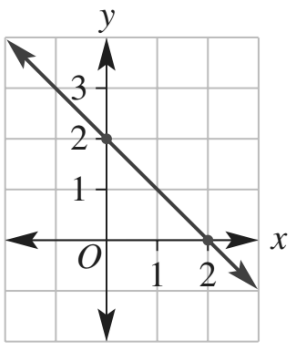
x	-2	-1	0	1	2
y					



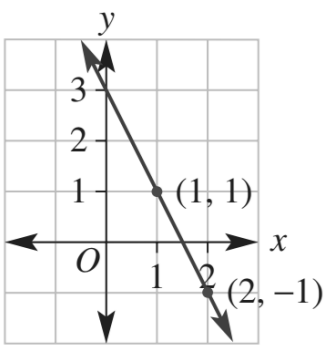
3 Match the rules A-D with the graphs a-d.

A $y = 2x + 1$ **B** $y = -x - 1$ **C** $y = -2x + 3$ **D** $y = -x + 2$

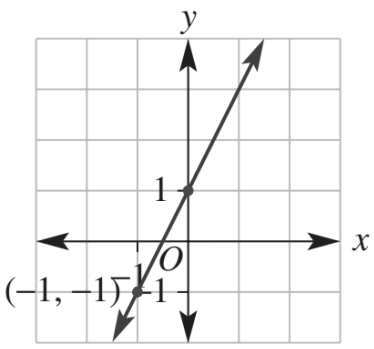
a



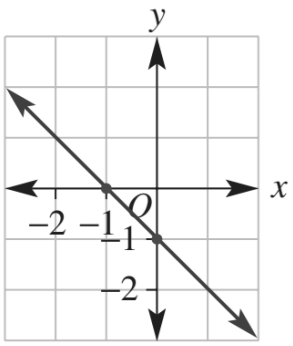
b



c



d

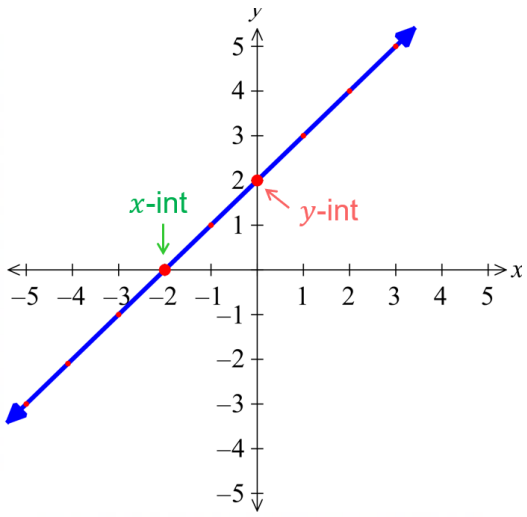


A – c, B – d, C – b, D – a

INTERCEPTS OF A LINE

- **x-intercept**: where the line meets the x axis ($\boldsymbol{y = 0}$)
- **y-intercept**: where the line meets the y axis ($\boldsymbol{x = 0}$)

		x-int					
$\boldsymbol{x}$	-3	-2	-1	0	1	2	3
$\boldsymbol{y}$	-1	0	1	2	3	4	5
				y-int			

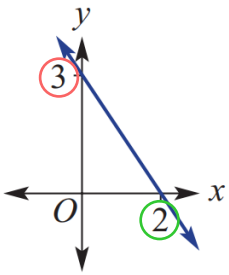


Example Identify x and y intercepts of a line.

Identify the x and y intercepts.

x-intercept: $(2,0)$

y-intercept: $(0,3)$



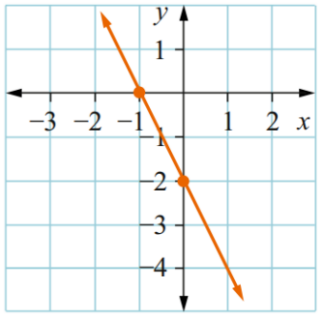
$\boldsymbol{x}$	-2	-1	0	1	2
$\boldsymbol{y}$	-6	-4	-2	0	2

x-intercept: $(1,0)$

y-intercept: $(0,-2)$

Identify the x and y intercepts of these lines.

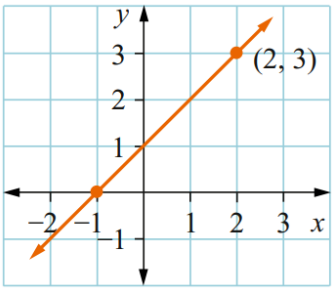
a



x intercept: ..... $(-1,0)$

y intercept: ..... $(0,-2)$

b



x intercept: ..... $(-1,0)$

y intercept: ..... $(0,1)$

c

x	-2	-1	0	1	2
y	2	0	-2	-4	-6

x intercept: ..... $(-1,0)$ y intercept: ..... $(0,-2)$

d

x	-4	-3	-2	-1	0
y	8	4	0	-4	-8

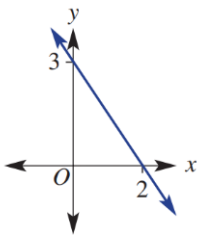
x intercept: ..... $(-2,0)$ y intercept: ..... $(0,-8)$

Key ideas

1. The ……-intercept is where the line meets the x axis. The …… coordinate is 0.
2. The ……-intercept is where the line meets the y axis. The …… coordinate is 0.

1 Identify the intercepts from these graphs.

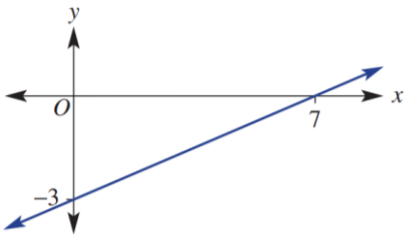
a



x-intercept: (2,0)

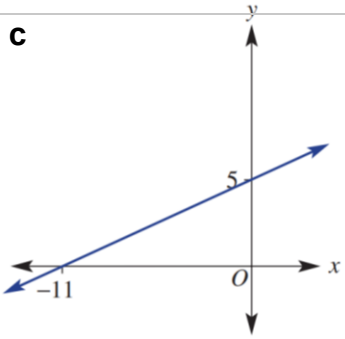
y-intercept: (0,3)

b



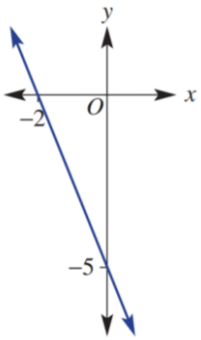
x-int: (7,0) y-int: (0,–3)

c



x-int: (–11,0) y-int: (0,5)

d



x-int: (–2,0) y-int: (0,–5)

2 Identify the x and y intercepts from these tables.

a

x	–3	–2	–1	0	1	2	3
y	4	3	2	1	0	1	2

x –int: ..... y –int: .....

x-int: (1,0) y-int: (0,1)

b

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5

x-int: (–2,0) y-int: (0,2)

c

x	–1	0	1	2	3	4
y	10	8	6	4	2	0

x-int: (4,0) y-int: (0,8)

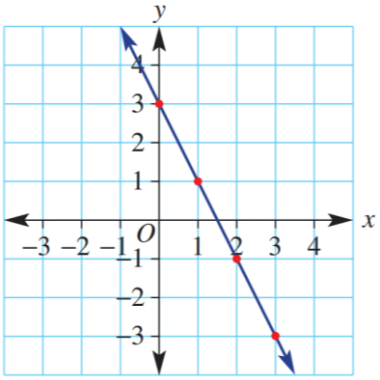
d

x	–5	–4	–3	–2	–1	0
y	0	2	4	6	8	10

x-int: (–5,0) y-int: (0,10)

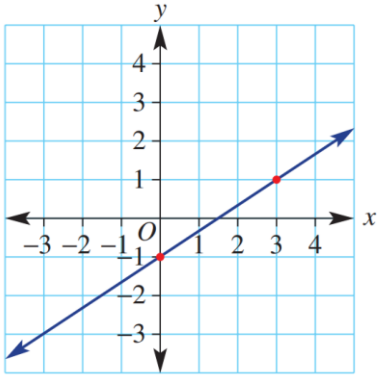
3 Identify the x and y-intercepts from these graphs.

a



x-int: (1.5,0) y-int: (0,3)

b



x-int: (1.5,0) y-int: (0,–1)

CHECKING IF A POINT LIES ON A LINE

- A point lies on a line if it **satisfies** the line's equation

1. Separate the **left-hand side LHS** and **right-hand side RHS** of the equation.
2. Substitute x and y values of the point into both sides.
3. Compare the LHS and RHS. If $LHS = RHS$, the point lies on the line.

Example Check whether a point lies on a line by substitution.

Decide whether $(2,5)$ lies on the line

$$y = 2x + 1$$

$\text{LHS} = y$	$\text{RHS} = 2x + 1$
$= 5$	$= 2(2) + 1$
	$= 5$

$\text{LHS} = \text{RHS}$

$(2,4)$ lies on the line $y = 2x$

Decide whether $(-2,5)$ lies on the line

$$y = 2x + 1$$

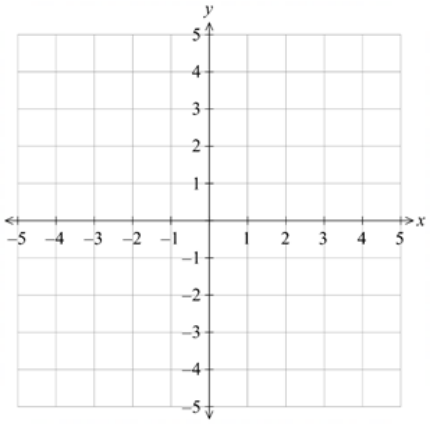
$\text{LHS} = y$	$\text{RHS} = 2x + 1$
$= 5$	$= 2(-2) + 1$
	$= -3$

$\text{LHS} \neq \text{RHS}$

$(-2,5)$ does not lie on the line $y = 2x$

Graph the equation $y = 2x + 1$ and check whether $(2,5)$ and $(-2,5)$ lie on the line.

x	-2	-1	0	1	2
y					



Decide whether $(-2,4)$ lies on the line

$$y = -x + 2$$

$\text{LHS} =$	$\text{RHS} =$

yes

Decide whether $(11,23)$ lies on the line

$$y = 35 - x$$

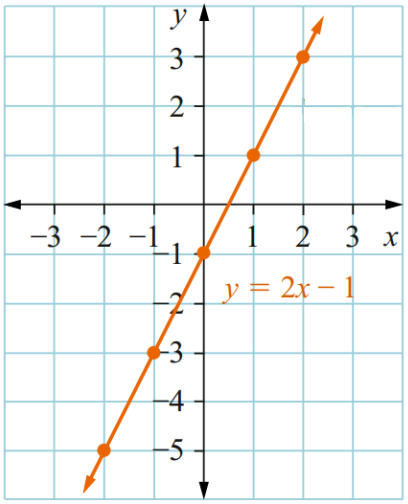
$\text{LHS} =$	$\text{RHS} =$

no

Key ideas

1. A point lies on the line if it …… the equation.

1 Does the point $(2,2)$ lie on the line $y = 2x - 1$?



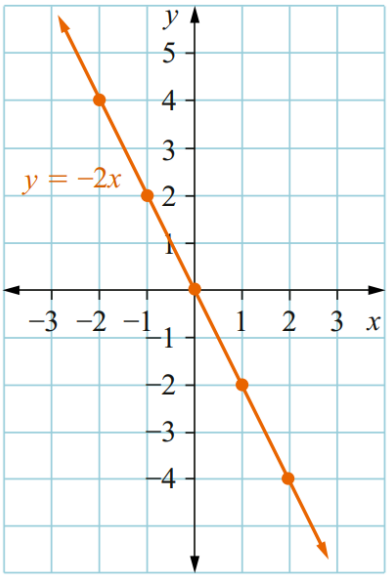
LHS $= y$	RHS $= 2x - 1$
$=$	$=$
	$=$

LHS …… RHS

The point $(2,2)$ does | doesn't lie on the line $y = 2x - 1$

no

2 Does the point $(-1, 2)$ lie on the line $y = -2x$?

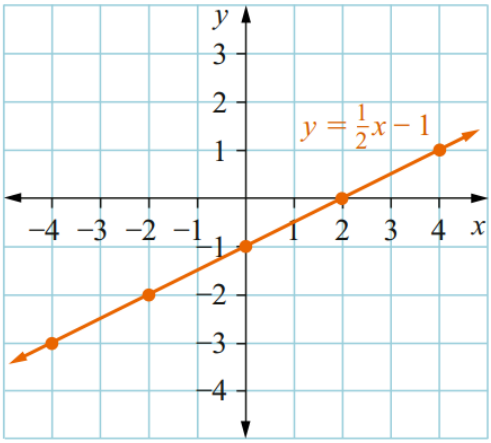


LHS $=$	RHS $=$
$=$	$=$
	$=$

LHS …… RHS

yes

3 Does the point $(-3, -2)$ lie on the line $y = \frac{1}{2}x - 1$?



LHS $=$	RHS $=$
$=$	$=$
	$=$

LHS …… RHS

no

4

a Does $(1,-3)$ lie on the line $y = x - 4$?

LHS =	RHS =
=	=

LHS …… RHS

yes

b Does $(0,2)$ lie on the line $y = 2x$?

LHS =	RHS =
=	=

LHS …… RHS

no

c Does $(4,1)$ lie on the line $y = \frac{x}{2}$?

no

d Does $(1,-2)$ lie on the line $y = -x - 3$?

no

e Does $(5,4)$ lie on the line $y = 1 - x$?

no

f Does $(-2,-1)$ lie on the line $y = 2x + 3$?

yes

g Does $(-4,-11)$ lie on the line $y = 2x - 3$?

yes

h Does $(-5,4)$ lie on the line $y = 3 - 2x$?

no

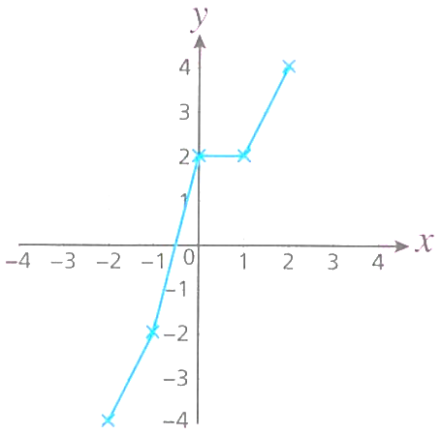
5 Emily says:

any line of the form $y = kx$ will go through the origin.

Explain why she is correct.

$x = 0, y = 0$ will always be a solution to $y = kx$

6 Rob has drawn the graph of $y = 2x$.



a Explain how you know he must have made a mistake.

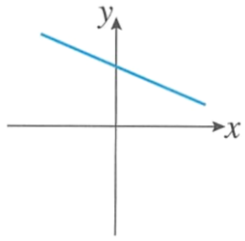
This is not a straight line.

b Which point is incorrect? Justify your answer.

(0,2), as when $x = 0, y = 0$

7 Ed has drawn the line with equation $y = -x$.

Explain how you know that Ed must have made a mistake.



This line must go through the origin as (0,0) is a solution.

8 A straight line has equation $y = 5x + 2a + 3$. The line passes through the point $(0,19)$.
Work out the value of a.

Substitute into equation

$19 = 5(0) + 2a + 3$

$2a = 16$

$a = 8$

9 A line has gradient 3 and passes through the point $(1,7)$. Find its equation $y = mx + c$.

$y = 3x + c$

$7 = 3(1) + c$

$c = 4$

$\therefore y = 3x + 4$