Student Name

Mathematics Stage 5 Paths

EQUATIONS C

Book 2 Rearrange literal equations

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-EQU-P-02 solves linear equations of more than 3 steps, monic and non-monic quadratic equations, and linear simultaneous equations

Rearrange literal equations

- Change the subject of a formula

REARRANGING FORMULAS: ONE-STEP BASIC OPERATIONS

- Rearranging a formula means to change the subject to another pronumeral.
- Isolate the pronumeral by applying inverse operations to both sides.
- Remember that the subject is always on the LHS.

Example Rearrange formulas (basic operations, one step).

Make n the subject.	How did we know to divide by Pr?
$I = Prn$ $\div Pr \quad \div Pr$ $\frac{I}{Pr} = n$ $n = \frac{I}{Pr}$	Prn means n is multiplied by ……, so we …… by Pr. Why did we swap the LHS and RHS so n is on the left? *The …… of a formula is always on the LHS.*

Rearrange to make x the subject.

a $y = x + k$

$x = y - k$

b $y = x - q$

$x = y + q$

c $y = x + \sqrt{rs}$

$x = y - \sqrt{rs}$

d $y = ax$

$x = \frac{y}{a}$

e $y = \frac{x}{abc}$

$x = abcy$

f $y = \frac{x}{\sqrt{z}}$

$x = y\sqrt{z}$

Key ideas

1. The …… of a formula is the pronumeral on its own on the left-hand-side.
2. Rearranging a formula means to change the …… to another pronumeral.

1 Make x the subject of these formulas.

a $y = x + a$

$x = y - a$

b $y = b + x$

$x = y - b$

c $y = x + abc$

$x = y - abc$

d $y = def + x$

$x = y - def$

e $y = x - a$

$x = y + a$

f $y = -b + x$

$x = y + b$

g $y = x - abc$

$x = y + abc$

h $y = -def + x$

$x = y + def$

2 Make x the subject of these formulas.

a $y = ax$

$x = \frac{y}{a}$

b $y = -ax$

$x = -\frac{y}{a}$

c $y = bcx$

$x = \frac{y}{bc}$

d $y = -bcx$

$x = -\frac{y}{bc}$

e $y = \frac{x}{a}$

$x = ay$

f $y = -\frac{x}{\sqrt{a}}$

$y = -y\sqrt{a}$

g $y = \frac{x}{bc}$

$x = bcy$

h $y = -\frac{x}{\sqrt{bc}}$

$x = -y\sqrt{bc}$

4 $V = IR$ is a formula used in physics. Rearrange the formula to make:

a I the subject

$I = \frac{V}{R}$

b R the subject

$I = \frac{V}{I}$

5 $P = \frac{F}{A}$ is a formula that connects pressure (P), force (F) and area (A)

a Express F in terms of P and A (make F the subject)

$F = AP$

b Express A in terms of P and F

$A = \frac{F}{P}$

3 The circumference of a circle is 40 cm. The formula for the circumference is $C = \pi d$

a Rearrange to get d as the subject.

$d = \frac{c}{\pi}$

b Find, to 1 decimal place, the diameter of the circle with circumference:

i 40 cm

12.7 cm

ii 82 cm

26.1 cm

REARRANGING FORMULAS: TWO-STEP BASIC OPERATIONS

- Identify the operations done to the subject.
- Apply inverse operations to undo the operations.

Example Rearrange formulas (basic operations, two steps).

Make x the subject.	What operations have been done to x?
$y = 4x - c$ $+c \quad +c$ $y + c = 4x$ $\div 4 \quad \div 4$ $\frac{y+c}{4} = x$ $x = \frac{y+c}{4}$	*x was ..... by 4,* *then c was .....* How did we isolate x? *First, ... c* *Then, ... by 4* *Then write the formula so x is on the .....*

Rearrange to make x the subject.

a $y = 5x + 7$

$x = \frac{y-7}{5}$

b $ab = \frac{x}{2} + y$

$x = 2(ab - y)$

c $P = 2x + 2y$

$x = \frac{P-2y}{2}$

1 Make x the subject of these formulas.

a $y = ax + b$

$x = \frac{y-b}{a}$

b $y = b^2 + ax$

$x = \frac{y-b^2}{a}$

c $y = ax + bcd$

$x = \frac{y-bcd}{a}$

d $y = d^2ef + ax$

$x = \frac{y-d^2ef}{a}$

e $y = ax - b$

$y = \frac{y+b}{a}$

f $y = -\sqrt{b} + ax$

$x = \frac{y+\sqrt{b}}{a}$

g $y = ax - bcd$

$x = \frac{y+bcd}{a}$

h $y = -\sqrt{bcd} + ax$

$x = \frac{y+\sqrt{bcd}}{a}$

2 Rearrange these formulae to make t the subject.

a $p = 3t + 7$

$t = \frac{p-7}{3}$ or $\frac{p}{3} - \frac{7}{3}$

b $p = 3t - 7$

$t = \frac{p+7}{3}$ or $\frac{p}{3} + \frac{7}{3}$

c $p = 3t + a$

$t = \frac{p-a}{3}$ or $\frac{p}{3} - \frac{a}{3}$

d $p = 3t - a$

$t = \frac{p+a}{3}$ or $\frac{p}{3} + \frac{a}{3}$

e $p = \frac{t}{2} + 7$

$t = 2(p + 7)$ or $2p - 14$

f $p = \frac{t}{2} - 7$

$t = 2(p + 7)$ or $2p + 14$

g $p = \frac{t}{2} + a$

$t = 2(p - a)$ or $2p - 2a$

h $p = \frac{t}{2} - a$

$t = 2(p + a)$ or $2p + 2a$

3 Make x the subject of these formulas.

a $y = \frac{x}{a} + c$

$x = a(y - c)$

b $y = -\frac{x}{a} + c$

$x = -a(y - c)$

c $y = \frac{x}{a^2} - c$

$x = a^2(y + c)$

d $y = -\frac{x}{a^2} - c$

$x = -a^2(y + c)$

e $y = \frac{x}{bc} + \sqrt{qrs}$

$x = bc\left(y - \sqrt{qrs}\right)$

f $y = -\frac{x}{bc} - \sqrt{qrs}$

$x = -bc\left(y + \sqrt{qrs}\right)$

4 Here are the equations of some straight lines. Rearrange them into the form $y = mx + c$

a $y + 3x = 7$

$y = -3x + 7$

b $2y + 8 = 4x$

$y = 2x - 4$

c $x = 2y + 10$

$y = \frac{x}{2} - 5$

d $3x = 5y - 10$

$y = \frac{3x}{5} + 2$

5 The formula to convert Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$.

a Rearrange this formula to make C the subject.

$C = \frac{5F - 160}{9}$

b Use your rearranged formula to convert these degrees Fahrenheit to Celsius

$350°F$

176.67°C

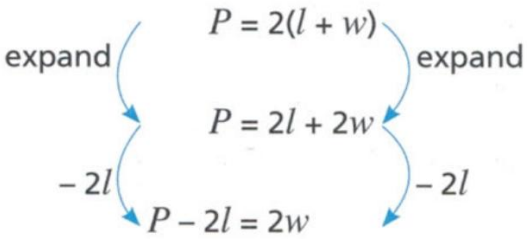
$400°F$

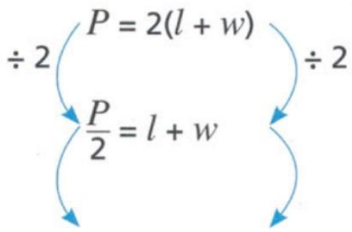
204.44°C

$100°F$

37.78°C

6 Complete these methods for making w the subject of the formula $P = 2(1 + w)$





Make t the subject of each of these formulae,

a $x = 3(t + 4)$

$t = \frac{x}{3} - 4$

b $x = 3(2t + 4)$

$t = \frac{x}{6} - 2$

c $x = 3(2t - 4)$

$t = \frac{x}{6} + 2$

d $x = 3(4 + 5t)$

$t = \frac{x}{15} - \frac{4}{5}$

e $x = 3(4 - 5t)$

$t = -\frac{x}{15} + \frac{4}{5}$

7 Fill in the table.

	$a =$	$b =$	$c =$
a	$a = b + c$	$b = a - c$	$c = a - b$
b	$a = b - c$	$b = a + c$	$c = b - a$
c	$a = \frac{b}{c}$	$b = ac$	$c = \frac{b}{a}$
d	$a = \frac{2b}{c}$	$b = \frac{ac}{2}$	$c = \frac{2b}{a}$
e	$a = 2b + c$	$b = \frac{a - c}{2}$	$c = a - 2b$
f	$a = 2b - c$	$b = \frac{a + c}{2}$	$c = 2b - a$
g	$a = 2b - 2c$	$b = \frac{a}{2} + c$	$c = b - \frac{a}{2}$

8 **2016 HSC Standard 2 Band 4**

Which of the following correctly expresses Q as the subject of $e = iR + \frac{Q}{C}$

A. $Q = Ce + CiR$

B. $Q = Ce - CiR$

C. $Q = \frac{e + iR}{C}$

D. $Q = \frac{e - iR}{C}$

B

9 **2022 HSC Standard 2 Band 4**

Which of the following correctly expresses x as the subject of $y = \frac{ax-b}{2}$

A. $x = \frac{2y + b}{a}$

B. $x = \frac{y + b}{2a}$

C. $x = \frac{2y}{a} + b$

D. $x = \frac{y}{2a} + b$

A

REARRANGING FORMULAS: POWERS & ROOTS

Example Rearrange formulas (powers & roots).

Worked Example	Your Turn
Make r the subject. $A = \pi r^2$ $\div \pi \quad \div \pi$ $\frac{A}{\pi} = r^2$ $\sqrt{\quad} \quad \sqrt{\quad}$ $\pm\sqrt{\frac{A}{\pi}} = r$ $r = \pm\sqrt{\frac{A}{\pi}}$	Make v the subject. $a = \frac{v^2}{r}$ $v = \pm\sqrt{ar}$
Make M the subject. $v = \sqrt{\frac{2GM}{r}}$ $\square^2 \quad \square^2$ $v^2 = \frac{2GM}{r}$ $\times r \quad \times r$ $rv^2 = 2GM$ $\div 2G \quad \div 2G$ $\frac{rv^2}{2G} = M$ $M = \frac{rv^2}{2G}$	Make r the subject $v = \sqrt{\frac{rF}{m}}$ $r = \frac{mv^2}{F}$

1 Rearrange these to make q the subject.

a $p = 2q + 4$

$q = \frac{p-4}{2}$

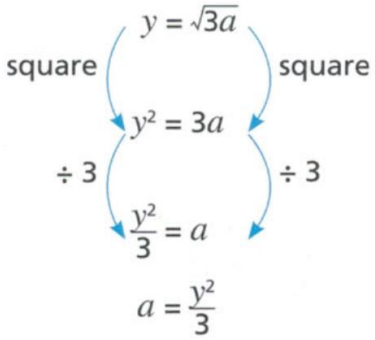
b $p = q^2 + 4$

$q = \pm\sqrt{p-4}$

c $p = 2q^2 + 4$

$q = \pm\sqrt{\frac{p-4}{2}}$

2 Consider the working out for making a the subject for $y = \sqrt{3a}$



Make a the subject of each of these formulae.

a $y = \sqrt{4a}$

$a = \frac{y^2}{4}$

b $y = \sqrt{4 + a}$

$a = y^2 - 4$

c $y = \sqrt{a - 4}$

$a = y^2 + 4$

d $y = \sqrt{4 - a}$

$a = 4 - y^2$

e $y = 4\sqrt{a}$

$a = \frac{y^2}{16}$

f $y = 4 + \sqrt{a}$

$a = (y-4)^2$

g $y = \sqrt{a} - 4$

$a = (y+4)^2$

h $y = 4 - \sqrt{a}$

$a = (4-y)^2$

3 Einstein worked out that energy (E), mass (m) and the speed of light (c) are connected by the formula $E = mc^2$. Use this formula to

a Express m in terms of E and c.

$m = \frac{E}{c^2}$

b Express c in terms of E and m.

$c = \pm\sqrt{\frac{E}{m}}$

4 Rearrange the formula $K = \frac{1}{2}mv^2$ to make:

a m the subject

$m = \frac{2k}{v^2}$

b v the subject

$v = \pm\sqrt{\frac{2k}{m}}$

5 Rearrange these equations to make a the subject.

a $2a^2 = c$

$a = \pm\sqrt{\frac{c}{2}}$

b $2a^2 + b = c$

$a = \pm\sqrt{\frac{c-b}{2}}$

c $2a^2 - b = c$

$a = \pm\sqrt{\frac{c+b}{2}}$

d $4a^2 - b = c$

$a = \pm\sqrt{\frac{c+b}{4}}$

e $64a^2 - b = c$

$a = \pm\sqrt{\frac{c+b}{64}}$

f $64a^3 - b = c$

$a = \sqrt[3]{\frac{c+b}{64}}$

g $\frac{\sqrt{a}}{2} = c$

$a = (2c)^2$

h $2\sqrt{a} = c$

$a = \left(\frac{c}{2}\right)^2$

i $2\sqrt{a} + b = c$

$a = \left(\frac{c-b}{2}\right)^2$

j $2\sqrt{3a} + b = c$

$a = \frac{\left(\frac{c-b}{2}\right)^2}{3}$

k $\sqrt[3]{a} = c$

$a = c^3$

l $2\sqrt[3]{a} = c$

$a = \left(\frac{c}{2}\right)^3$

6 Rearrange these equations to make the variable in the bracket the subject.

a $P = 2a + b$ (a)

$a = \frac{p-b}{2}$

b $P = 2l + 2b$ (b)

$b = \frac{p-21}{2}$

c $M = \frac{a+b}{2}$ (a)

$a = 2M - b$

d $D = b^2 - 4ac$ (c)

$c = \frac{D-b^2}{-4a}$

e $d = st$ (t)

$t = \frac{d}{s}$

f $A = \pi r^2$ (r)

$r = \pm\sqrt{\frac{A}{\pi}}$

g $V = \pi r^2 h$ (h)

$h = \frac{V}{\pi r^2}$

h $c^2 = a^2 + b^2$ (b)

$b = \pm\sqrt{c^2 - a^2}$

i $Q = \sqrt{2gh}$ (h)

$h = \frac{Q^2}{2g}$

j $y = \sqrt{9 - x^2}$ (x)

$x = \pm\sqrt{9 - y^2}$

k $A = \frac{h}{2}(a + b)$ (a)

$a = \frac{2A}{h} - b$

l $s = ut + \frac{1}{2}at^2$ (a)

$a = \frac{2(s-ut)}{t^2}$

7 **2012 HSC Standard 2 Band 4**

Which of the following correctly expresses c as the subject of $E = mc^2 + p$?

(A) $c = \pm\sqrt{\dfrac{E-p}{m}}$

(B) $c = \pm\dfrac{\sqrt{E-p}}{m}$

(C) $c = \pm\sqrt{\dfrac{E}{m}-p}$

(D) $c = \pm\sqrt{\dfrac{E}{m}-p}$

A

8 **HSC Standard 2 Sample Question Band 4**

Make r the subject of the equation $V = \frac{4}{3}\pi r^3$

$r = \sqrt[3]{\dfrac{3V}{4\pi}}$

9 **2017 HSC Standard 2 Band 5**

Make y the subject of the equation $x = \sqrt{yp-1}$

$y = \dfrac{x^2+1}{p}$

10 Rearrange the following to make a the subject.

a $y = \sqrt{\frac{a}{4}}$

$a = 4y^2$

b $y = \sqrt{\frac{a}{4}} + 1$

$a = 4(y-1)^2$

c $y = \sqrt{\frac{a^2}{4}+1}$

$a = \pm\sqrt{4(y^2-1)}$

d $y = \sqrt{a^2+\frac{1}{4}}$

$a = \pm\sqrt{y^2-\frac{1}{4}}$

11 Rearrange each of these formulae to make x the subject.

a $y = \frac{x}{5}$ $x = 5y$	**b** $y = \frac{x}{5} + 1$ $x = 5(y - 1)$	**c** $y = \frac{x + 1}{5}$ $x = 5y - 1$	**d** $y = x + \frac{1}{5}$ $x = y - \frac{1}{5}$	**e** $y = \frac{3x - 1}{5}$ $x = \frac{5y + 1}{3}$
f $y = \frac{3x^2 - 1}{5}$ $x = \pm\sqrt{\frac{5y + 1}{3}}$	**g** $y = \frac{1 - 3x}{5}$ $x = -\frac{5y - 1}{3}$	**h** $y = \frac{1 - 3x^2}{5}$ $x = \pm\sqrt{-\frac{5y - 1}{3}}$	**i** $y = \frac{(3x - 1)^2}{5}$ $x = \frac{1 \pm \sqrt{5y}}{3}$	**j** $y = 3 + \frac{2(x + 1)}{5}$ $x = \frac{5(y - 3)}{2} - 1$

12 Fill in the table.

	$a =$	$b =$	$c =$
a	$a = 2(b^2 - c)$	$b = \pm\sqrt{c + \frac{a}{2}}$	$c = b^2 - \frac{a}{2}$
b	$a = \pm\sqrt{c^2 - b^2}$	$b = \pm\sqrt{c^2 - a^2}$	$c = \sqrt{a^2 + b^2}$
c	$a = 2(b - \sqrt{c})$	$b = \frac{a}{2} + \sqrt{c}$	$c = \left(b - \frac{a}{2}\right)^2$
d	$a = \frac{2b - 2\sqrt{c}}{3}$	$b = \frac{3a + 2\sqrt{c}}{2}$	$c = \left(-\frac{3a - 2b}{2}\right)^2$

13 In maths and physics, the "equations of motion" relate the displacement of an object (s) with its initial velocity (u), final velocity (v), acceleration (a) and time (f).

a Acceleration is defined as "change in velocity divided by time".
Form an equation for a in terms of v, u and t.

$$a = \frac{v-u}{t}$$

b Show that your answer to part **a** can be rearranged to give $v = u + at$.

$$a = \frac{v-u}{t}$$
$$at = v - u$$
$$v = u + at$$

c Another equation of motion is $v^2 = u^2 + 2as$. Rearrange this formula to make:

a the subject	s the subject	u the subject
$a = \frac{v^2-u^2}{2s}$	$s = \frac{v^2-u^2}{2a}$	$u = \pm\sqrt{v^2-2as}$

d Yet another equation of motion is $S = \frac{1}{2}(u+v)t$.
Use the formula in part **b** to show that this leads to $S = ut + \frac{1}{2}at^2$

$$s = \frac{1}{2}(u+v)t$$
$$= \frac{1}{2}(u+(u+at))t$$
$$= \frac{1}{2}(2u+at)t$$
$$= \frac{t}{2}(2u+at)$$
$$= ut + \frac{1}{2}at^2$$

14 Rearrange each formula so that the pronumeral in square brackets is the subject.

a $a = c\sqrt{b+2} - d$ $[b]$

$$b = \left(\frac{a+d}{c}\right)^2 - 2$$

b $t = a + (n-1)d$ $[n]$

$$n = \frac{t-a}{d} + 1$$

15 Rearrange these equations to make a the subject.

a $2(a+b)^2 = c$

$a = -b \pm \sqrt{\frac{c}{2}}$

b $2(a-2b)^2 + b = c$

$a = 2b \pm \sqrt{\frac{c-b}{2}}$

c $2(3a-bc)^2 - b = c$

$a = \frac{bc \pm \sqrt{\frac{c+b}{2}}}{3}$

d $4(a+def)^2 - b = c$

$a = \pm\sqrt{\frac{c+b}{4}} - def$

e $m(a-de^3)^2 - b = c$

$a = de^2 \pm \sqrt{\frac{c+b}{m}}$

f $p(a+m)^3 - yb = c$

$a = \sqrt[3]{\frac{c+yb}{p}} - m$

g $\frac{\sqrt{a+b}}{2} = c$

$a = 4c^2 - b$

h $2\sqrt{a-de} = c$

$a = \frac{c^2}{4} + de$

i $2\sqrt{a-gh} + b = c$

$a = \left(\frac{c-b}{2}\right)^2 + gh$

j $2\sqrt{3a-4} + b = c$

$a = \frac{\left(\frac{c-b}{2}\right)^2 + 4}{3}$

k $\sqrt[3]{a-b} = c$

$a = c^3 + b$

l $2\sqrt[3]{\frac{a}{b}} = c$

$a = \frac{bc^3}{8}$

REARRANGING FORMULAS: SUBJECT IN DENOMINATOR

- To remove the subject from the denominator, either:
- Multiply both sides by the denominator OR
- If fraction = fraction
  - Take the reciprocal (flip) of both sides
  - Cross multiply
- If multiplying by a binomial denominator, you must multiply every term.

Example Rearrange formulas (subject in denominator).

Worked Example	Your Turn
Make x the subject. $y = \frac{6}{x}$ $\times x \quad \times x$ $xy = 6$ $\div y \quad \div y$ $x = \frac{6}{y}$	Make a the subject: $\frac{3}{a} = 17x$ $a = \frac{3}{17x}$
Make t the subject. $A = \frac{m}{t+1}$ $\times (t+1) \quad \times (t+1)$ $A(t+1) = m$ $t + 1 = \frac{m}{A}$ $t = \frac{m}{A} - 1$	Make x the subject. $t^2 = \frac{i}{x-1}$ $x = \frac{i+t^2}{t^2}$ or $x = \frac{i}{t^2} + 1$
Make v the subject. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ $-\frac{1}{u} \quad -\frac{1}{u}$ $\frac{1}{f} - \frac{1}{u} = \frac{1}{v}$ $\frac{u-f}{fu} = \frac{1}{v}$ *reciprocate both sides* $\frac{fu}{u-f} = v$ $v = \frac{fu}{u-f}$	Make x the subject. $\frac{2}{z} = \frac{1}{x} + \frac{1}{y}$ $x = \frac{zy}{2y-z}$

1 Rearrange these equations to make the variable in the bracket the subject.

a $D = \frac{m}{v}$ $[v]$

$v = \frac{m}{D}$

b $T = \frac{2\pi}{n}$ $[n]$

$n = \frac{2\pi}{T}$

c $E = \frac{p^2}{2m}$ $[m]$

$m = \frac{p^2}{2E}$

d $E = \frac{m}{2r^2}$ $[r]$

$r = \pm\sqrt{\frac{m}{2E}}$

e $a = \sqrt{\frac{x}{y}}$ $[y]$

$y = \frac{x}{a^2}$

f $w = \sqrt{\frac{3V}{\pi h}}$ $[h]$

$h = \frac{3V}{\pi w^2}$

g $\frac{p}{q+r} = t$ $[r]$

$r = \frac{p}{t} - q$

h $u = 1 + \frac{3}{v}$ $[v]$

$v = \frac{3}{u-1}$

i $\frac{1}{x}-\frac{1}{y}=\frac{5}{n}$ $[n]$

$n=\frac{5xy}{y-x}$

j $\frac{1}{x}-\frac{1}{y}=\frac{5}{n}$ $[y]$

$y=\frac{nx}{n-5x}$

k $t=\sqrt{\frac{M}{M-m}}$ $[m]$

$m=M-\frac{M}{t^2}$

l $T=2\pi\sqrt{\frac{W}{gF}}$ $[F]$

$F=\frac{4\pi^2W}{gT^2}$

m $M=\sqrt{\frac{ab}{t}+a^2}$ $[t]$

$t=\frac{ab}{M^2-a^2}$

n $x=a\sqrt{\frac{k}{ph+y}}$ $[h]$

$h=\frac{ka^2}{px^2}-\frac{y}{p}$

REARRANGING FORMULAS: SUBJECT APPEARS TWICE

- When a subject appears twice, you will need to move them all to one side, then factorise.

Example Rearrange formulae (subject appears twice).

Worked Example	Your Turn
Make x the subject. $ax + bx = 3$ $x(a + b) = 3$ $\div (a + b) \quad \div (a + b)$ $x = \frac{3}{a + b}$	Make a the subject. $ak + am = 5$ $\frac{5}{k + m}$
Make x the subject. $ax + 2y = 5y + mx$ $-mx \quad -mx$ $ax - mx + 2y = 5y$ $-2y \quad -2y$ $ax - mx = 3y$ $x(a - m) = 3y$ $\div (a - m) \quad \div (a - m)$ $x = \frac{3y}{a - m}$	Make a the subject. $ab + 3y = 7y + ak$ $a = \frac{4y}{b - k}$
Make x the subject. $y = \frac{x + 1}{x + 2}$ $\times (x + 2) \quad \times (x + 2)$ $y(x + 2) = x + 1$ $xy + 2y = x + 1$ $-x \quad -x$ $xy - x + 2y = 1$ $-2y \quad -2y$ $xy - x = 1 - 2y$ $x(y - 1) = 1 - 2y$ $\div (y - 1) \quad \div (y - 1)$ $x = \frac{1 - 2y}{y - 1}$	Make x the subject. $y = \frac{2x + 5}{x - 1}$ $x = \frac{y + 5}{y - 2}$

1 Rearrange these equations to make a the subject.

a $2a = b$

$a = \frac{b}{2}$

b $\frac{a}{2} = b$

$a = 2b$

c $a + 2 = b$

$a = b - 2$

d $a - 2 = b$

$a = b + 2$

e $\frac{a}{c} = b$

$a = bc$

f $\frac{a}{c} + 2 = b$

$a = c(b - 2)$

g $ac = b$

$a = \frac{b}{c}$

h $ac - 2 = b$

$a = \frac{b+2}{c}$

i $ac - d = b$

$a = \frac{b+d}{c}$

j $\frac{a+2}{c} = b$

$a = bc - 2$

k $\frac{2a}{c} = b$

$a = \frac{bc}{2}$

l $\frac{d(a+2)}{c} = b$

$a = \frac{bc}{d} - 2$

i $2a = b + a$

$a = b$

j $2a + c = b + a$

$a = b + c$

k $2a + c = b - a$

$a = \frac{b-c}{3}$

l $ae + c = b + a$

$a = \frac{b-c}{e-1}$

m $ae + c = b + af$

$a = \frac{b-c}{e-f}$

n $ae + c = b - af$

$a = \frac{b-c}{e+f}$

2 Make x the subject of these formulas.

a $5x + b = 2x + a$

$x = \frac{a-b}{3}$

b $5x + bd = 2x + ac$

$x = \frac{ac-bd}{3}$

c $bx + 5 = ax + 2$

$x = \frac{-3}{b-a}$

d $bx + 5d = ax + 2c$

$x = \frac{2c-5d}{b-a}$

e $5(x + b) = 2(x + a)$

$x = \frac{2a-5b}{3}$

f $5(x + bd) = 2(x + ac)$

$x = \frac{2ac-5bd}{3}$

g $b(x + 5) = a(x + 2)$

$x = \frac{2a-5b}{b-a}$

h $b(x + 5d) = a(x + 2c)$

$x = \frac{2ac-5bd}{b-a}$

3 Make a the subject in each of these equations.

a $a(b + 1) = c$

$a = \frac{c}{b+1}$

b $ab + a = b$

$a = \frac{b}{b+1}$

c $\frac{1}{a} + b = c$

$a = \frac{1}{c-b}$

d $\frac{a+b}{a-b} = 2$

$a = 3b$

e $\frac{1}{a} + \frac{1}{b} = 1$

$a = \frac{b}{b-1}$

f $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

$a = \frac{cb}{b-c}$

4 Rearrange these equations to make a the subject.

a $ab + ac = 3$

$a = \frac{3}{b+c}$

b $3a + ay = x$

$a = \frac{x}{3+y}$

c $a(3 + y) + 2a = x$

$a = \frac{x}{5+y}$

d $2a = x + ak$

$a = \frac{x}{2-k}$

e $ay + k = 3a$

$a = -\frac{k}{y-3}$

f $ay - x = ab$

$a = \frac{x}{y-b}$

g $ay + b = 3b + ax$

$a = \frac{2b}{y-x}$

h $m(a + y) = x(a + b)$

$a = \frac{bx-my}{m-x}$

i $\frac{y-a}{x+a} = b$

$a = \frac{y-bx}{1+b}$

j $a - n = \frac{a+2}{n}$

$a = \frac{n^2+2}{n-1}$

k $\sqrt{\frac{a+x}{a-x}} = 2$

$a = \frac{5x}{3}$

l $\sqrt{\frac{m(a+n)}{a}} = p$

$a = \frac{-mn}{m-p^2}$

5 Make x the subject.

a $\frac{ax+b}{4} = \frac{x+c}{3}$

$x = \frac{4c-3b}{3a-4}$

b $\frac{x}{a} - \frac{x}{2a} = b$

$x = 2ab$

c $\frac{ax}{b} - \frac{cx}{2} = d$

$x = \frac{2bd}{2a-bc}$

d $\frac{x-a}{b} = \frac{x}{c}$

$x = \frac{ca}{c-b}$

e $\frac{x+a}{b} = \frac{d+e}{c}$

$x = \frac{bd+be-ac}{c}$

f $\frac{x+a}{3b} + \frac{x-a}{2b} = 1$

$x = \frac{6b+a}{5}$

g $\frac{a}{x} = \frac{b}{c}$

$x = \frac{ac}{b}$

h $\frac{2a-b}{a} + \frac{a}{x} = a$

$x = \frac{a^2}{a^2-2a+b}$

i $\frac{a}{x} + b = \frac{c}{x}$

$x = -\frac{a-c}{b}$

j $\frac{ax-b}{x-b} = c$

$x = \frac{b-bc}{a-c}$

k $\frac{cx+b}{x+a} = d$

$x = \frac{ad-b}{c-d}$

l $\frac{1}{x-a} = \frac{1}{ax+b}$

$x = \frac{-a-b}{a-1}$

m $y = \frac{5x+5b}{x+a}$

$x = \frac{5b-ay}{y-5}$

n $y = \frac{5x+5b}{3x-3a}$

$x = \frac{3ay+5b}{3y-5}$

o $y - 1 = \frac{9-x}{2x-3}$

$x = \frac{6+3y}{2y-1}$

6 This is the thin lens equation used in the study of optics.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Where f is the focal length of a lens, u is distance to the image, and v is distance to the object.

a Show that $u=\frac{vf}{v-f}$

$$\frac{1}{u}=\frac{1}{f}-\frac{1}{v}$$
$$\frac{1}{u}=\frac{v-f}{fv}$$
$$u=\frac{fc}{v-f}$$

b Briefly explain why $v \neq f$

If $v=f$, then $v-f=0$, and you can't divide by zero.

c Let $x_1=u-f$ and $x_2=v-f$. Show that $x_1x_2=f^2$

$$x_1x_2=(u-f)(v-f)$$
$$=uv-fu-fv-f^2$$
Substitute $u=\frac{vf}{v-f}$
$$=\left(\frac{vf}{v-f}\right)v-f\left(\frac{vf}{v-f}\right)-fv+f^2$$
$$=\frac{v^2f}{v-f}-\frac{f^2v}{v-f}-fv+f^2$$
$$=\frac{vf(v-f)}{v-f}-fv+f^2$$
$$=vf-fv+f^2$$
$$=f^2$$