Student Name

Mathematics Stage 4

LINEAR RELATIONSHIPS

Book 2 Plot linear relationships on the Cartesian planes

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA4-LIN-C-01 creates and displays number patterns and finds graphical solutions to problems involving linear relationships

Plot linear relationships on the Cartesian plane

- Construct a geometric pattern and record the results in a table of values
- Represent a given number pattern (including decreasing patterns) using a table of values
- Describe a number pattern in words and generate an equation using algebraic symbols
- Apply an equation generated from a pattern to calculate the corresponding value for a smaller or larger number
- Recognise that a linear relationship can be represented by a number pattern, an equation (or a rule using algebraic symbols), a table of values, a set of pairs of coordinates and a line graphed on a Cartesian plane, and move flexibly between these representations
- Explain that there are an infinite number of ordered pairs that satisfy a given linear relationship by extending a line joining a set of points on the Cartesian plane
- Compare similarities and differences of multiple straight-line graphs on the same set of axes using graphing applications
- Describe linear relationships in real-life contexts and solve related problems

SUMMARY

Plot Linear Relationships on a Cartesian Plane

Linear Relationship

A relationship between two variables is linear if is coordinates form a straight-line graph.

Table of Values

Number of shapes x	1	2	3	4
Number of matches y	5	9	13	17

Equation

Number of matches = 4 × number of shapes + 1

$$y = 4x + 1$$

Coordinates

$(1,5)$ $(2,9)$ $(3,13)$ $(4,17)$

Straight-line Graph

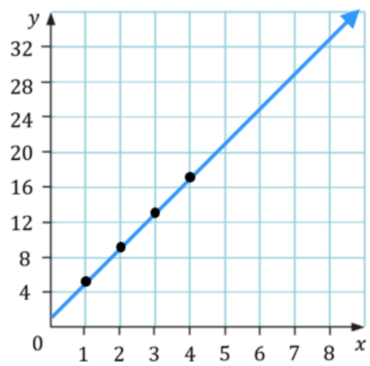


Table of Values

x	-2	-1	0	1	2
y	-7	-3	1	5	9

columns are (x, y) pairs

Coordinates

$(-2,-7)$ $(-1,-3)$ $(0,1)$ $(1,5)$ $(2,9)$

points are coordinates

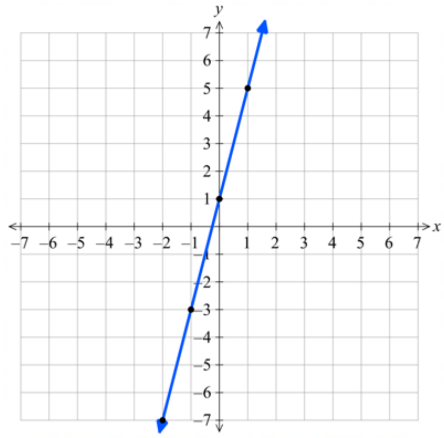
every column satisfies the equation

x	0		n	$n+1$	$n+2$	$n+3$
y	c					

$+m$ $+m$ $+m$ $+m$

substitute x values into equation to find y values

Straight Line



Linear Equation $y = mx + c$

$y = 4x + 1$

coefficient of x
number x is multiplied by
change in y as x increases by 1

constant term
number to add or subtract
value of y when $x = 0$

Problem-solving using the Equation

If we want to make 25 shapes, how many matches do we need?
Let $x = 25$

$y = 4(25) + 1$

$= 101$

We need 101 matches to make 25 pentagons.

If we have 73 matches, how many shapes can we make?
Let $y = 73$

$73 = 4x + 1$

$72 = 4x$

$18 = x$

We can make 18 pentagons with 73 matches.

Relationship between Equation and Line

m: coefficient of x
Gradient, or slope, of the line.
$m > 0$, increasing.
$m < 0$, decreasing.
If two lines have the same gradient, then they are parallel.

c: constant term
y-intercept of the line.

Equation → Table → Coordinates → Line

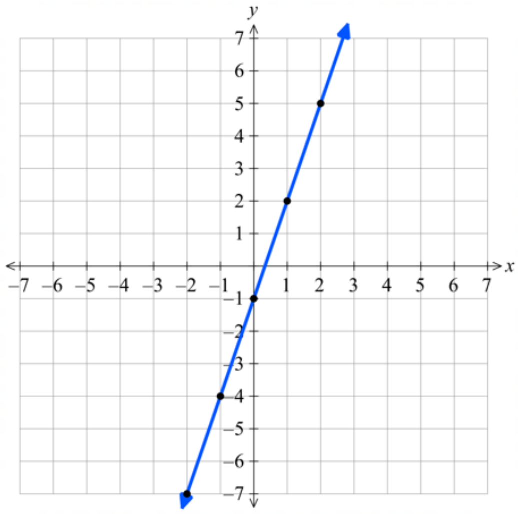
1. Substitute x values into the equation to find y values.
2. Plot the columns as points (x, y) on a Cartesian plane.
3. Draw and extend a line through the points.

$y = 3x - 1$

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

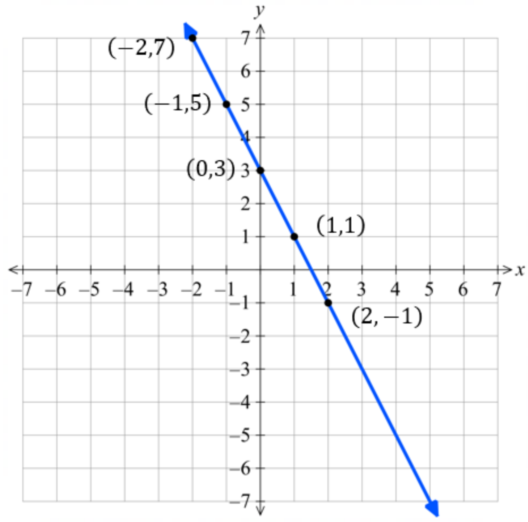
$3(-2) - 1$ $\quad$ $3(-1) - 1$ $\quad$ $3(0) - 1$ $\quad$ $3(1) - 1$ $\quad$ $3(2) - 1$

$(-2, -7)$ $(-1, -4)$ $(0, -1)$ $(1, 2)$ $(2, 3)$



Line → Coordinates → Table → Equation

1. Identify coordinates of points on the line.
2. Construct a table of values with consecutive x values.
3. Find the equation $y = mx + c$



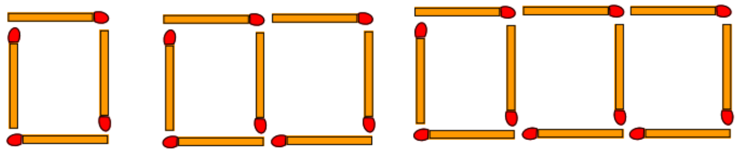
x	-2	-1	0	1	2
y	7	5	3	1	-1

-2 $\quad$ -2 $\quad$ -2 $\quad$ -2

$y = -2x + 3$

REVIEW OF PRIOR KNOWLEDGE

1 Consider the pattern. Draw the next term in the pattern and fill in the table.



Number of shapes	1	2	3	4	5
Number of matches					

2 What operations might these function machines be representing?

a $7 \rightarrow \boxed{?} \rightarrow 10$

b $7 \rightarrow \boxed{?} \rightarrow 70$

c $7 \rightarrow \boxed{?} \rightarrow 1$

d $7 \rightarrow \boxed{?} \rightarrow 21$

e $6 \rightarrow \boxed{?} \rightarrow 36$

f $6 \rightarrow \boxed{?} \rightarrow 150$

3 Find possible operations for these two-step functions. The first is multiplication.

a $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 10$

b $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 20$

c $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 3$

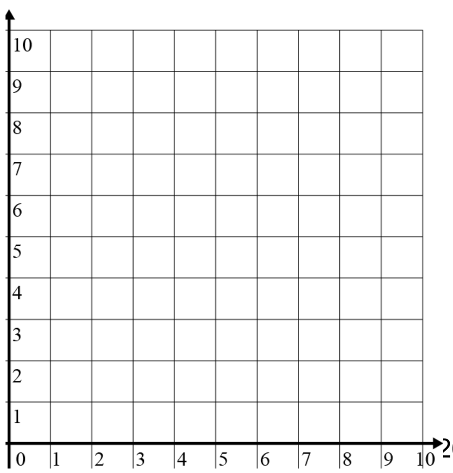
d $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 2$

e $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 0$

f $3 \rightarrow \boxed{\times\ ?} \rightarrow \boxed{?} \rightarrow 100$

4 Plot these four points on the Cartesian Plane.

x-coordinate	0	1	2	3
y-coordinate	9	6	3	0



5 Complete these tables of values.

a

x	0	1	2	3	4
$x+5$					

b

x	0	1	2	3	4
$2x-1$					

c

x	-2	-1	0	1	2
$3x+1$					

d

x	-2	-1	0	1	2
$-2x$					

e

x	-2	-1	0	1	2
$-2x+5$					

f

g

x	-2	-1	0	1	2
$-6x-3$					

6 Determine the function rule shown by the table. The rule must apply to every column.

a

input	10	11	12
output	13	14	15

$\text{input} + 3 = \text{Output}$

b

input	5	6	7
output	15	18	21

c

input	8	9	10
output	5	6	7

d

input	20	14	6
output	18	12	4

e

input	8	10	12
output	4	5	6

f

input	4	5	6
output	8	10	12

7 These rules all involve two operations. Find the rule for each table.

a

input	1	2	3	4	5
output	5	9	13	17	21

$\text{Input} \times \ldots\ldots + \ldots\ldots\ldots = \text{Output}$

b

input	4	5	6	7	8
output	5	7	9	11	13

c

input	10	8	3	1	14
output	49	39	14	4	69

d

input	6	18	30	24	66
output	3	5	7	6	13

8 Using a calculator, or otherwise, substitute $x = 8$ into these equations to find y

a $y = 3x + 7$	**b** $y = 4x - 11$	**c** $y = -4x + 5$	**d** $y = \frac{x}{2} + 2$
e $y = -3x + 1$	**f** $y = -12x - 5$	**g** $y = 3x - 1$	**h** $y = -3x + 4$

9 Solve these equations.

a $18 = 4x + 2$	**b** $22 = 3x + 7$	**c** $12 = 2x - 5$	**d** $16 = 5x + 1$
e $13 = 4x + 1$	**f** $20 = 6x - 4$	**g** $2 = 3x + 8$	**h** $-5 = -2x + 3$

PATTERNS, EQUATIONS, GRAPHS

- A **sequence** is an ordered list of items showing a pattern between two quantities.
- We can show relationships between two quantities as:
  - A table of values
  - An equation
  - Coordinates
- In a **linear** sequence, the coordinates form a **straight-line**.

Example Express a spatial sequence as a table, equation, and graph.

Consider this pattern of pentagons made from matches:

a Complete the table of values.

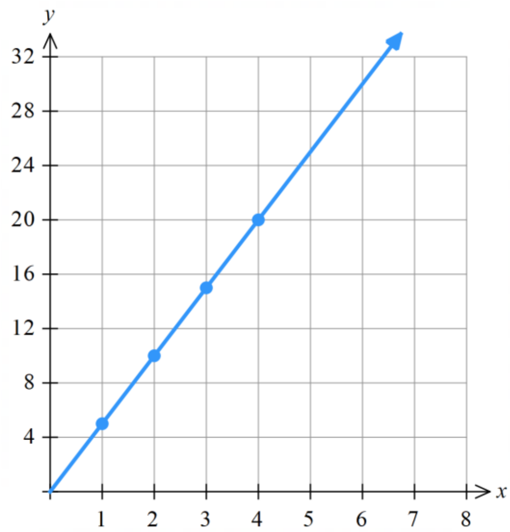
Number of shapes x	1	2	3	4
Number of matches y	5	10	15	20

b Determine an equation that relates the two quantities.

Number of matches = 5 × number of shapes.

$$y = 5x$$

c Graph the relationship on the Cartesian Plane.



Why do we use variables x and y instead of writing "number of matches" and "number of shapes"?

It is much ……………… to write x and y

How do we determine the equation from the table of values?

The equation describes what ………………… to do to x to get y, and must work for every ……………… of the table.

Why is it called a "linear" relationship?

When plotted on a Cartesian plane, the points form a …………………

1

a Complete the table of values.

Number of shapes x	1	2	3
Number of matches y			

4, 8, 12

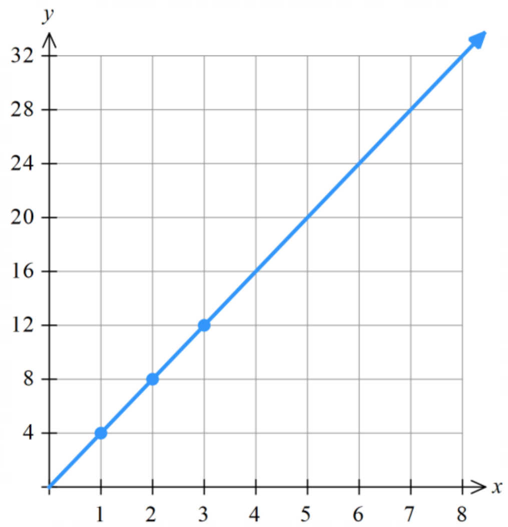
b Determine an equation that relates the two quantities.

Number of matches = ...

y = ...

$y = 4x$

c Graph the relationship on the Cartesian Plane.



2

Table:

x	1	2	3
y			

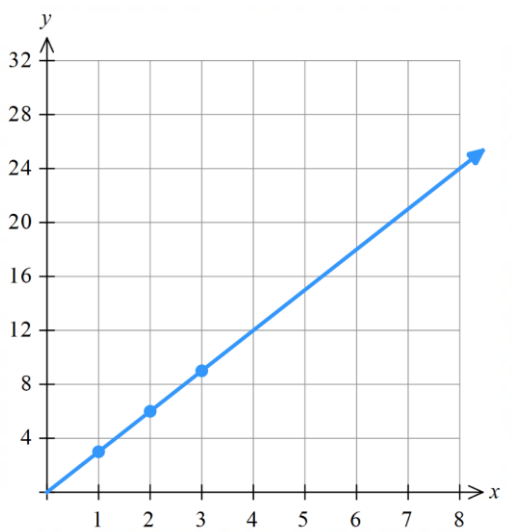
3, 6, 9

Equation:

....

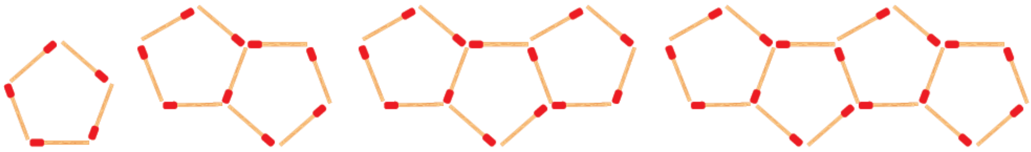
$y = 3x$

Graph:



Example Express a spatial sequence as an equation and graph.

Consider this pattern of pentagons made from matches:



a Complete the table of values.

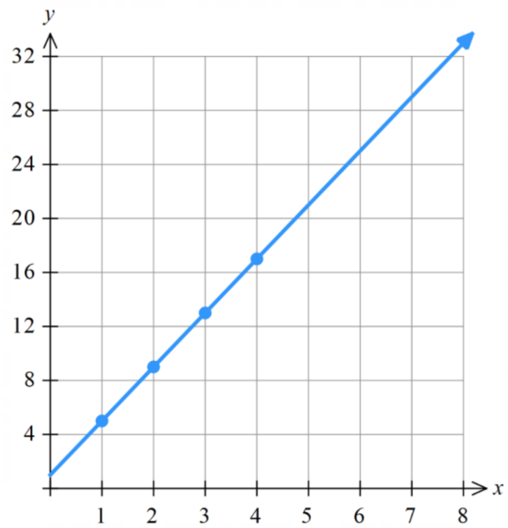
Number of shapes x	1	2	3	4
Number of matches y	5	9	13	17

b Determine an equation that relates the two quantities.

Number of matches = 4 × number of shapes + 1

$$y = 4x + 1$$

c Graph the relationship on the Cartesian Plane



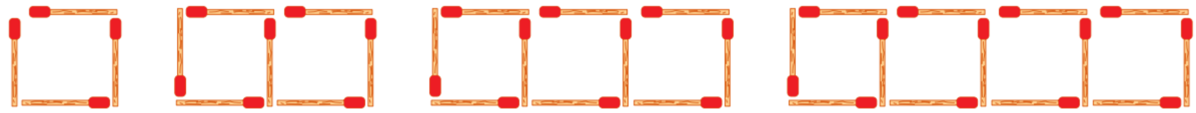
How do we determine the equation from the table of values?

The equation describes what ………………… to do to x to get y, and must work for every ……………… of the table.

What is different about this pattern, compared with the previous ones?

The equation involves ………… operations, so the equation has the form $y =$ ……………

1



a Complete the table of values.

Number of shapes x	1	2	3	4
Number of matches y				

4, 7, 10 ,13, 16

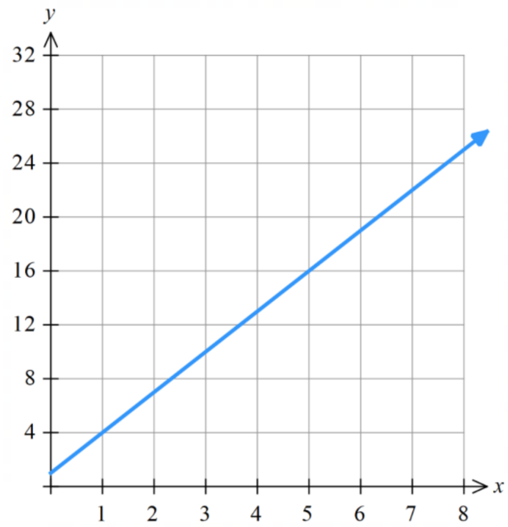
b Determine an equation that relates the two quantities.

Number of matches = ...

$y =$ ....

$y = 3x + 1$

c Graph the relationship on the Cartesian Plane.



2

Table:

x	1	2	3	4	5
y					

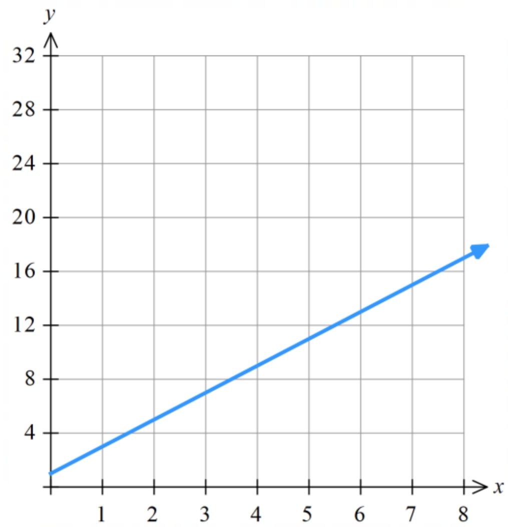
3, 5, 7, 9 , 11

Equation:

....

$y = 2x + 1$

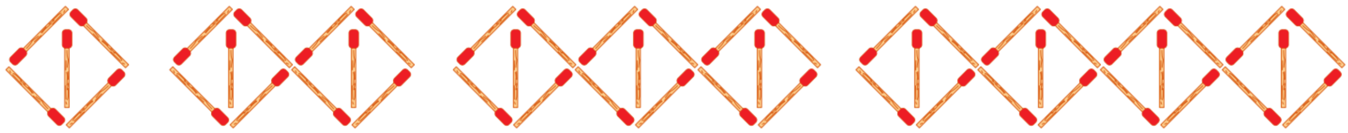
Graph:



Key ideas

1. A spatial sequence is a list of shapes that change according to some ....
2. A relationship shown by a sequence can be described using an ....
3. To find the equation, we look at the .... of a table of values.

1



a Complete the table of values.

Number of shapes x	1	2	3	4
Number of matches y				

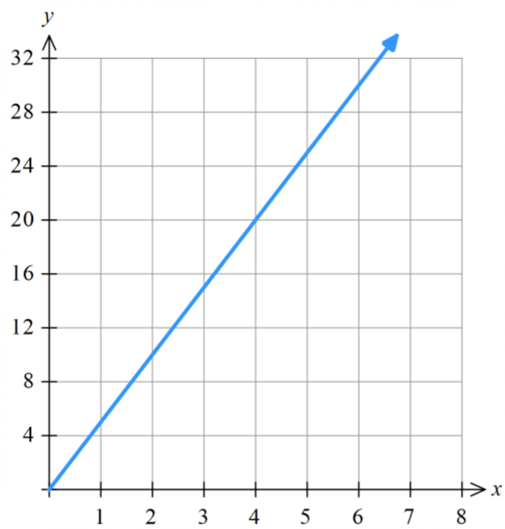
5, 10, 15, 20

b Determine an equation that relates the two quantities.

Number of matches = ...

$y =$ ....

c Graph the relationship on the Cartesian Plane.



$y = 5x$

2

Table:

x	1	2	3
y			

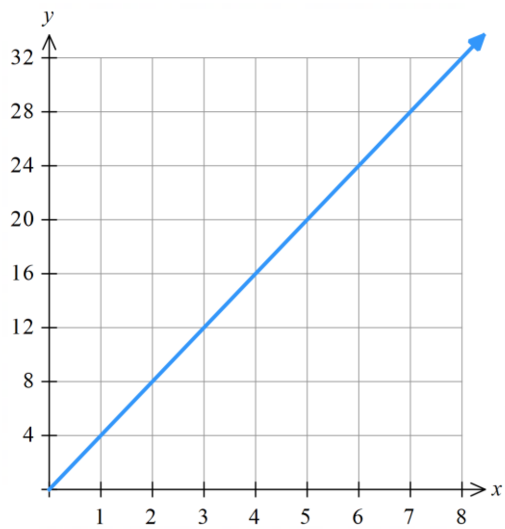
$4, 8, 12$

Equation:

....

$y = 4x$

Graph:



3

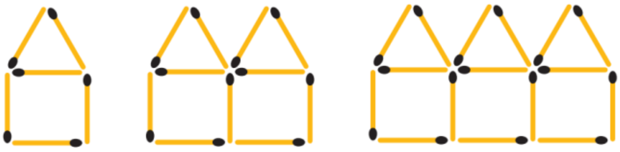


Table:

x	1	2	3
y			

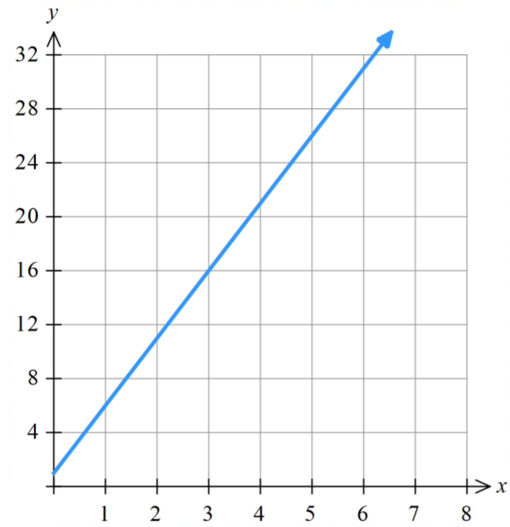
6, 11, 16

Equation:

....

$y = 5x + 1$

Graph:



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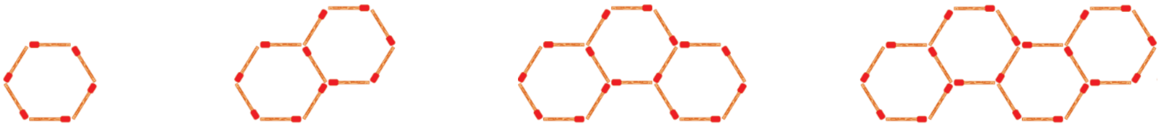


Table:

x	1	2	3	4
y				

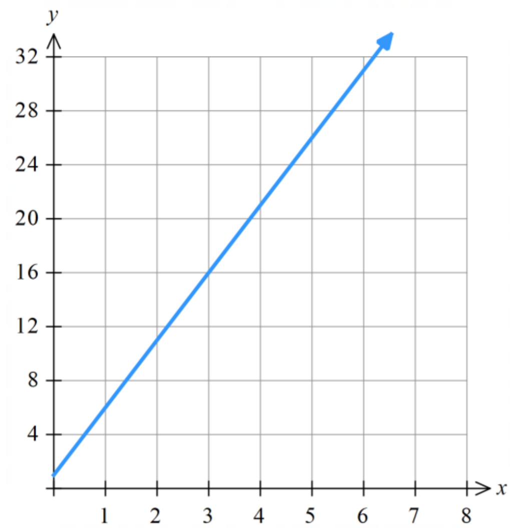
6, 11, 16, 21

Equation:

....

$y = 5x + 1$

Graph:



5 What do you notice about the equation of Q3 and Q4?

....

They are the same.

6

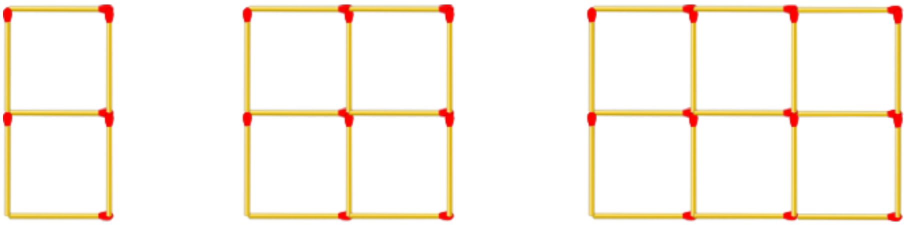


Table:

x	1	2	3
y			

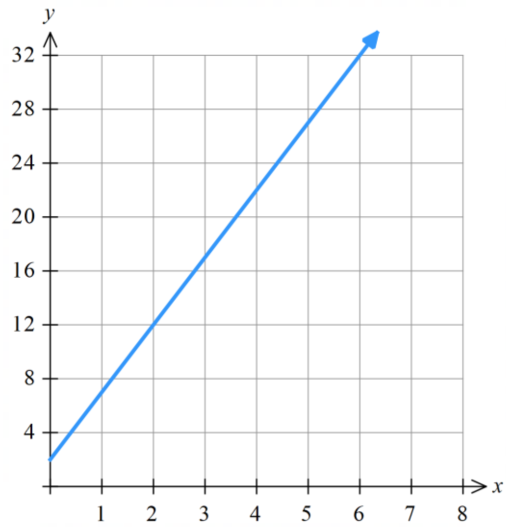
7, 12, 17

Equation:

....

$y = 5x + 2$

Graph:



7

Table:

x	1	2	3
y			

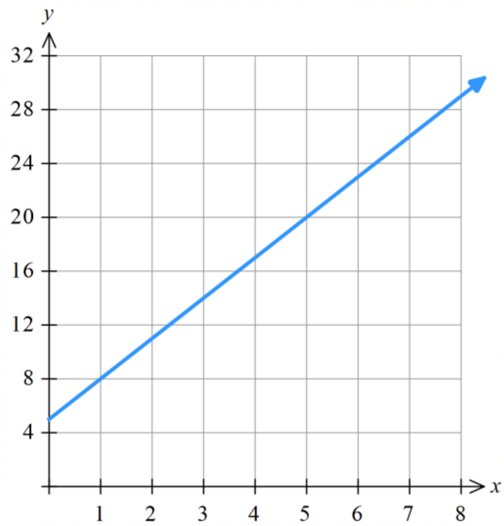
8, 11, 14

Equation:

....

$y = 3x + 5$

Graph:



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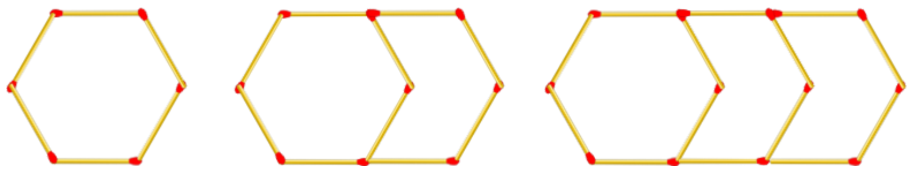


Table:

x	1	2	3
y			

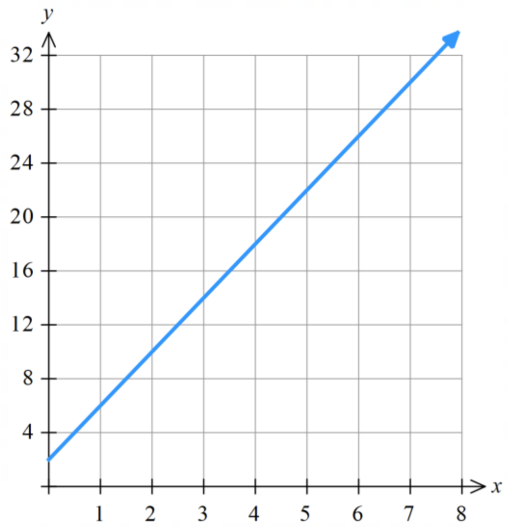
6, 10, 14

Equation:

……………………………………………………………

$y = 4x + 2$

Graph:



9

Table:

x	1	2	3
y			

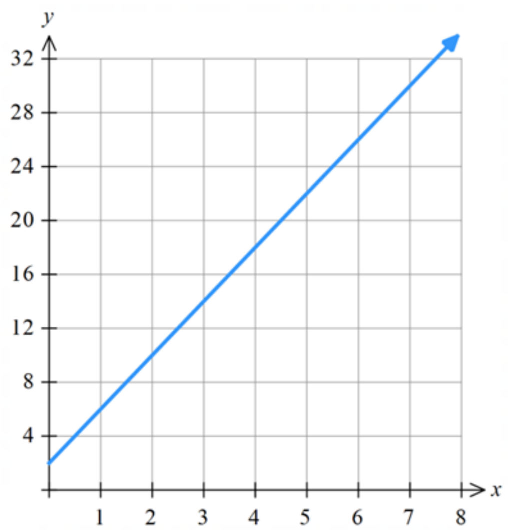
6, 10, 14

Equation:

……………………………………………………………

$y = 4x + 2$

Graph:



10 x is “number of tables”
y is “number of chairs”

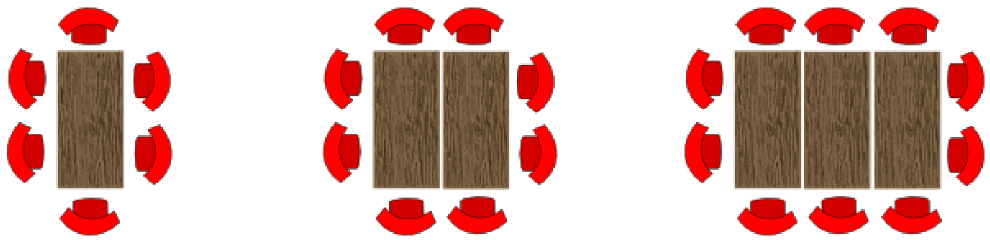


Table:

x	1	2	3
y			

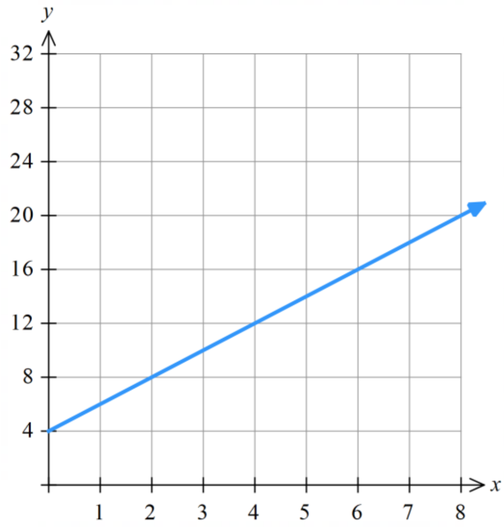
Equation:

....

6, 8, 10

$y = 2x + 4$

Graph:



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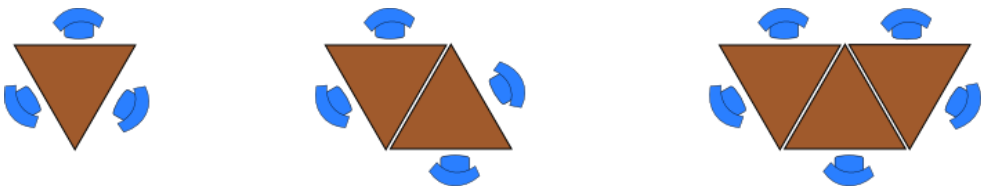


Table:

x	1	2	3
y			

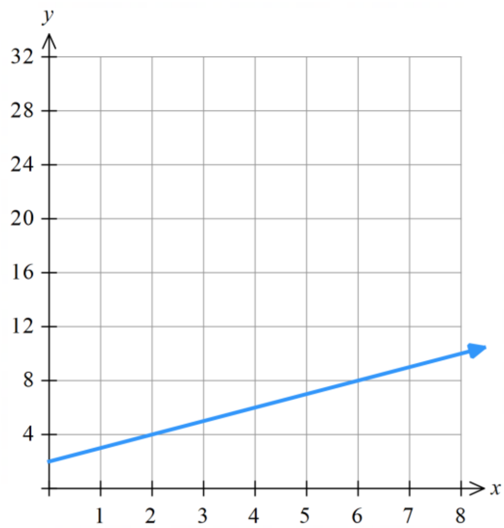
Equation:

....

3, 4, 5

$y = x + 2$

Graph:



CONSTRUCTING A TABLE OF VALUES

- An equation shows the relationship between x and y in a table of values.
  1. To find a y value, substitute an x value into the equation.

Example Construct a table of values.

$y = x - 2$

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0
Working out	$(-2) - 1 =$	$(-1) - 1 =$	$(0) - 1 =$	$(1) - 1 =$	$(2) - 1 =$

$y = 3x + 1$

x	-2	-1	0	1	2
y	-5	-2	1	4	7
Working out	$3(-2) + 1 =$	$3(-1) + 1 =$	$3(0) + 1 =$	$3(1) + 1 =$	$3(2) + 1 =$

Complete these tables of values.

a $y = 2x$

x	-2	-1	0	1	2
y					

$-4, -2, 0, 2, 4$

b $y = x - 3$

x	-2	-1	0	1	2
y					

$-5, -4, -3, -2, -1$

c $y = 2x + 4$

x	-2	-1	0	1	2
y					

$0, 2, 4, 6, 8$

d $y = 3x - 1$

x	-2	-1	0	1	2
y					

$-7, -4, -1, 2, 5$

e $y = -3x$

x	-2	-1	0	1	2
y					

$6, 3, 0, -3, -6$

f $y = 5 - x$

x	-2	-1	0	1	2
y					

$7, 6, 5, 4, 3$

Key ideas

1. If we have an ……………, we can show the relationship between x and y using a table of values by ……………… an x value into the equation to find the y value.

1 Complete these tables of values.

a $y = x + 2$

x	0	1	2	3	4
y					
	$0+2$	$1+2$	$2+2$	$3+2$	$4+2$

2, 3, 4, 5, 6

b $y = 4x$

x	0	1	2	3	4
y					
	4×0	4×1	4×2	4×3	4×4

0, 4, 8, 12, 16

c $y = 3x + 6$

x	0	1	2	3	4
y					
	$3\times 0+6$	$3\times 1+6$	$3\times 2+6$	$3\times 3+6$	$3\times 4+6$

6, 9, 12, 15, 18

d $y = 8 - 2x$

x	0	1	2	3	4
y					
	$8-2\times 0$	$8-2\times 1$	$8-2\times 2$	$8-2\times 3$	$8-2\times 4$

8, 6, 4, 2, 0

e $y = \frac{1}{2}x + 7$

x	0	1	2	3	4
y					
	$\frac{1}{2}\times 0+7$	$\frac{1}{2}\times 1+7$	$\frac{1}{2}\times 2+7$	$\frac{1}{2}\times 3+7$	$\frac{1}{2}\times 4+7$

7, 7.5, 8, 8.5, 9

2 Complete these tables of values. There is no need to write the substitution.

a $y = x + 2$

x	0	1	2	3
y				

2, 3, 4, 5

b $y = 2x$

x	0	1	2	3
y				

0, 2, 4, 6

c $y = x - 2$

x	0	1	2	3
y				

–2, –1, 0, 1

d $y = 2x + 4$

x	0	1	2	3
y				

4, 6, 8, 10

e $y = \frac{x}{2}$

x	0	1	2	3
y				

$0, \frac{1}{2}, 1, \frac{3}{2}$

f $y = \frac{x}{2} - 1$

x	0	1	2	3
y				

$-1, -\frac{1}{2}, 0, \frac{1}{2}$

g $y = 3x$

x	−2	−1	0	1	2
y					

–6, –3, 0, 3, 6

h $y = 3x - 2$

x	−2	−1	0	1	2
y					

–8, –5, –2, 1, 4

i $y = -2x$

x	−2	−1	0	1	2
y					

4, 2, 0, –2, –4

j $y = -2x + 5$

x	−2	−1	0	1	2
y					

9, 7, 5, 3, 1

SKETCH A LINE FROM A TABLE OF VALUES

- We can show each $\boldsymbol{x}$ $\boldsymbol{y}$ **pair** from a table of values as **coordinates**.
  1. Construct a table of values by substituting x values into the equation to find y.
  2. Plot the (x, y) points on a Cartesian plane.
  3. Draw a straight line through the points and extend it.

Example Graph a linear relationship using a table of values.

Draw the graph of the equation $y = 5 - 2x$

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{y}$	5	3	1	−1

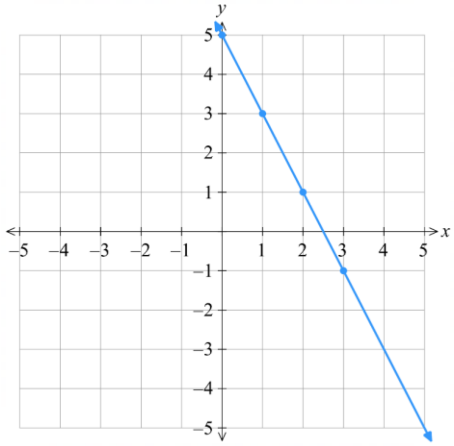
Coordinates:

(0, 5)

(1, 3)

(2, 1)

(3, −1)



How do we get coordinates from a table of values?

We look at the ………………… of the table of values to get the x and y values.

How many coordinates do we need to draw a line?

We need at least ……….., but 3 or more is often useful to see if we've made a mistake.

Draw the graph of:

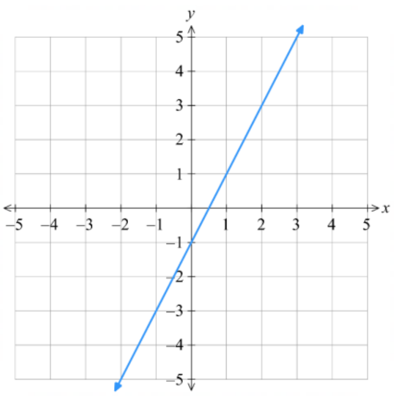
a $y = 2x - 1$

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{y}$				

Coordinates:

……………………………………………………

……………………………………………………

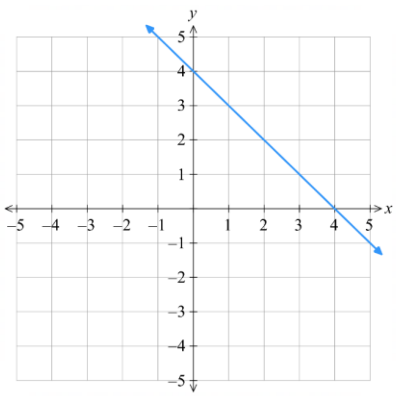


b $y = 4 - x$

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{y}$				

……………………………………………………

……………………………………………………



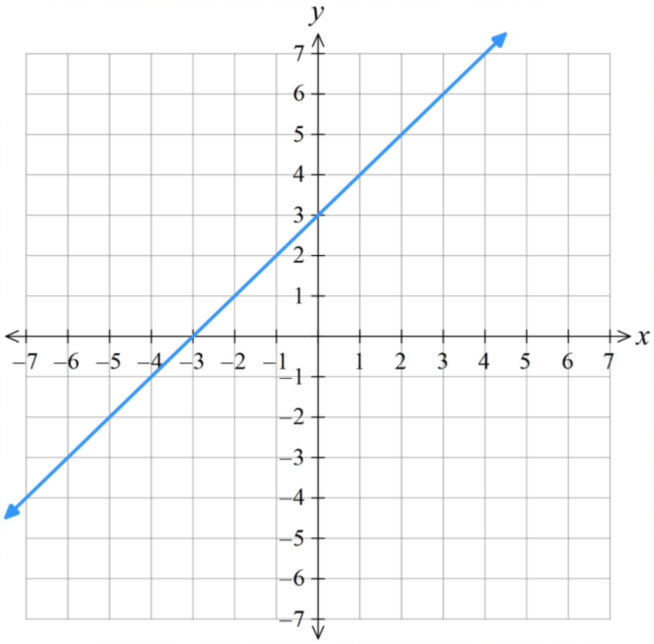
Key ideas

1. If we have a table of values, then we can represent each x y pair as a …………………

1 Complete the table of values and graph the result.

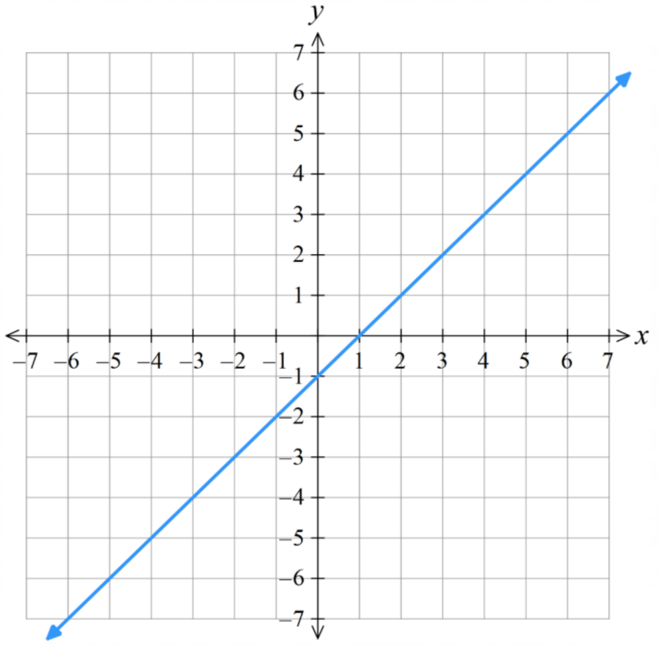
a $y = x + 3$

x	-2	-1	0	1	2
y					



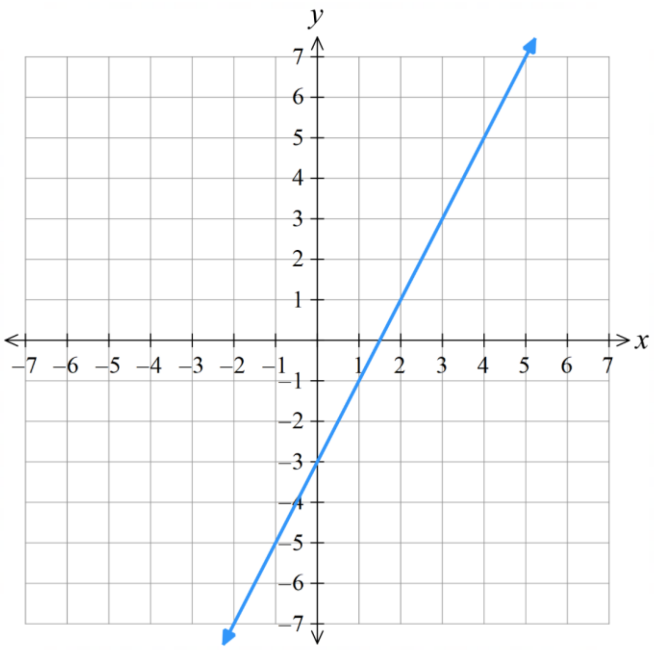
b $y = x - 1$

x	-2	-1	0	1	2
y					



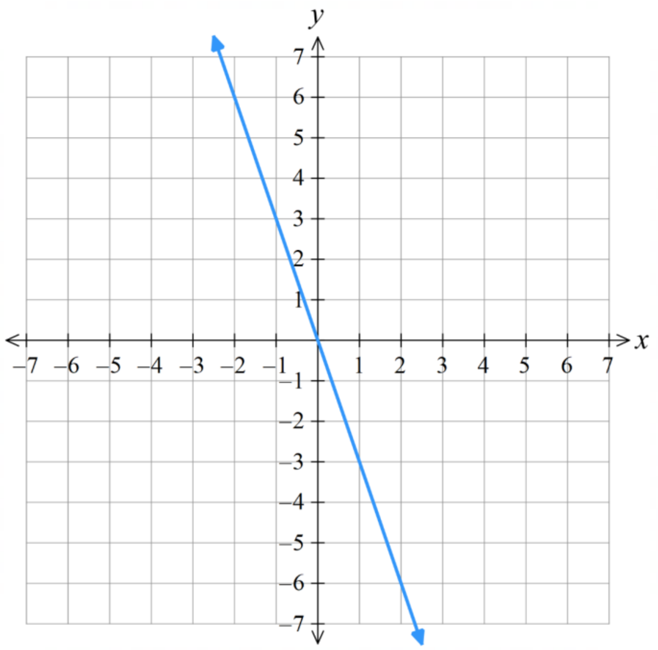
c $y = 2x - 3$

x	-2	-1	0	1	2
y					



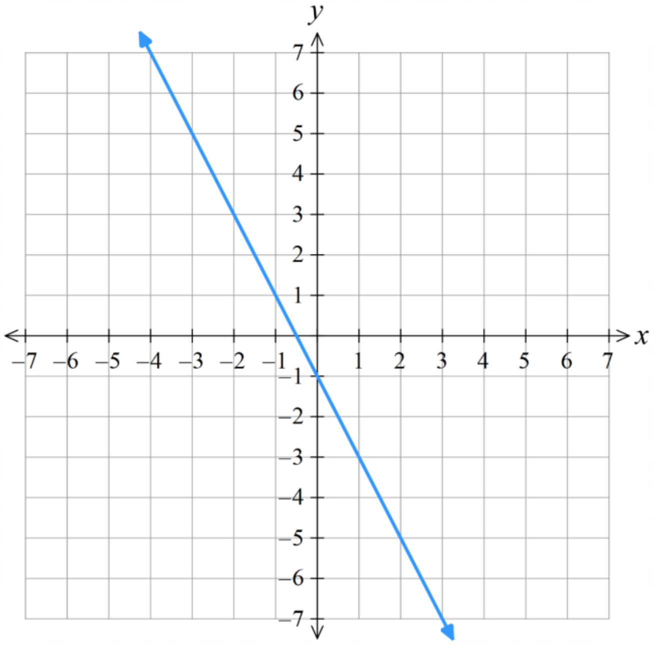
d $y = -3x$

x	-2	-1	0	1	2
y					



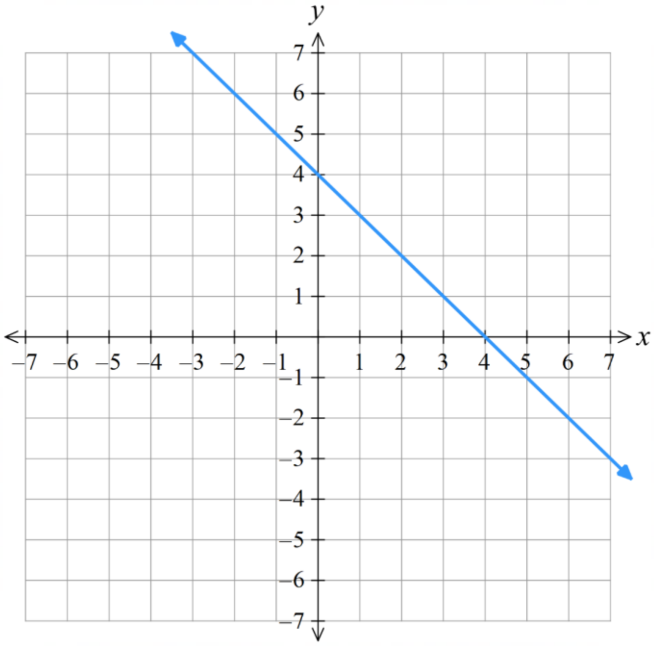
e $y = -2x - 1$

x	-2	-1	0	1	2
y					



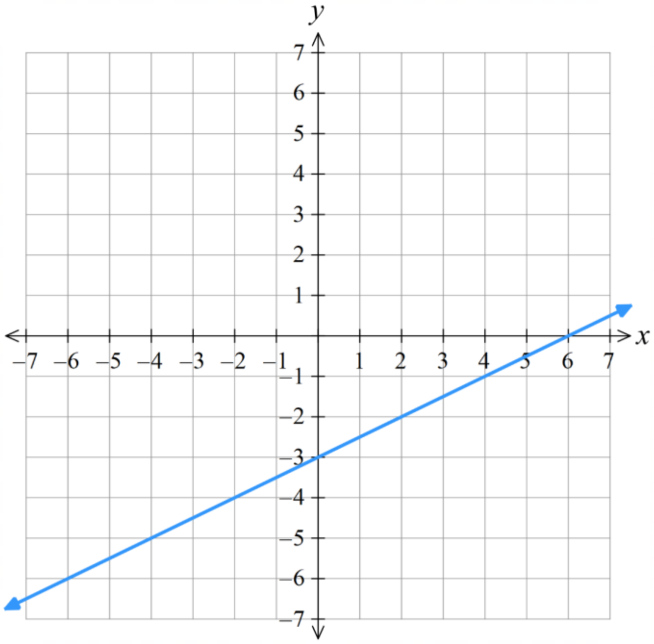
f $y = -x + 4$

x	-2	-1	0	1	2
y					



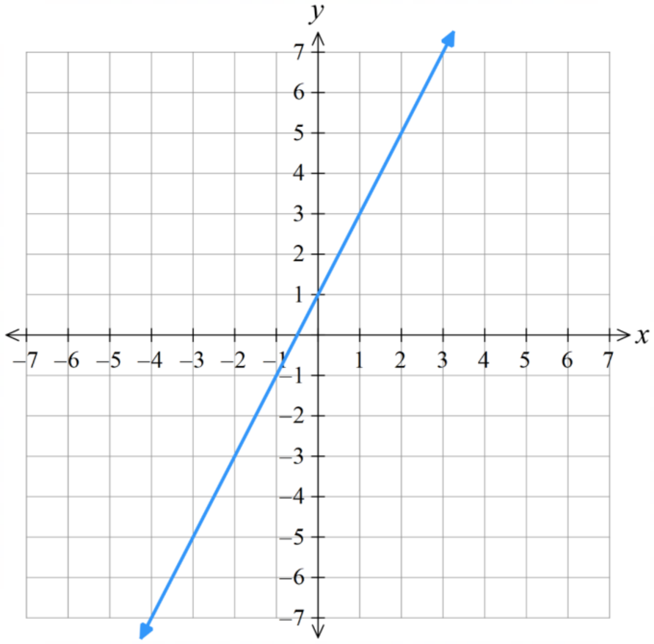
g $y = 0.5x - 3$

x	-2	-1	0	1	2
y					



h $y = 1 + 2x$

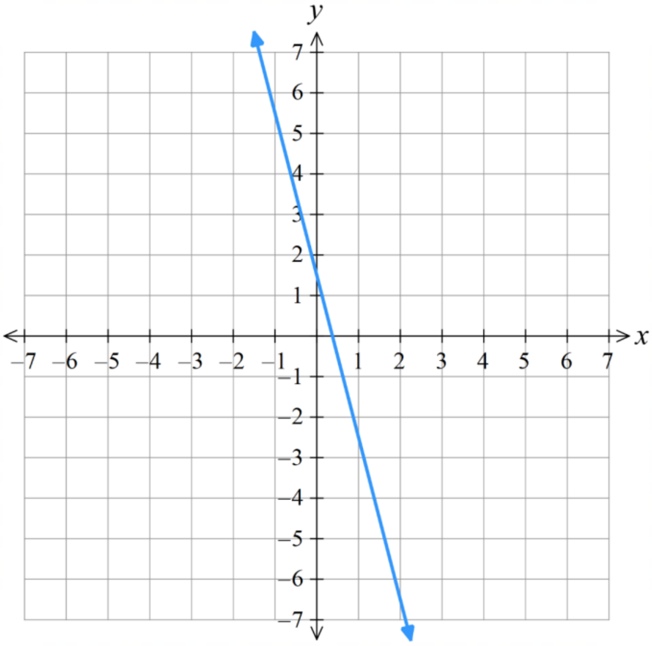
x	-2	-1	0	1	2
y					



2 Complete the table of values and graph the result.

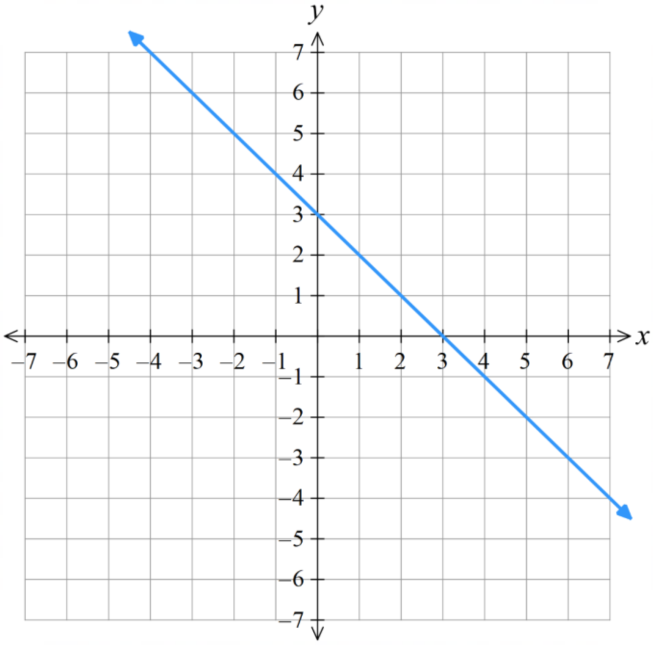
a $y = 1.5 - 4x$

x	-2	-1	0	1	2
y					



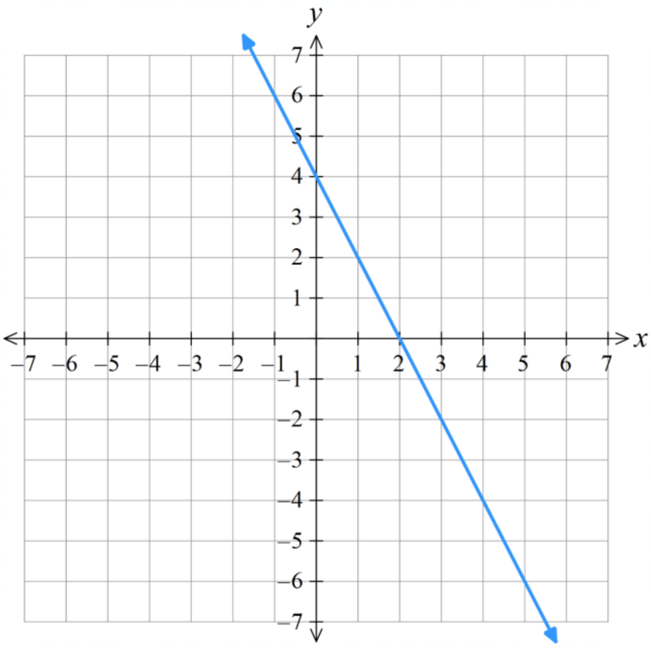
b $y = 3 - x$

x	-2	-1	0	1	2
y					



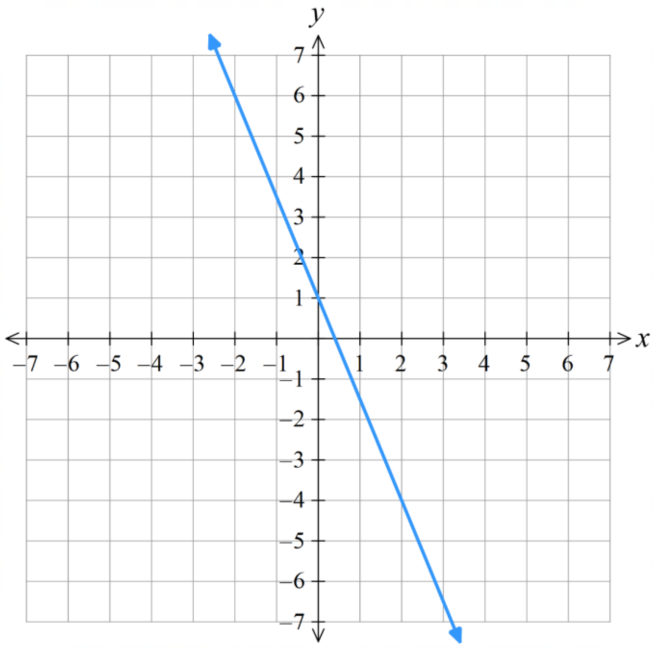
c $y = 4 - 2x$

x	-2	-1	0	1	2
y					



d $y = 1 - \frac{5}{2}x$

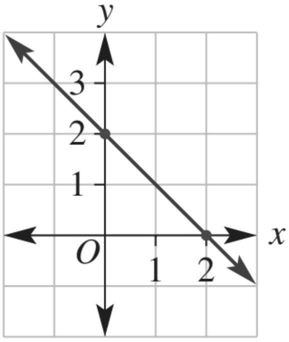
x	-2	-1	0	1	2
y					



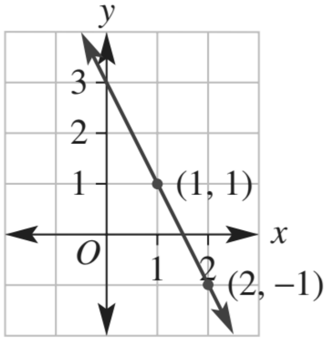
3 Match the rules A-D with the graphs a-d.

A $y = 2x + 1$ **B** $y = -x - 1$ **C** $y = -2x + 3$ **D** $y = -x + 2$

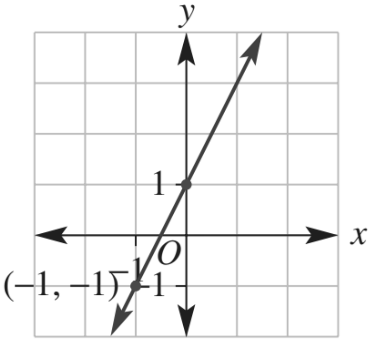
a



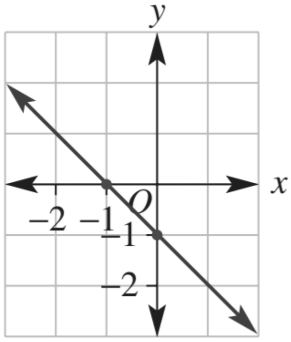
b



c



d



A – c, B – d, C – b, D – a

FINDING THE RULE $y = x + c$ OR $y = mx$

- We can represent x y pairs in a table of values as an equation linking x and y.
- The equation must work for every column of the table.

Example Find the rule from a table of values.

<table><tr><td>x</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td></tr><tr><td>y</td><td>12</td><td>13</td><td>14</td><td>15</td><td>19</td></tr></table> Every y value is 9 more than the x value. $y = x + 9$	Looking at the first column, you could also multiply x by 4 to get y. Why is the equation $y = x + 9$? *The equation must work for every .... of the table.*
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td></tr><tr><td>y</td><td>7</td><td>14</td><td>21</td><td>28</td><td>70</td></tr></table> Every y value is 7 times the x value. $y = 7x$	Complete the sentence: *The equation describes what .... to do to x to get y, and must work for every .... of the table.*

a Determine the rule shown in the table, then complete the table.

x	2	5	6	1	8
y	6	9	10		

$y =$ ....

$y = x + 4$ | 5, 12

b

x	12	5	8	0	23
y	36	15	24		

...... = ...

$y = 3x$ | 0, 69

c

x	2	3	10	7	12
y	3	5	19		

....

$y = 2x - 1$ | 13, 23

1 Determine the rule shown in the table. Every equation involves a single operation only.

a

x	5	6	7
y	15	18	21

$y = 3x$

b

x	10	11	12
y	13	14	15

$y = x + 3$

c

x	8	9	10
y	5	6	7

$y = x - 3$

d

x	20	14	6
y	18	12	4

$y = 24 - x$

e

x	8	10	12
y	4	5	6

$y = \frac{x}{2}$

f

x	4	5	6
y	8	10	12

$y = 2x$

g

x	4	3	2
y	6	5	4

$y = 10 - x$

h

x	20	14	6
y	15	9	1

$y = 21 - x$

i

x	8	10	12
y	13	15	17

$y = x + 5$

j

x	1	2	3	4	5
y	4	8	12	16	20

$y = 4x$

k

x	10	8	3	1	14
y	21	19	14	12	25

$y = x + 11$

l

x	6	18	30	24	66
y	1	3	5	4	11

$y = \frac{x}{6}$

2 Describe the relationship between the x and y values of each point on the line.

a $y = x + 1$ The y value is one more than the x value.

b $y = x + 2$ The y value is .... the x value.

Two more than

c $y = x - 1$ The y value is .... the x value.

1 less than

d $y = x - 4$ The y value is .... the x value.

4 less than

e $y = x + 4$ The y value is .... the x value.

Four more than

f $y = 2x$ The y value is two times the x value.

g $y = 3x$ The y value is .... the x value.

Three times

h $y = -3x$ The y value is .... the x value.

Negative three times

i $y = \frac{x}{2}$ The y value is .... the x value.

One half

j $y = -\frac{x}{2}$ The y value is .... the x value.

Negative one half

3 Determine the equation of the line shown by these tables of values, then complete the table.

a

x	2	5	6	1	8
y	6	9	10		

y = x + 4
Missing values: 5, 12

b

x	12	5	8	0	12
y	36	15	24		

y = 3x
Missing values: 0, 36

c

x	4	3	5	1	0
y	-8	-6	-10		

y = –2x
Missing values: –2, 0

d

x	6	18	30	24	66
y	1	3	5		

$y = \frac{x}{6}$
Missing values: 4, 11

e

x	8	10	12	20	-20
y	4	5	6		

$y = \frac{x}{2}$
Missing values: 10, –10

f

x	20	14	6	15	1
y	15	9	1		

y = x – 6
Missing values: 9, –5

4 First figure out the rule, then complete the rest of the table.

a

x	4	10	13	24			5	11
y			39		9	42	15	

$y = 3x$
12, 30, 72, 3, 14, 33

b

x	12	93	14	17		10		
y	3			8	12	1	34	0

$y = x - 9$
$84, 5, 21, 43, 9$

5

a In which of these coordinate pairs is the y value twice the x value?

(2, 4) (4, 2) (6, –3) (3, –6) (–3, –6)

(0, 0) (19, 38) (–12, –6) (50, 25) (–20, –40)

(2,4), (–3, –6), (0,0), (19,38), (–20, –40)

b The points you identified in part **a** lie on what line? Give the equation.

$y = 2x$

c Will (8, 16) lie on $y = 2x$?

Yes

d Will (9, 20) lie on $y = 2x$?

No

6 All of these points lie on the same line. (1, 10) (3, 30) (-10,-100) (-5,-50)
Complete this sentence:

a In each pair of coordinates, the y value is ………………… the x value.

10 times

b Write the equation of the straight line that passes through all four points.

$y = 10x$

7 Here are some pairs of coordinates.

(4, 19)	(19, 4)	(0, –15)	(15, 0)	(–15, 0)
(23, 38)	(3.8, 2.3)	(100, 85)	(–17, –2)	(12, –3)

a Which of the points lie on the line $y = x + 15$?

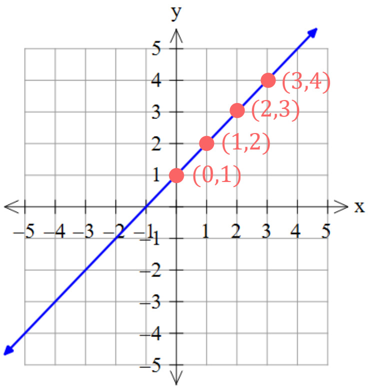
(4, 19), (–15, 0), (23, 38), (–17, –2)

b Which of the points lie on the line $y = x - 15$?

(19, 4), (0,–15), (15, 0), (100, 85), (12, –3)

8 These three straight line graphs have equations of the form $y = x + c$ or $y = mx$

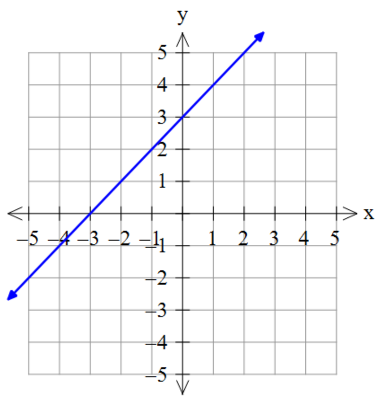
a



At each point on the line, the y-value is 1 more than the x-value. The equation of the line is ………………

$y = x + 1$

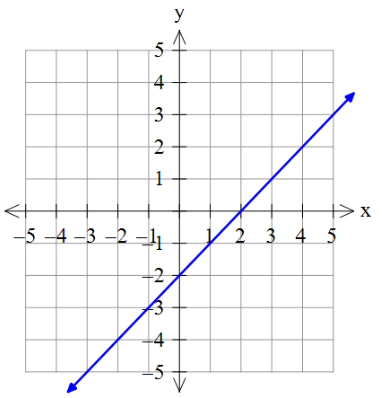
b



At each point on the line, the y-value is ………………… the x-value. The equation of the line is ………………

Three more than; $y = x + 3$

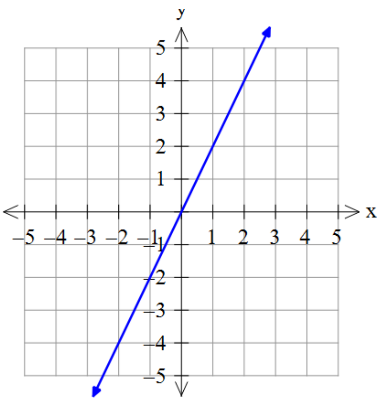
c



At each point on the line, the y-value is ………………… the x-value. The equation of the line is ………………

2 less than, $y = x - 2$

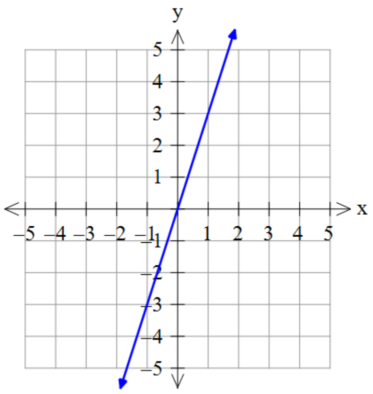
d



At each point on the line, the y-value is 2 times the x-value. The equation of the line is ………………

$y = 2x$

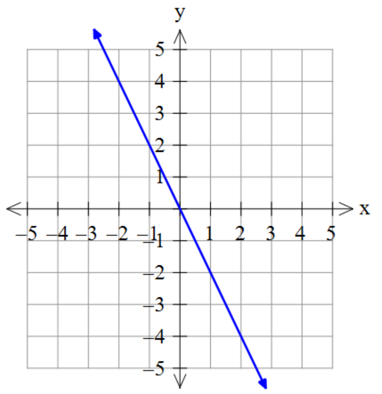
e



At each point on the line, the y-value is ………………… the x-value. The equation of the line is ………………

Three times, $y = 3x$

f



At each point on the line, the y-value is ………………… the x-value. The equation of the line is ………………

Negative two times, $y = -2x$

FINDING THE RULE $y = mx + c$

- A **linear relationship** has an equation in the form $y = mx + c$.

$$y = mx + c$$

coefficient of x
number x is multiplied by
change in y as x increases by 1

constant term
number to add or subtract
value of y when $x = 0$

x	0		n	$n+1$	$n+2$	$n+3$
y	c					

$+m$ $+m$ $+m$ $+m$

Example Identify m from a table of values.

a

x	2	3	4
y	8	12	16

$+4$ $+4$

$m =$ ....

b

x	5	6	7
y	30	36	35

$m =$ ....

c

x	9	10	11
y	20	18	16

$m =$ ....

d

x	201	202	203
y	18	15	12

$m =$ ....

e

x	1	2	3
y	905	910	915

$m =$ ....

f

x	7	2	11
y	25	15	33

Why can't we find m?

Example Identify c from a table of values.

a

x	0	1	2
y	8	9	10

$c =$ ....

b

x	0	4	18
y	11	3	-25

$c =$ ....

c

x	-2	0	15
y	15	23	83

$c =$ ....

d

x	0	22	8
y	-5	127	43

$c =$ ....

e

x	5	-3	0
y	17	-15	-3

$c =$ ....

f

x	-7	14	8
y	51	-96	-54

Why cant we find c?

- If we have a table where the value of x increases by 1, and the value of y when $x = 0$, then we can use this method to find the equation $y = mx + c$:
  1. Find $\boldsymbol{m}$: This is how much y changes by as x increases by 1.
  2. Find $\boldsymbol{c}$: This is the value of y when $x = 0$.
  3. Write the equation $y = mx + c$ and check it works for every column.

Example Express a table of values as an equation.

Determine the rule that relates x and y

a

x	0	1	2	3	4	5
y	1	3	5	7	9	11

$+2$ $+2$ $+2$ $+2$ $+2$

$m = 2,\ c = 1$

$y = 2x + 1$

Every linear relationship has the form $y = mx + c$.

What does m represent?

m is the change in … as x increases by ….

What does c represent?

c is the value of … when $x =$ ….

b

x	0	1	2	3	4	5
y	7	4	1	-2	-5	-8

-3 -3 -3 -3 -3

$m = -3,\ c = 7$

$y = -3x + 7$

In this example, why is m a negative number?

As x increases by 1, the y value is …………… by 3.

c

x	-2	-1	0	1	2	3
y	-10	-6	-2	2	6	10

$+4$ $+4$ $+4$ $+4$ $+4$

$m = 4,\ c = -10$

$y = 4x - 2$

In this example, why is $c = -2$?

c is the value of y when $x =$ ……, not always the leftmost value on the table.

Determine the equation that relates x and y, then double check it works for each column.

a

x	0	1	2	3	4	10
y	2	6	10	14	18	42

$+4$ $+4$ $+4$ $+4$ $+4$

$m =$ ……….. $c =$ ………..

$y =$ ………..

$y = 4x + 2$

b

x	0	1	2	3	4	10
y	0	-2	-4	-6	-8	-20

$m =$ ……….. $c =$ ………..

$y =$ ………..

$y = -2x$

c

x	0	1	2	3	4	10
y	3	1	-1	-3	-5	-17

$m =$ ……….. $c =$ ………..

$\ldots =$ ………..

$y = -2x + 3$

d

x	-1	0	1	2	3	10
y	-2	1	4	7	10	31

$m =$ ……….. $c =$ ………..

$\ldots =$ ………..

$y = 3x + 1$

e

x	0	1	2	3	4	10
y	-6	-3	0	3	6	24

$y = 3x - 6$

f

x	-2	-1	0	1	2	3
y	-13	-8	-3	2	7	12

$y = 5x - 3$

Key ideas

1. There are two methods for finding the rule from a table of values.
   a. Guess and check the ……………… done to x to get y.
   b. Recognise that every linear relationship is in the form $y = mx + c$
      i. m, called the …………………… of x,
         is the change in y as x …………………… by ………
      ii. c, called the ……………… term, is the value of y when $x =$ ……
2. The second method is not as useful when x is not increasing by ……,
   or if $x =$ …… is not on the table.

1 Determine the change in y as x increases by 1. This is m. Find the y-value at $x = 0$; this is c. Write the equation linking x and y with this information.

a

x	0	1	2	3	4
y	5	7	9	11	13

+2 +2 +2 +2

$m = 2$ $c = 5$

$y = 2x + 5$

b

x	0	1	2	3	4
y	-2	1	4	7	10

$m = \ldots\ldots$ $c = \ldots\ldots$

$y = \ldots\ldots x + \ldots\ldots$

$y = 3x - 2$

c

x	0	1	2	3	4
y	0	5	10	15	20

$m = \ldots\ldots$ $c = \ldots\ldots$

$y = \ldots\ldots x + \ldots\ldots$

$y = 5x$

d

x	0	1	2	3	4
y	-9	-10	-11	-12	-13

$m = \ldots\ldots$ $c = \ldots\ldots$

$y = \ldots\ldots\ldots\ldots\ldots$

$y = -x - 9$

e

x	-2	-1	0	1	2
y	5	6	7	8	9

$y = 4x + 7$

f

x	-2	-1	0	1	2
y	-2	-1	0	1	2

$y = x$

2 Find the equation shown in these tables.

a

x	-2	-1	0	1	2
y	0	2	4	6	8

$y = 2x + 4$

b

x	-3	-2	-1	0	1
y	4	3	2	1	0

$y = -x + 1$

c

x	-1	0	1	2	3
y	8	6	4	2	0

$y = -2x + 6$

d

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

$y = 3x - 1$

3 Find the equation for each table, then complete the table.

a

x	0	1	2	3	5	8
y	3	6	9	12		

Equation: ....

y = 3x + 3
Missing values: 18, 27

b

x	0	1	2	3	4	7
y	1	4	7	10		

Equation: ....

y = 3x + 1
Missing values: 13, 22

c

x	0	1	2	3	6	-2
y	5	8	11	14		

Equation: ....

y = 3x + 5
Missing values: 23, –1

d

x	0	1	2	3	40	-3
y	0	0.5	1	1.5		

Equation: ....

y = x/2
Missing values: 20, –1.5

e

x	0	1	2	3	23	-7
y	-1	-3	-5			

Equation: ....

y = –2x – 1
Missing values: –7, –47, 13

f

x	-1	0	1	2	8	22
y		13	8	3		

Equation: ....

y = –5x + 13
Missing values: 18, –27, –97

g

x	-1	0	1	2	9	-4
y		20	10	0		

Equation: ....

y = –10x + 20
Missing values: 30, –70, 60

h

x	0	1	2	3	12	10
y	-0.5	-1	-1.5			

Equation: ....

$y = -\frac{x}{2} - \frac{1}{2}$
Missing values: –2, –6.5, –5.5

4 Extend the table so that $x = 0$ is shown, then find for equation of the line.

a

0	1	2	x	3	4	5	6	7
3	5	7	y	9	11	13	15	17

-2 -2 -2 +2 +2 +2 +2

$m =$ ...... $c =$ ......

$y =$ ....

$y = 2x + 3$

b

		x	2	3	4	5	6
		y	2	5	8	11	14

$m =$ ...... $c =$ ......

$y =$ ....

$y = 3x - 4$

c

				x	4	5	6	7
				y	11	13	15	17

$y = 2x + 3$

d

x	-6	-5	-4	-3			
y	-9	-6	-3	0			

$y = 3x + 9$

5 Find the equation that describes this table of values. Select A, B, C, or D.

x	−1	0	1	2
y	3	1	−1	−3

A $y = 2x + 1$ **B** $y = 2x - 1$ **C** $y = x - 2$ **D** $y = -2x + 1$

D

6 Which table of values matches $y = 3x - 1$? Select A, B, C, or D.

A

x	1	2	3	4	5
y	2	3	4	5	6

B

x	1	2	3	4	5
y	2	5	8	11	14

C

x	1	2	3	4	5
y	4	7	10	13	16

D

x	1	2	3	4	5
y	0	3	6	9	12

B

7 Determine the equation shown in the table.

a

x	1	2	3	4
y	6	10	14	18

y = 4x + 2

b

x	1	2	3	4
y	0	4	8	12

y = 4x – 4

c

x	2	3	4	5
y	17	27	37	47

y = 10x – 3

d

x	4	5	6	7
y	4	6	8	10

y = 2x – 4

e

x	5	6	7	8	9
y	−12	−14	−16	−18	−20

y = –2x – 2

f

x	−5	−4	−3	−2	−1
y	−13	−11	−9	−7	−5

y = 2x – 3

g

x	−6	−5	−4	−3	−2
y	10	9	8	7	6

y = –x + 4

8 Determine the equation for each table, then complete the last two columns.

a

x	1	2	3	4		7	9
y	2	7	12	17			

y = 5x – 3
Missing values: 32, 42

b

x	3	4	5	6		9	11
y	8	10	12	14			

y = 2x + 2
Missing values: 20, 24

c

x	2	3	4	5		0	12
y	2	4	6	8			

y = 2x – 2
Missing values: –2, 22

d

x	3	4	5	6		9	50
y	31	41	51	61			

y = 10x + 1
Missing values: 91, 501

e

x	3	4	5	6		12	−2
y	14	19	24	29			

y = 5x – 1
Missing values: 59, –11

f

x	5	6	7	8		9	12
y	4	6	8	10			

y = 2x – 6
Missing values: 12, 18

g

x	1	2	3	4		7	−3
y	5	9	13	17			

y = 4x + 1
Missing values: 29, –11

h

x	4	5	6	7		0	−3
y	1	3	5	7			

y = 2x – 7
Missing values: –7, –13

9 These x values don't increase by 1. You will need to divide to find m.

a

x	-4	-2	0	2	4
y	-19	-11	-3	5	13

$m = 8 \div 2$ $c =$ ....

$=$ ....

$y =$ ....

$y = 4x - 3$

b

x	0	5	10	15	20
y	-3	22	47	72	97

$m =$ ......$\div 5$ $c =$ ....

$=$ ....

$y =$ ....

$y = 5x - 3$

c

x	-6	-3	0	3	6
y	8	-1	-10	-19	-28

$y = 3x - 10$

d

x	0	2	4	6	8
y	-2	-14	-26	-38	-50

$y = -6x - 2$

e

x	-40	-30	-20	-10	0
y	195	155	115	75	35

$y = -4x + 35$

10 2006 HSC Maths Standard 2 Band 4

What equation represents the relationship between x and y in this table?

x	0	2	4	6	8
y	1	2	3	4	5

$y = \frac{x}{2} + 1$

11 These tables need to be extended and the x-value steps to be considered.

a

		x	4	6	8	10	12
		y	17	23	29	35	41

$y = 3x + 5$

b

				x	12	15	18	21
				y	16	25	34	43

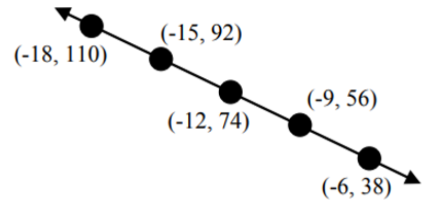
$y = 3x - 20$

c

			x	15	20	25	30	35
			y	20	30	40	50	60

$y = 2x - 10$

d



x							
y							

$y = -6x + 2$

12 Determine the rule shown in the table. Notice that the x values are not increasing by 1.

a

x	-4	-2	0	2	4
y	-5	-1	3	7	11

$y = 2x + 3$

b

x	-3	-1	1	3	5
y	-10	-4	2	8	14

$y = 3x - 1$

c

x	-6	-3	0	3	6
y	15	9	3	-3	-9

$y = -2x + 3$

d

x	-10	-8	-6	-4	-2
y	20	12	4	-4	-12

$y = -4x - 20$

13 Describe the relationship between the x and y values at each point on these lines.

a $y = 5x + 3$

To obtain the y-value, we multiply x by 5 then add 3

b $y = 2x + 1$

To obtain the y-value, we multiply x by ………… then ………………

2, add 1

c $y = 4x - 1$

To obtain the y-value, we multiply x by ………… then ………………

4, subtract 1

d $y = -3x - 9$

To obtain the y-value, we multiply x by ………… then ………………

Negative 3, subtract 9

e $y = \frac{x}{2} + 7$

To obtain the y-value, we **divide** x by ………… then ………………

2, add 7

f $y = \frac{x}{3} - 12$

To obtain the y-value, we ………… x by ………… then ………………

divide, 3, subtract 12

14 Remember that every point on a line is a solution to the equation.
If you know one of either m or c, then you can find the equation by substitution.

Example 1

x	7	8	9	10
y	10	13	16	19

$m = 3$

$y = 3x + c$

Sub in any point: $(7,10)$

$10 = 3(7) + c$

$10 = 21 + c$

$c = -11$

$\therefore y = 3x - 11$

Example 2

x	0	8	47	–7
y	5	–11	–89	19

$c = 5$

$y = mx + 5$

Sub in any point, *except (0,5)*: $(8, -11)$

$-11 = m(8) + 5$

$-11 = 8m + 5$

$-16 = 8m$

$m = -2$

$\therefore y = -2x + 5$

It is a good idea to double check your rule by using the columns.

15 Work out the rule for these tables.

a

x	9	10	11	12
y	1	5	9	13

$y = 4x - 35$

b

x	-2	12	0	9
y	4	-38	-2	-29

$y = -3x - 2$

c

x	-10	-9	-8	-7
y	13	15	17	19

$y = 2x + 33$

d

x	10	12	14	16
y	10	11	12	13

$y = \frac{x}{2} + 5$

16 Find the rule for these tables of values.

a

x	-2	-1	0	1	2
y	0	2	4	6	8

$y = 2x + 4$

b

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

$y = 3x - 1$

c

x	-3	-2	-1	0	1
y	4	3	2	1	0

$y = -x + 1$

d

x	-1	0	1	2	3
y	8	6	4	2	0

$y = -2x + 6$

e

x	1	2	3	4	5
y	5	9	13	17	21

$y = 4x + 1$

f

x	-5	-4	-3	-2	-1
y	-13	-11	-9	-7	-5

$y = 2x - 3$

g

x	5	6	7	8	9
y	-12	-14	-16	-18	-20

$y = -2x - 2$

h

x	-6	-5	-4	-3	-2
y	10	9	8	7	6

$y = -x + 4$

i

x	-4	-2	0	2	4
y	-5	-1	3	7	11

$y = 2x + 3$

j

x	-3	-1	1	3	5
y	-10	-4	2	8	14

$y = 3x - 1$

k

x	-6	-3	0	3	6
y	15	9	3	-3	-9

$y = -2x + 3$

l

x	-6	-3	0	3	6
y	15	9	3	-3	-9

$y = -2x + 3$

m

x	0	2	4	6
y	-10	-16	-22	-28

$y = -3x - 10$

n

x	-5	-4	-3	-2
y	29	24	19	14

$y = -5x + 4$

o

x	1	3	5	7
y	1	2	3	4

$y = \frac{x}{2} + \frac{1}{2}$

p

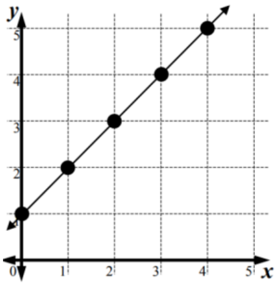
x	-14	-13	-12	-11
y	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4

$y = -\frac{x}{2} - \frac{3}{2}$

FINDING THE RULE FROM A GRAPH

FOUNDATION

1 Change the points to a table of values then find the equation of the line.



x					
y					

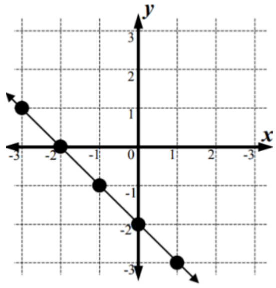
$m = \dots\dots$

$c = \dots\dots$

$y = \dots\dots\dots\dots\dots$

$y = x + 1$

a



x					
y					

$m = \dots\dots$

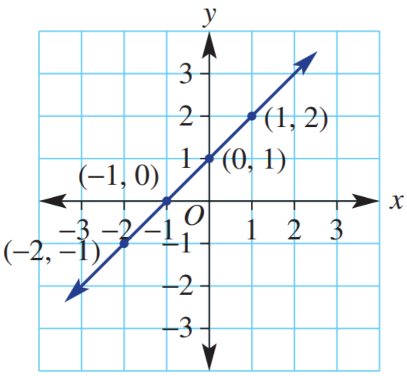
$c = \dots\dots$

$y = \dots\dots\dots\dots\dots$

$y = -x - 2$

2 By first constructing a table of values, find the equations of these lines.
Ensure your x values are in ascending order.

a



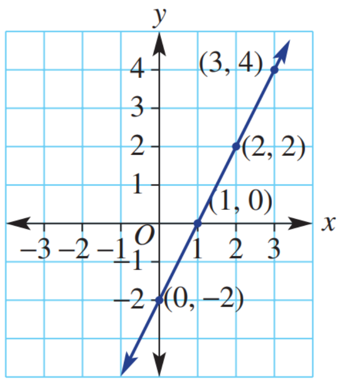
x	-2	-1	0	1
y				

$m = \dots\dots\dots$ $c = \dots\dots\dots$

$y = \dots\dots\dots\dots\dots$

$y = x + 1$

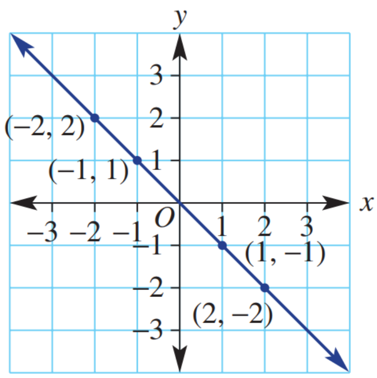
b



x				
y				

$y = 2x - 2$

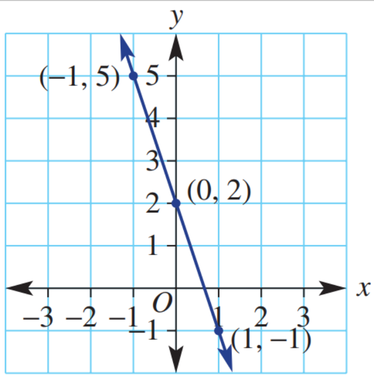
c



x	-2	-1	0	1
y				

$y = -3x + 2$

d

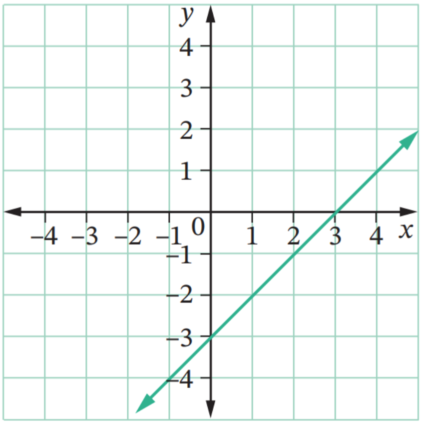


x			
y			

$y = -x$

3 Complete the table for each graph, then find the equation.

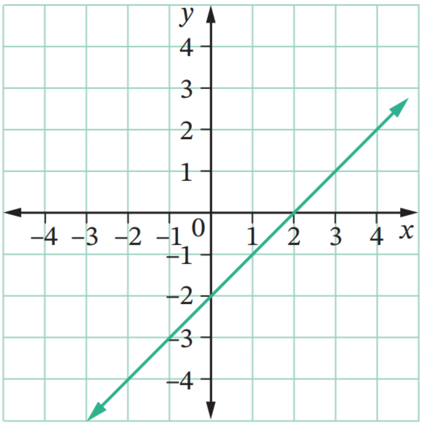
a



x	−1	0	1	2
y				

$y = x - 3$

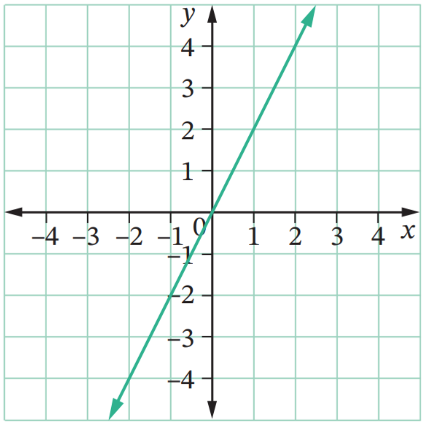
b



x	−2	−1	0	1
y				

$y = x - 2$

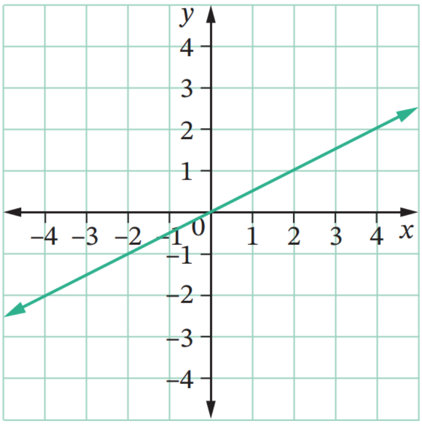
c



x	−1	0	1	2
y				

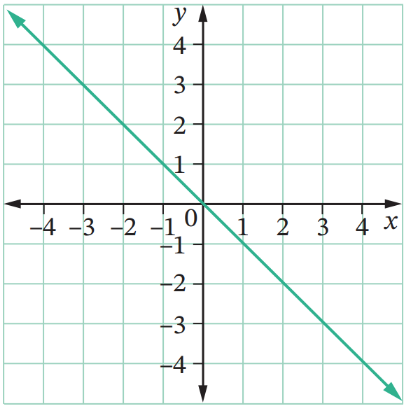
$y = 2x$

d



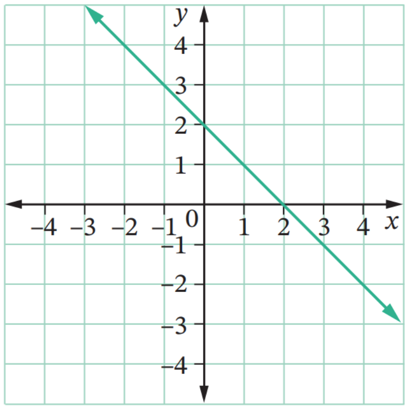
$y = \frac{x}{2}$

e



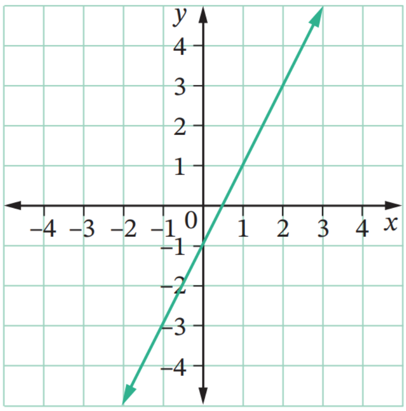
$y = -x$

f



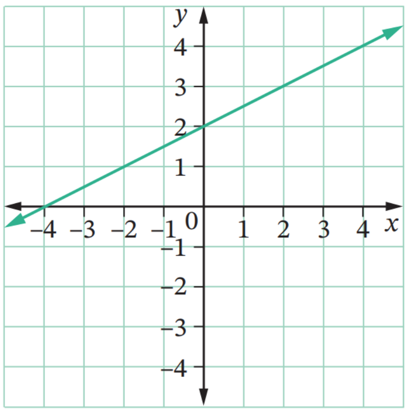
$y = -x + 2$

g



$y = 2x - 1$

h

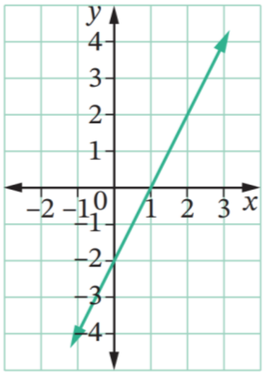


$y = \frac{x}{2} + 2$

4 What is the equation of this line? Select A, B, C, or D.

A $y = -x + 2$ **B** $y = 2x - 2$

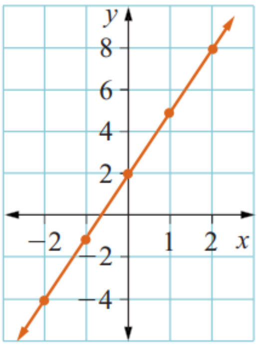
C $y = 2 - 2x$ **D** $y = x - 2$



B

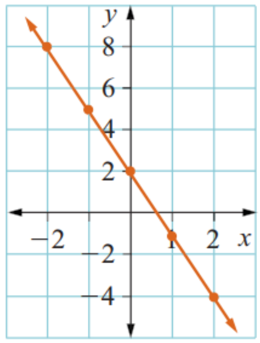
5 By first constructing a table of values, find the equations of these lines.
Notice that the y-axis scale is different to the x-axis scale.

a



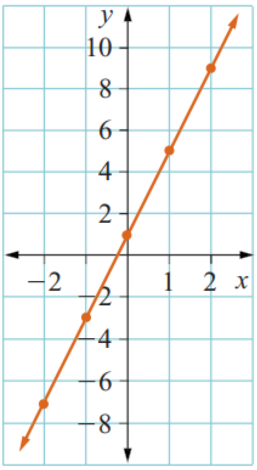
$y = 3x + 2$

b



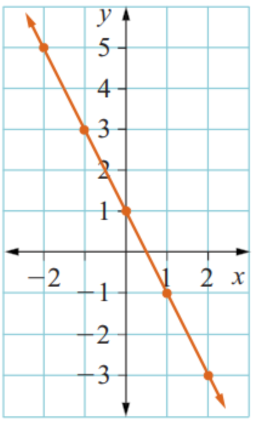
$y = -3x + 2$

c



$y = 4x + 1$

d



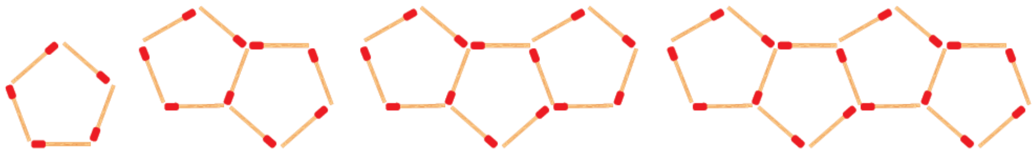
$y = -2x + 1$

APPLYING THE EQUATION

- We can use the equation to make predictions by substituting the known variable into the equation, then finding the value of the unknown variable.

Example Apply the equation to calculate the value of a term.

Consider this pattern of pentagons made from matches:



a Complete the table of values.

Number of shapes x	0	1	2	3	4	5
Number of matches y	1	5	9	13	17	21

b Determine an equation that relates the two quantities.

Number of matches = 4 × number of pentagons + 1

$y = 4x + 1$

c If we want to make 25 pentagons, how many matches do we need?

Let $x = 25$

$y = 4x + 1$

$= 4(25) + 1$

$= 101$ We need 101 matches to make 25 pentagons.

d If we have 73 matches, how many pentagons can we make?

Let $y = 73$

$73 = 4x + 1$

$72 = 4x$

$18 = x$ We can make 18 pentagons with 73 matches.

Describe two methods we could use to determine the equation $y = 4x + 1$

One method is to guess-and-check the ……………… done to x to get y for every ……………

The other method is to use $y = mx + c$; *m is the change in … as x increases by ….*

c is the value of … when $x =$ ….

In part **c**, why did we let $x = 25$?

25 pentagons means $x = 25$, since x represents the ……………………………………………

In part **d**, why did we let $y = 73$?

73 matches means $y = 73$, since y represents the ……………………………………………

a

a Complete the table of values.

Number of triangles x	1	2	3	4	5
Number of matches y					

3, 5, 7, 9, 11

b Determine an equation that relates the two quantities.

Number of matches = ……………………………………………

$y =$ ………………

$y = 2x + 1$

c If we want to make 36 triangles,
how many matches do we need?

Let …… = ………

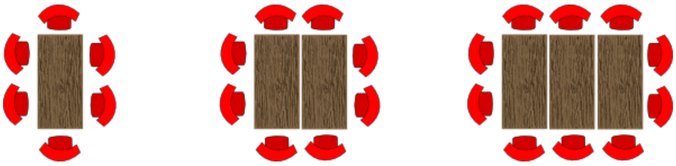
73

d If we have 27 matches,
how many triangles can we make?

Let …… = ………

13

b Find the equation relating x and y, then use the equation to find the missing values.



a Complete the table of values.

Number of tables x	1	2	3
Number of chairs y			

6, 8, 10

b Determine an equation that relates the two quantities.

Number of chairs = ……………………………………………

$y =$ ………………

$y = 2x + 4$

c If we want 36 tables,
how many chairs do we need?

Let …… = ………

76

d If we have 42 chairs,
how many tables can we have?

Let …… = ………

19

1

a Complete the table of values.

Number of shapes x	1	2	3	4	5
Number of matches y					

b Determine an equation that relates the two quantities.

Number of matches = ...

$y =$ ....

$y = 3x$

c How many matches are needed for a pattern with 14 triangles?

Let ...... = ...

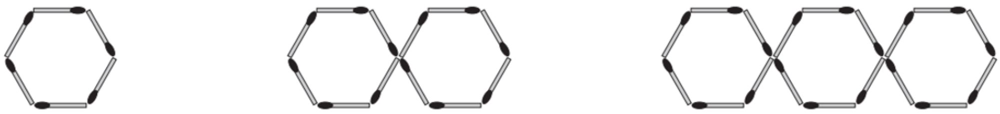
....

....

....

$y = 42$

2



a Complete the table of values.

Number of shapes x	1	2	3	4	5
Number of matches y					

b Determine an equation that relates the two quantities.

Number of matches = ...

$y =$ ....

$y = 6x$

c How many matches are needed for a pattern with 9 hexagons?

Let ...... = ...

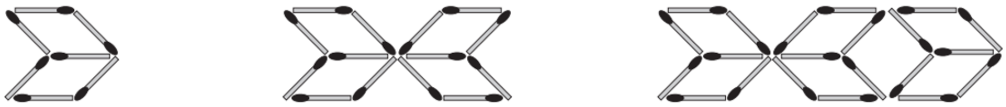
....

....

....

$y = 54$

3



a Complete the table of values.

x	1	2	3	4	5
y					

b Determine an equation that relates the two quantities.

$y = 7x$

c How many matches are needed for a pattern with 10 shapes?

Let …… = ………

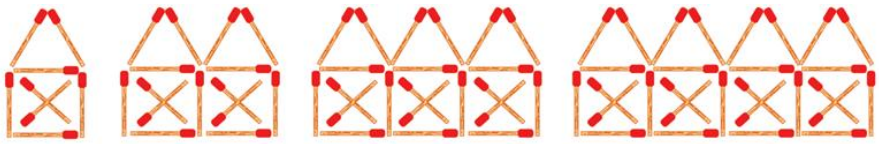
…………………………

…………………………

………………………… ……………………………………………………………………………

$y = 70$

4



a Complete the table of values.

x	1	2	3	4	5
y					

b Determine an equation that relates the two quantities.

$y = 7x + 1$

c How many matches are needed for a pattern with 30 "houses"?

Let …… = ………

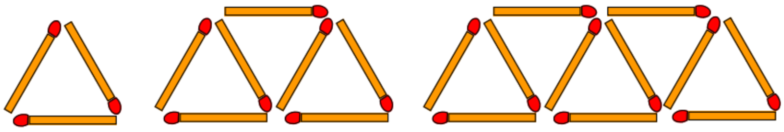
…………………………

…………………………

………………………… ……………………………………………………………………………

$y = 211$

5



1 wide 2 wide 3 wide

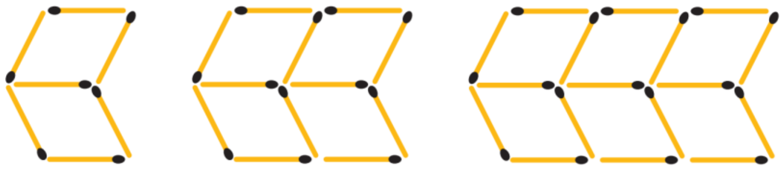
By first finding the equation of the pattern, determine how many matches are needed to make a shape 40 wide.

x					
y					

$y = 4x - 1$

159 matches

6



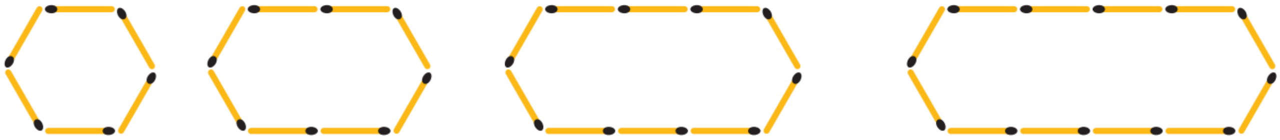
1 shape 2 shapes 3 shapes

By first finding the equation of the pattern, determine how many matches are needed to make 17 shapes.

$y = 5x + 2$

87 matches

7



1 wide 2 wide 3 wide 4 wide

a Determine the equation of the pattern.

$y = 2x + 4$

b Determine how many matches are needed to make a shape 27 wide.

58

c Determine the width of a shape that can be made with 136 matches.

66

8 A pattern of towers is being assembled using blocks shown.

a Determine the equation of the pattern, relating number of blocks (y) to height (x).

$y = 2x - 1$

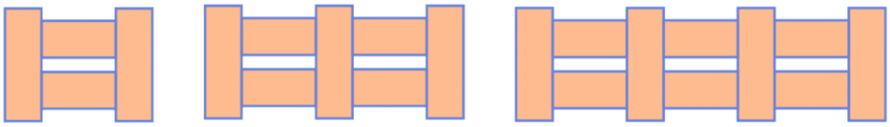
b Determine how many blocks would be needed for a tower 43 high.

85

c Determine the height of a tower that can be made with 133 blocks.

67

9 Consider this fence.



a Determine how many wooden planks are needed to make 20 fence sections.

61

b Determine how many fence sections can be made with 34 wooden planks.

11

2 Consider this table arrangement.

a Determine how many chairs are needed for 10 tables in a straight line.

32

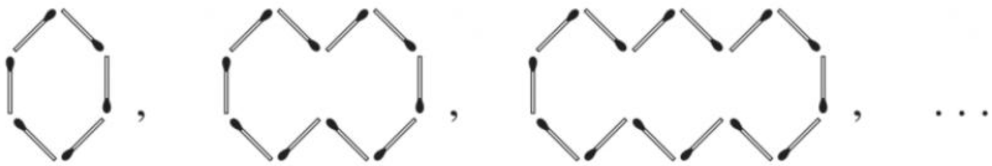
c The tables are to be arranged in a single straight line for a picnic.

How many tables are needed to seat 65 people?

21

10 2007 HSC Maths Standard 2 Band 6

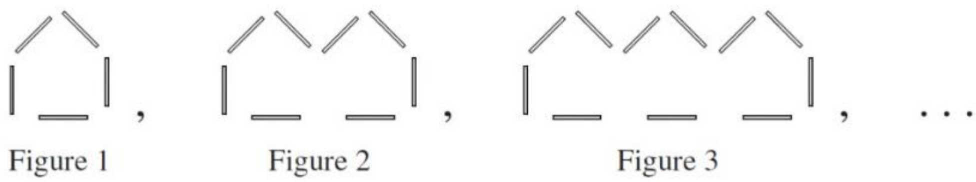
Chris started to make this pattern of shapes using matchsticks.



If the pattern of shapes is continued, which shape would use exactly 486 matchsticks?

121st shape

11 **2011 HSC Maths Standard 2 Band 5**



Is it possible to create a figure in this pattern using exactly 543 sticks?

Show suitable calculations to support your answer.

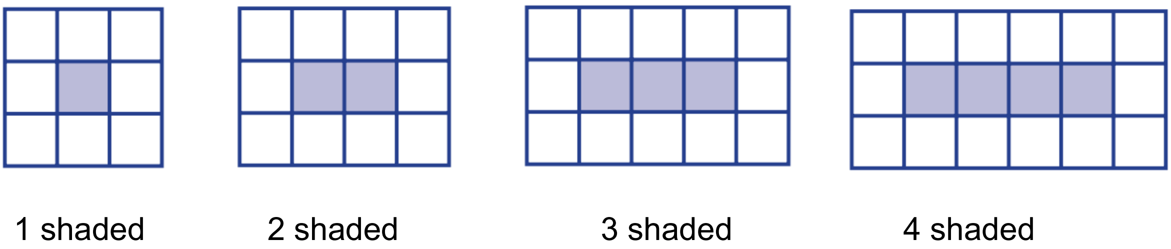
$y = 3x + 2$

$543 = 3x + 2$

$x = 180 \text{ r}1$

No, as the figure is not a whole number.

12 A tiling pattern is made from surrounding a row a shaded tiles with unshaded tiles.



a Determine the total number of tiles needed to make a pattern with 23 shaded tiles.

75

b Determine the number of shaded tiles in a pattern with 45 total tiles.

13

c Determine how many *unshaded* tiles would be in a pattern with 17 shaded tiles.

40

d Determine how many *unshaded* tiles would be in a pattern with 72 total tiles.

50

Example Represent linear relationships as equations, tables, and graphs.

Sequence:

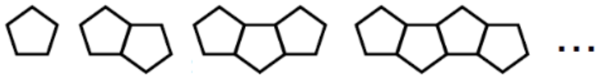


Table:

Pentagons x	1	2	3	4
Matches y	5	9	13	17

Equation:

Matches = 4 × Pentagons + 1

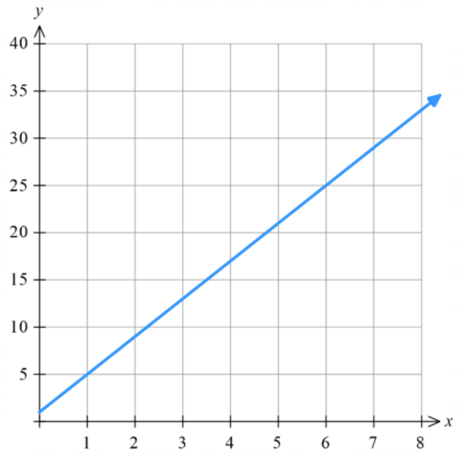
$y = 4x + 1$

Coordinates:

(1, 5)
(2, 9)
(3, 13)
(4, 17)

Graph:

Matches



Pentagons

Using the graph, how many matches are needed for 8 pentagons? ……………………

Using the equation, how many matches are needed for 8 pentagons?

Using the graph, how many pentagons can be made with 29 matches? ……………………

Using the equation, how many pentagons can be made with 29 matches?

What do you think are the advantages and disadvantages of solving equations using the graph?

Advantages: It is much faster as you don't need to …………… an equation.

Disadvantages: It can be …………………

Sequence:

Table:

Squares x	0	1	2	3	4
Matches y	1	4	7	10	13

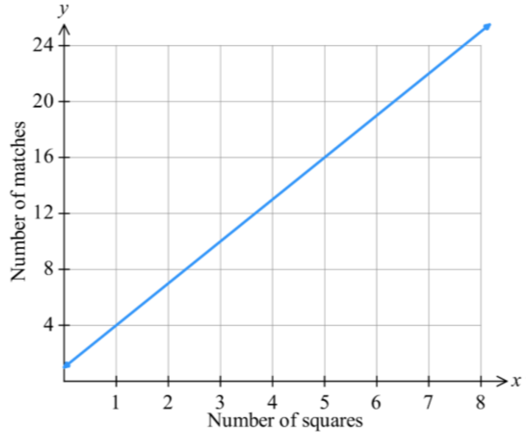
Equation:

$y = 3x + 1$

Coordinates:

(0, 1) (1, 4) (2, 7) (3, 10) (4,13)

Graph:



Sequence:

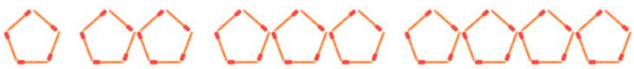


Table:

x	0	1	2	3	4
y	0	5	10	15	20

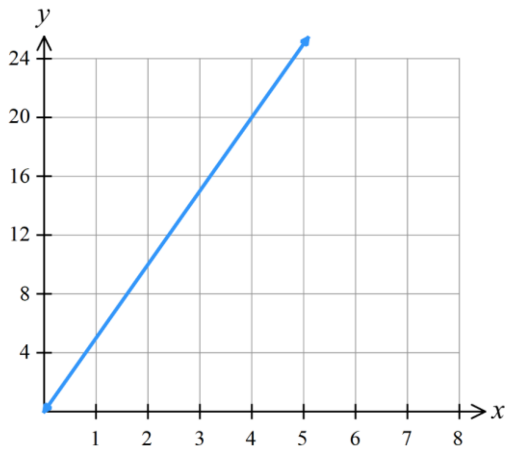
Equation:

$y = 5x$

Coordinates:

(0,0) (1,5), (2,10), (3,15) (4,20)

Graph:



1

Sequence:

1st term 2nd term 3rd term 4th term

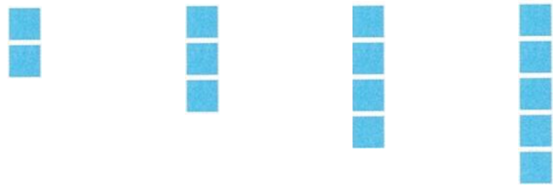


Table:

Term x	0	1	2	3	4	5	43
Value y	1	2	3	4	5	6	44

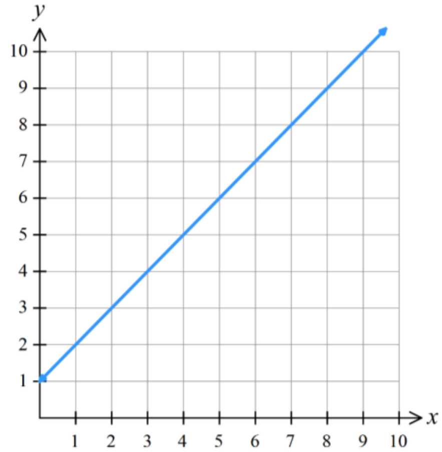
Equation:

$y = x + 1$

Coordinates:

(0,1) (1,2) (2,3) (4,5) (43,44)

Graph:



2

Sequence:

1st term 2nd term 3rd term 4th term

Table:

x	0	1	2	3	4	5	17
y	–2	0	2	4	6	8	32

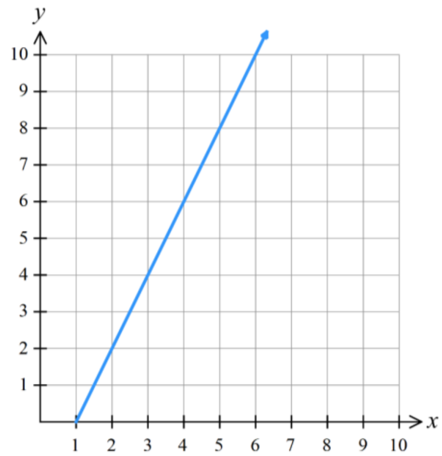
Equation:

$y = 2x - 2$

Coordinates:

(0,–2), (1,0), (2,2), (3,4), (4,6), (5,8), (17,32)

Graph:



3

Sequence:

1st term 2nd term 3rd term

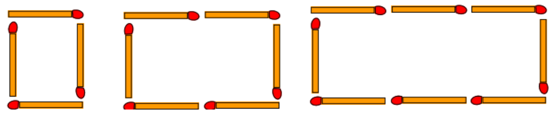


Table:

x	0	1	2	3	4	5	18
y	2	4	6	8	10	12	38

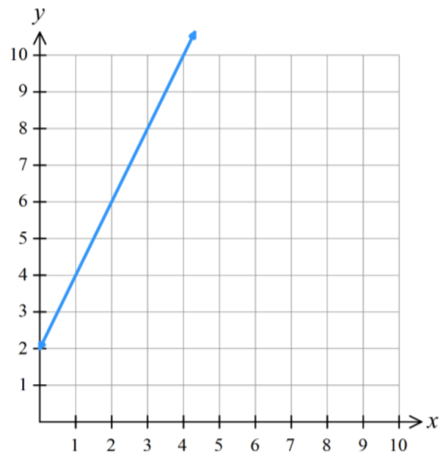
Equation:

$y = 2x + 2$

Coordinates:

(0,2) (1,4) (2,6) (3,8) (4,10) (5,12) (18,38)

Graph:



4

Sequence:

1st term 2nd term 3rd term 4th term

Table:

x	0	1	2	3	4	5	17
y	7	6	5	4	3	2	−10

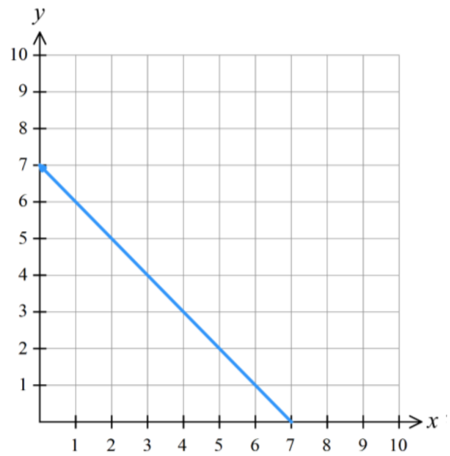
Equation:

$y = -x + 7$

Coordinates:

(0,7) (1,6) (2,5) (3,4) (4,3) (5,2) (17,–10)

Graph:



5

Sequence:

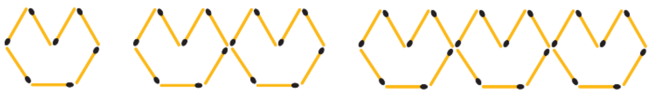


Table:

x	0	1	2	3	4	10	8
y	0	7	14	21	28	70	56

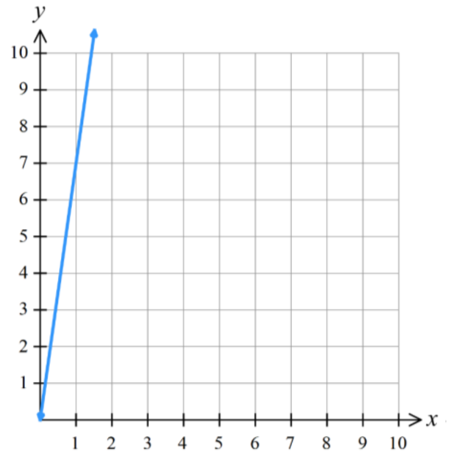
Equation:

$y = 7x$

Coordinates:

(0,0) (1,7) (2,14) (3,21) (4,28) (10,70) (8,56)

Graph:



6

Sequence:

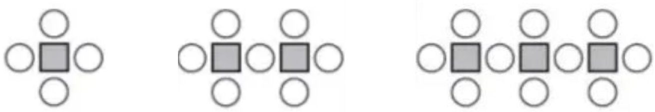


Table:

x	0	1	2	3	4	5	9
y	1	4	7	10	13	16	28

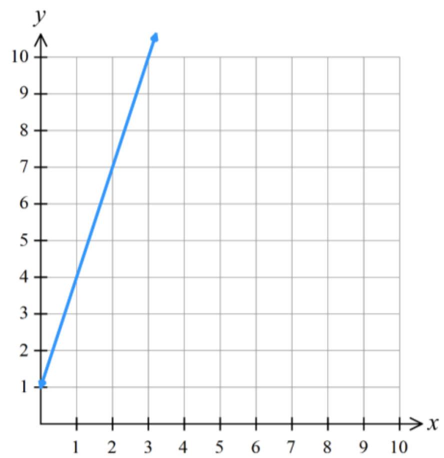
Equation:

$y = 3x + 1$

Coordinates:

(0,1) (1,4) (2,7) (3,10) (4,13) (5,16) (9,28)

Graph:



7 For this question, fill in every section by first looking at the table.

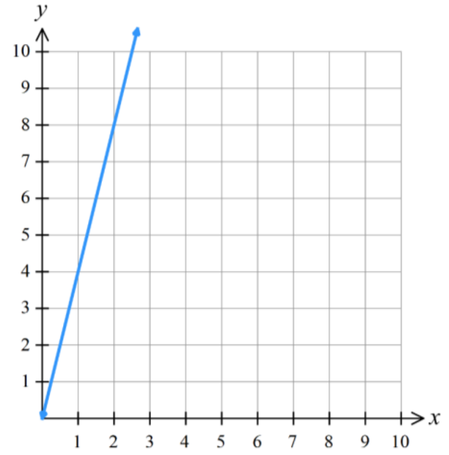
Table:

x	0	1	2	3	4	21	19
y	0	4	8	12	16	84	76

Equation:

$y = 4x$

Graph:



8 For this question, fill in every section by first looking at the table.

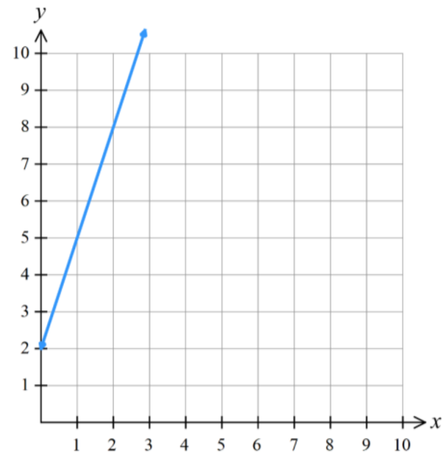
Table:

x	0	1	2	3	4	81	60
y	2	5	8	11	14	245	182

Equation:

$y = 3x + 2$

Graph:



9 For this question, fill in every section by first looking at the table.

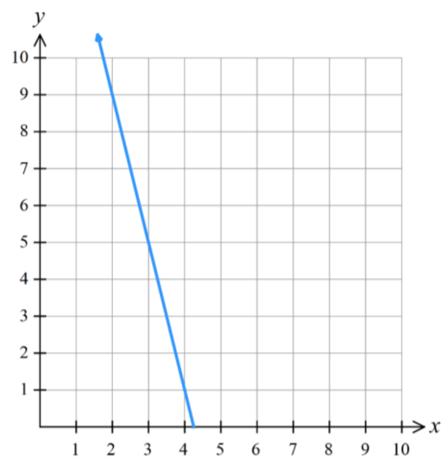
Table:

x	0	1	2	3	4	-4	-17
y	17	13	9	5	1	33	85

Equation:

$y = -4x + 17$

Graph:



10 For this question you fill in every section by first looking at the graph.

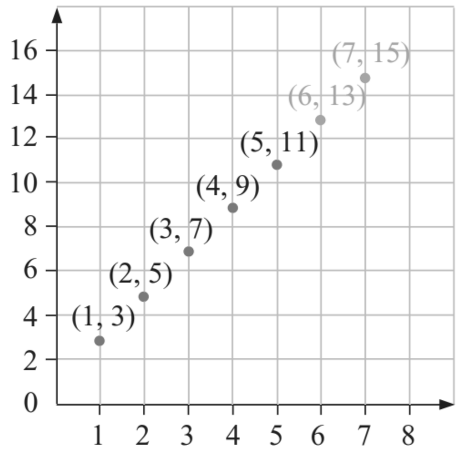
Table:

x	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15

Equation:

$y = 2x + 1$

Graph:



11 For this question you fill in every section by first looking at the graph.

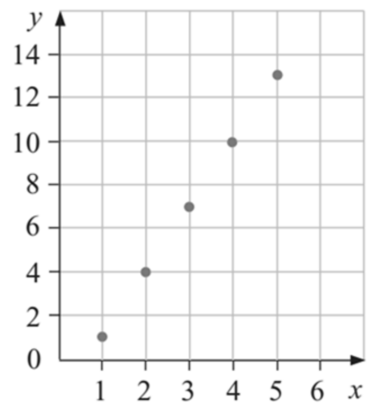
Table:

x	1	2	3	4	5
y	1	4	7	10	13

Equation:

$y = 3x - 2$

Graph:



12 For this question you fill in every section by first looking at the graph.

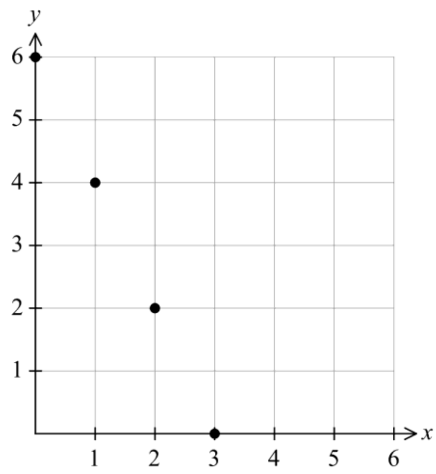
Table:

x	0	1	2	3
y	6	4	2	0

Equation:

$y = -2x + 6$

Graph:



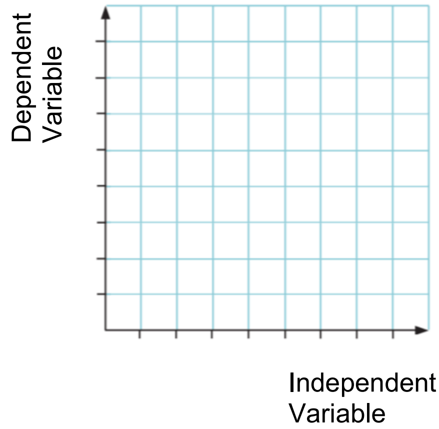
LINEAR RELATIONSHIPS IN REAL LIFE

VARIABLES

- A **variable** is a quantity that can change.
- Variables are usually represented by a pronumeral.

Examples:

Weight, cost, time, distance, speed, x, y



- The **dependent variable** depends on another variable.
  - It is the **second row** of a table of values.
  - It is the y **axis** of a graph.
- The **independent variable** does not depend on anything.
  - It is the **first row** of a table of values.
  - It is the x **axis** of a graph.

Key ideas

1. The independent variable is the first ………… of a table of values and always goes on the …… axis.
2. The dependent variable is the second …………… of a table of values always goes on the …… axis.
3. In the last section:
   a. The number of matches was the ………………… variable.
   b. The number of shapes was the ………………… variable.
4. For *age* and *height*:
   a. ………… is the dependent variable, since it changes based on ……………
   b. ………… is the independent variable, since it doesn't depend on anything.
5. If you graph the dependent and independent variable, and you can draw a straight line through all the coordinates, then we say that the variables have a …………… relationship.

Example Model a scenario using a linear equation.

Scenario:

Chocolates are sold for $2 per kilogram.

Independent variable mass in kg

Dependent variable cost in $

Table:

Mass x	0	1	2	3
Cost y	0	2	4	6

Equation:

Cost = 2 × mass

$y = 2x$

Find the cost of 7 kg of chocolate.

$y = 2(7) = \$14$

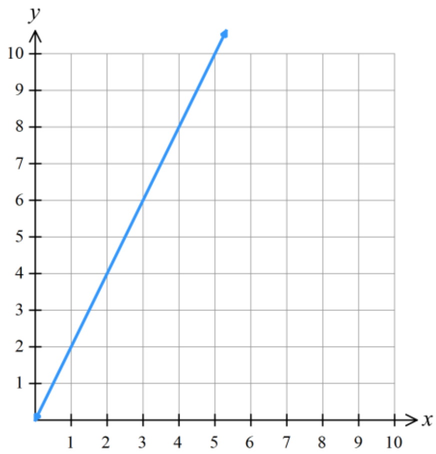
Find how much chocolate you can buy for $13.

$13 = 2x$

$x = 6.5$ kg

Graph:

Cost ($)



Mass (kg)

Scenario:

Watermelons cost $2.50 per kilogram.

Independent variable Mass (kg)

Dependent variable Cost ($)

Table:

…………… x Mass (kg)	0	1	2	3
…………… y Cost ($)	0	2.5	5	7.5

Equation:

$y = 2.5x$

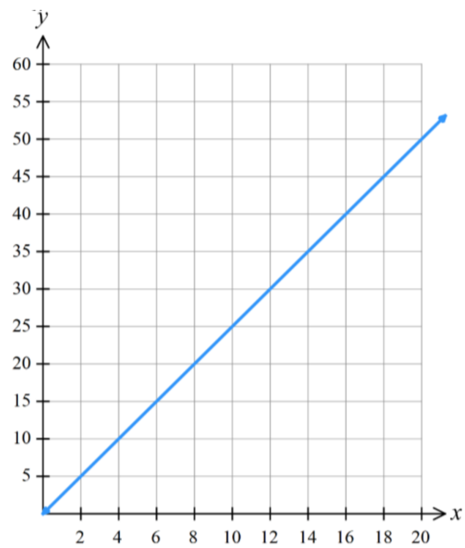
How many kilograms of watermelon can be purchased for $37.50?

15 kg

Calculate the cost of 50 kg of watermelon.

$125

Graph:



Example Model a scenario using a linear equation.

Scenario:

A taxi has a \$10 hiring fee and \$4 per kilometre.

Independent variable distance in km

Dependent variable cost in \$

Table:

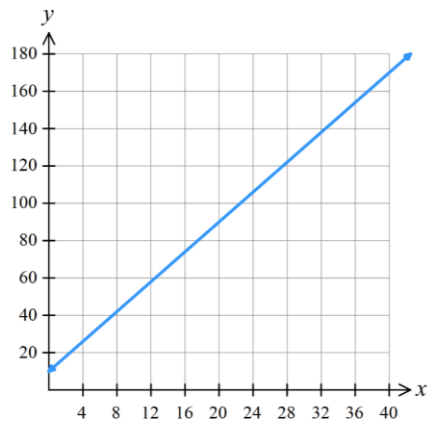
Distance x	0	10	20	30	40
Cost y	10	50	90	130	170

Equation:

$\text{Cost} = 10 + 4 \times x$

$y = 4x + 10$

Graph:



Find the cost of a journey of 25 km \$110, using the graph.

How far can you travel for \$70? 15 km, using the graph.

Scenario:

The cost of a taxi is $5 to start and $2.50 per kilometre.

Independent variable

Distance in km

Dependent variable

Cost in $

Table:

....	x	0	10	20	40
....	y				

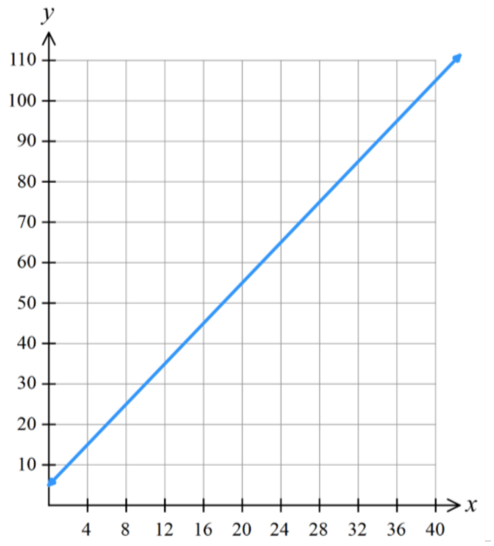
5, 30, 55, 105

Equation:

....

$y = 2.5x + 5$

Graph:



Use the graph to find how far you could travel for $60.

22 km

Use the graph to find how much it costs to travel 15 km.

Around $42

Use your equation to calculate the cost of travelling 15 km.

$42.50

Why might your graph be inaccurate?

It is hard to accurately read the axis scales.

Use your equation to calculate the cost of travelling 43.5 km.

$113.75

How far could you travel for $100?

38 km

1

Scenario:

Oranges are sold for $3.00 per kilogram

Independent variable

Mass (kg)

Dependent variable

Cost ($)

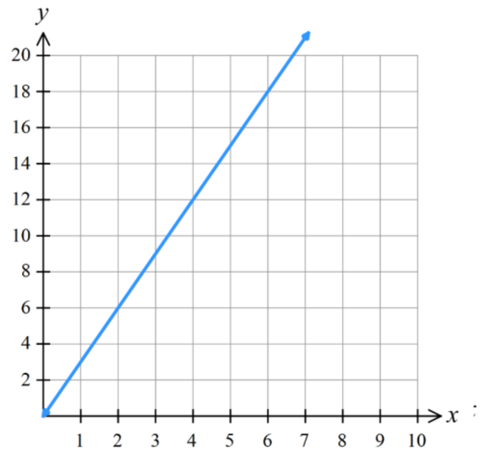
Table:

.... x Mass (kg)	0	1	2	5	10
.... y Cost ($)	0	3	6	15	30

Equation:

$y = 3x$

Graph:



a find the cost of 17 kg of oranges.

$51

b find how many kilograms of oranges could be purchased for $21.90

7.3 kg

2

Scenario:

Washing powder is sold for $1.40 per kg.

Independent variable

Mass (kg)

Dependent variable

Cost ($)

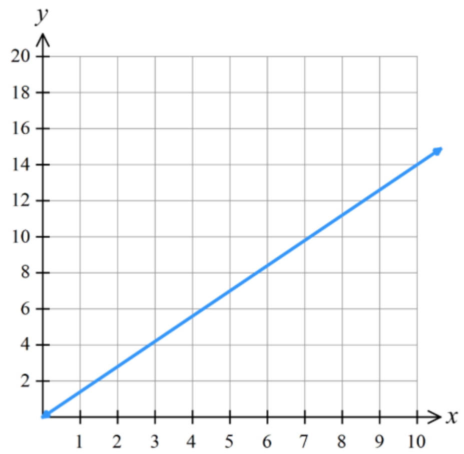
Table:

.... x Mass (kg)	0	1	2	5	10
.... y Cost ($)	0	1.4	2.8	7	14

Equation:

$y = 1.4x$

Graph:



a find the cost of 12.5 kg of washing powder.

$17.50

b find how much washing powder can be bought for $31.50.

21 kg

3

Scenario:

The cost of travelling in a hire car is $10 booking fee plus $3.20 per kilometre.

Independent variable

Distance (km)

Dependent variable

Cost ($)

Graph:

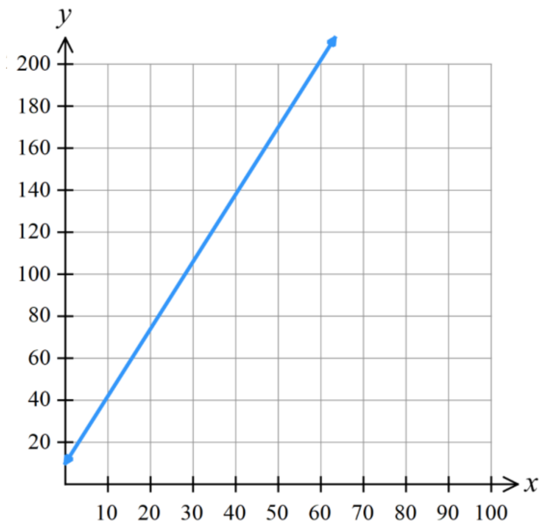


Table:

.... x Distance (km)	0	10	20	50	100
.... y Cost ($)	10	42	74	170	330

Equation:

$y = 3.2x + 10$

a Find the cost of a 25 km journey.

$90

b How far can the car be driven for $50?

12.5 km

4

Scenario:

The cost of hiring a plumber is $80 call out fee and $240 per hour.

Independent variable

Time (hours)

Dependent variable

Cost ($)

Graph:

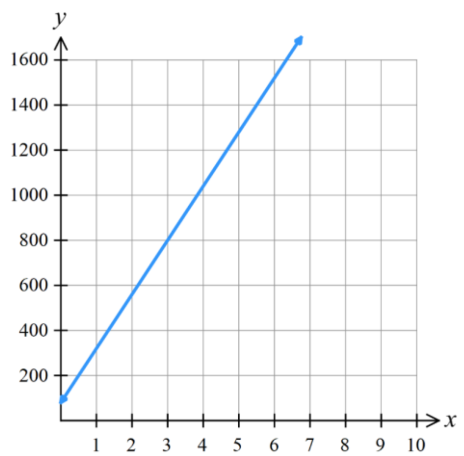


Table:

.... x Time (h)	0	1	2	3	4
.... y Cost ($)	80	320	560	800	1040

Equation:

a How much does it cost for a 3.5-hour job?

$520

b How long can you afford the plumber for if you have $200?

30 minutes

5

Scenario:

A car uses 3 L of fuel per hour.
Its fuel tank initially holds 32 L.
Independent variable

Time (h)

Dependent variable

Fuel (L)

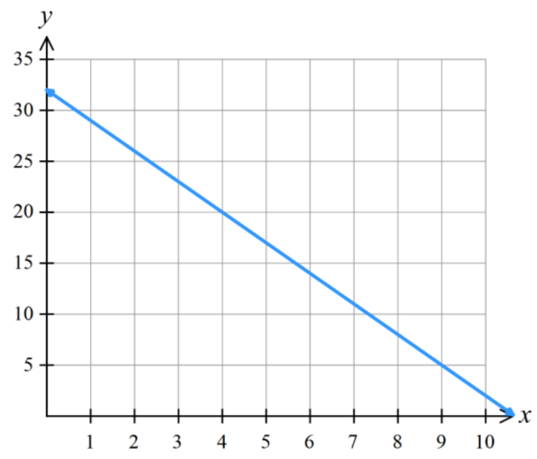
Table:

………… x Time (h)	0	1	2	3	4
………… y Fuel (L)	32	29	26	23	20

Equation:

$y = -3x + 32$ or $y = 32 - 3x$

Graph:



a How much fuel would be left after 4 hours 30 minutes?

18.5 L

b When exactly, to the nearest minute, will the car run out of fuel?

10 h 40 min

6

Scenario:

A candle starts off 3 cm tall and decreases by half a centimetre every minute.
Independent variable

Time (min)

Dependent variable

Height (cm)

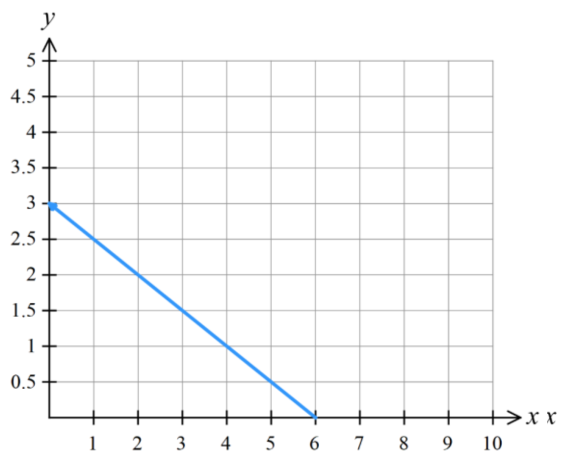
Table:

………… x Time (min)	0	1	2	3	4
………… y Height (cm)	3	2.5	2	1.5	1

Equation:

$y = -\frac{x}{2} + 3$ or $y = 3 - \frac{x}{2}$

Graph:



a How tall is the candle after 15 minutes?

–4.5 cm

b What can you say about the limitations of linear models?

This model is only valid for positive values of x and y

7 Write an equation for the following situations. Make the first listed pronumeral the subject.

a $\$D$ given c cents

$$D = \frac{c}{100}$$

b d cm given e metres

$$d = 100e$$

c The discounted price $\$D$ that is 30% off the marked price $\$M$

$$D = 0.7M$$

d The value of an investment $\$V$ that is 15% more than the initial amount $\$P$

$$V = 1.15P$$

e The cost $\$C$ of hiring a car at $50 upfront plus $18 per hour for t hours

$$C = 50 + 18t$$

f The distance d km remaining in a 42 km marathon after t hours if the running speed is 14 km/h

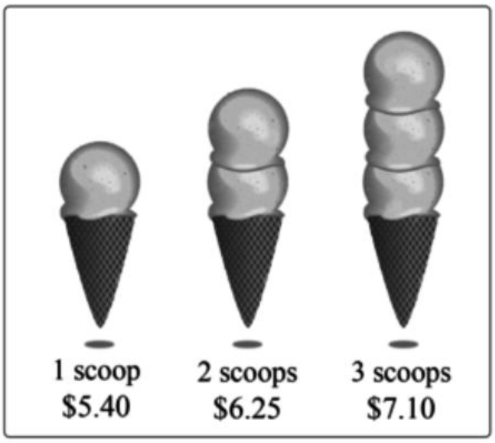
$$d = 42 - 14t$$

g The price $\$P$ of a bottle of soft drink if b bottles cost $\$c$

$$P = \frac{c}{b}$$

8 **NAPLAN A**

Each extra scoop of ice cream costs the same amount of money.
How much will an ice cream with 7 scoops cost?



1 scoop costs $0.85

The equation is $y = 0.85x + 4.55$

$\therefore \$10.50$

9 NAPLAN A

Sabre is saving to buy a new skateboard. After one week she has saved $11.
She then saves the same amount of money each week.

Week	1	2	3	4	5
Total amoun saved	$11	$18	$25	$32	$39

How much money will she have saved by the end of week 10?

$y = 7x + 4$

$74

10 NAPLAN A

A plumber calculates the price of a job using a call out fee and an amount per hour.
This table shows some of the job prices. Determine the call out fee and amount per hour.

Hours	2	4	6	8
Job price	$130	$202	$274	$346

Amount per hour: $36

Call out fee: $58

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A plumber charges a call-out fee of $90 as well as $2 per minute while working.

Suppose the plumber works for t hours.

Which equation expresses the amount the plumber charges ($$C$) related to time ($t$ hours)?

A. $C = 2 + 90t$

B. $C = 90 + 2t$

C. $C = 120 + 90t$

D. $C = 90 + 120t$

D

COMPARING LINEAR RELATIONSHIPS

- We have special words to describe parts of a linear equation:

$$y = mx + c$$

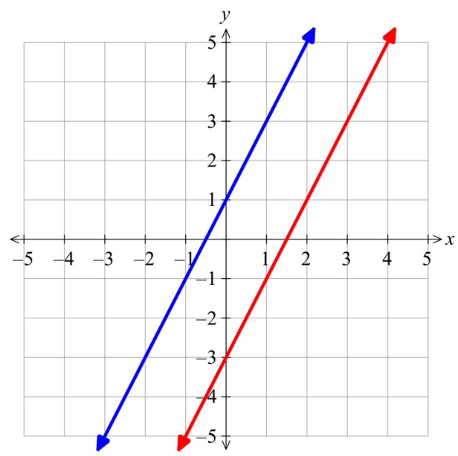
Coefficient of x — Gradient

Constant term — y-intercept

- From the equation, you can tell a lot of things about a graph.

If two equations have the **same coefficient**, then the lines are **parallel**.

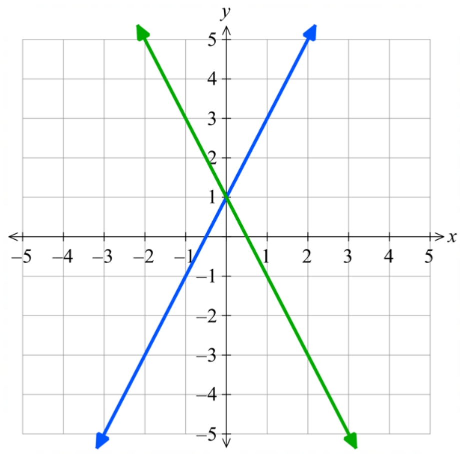
$$y = 2x + 1$$
$$y = 2x - 3$$



$y = 2x + 1$ $\quad$ $y = 2x - 3$

If two equations have the **same constant term,** then the ***y*-intercepts** are the same.

$$y = -2x + 1$$
$$y = 2x + 1$$

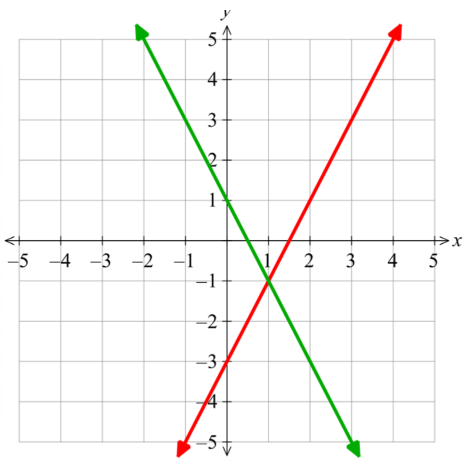


$y = -2x + 1$ $\quad$ $y = 2x + 1$

If the **coefficient of x is positive**,
the graph **slopes upwards** from left to right;
increasing graph.

If the **coefficient of x is negative**,
the graph **slopes downwards** from left to right;
decreasing graph.

$$y = 2x - 3$$
$$y = -2x + 1$$

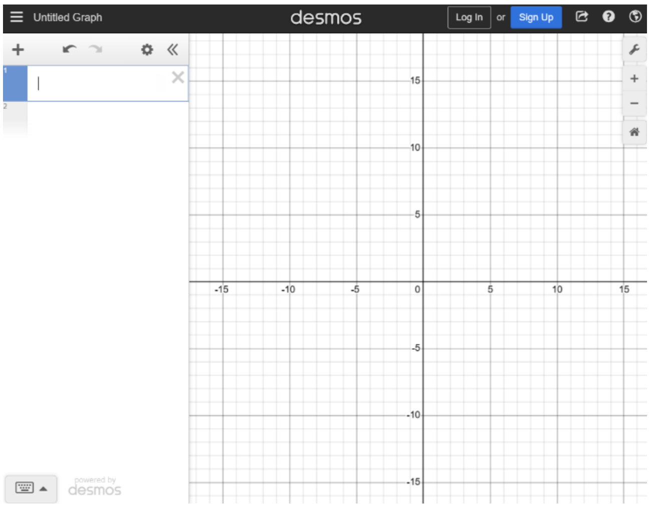


$y = 2x - 3$ $\quad$ $y = -2x + 1$

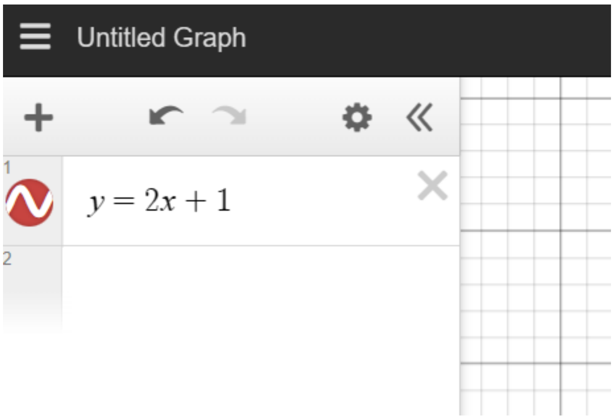
$y = mx + c$: https://www.geogebra.org/m/vzxvwrpw

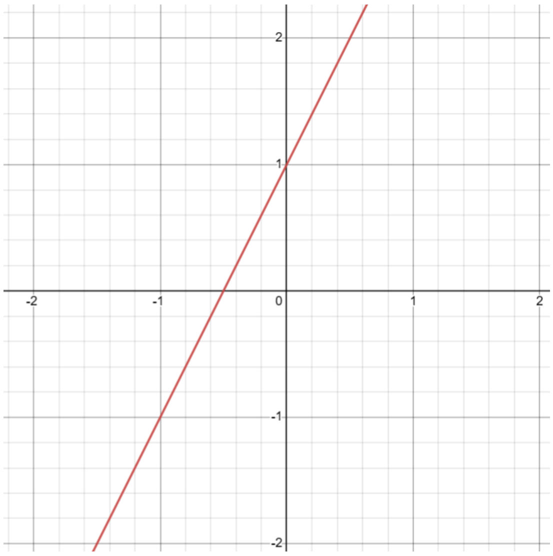
Investigation Graphing software

Go to
www.desmos.com/calculator
your screen should look like the right.

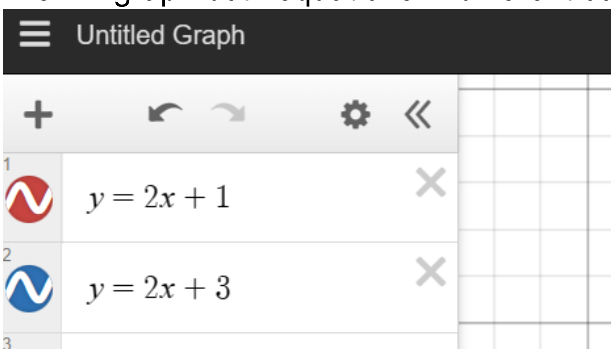


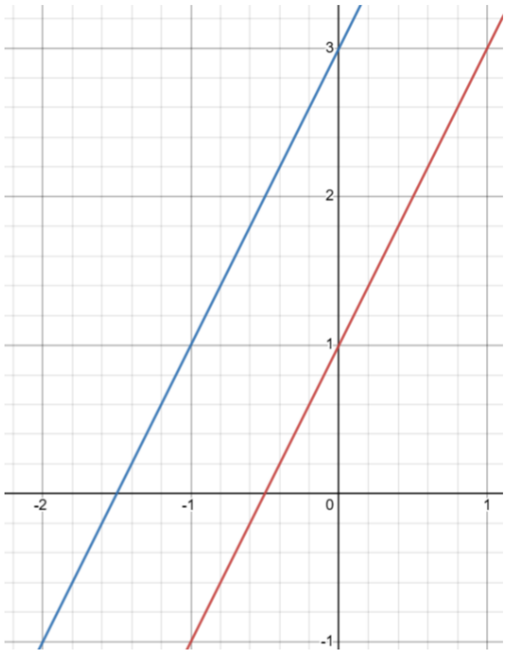
On the left, type in $y = 2x + 1$
This will graph the equation. Use the mouse wheel to zoom in and out and hold down left click to move around.





Below the first equation, type $y = 2x + 3$
This will graph both equations in different colours.





You can use the **X** button to remove the graphs.

Using DESMOS, draw a sketch and explain the similarities and differences between the set of lines.

Use these words to describe the lines: **gradient, parallel, increasing/decreasing, y-intercept**

Use these words to describe the equations: **coefficient, constant term**

1

$y = 4x + 1$

$y = 4x - 3$

$y = 4x$

These lines are all ………………… because the ………………… of x are the same.

Parallel, coefficient

The lines have different …………………… because the constant terms are different.

y-intercept

2

$y = 2x + 2$

$y = 4x + 2$

$y = 7x + 2$

The lines have the same ………………… because the …………… ………… are the same.

y-intercept, constant term

The lines have different ………………… because the …………………… of x are not the same.

gradient, coefficient

3

$y = 3x + 1$

$y = -3x + 1$

The lines both have the same ……………………………………………………………………………

y-intercept

because ……………………………………………………………………………………………

the constant terms are the same

The lines have different …………………………………………………………………………….

Gradient

because ……………………………………………………………………………………………

the coefficients of x are different

4

$y = 2x$

$y = -2x$

$y = 2x$ is an increasing graph because the …………………… of x is positive.

coefficient

$y = -2x$ is a ………….. graph because ………………………………………………………

Decreasing, coefficient of x is negative

5

$y = -2x + 2$

$y = -2x + 1$

$y = -2x - 3$

The lines are all ………………… because they have the same ……………… of x.

Parallel, same

The lines are all decreasing because their coefficients are ……………………

negative

The lines have different ……………… because ………………… ……….. are different.

y-intercept, constant terms are different

6 Complete each statement:

a All equations of a line have the form: ……………………

$y = mx + C$

b m is the c……………… of x and represents the g………………, or steepness of the line.

Coefficient, gradient

c c is the c………………… t……………… and represents the …-…………………, or where the line cuts the ……-axis.

Constant term, y-intercept, y

d The equations of parallel lines have the same ………………………

Coefficient of x

e Equations with the same ……………… ……… have graphs with the same y-intercept.

Constant term

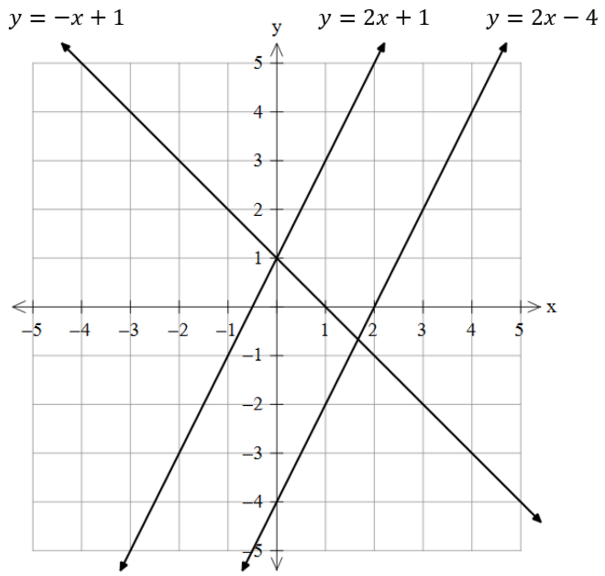
f Linear equations with a positive coefficient of x have graphs that are ………………………

increasing

g Linear equations with a ……….. coefficient of x have graphs that are decreasing.

negative

7 Consider these three lines.



a Which two lines are parallel? What part of the equation tells you this?

$y = 2x + 1, y = 2x - 4$

Coefficients of x are the same

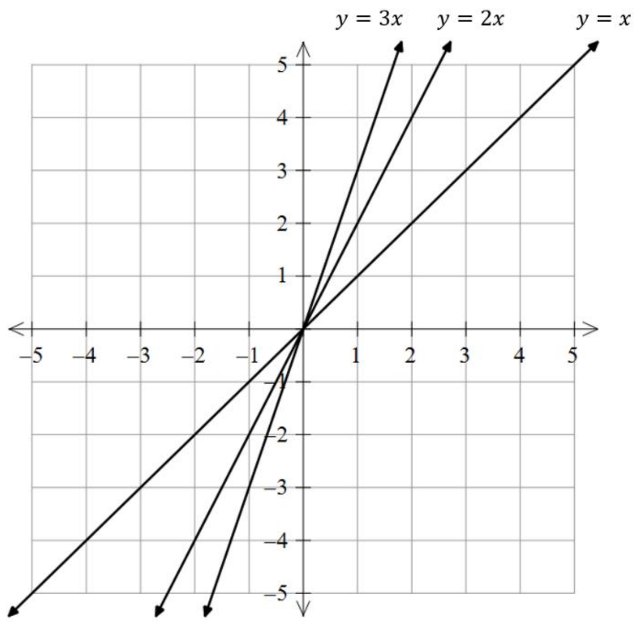
b Which two lines have the same y-intercept? What part of the equation tells you this?

$y = -x + 1, y = 2x + 1$

Constant terms are the same

8 Consider these lines.

a



Which line is the steepest? ………………

$y = 3x$

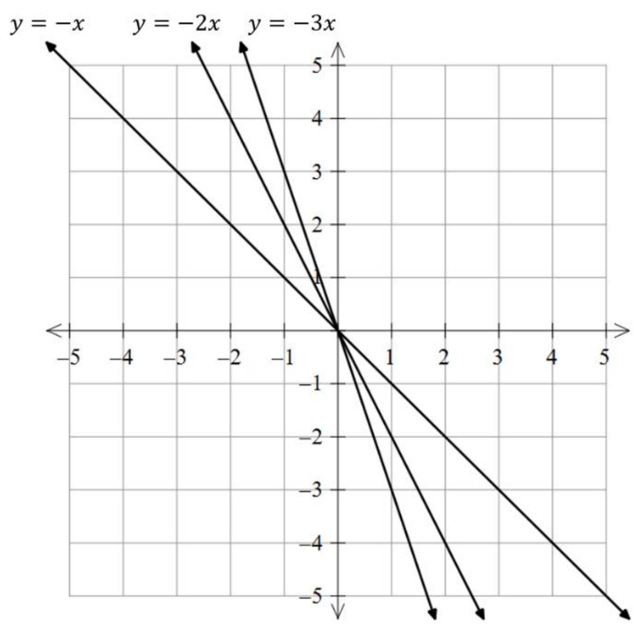
Which line is the least steep? ………………

$y = x$

How does the coefficient of x tell you the steepness of a graph?

The larger the magnitude of the coefficient of x, the steeper

b



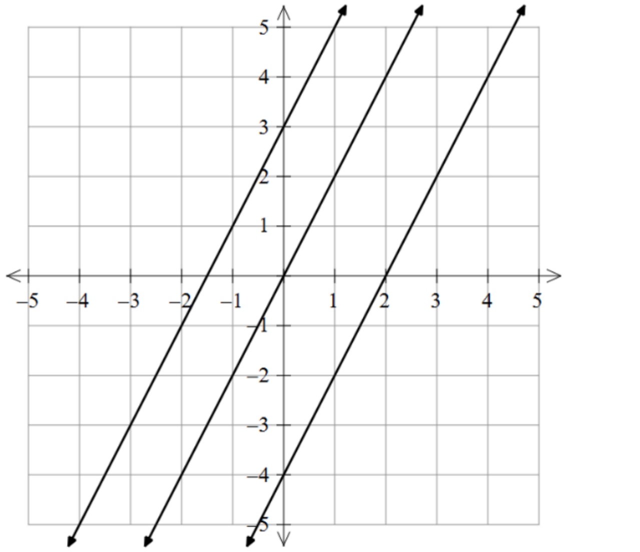
All the graphs in part **a** are increasing.

All the graphs shown here are decreasing.

What part of the equation tells you whether a graph is increasing or decreasing?

The sign of the coefficient of x

c



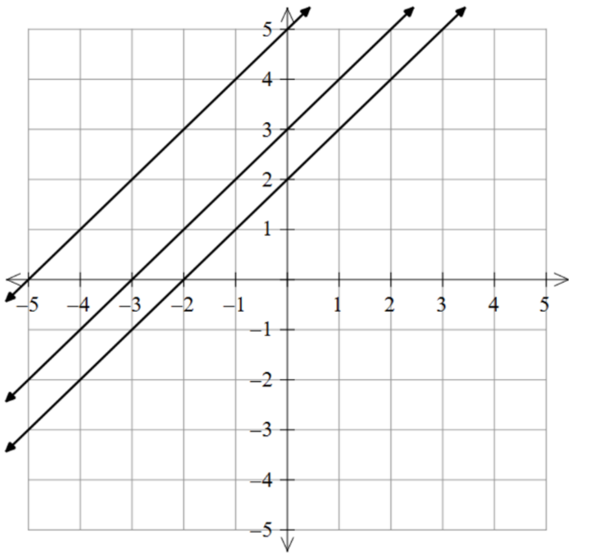
All of the lines are parallel.
What part of the equation tells you whether two or more lines will be parallel?

The coefficient of x are the same

Write down the equation of any other line that is parallel to these lines.

Any other line in the form $y = 2x + c$

d



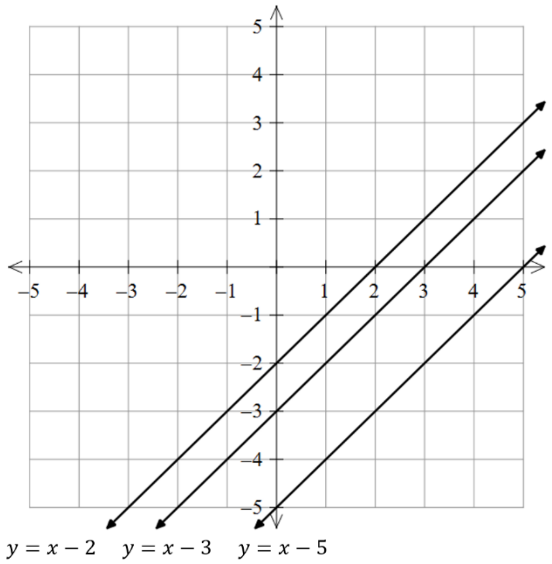
All these lines have a positive y-intercept.
Describe what this means.

The lines meet the y-axis above the x-axis.

What part of the equation tells you the y-intercept will be positive?

The constant terms are positive

e



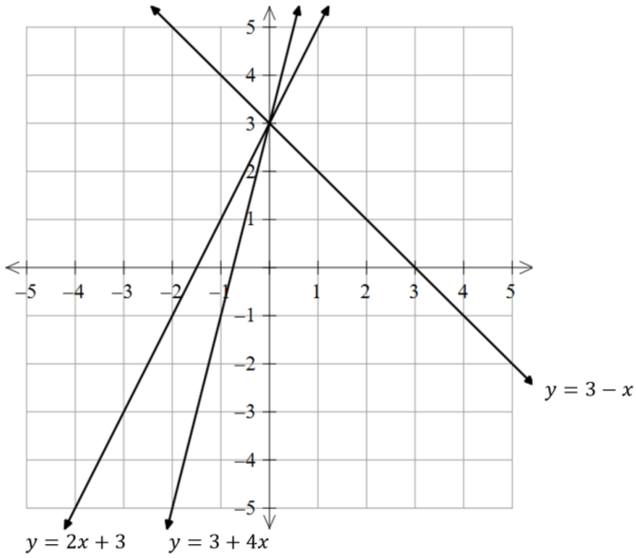
All these lines have a negative y-intercept.
Describe what this means.

The lines meet the y-axis below the x-axis.

What part of the equation tells you the y-intercept will be negative?

The constant terms are negative

f



All of the lines have the same y-intercept.
Explain what part of the equation tells you this.

The constant terms are the same

Write down the equation of any other line that has the same y-intercept.

Any other line in the form $y = mx + 3$

9 Consider the lines:

$$y = 2x + 3, \qquad y = 2x - 4, \qquad y = -x + 1, \quad y = 4x + 3$$

a Which lines are parallel?

$y = 2x + 3, y = 2x - 4.$

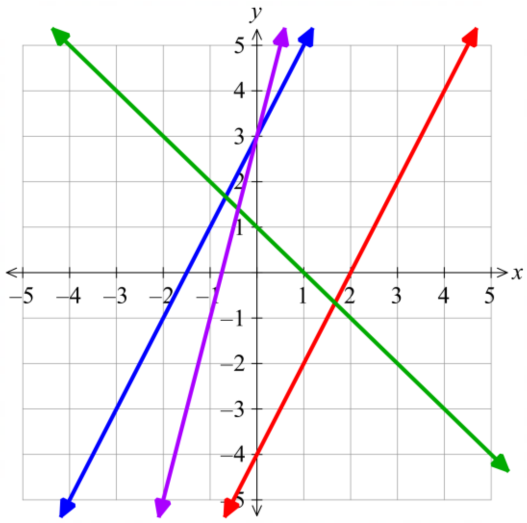
b Which lines are increasing?

$y = 2x + 3, y = 2x - 4, y = 4x + 3$

c Which lines have the same y-intercept?

$y = 2x + 3, y = 4x + 3$

d Label the four lines shown using the equations above.



Green: $y = -x + 1$

Blue: $y = 2x + 3$

Purple: $y = 4x + 3$

Red: $y = 2x - 4$

10 Consider the graph of $\boldsymbol{y = x + 2}$. Using DESMOS, or the rules, describe what is the same and what is different between that graph and these graphs.

Comparison	Same	Different
$y = x + 2$ $y = x + 1$	Gradient	y-intercept
$y = x + 2$ $y = 2x + 2$	y-intercept	Gradient
$y = x + 2$ $y = -x + 2$	y-intercept	Gradient
$y = x + 2$ $y = x$	Gradient	y-intercept
$y = x + 2$ $y = x - 2$	Gradient	y-intercept

CHECK YOUR UNDERSTANDING

Construct table of values

1 $y = 3x$ What should replace the star?

x	0
y	★

a 3 **b** 0 **c** 30 **d** 1

2 $y = 2 + 3x$ What should replace the star?

x	9
y	★

a 45 **b** 14 **c** 41 **d** 29

3 $y = 2 + 3x$ What should replace the star?

x	-9
y	★

a -45 **b** -25 **c** -37 **d** -29

4 $y = 2 + 3x$ What should replace the star?

x	0
y	★

a 2 **b** 3 **c** 5 **d** 32

Finding the rule

1 The equation of the relationship below is $y = mx + c$

x	0	1	2	3	4
y	3	7	10	13	16

What is the value of m?

a 7 **b** 5 **c** 3 **d** 4

What is the value of c?

a 7 **b** 5 **c** 3 **d** 4

2 The equation of the relationship below is $y = mx + c$

x	0	1	2	3	4
y	26	20	14	8	2

What is the value of m?

a 5 **b** 6 **c** 26 **d** -6

What is the value of c?

a 5 **b** 6 **c** 26 **d** -6

3 Which equation represents this relationship?

x	0	1	2	3	4
y	−1	4	9	14	19

a $y = 5x + 1$ **b** $y = x + 5$ **c** $y = 5x - 1$ **d** $y = 5x + 4$

4 Which equation represents this relationship?

x	0	1	2	3	4
y	6	5	4	3	2

a $y = 5x$ **b** $y = -5x$ **c** $y = 5 - x$ **d** $y = 6 - x$

5 Which equation represents this relationship?

x	1	2	3	4
y	9	14	19	24

a $y = 5x + 4$ **b** $y = 5x + 5$ **c** $y = 5x + 6$ **d** $y = 5x + 9$

6 Which equation represents this relationship?

x	1	2	3	4
y	98	92	86	80

a $y = -6x + 104$ **b** $y = -6x + 92$ **c** $y = -6x$ **d** $y = -6x + 92$