

Student Name

Mathematics Stage 5 Paths

VARIATION & RATES OF CHANGE A

Book 1 Identify and describe problems involving direct
and inverse variation

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Direct and Inverse Variation2

OVERVIEW

SYLLABUS CONTENT

MA5-RAT-P-01 identifies and solves problems involving direct and inverse variation and their graphical representations

Identify and describe problems involving direct and inverse variation

- Describe typical examples of direct variation/proportion
- Apply the language of direct variation to everyday contexts: y is directly proportional to x , y is proportional to x , y varies directly as x
- Identify and represent direct variation/proportion as $y \propto x$ (y is proportional to x) or $y = kx$, where k is the constant of variation
- Describe typical examples of inverse (indirect) variation
- Apply the language of inverse variation to everyday contexts: y is inversely proportional to x , y is proportional to the reciprocal of x , y varies inversely as x
- Identify and represent inverse variation/proportion as $y \propto \frac{1}{x}$ (y is inversely proportional to x) or $y = \frac{k}{x}$, where k is the constant of variation

REVIEW OF PRIOR KNOWLEDGE

DIRECT AND INVERSE VARIATION

- Two quantities are **directly proportional** if:
 - An **increase** in one quantity causes a proportional **increase** in the other.
 - If you triple one quantity, the second quantity is also tripled.
 - A **decrease** in one quantity causes a proportional **decrease** in the other.
 - If you halve one quantity, the other quantity is also halved.
- Two quantities are **inversely proportional** if:
 - An **increase** in one quantity causes a proportional **decrease** in the other.
 - If you triple one quantity, the second quantity is divided by 3.
 - A **decrease** in one quantity causes a proportional **increase** in the other.
 - If you halve one quantity, the second quantity is doubled.

Example Identify examples of direct and inverse proportion.

Determine whether these variables are in direct proportion, inverse proportion, or unrelated.	
a The numbers of hours worked, and the amount of money earned for a wage worker.	b The weight of tomatoes purchased and the total amount paid.
c The number of painters and the time it takes to paint a room.	d The speed of a car and the time taken to travel a certain distance.
e A person's height and the number of pets owned.	f The side length of a square and the perimeter of the square.
g The height of a girl and her performance in mathematics.	h The flow rate of water from a tap and the time taken to fill a bucket.
i The number of nights at a hotel and the total cost of the stay	j The amount of money borrowed and the amount of simple interest.
k The number of people digging and the time taken to dig a hole.	l The number of flowers in a room and the number of pens.

NOTATION AND LANGUAGE OF DIRECT VARIATION

Direct Variation	Inverse Variation
$y = kx$	$y = \frac{k}{x}$
$y \propto x$	$y \propto \frac{1}{x}$
y is proportional to x	y is proportional to $\frac{1}{x}$
y is directly proportional to x	y is inversely proportional to x
y varies directly with x	y varies inversely with x

Example Identify direct variation.

All direct variation equations are in the form $y = kx$.

Identify which of the following relationships show direct variation.

a $W = 15t$

b $P = 3s$

c $P = \frac{3}{s}$

d $y = 0.5x$

e $y = \frac{1}{2}x$

f $y = \frac{x}{2}$

g $C = 100 + 5n$

h $y = 3x + 5$

i $C = 3.4 + 2.45d$

j $n = 4.65g$

k $h = \frac{3x}{2}$

l $l = \frac{3}{2x}$

Example Identify inverse variation.

All inverse variation equations are in the form $y = \frac{k}{x}$.

Identify which of the following relationships show inverse variation.

a $W = \frac{15}{t}$

b $P = \frac{t}{15}$

c $P = \frac{3}{s}$

d $y = \frac{x}{2}$

e $y = \frac{0.5}{x}$

f $y = \frac{1}{2x}$

g $C = \frac{1}{2} + \frac{1}{x}$

h $y = \frac{3}{x} - 5$

i $C = \frac{5924}{x}$

j $n = -\frac{15}{g}$

k $h = \frac{3}{2x}$

l $l = \frac{3}{x}$

1 Decide if these pairs of variables are in direct proportion (D) or inverse proportion (I).

a Wage earned and
Number of hours worked.

D

b Height of a prism and volume.

D

c Time to complete a job and
Number of helpers.

I

d Speed and Time.

I

e Distance and Time.

D

f Distance and Speed.

D

g Revenue and Number of sales.

D

h Time to paint a house and
Number of painters.

I

2 Are the variables in these scenarios examples of direct variation, inverse variation, or neither?

a The equation for the perimeter of an
equilateral triangle is $P = 3x$,
where x represents the side length.

D

b The equation for the circumference of a
circle is $P = 2\pi r$,
where r represents the radius.

D

c The equation for the distance travelled at
a speed of 60 km/h is $d = 60t$,
where t is time in hours.

D

d The equation for the time required to
travel 60 km is $t = \frac{60}{s}$,
where s is speed in km/h

I

e The number of days required for a
construction job to be completed is
 $D = \frac{1569}{W}$,
where W is the number of workers.

I

f The total amount of money paid for
potatoes is $C = 1.54m$,
where m is mass of the potatoes in kg.

D

g The amount of time required to fill a
bucket of water is $t = \frac{20}{r}$,
where r is the flow rate of water.

I

h The cost of a taxi ride is $C = 15 + 3d$,
where d is the of kilometres travelled.

N

i The cost per person of hiring a yacht for
a party is $C = \frac{2000}{n}$,
where n is the number of people.

I

j The volume of a solid is $V = 45h$,
where h is the perpendicular height.

D

- 3** When a wage worker works for 1 hour, they earn \$30.
- a** If they work 2 hours, they will earn \$.....
 - b** If they work 3 hours, they will earn \$.....
 - c** This is an example of variation.

\$60, \$90, direct

- 4** \$10 AUD can buy ¥800 Japanese Yen.
- a** If you have \$30, this is ¥..... Japanese Yen.
 - b** If you have \$5, this is ¥..... Japanese Yen.
 - c** If you have \$....., this is ¥7200 Japanese Yen

¥2400, ¥400, \$90

- 5** It takes 1 farmer 24 days to harvest a crop.
- a** If there are 2 workers, the job can be completed in days.
 - b** If there are 3 workers, the job can be completed in days.
 - c** This is an example of variation.

12, 8, inverse

- 6** 6 hose pipes are used to fill a swimming pool in 3 hours.
- a** 12 hose pipes would take hours.
 - b** 1 hose pipe would take hours.
 - c** hose pipes would take 6 hours.

1.5, 18, 3

- 7** A plane cruising at constant speed takes 2 hours to travel 1600 km.
- a** If the plane cruises for 4 hours, it would travel km.
 - b** If the plane cruises for 10 hours, it would travel km.
 - c** If the plane cruises for hours, it would travel 4800 km.

3200, 8000, 6

- 8** A car driving at a constant at 60 km/hr takes 6 hours to complete a journey.
- a** If the car was driving at 120 km/hr, it would take hours to complete the journey.
 - b** If the car was driving at 30 km/hr, it would take hours to complete the journey.
 - c** If the car was driving at km/hr, it would take 24 hours to complete the journey.

3, 12, 15