

Student Name

Mathematics Stage 5 Paths

ALGEBRAIC TECHNIQUES C

Book 1 Operate with algebraic fractions involving
binomial numerators and numerical
denominators

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OVERVIEW

SYLLABUS CONTENT

MA5-ALG-P-02 selects and applies appropriate algebraic techniques to operate with algebraic fractions, and expands, factorises and simplifies algebraic expressions

Operate with algebraic fractions involving binomial numerators and numerical denominators

- Add and subtract algebraic fractions with binomial numerators and numerical denominators

REVIEW OF PRIOR KNOWLEDGE

1 Expand and simplify:

a $3 + 2(x + 4)$

b $2 + 5(3x - 1)$

c $3 + 4(x - 2)$

d $3(x + 2) + 2(x + 3)$

e $3(x + 2) + 2(x - 3)$

f $p(p + 2) + 5(p + 3)$

g $5(y + 5) + 3(2 + y)$

h $5(3m + 2) + 4m$

i $3(1 - 5e) + 6e$

2 Expand and simplify:

a $-2(x + 3)$

b $-5(2x - 7)$

c $-(3x - 4)$

d $3 - 4(x - 2)$

e $1 - 5(x + 4)$

f $5 - 2(x - 6)$

g $9 - (x - 3)$

h $5 - (3 + 2x)$

i $4w - 2(5 + 2w)$

j $3(2d + 3) - 5(d + 4)$

k $2a - 4(3 - a)$

l $3(x + 2) - 2(x - 3)$

ADDING FRACTIONS WITH BINOMIAL NUMERATORS

EQUIVALENT FRACTIONS WITH BINOMIAL NUMERATORS

When creating equivalent algebraic fractions:

- Multiply both numerator and denominator by the same factor.
- Remember to multiply *each* term in the numerator by your factor.

Equivalent Fractions
$\frac{a+b}{c} \times \frac{n}{n} = \frac{n(a+b)}{cn}$

Example Produce equivalent fractions involving binomial numerator.

$\frac{4x+7}{3} = \frac{\quad}{15}$ $= \frac{4x+7 \times 5}{3 \times 5}$ $= \frac{5(4x+7)}{15}$ $= \frac{20x+35}{15}$	$\frac{3x+9}{18} = \frac{\quad}{6}$ $= \frac{3x+9 \div 3}{18 \div 3}$ $= \frac{x+3}{6}$
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Determine the equivalent fraction.

a
$$\frac{2x+1}{3} = \frac{\quad}{12}$$

$$= \frac{2x+1 \times 4}{3 \times 4}$$

b
$$\frac{5x-8}{2} = \frac{\quad}{6}$$

c
$$\frac{6x-8}{4} = \frac{\quad}{2}$$

d
$$\frac{12x+14}{6} = \frac{\quad}{3}$$

e
$$\frac{11x+2}{4} = \frac{\quad}{-12}$$

f
$$\frac{3x-20}{6} = \frac{\quad}{30}$$

$$\frac{6x+7}{3}$$

$$\frac{-33x-6}{-12}$$

$$\frac{15x-100}{30}$$

Key ideas

1. When making equivalent fractions, if there is more than one term in the numerator, we need to multiply term.

ADDING FRACTIONS WITH BINOMIAL NUMERATORS

1. Rewrite the fractions with the **lowest common multiple** of the denominators.
2. Add the numerators.
3. Expand and simplify.

Example Add algebraic fractions with binomial numerators.

1. same denominators $\frac{y+1}{5} + \frac{2y}{3}$ $\frac{3(y+1)}{15} + \frac{5(2y)}{15}$ 2. add numerators $\frac{3(y+1)+5(2y)}{15}$ 3. simplify $\frac{3y+3+10y}{15}$ $= \frac{13y+3}{15}$	Why did we make the denominators 15? 15 is the of 3 and 5 How did we change denominator of $\frac{y+1}{5}$ to 15? We the numerator and denominator by
1. same denominators $\frac{x+3}{2} + \frac{x-2}{5}$ $\frac{5(x+3)}{10} + \frac{2(x-2)}{10}$ 2. add numerators $\frac{5(x+3)+2(x-2)}{10}$ 3. simplify $\frac{5x+15+2x-4}{10}$ $= \frac{7x+11}{10}$	How do we know what number to change the denominator to? Change the denominator to the of the denominators. What do we need to remember when we have binomial numerators? each term.

Simplify these expressions.

a $\frac{x+6}{2} + \frac{x}{5}$

$$= \frac{(\quad)}{10} + \frac{\quad}{10}$$

b $\frac{x+3}{3} + \frac{x}{4}$

$$= \frac{(\quad)}{12} + \frac{\quad}{12}$$

c $\frac{2x+5}{5} + \frac{x}{10}$

d $\frac{x+1}{2} + \frac{x+3}{5}$

$$\frac{7x+30}{10}$$

e $\frac{y+4}{5} + \frac{y-3}{6}$

$$\frac{7x+12}{12}$$

f $\frac{2x-3}{6} + \frac{x+2}{8}$

$$\frac{x+2}{2}$$

$$\frac{7x+11}{10}$$

$$\frac{11y+9}{30}$$

$$\frac{11x-6}{24}$$

1 Complete these simplifications.

a

$$\frac{x}{4} + \frac{2x}{3}$$

$$= \frac{\square}{12} + \frac{\square}{12}$$

$$= \frac{\square}{12}$$

$$\frac{11x}{12}$$

b

$$\frac{5x}{7} - \frac{2x}{5}$$

$$= \frac{\square}{35} - \frac{\square}{35}$$

$$= \frac{\square}{35}$$

$$\frac{11x}{35}$$

c

$$\frac{x+1}{2} + \frac{2x+3}{4}$$

$$= \frac{\square(x+1)}{4} + \frac{2x+3}{4}$$

$$= \frac{\square + 2x + 3}{4}$$

$$= \frac{\square}{4}$$

$$\frac{4x+5}{4}$$

2 Simplify these algebraic fractions.

a

$$\frac{m}{3} + \frac{m+5}{4}$$

$$\frac{7m+15}{12}$$

b

$$\frac{4p-3}{5} - \frac{2p}{7}$$

$$\frac{18p-21}{35}$$

c

$$\frac{p}{3} + \frac{p+2}{4}$$

$$\frac{7p+6}{12}$$

d

$$\frac{x}{2} + \frac{3x}{4}$$

$$\frac{5x}{4}$$

e

$$\frac{y+1}{5} + \frac{2y}{3}$$

$$\frac{13y+3}{15}$$

f

$$\frac{a+2}{3} - \frac{a}{4}$$

$$\frac{a+8}{12}$$

g

$$\frac{k+4}{3} + \frac{k-2}{5}$$

$$\frac{8k+14}{15}$$

h

$$\frac{x+2}{3} + \frac{x+1}{4}$$

$$\frac{7x+11}{12}$$

i

$$\frac{x-3}{4} + \frac{x+2}{2}$$

$$\frac{3x+1}{4}$$

j

$$\frac{2x+1}{2} + \frac{x-2}{3}$$

$$\frac{8x-1}{6}$$

k

$$\frac{3y-5}{4} + \frac{2y-7}{10}$$

$$\frac{19y-39}{20}$$

l

$$\frac{2x+3y}{6} + \frac{5x-2y}{12}$$

$$\frac{9x+4y}{12}$$

m

$$\frac{5x+3}{14} + \frac{2x-2}{3}$$

$$\frac{43x-19}{42}$$

n

$$\frac{3-x}{14} + \frac{x-1}{7}$$

$$\frac{x+1}{14}$$

o

$$\frac{2x-3y}{8} + \frac{2y+x}{7}$$

$$\frac{22x-5y}{56}$$

3 Describe the mistake and provide the correct solution.

a

$$\frac{x+1}{5} + \frac{x}{2} = \frac{2x+1}{10} + \frac{5x}{10}$$

$$= \frac{7x+1}{10} \quad \text{✗}$$

Mistake:

Correct solution:

$$\frac{7x+2}{10}$$

b

$$\frac{5x}{3} + \frac{x-1}{2} = \frac{10x}{6} + \frac{3x-1}{6}$$

$$= \frac{13x-1}{6} \quad \text{✗}$$

Mistake:

Correct solution:

$$\frac{13x-3}{6}$$

4 A student thinks that the common denominator to use when simplifying $\frac{x+1}{2} + \frac{2x-1}{4}$ is 8.

a Complete the addition by making both denominators 8.

b Complete the simplification using the actual LCD of 4.

c You should have the same answer for both methods. Which do you prefer and why?

.....

5 Simplify these algebraic fractions.

a

$$\frac{2x}{5} - \frac{3x}{2} - \frac{x}{3}$$

b

$$\frac{x}{4} - \frac{2x}{3} + \frac{5x}{6}$$

c

$$\frac{5x}{8} - \frac{5x}{6} + \frac{3x}{4}$$

d

$$\frac{x+1}{4} + \frac{2x-1}{3} - \frac{x}{5}$$

$$\frac{-43x}{30}$$

$$\frac{43x-5}{60}$$

e

$$\frac{2x-1}{3} - \frac{2x}{7} + \frac{x-3}{6}$$

$$\frac{5x}{12}$$

$$\frac{23x-35}{42}$$

f

$$\frac{1-2x}{5} - \frac{3x}{8} + \frac{3x+1}{2}$$

$$\frac{13x}{24}$$

$$\frac{28+29x}{40}$$

WORKING WITH NEGATIVE FRACTIONS

- A negative symbol in line with the fraction bar has several possible interpretations; you can expand the negative into either the numerator or denominator, but not both.

Negative Fractions
$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$
$-\frac{a+b}{c} = \frac{-a-b}{c} = \frac{a+b}{-c}$

Example Working with negative fractions

Write as many equivalent fractions as possible.

a $-\frac{3}{5} =$

b $-\frac{x+y}{3} =$

c $-\frac{x-y}{3} =$

d $\frac{x-y}{3} =$

e $-\frac{x+y}{a+b} =$

f $-\frac{x-y}{a+b} =$

g $-\frac{x+y}{a-b} =$

h $-\frac{x-y}{a-b} =$

True or false?

i $\frac{a+b}{c} = \frac{-a-b}{-c}$

j $\frac{a+b}{-c} = \frac{-a+b}{c}$

k $-\frac{a+b}{c} = \frac{-(a+b)}{c}$

l $-\frac{a-b}{c} = \frac{b-a}{c}$

m $-\frac{a+b}{c} = \frac{-a-b}{-c}$

n $\frac{a-b}{c-d} = \frac{b-a}{d-c}$

SUBTRACTING FRACTIONS WITH BINOMIAL NUMERATORS

1. Rewrite the fractions with the lowest common multiple of the denominators.
2. Subtract the numerator.
3. Expand and simplify.

Example Subtract algebraic fractions with binomial numerators.

1. same denominators $\frac{x-1}{3} - \frac{x+4}{5}$ $\frac{5(x-1)}{15} - \frac{3(x+4)}{15}$ 2. subtract $\frac{5(x-1)-3(x+4)}{15}$ 3. simplify $\frac{5x-5-3x-12}{15}$ $= \frac{2x-17}{15}$	In the third step, explain why we have $-3x - 12$. <i>We are subtracting every of the second fraction, so we each term of $x + 4$ by</i>
1. same denominators $\frac{2x-3}{6} - \frac{3-x}{5}$ $\frac{5(2x-3)}{30} - \frac{6(3-x)}{30}$ 2. subtract $\frac{5(2x-3)-6(3-x)}{30}$ 3. simplify $\frac{10x-15-18+6x}{30}$ $= \frac{16x-33}{30}$	In the third step, why do we have $-18 + 6x$? <i>Expanding $-6(3 - x) =$</i>

Simplify these expressions.

a

$$\frac{x-4}{2} - \frac{x+2}{3}$$

$$= \frac{3(\quad)}{6} - \frac{2(\quad)}{6}$$

$$= \frac{3(\quad) - 2(\quad)}{6}$$

$$\frac{x-16}{6}$$

b

$$\frac{x+1}{2} - \frac{2x+3}{5}$$

$$\frac{x-1}{10}$$

c

$$\frac{2x+1}{4} - \frac{2-x}{3}$$

$$\frac{10x-5}{12}$$

Key ideas

1. When subtracting fractions, remember to take the of each term in the numerator.

1 Simplify these algebraic fractions.

a $\frac{9w}{10} - \frac{w+5}{3}$

$$\frac{17w - 50}{30}$$

b $\frac{5a}{9} - \frac{2a+5}{6}$

$$\frac{4a - 15}{18}$$

c $\frac{2a}{6} - \frac{a-5}{8}$

$$\frac{5a + 15}{24}$$

d $\frac{x+3}{4} - \frac{x+2}{3}$

e $\frac{3-x}{5} - \frac{x+4}{2}$

f $\frac{5x-1}{4} - \frac{2+x}{8}$

$$\frac{-x + 1}{12}$$

$$\frac{-7x - 14}{10}$$

$$\frac{9x - 4}{8}$$

g $\frac{1+3x}{4} - \frac{2x+3}{6}$

h $\frac{x+5}{5} - \frac{x-1}{2}$

i $\frac{5x-9}{7} - \frac{2-x}{3}$

$$\frac{5x - 3}{12}$$

$$\frac{-3x + 15}{10}$$

$$\frac{22x - 41}{21}$$

j $\frac{4x+3}{3} - \frac{5-2x}{9}$

k $\frac{2x-1}{4} - \frac{1-3x}{14}$

l $\frac{3x-2}{8} - \frac{4x-3}{7}$

$$\frac{14x + 4}{9}$$

$$\frac{20x - 9}{28}$$

$$\frac{-11x + 10}{56}$$

2 Read the example carefully and answer the questions.

$$\begin{aligned} \frac{7x}{2} - \frac{x-2}{5} &= \frac{35x}{10} - \frac{2(x-2)}{10} \\ &= \frac{35x - 2x - 4}{10} \\ &= \frac{33x - 4}{10} \end{aligned}$$



Explain why $-2x - 4$ is incorrect.

$$-2(x-2) = -2x + 4$$

Hence simplify the fractions correctly.

$$\frac{33x + 4}{10}$$

3 Simplify these algebraic fractions.

a $\frac{x+1}{2} + \frac{x+3}{5}$

b $\frac{x+3}{3} + \frac{x-4}{4}$

c $\frac{a-2}{7} + \frac{a-5}{8}$

$$\frac{7x+11}{10}$$

$$\frac{7x}{12}$$

$$\frac{15a-51}{56}$$

d $\frac{y+4}{5} + \frac{y-3}{6}$

e $\frac{m-4}{8} + \frac{m+6}{5}$

f $\frac{x-2}{12} + \frac{x-3}{8}$

$$\frac{11y+9}{30}$$

$$\frac{13m+28}{40}$$

$$\frac{5x-13}{24}$$

g $\frac{y+4}{5} - \frac{y-3}{6}$

h $\frac{m-4}{8} - \frac{m+6}{5}$

i $\frac{x-2}{12} - \frac{x-3}{8}$

$$\frac{y+41}{30}$$

$$\frac{-3m-68}{40}$$

$$\frac{-x+5}{24}$$

j $\frac{2b-3}{6} + \frac{b+2}{8}$

k $\frac{3x+8}{6} + \frac{2x-4}{3}$

l $\frac{2y-5}{7} + \frac{3y+2}{14}$

$$\frac{11b-6}{24}$$

$$\frac{7x}{6}$$

$$\frac{7y-8}{14}$$

m $\frac{2t-1}{8} + \frac{t-2}{16}$

n $\frac{2x+1}{2} - \frac{x-2}{3}$

o $\frac{3x+1}{10} + \frac{2x+1}{10}$

$$\frac{5t-4}{16}$$

$$\frac{4x+7}{6}$$

$$\frac{5x+2}{10}$$

p $\frac{5x+3}{10} + \frac{2x-2}{4}$

q $\frac{4-x}{3} + \frac{2-x}{7}$

r $\frac{3-x}{14} - \frac{x-1}{7}$

$$\frac{5x-1}{5}$$

$$\frac{34-10x}{21}$$

$$\frac{5-3x}{14}$$

4 Simplify these algebraic fractions.

a $\frac{3(a+b)}{4} - \frac{a-b}{6}$

b $\frac{3x+2}{3x} + \frac{7}{12}$

c $\frac{2x-1}{4} + \frac{2-3x}{10x}$

d $\frac{a+1}{6a} + \frac{a-4}{2a}$

e $\frac{7}{12} - \frac{3x+2}{3x}$

f $\frac{6b+7}{4b} - \frac{3b-2}{5b}$

g $\frac{8c+6}{5c} + \frac{7c-3}{6c^2}$

h $\frac{9d-4}{6d} - \frac{2d+5}{d^2}$

i $\frac{2x+3}{2x} - \frac{1-3x}{x^2}$

j $\frac{4a-2}{3a^2} + \frac{5}{4a^3}$

k $\frac{8c+6}{5c^3} + \frac{7c-3}{6c^2}$

l $\frac{4-d}{6d^2} - \frac{9d-4}{7d^3}$

$\frac{16a^2 - 8a + 15}{12a^3}$

$\frac{35c^2 + 33c + 36}{30c^2}$

$\frac{-7d^2 - 26d + 24}{42d^3}$