Student Name

Mathematics Stage 5 Paths

VARIATION & RATES OF CHANGE A

Book 2 Identify and describe graphs involving direct and inverse variation

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-RAT-P-01 identifies and solves problems involving direct and inverse variation and their graphical representations

Identify and describe graphs involving direct and inverse variation

- Recognise and describe direct variation from graphs, noting that the graph of $y = kx$ is a straight line passing through the origin, with its gradient k being the constant of variation
- Recognise and describe inverse variation from graphs, noting that the graph of $y = \frac{k}{x}$ is a curve

REVIEW OF PRIOR KNOWLEDGE

GRAPHS OF DIRECT VARIATION

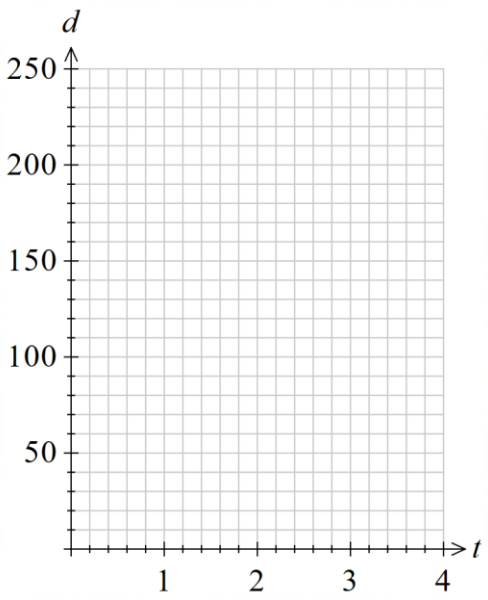
- Two quantities are **directly proportional** if:
  - An **increase** in one quantity causes a proportional **increase** in the other.
  - If you triple one quantity, the second quantity is also tripled.
  - A **decrease** in one quantity causes a proportional **decrease** in the other.
  - If you halve one quantity, the other quantity is also halved.

- The equation of a direct variation relationship is $y = kx$.
- A direct variation graph is a straight line passing through the origin.
- The constant of variation $\boldsymbol{k}$ is the **gradient**. It represents the **rate of $\boldsymbol{y}$ per $\boldsymbol{x}$**

Investigation Identify a direct variation relationship as a straight line graph.

A train is travelling at a constant speed of 75 km/h.
The distance d travelled in kilometres is directly proportional to time t in hours.
Complete the table of values and sketch the graph.

Time (h) t	0	1	2	3
Distance (km) d				



What type of graph is this?

This is a ……-line, going through the ……

Find the gradient, which is the constant of variation k.

$m = \frac{\text{rise}}{\text{run}} =$

What does k represent in this context?

…… per ……

Write an equation relating d and t:

$y = kx$

…… = ……

Key ideas

1. A graph of a direct variation relationship is a ……-line going through the ……
2. The gradient is the …… …… ……, k

DECIDING IF A DIRECT VARIATION RELATIONSHIP EXISTS

Constant of Variation (Direct)
$y = kx$ $\frac{y}{x} = k$ One quantity divided by the other will always be a constant value. This is the gradient.

Example Determine whether a direct variation relationship exists.

The distance d of a marathon runner over time t is shown on the table.
Determine if a direct variation relationship exists, if it does, write the equation $y = kx$.

x	30	108	156	204
y	5	18	26	34
$\frac{y}{x}$	$\frac{5}{30} = \frac{1}{6}$	$\frac{18}{108} = \frac{1}{6}$	$\frac{26}{156} = \frac{1}{6}$	$\frac{34}{204} = \frac{1}{6}$

$y = \frac{1}{6}x$

How did we determine that a direct variation relationship exists?

We found the ..... of $\frac{t}{d}$ for each column on the table and found that it was a ..... value.

How did we know the relationship is $y = \frac{1}{6}x$?

The constant of proportionality k is ....., using $y = kx$, the equation is .....

Determine if a direct variation relationship exists, if it does, write the equation $y = kx$.

a

x	15	25	35	70
y	30	50	70	140
$\frac{y}{x}$				

yes, $y = 2x$

b

x	5	9	12	14
y	7	11.2	19.6	26.6

no

c

x	1	2	3	4
y	19	16	11	4

no

d

m	5	12	14	18
C	18	43.2	50.4	64.8

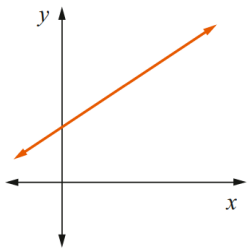
yes, $y = 3.6x$

Key ideas

1. Direct variation relationships are in the form $y =$ ....., this can be rearranged to $k =$ .....
2. We can check if two quantities are in direct variation by dividing one quantity by the other and see if the ratio is ..... for all pairs of values.

1 Which of the graphs show a direct variation relationship?

A



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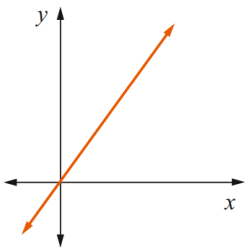

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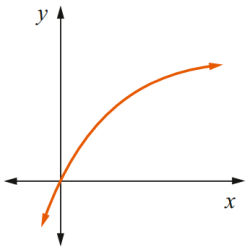

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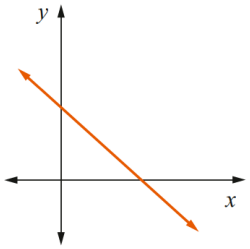


B

2 Decide whether x and y are in direct proportion. If they are, write the equation $y = kx$.

a

x	1	2	5
y	4	8	21
$\frac{x}{y}$			

no

b

x	2	3	6
y	14	21	42

yes

c

x	-4	-1	3
y	-8	-2	6

yes

d

x	5	30	92
y	12	72	220.8

yes

e

x	3	10	23
y	7	21	70

no

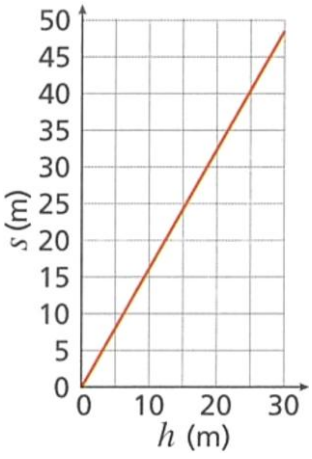
f

x	8	11	36
y	11	27.5	90

no

3 The table shows the lengths of the shadows of three trees.

Height of tree (h m)	5	15	20
Length of shadow (s m)	8	24	32



a Calculate the ratio $\frac{s}{h}$ for each tree.

1.6

b Explain how you know the length of the shadow is proportional to the height of the tree by referring to:

i the graph

straight line passing through origin

ii the calculations on table

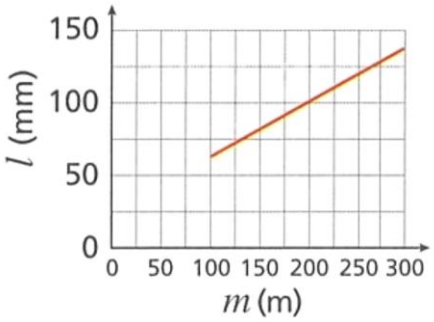
$\frac{s}{h}$ is a constant value, 1.6

c Write the equation $y = kx$ showing the relationship between s and h .....

$s = 1.6h$

4 In a science experiment, different masses are attached to the end of a spring and the length of the spring is measured. The results are shown in the table.

Mass (m g)	100	150	200	250	300
Length of spring (l mm)	60	80	100	120	140



Is the length of the spring directly proportional to the mass of the weight?

no

Explain how you know:

a by extending the graph

when extended, the graph does not pass through the origin

b by doing calculations using the numbers in the table.

the ratio of $\frac{l}{m}$ is not constant

5 A 750ml bottle of water costs 80 c. A 2 litre bottle of water costs $2.
Show that the cost of water is not directly proportional to the amount of water in the bottle.

Using $\frac{\text{Cost (\$)}}{\text{Volume (L)}}$

$\frac{0.8}{0.75} = 1.0\dot{6}$

$\frac{2}{2} = 1$

GRAPHS OF INVERSE VARIATION

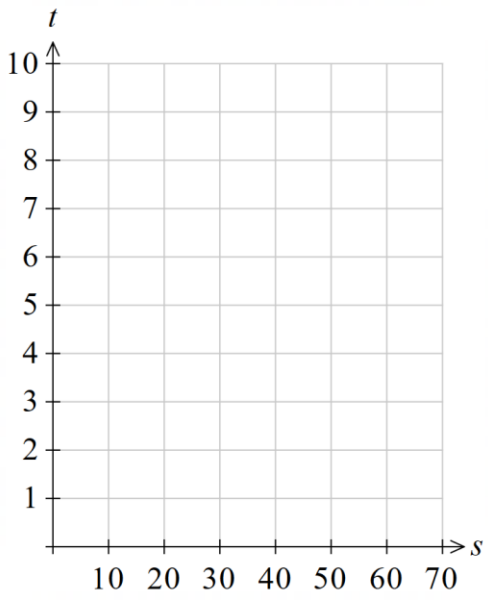
- Two quantities are **inversely proportional** if:
  - An **increase** in one quantity causes a proportional **decrease** in the other.
  - If you triple one quantity, the second quantity is divided by 3.
  - A **decrease** in one quantity causes a proportional **increase** in the other.
  - If you halve one quantity, the second quantity is doubled.

- The equation of an inverse variation relationship is $y = \frac{k}{x}$.
- An inverse variation graph is a smooth **curve**;
  a **hyperbolic decreasing** curve that never touches the axes.

Investigation Identify an inverse variation relationship as a curve.

A car needs to travel a journey of 60 km.
The time t required in hours is inversely proportional to speed s in km per hour.
Complete the table of values and sketch the graph.

Speed (km/h) s	0	10	20	30	60
Time (h) t					



What type of graph is this?

This is a h..... d..... c.....

Is there a point on the graph for $t = 0$?

Is there a point on the graph for $s = 0$?

Key ideas

1. Graphs of inverse variation relationships are a h..... d.....
   c.....
2. The graph of an inverse variation relationship never touches the .....

DECIDING IF AN INVERSE VARIATION RELATIONSHIP EXISTS

Constant of Variation (Inverse)
$y = \frac{k}{x}$ $xy = k$ One quantity multiplied by the other will always be a constant value.

Example Determine whether an inverse variation relationship exists.

The time t taken to travel a certain distance d is shown on the table.
Determine whether an inverse variation relationship exists.

s	1	2	5	10
t	60	30	12	6
st	$1 \times 60 = 60$	$2 \times 30 = 60$	$5 \times 12 = 60$	$10 \times 6 = 60$

How did we determine that a direct variation relationship exists?
We found the ..... of st for each column on the table and found that it was a ..... value.

Write an equation that describes the relationship between the variables.

Determine whether the variables in the tables given below are in inverse proportion.

a

x	1	2	4	6
y	48	24	12	8
xy				

yes

b

x	1	2	3	4
y	3	6	9	12

no

c

x	1	26	40
y	0.2	5.2	8

no

d

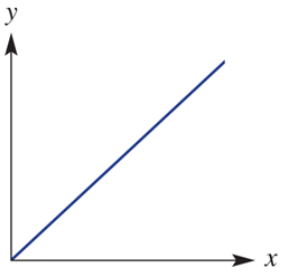
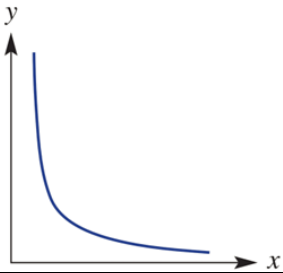
m	1	2	4	8
C	16	8	4	2

yes

Key ideas

1. Inverse variation relationships are in the form $xy = k$, where k is the ..... of proportionality.
2. To check whether two variables are in inverse variation, we can multiply one quantity by the other and see if the product is ..... for all pairs of values.

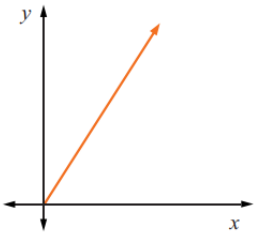
DISTINGUISHING DIRECT AND INVERSE VARIATION

Direct Variation	Inverse Variation
$y = kx$	$y = \frac{k}{x}$
Straight line passing through the origin.	A "hyperbolic" decreasing curve that doesn't touch the axes.
	

Example Identify whether a graph shows direct or inverse proportion

Identify these graphs as direct variation, inverse variation, or neither.

a



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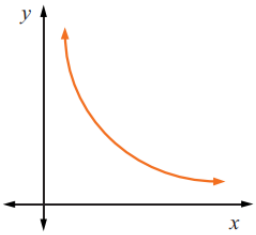
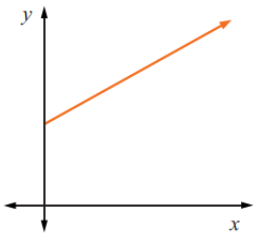


c



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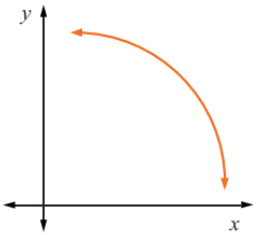

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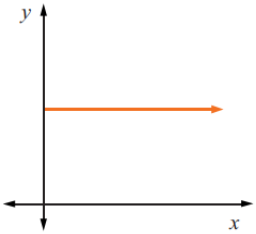

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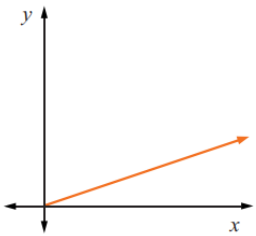

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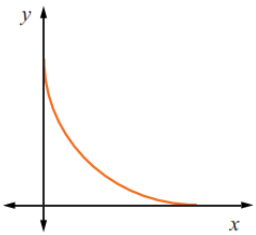
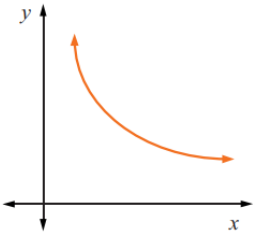
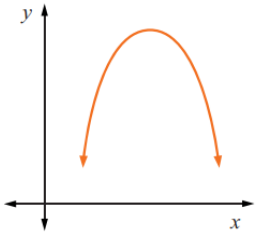


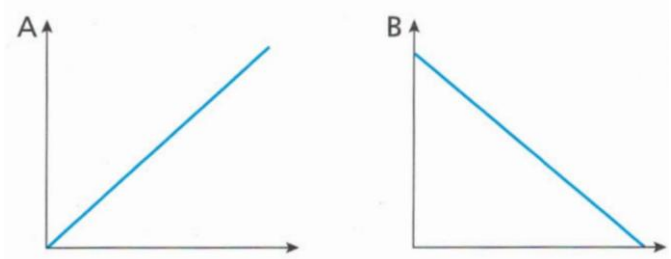
h



i



1 Ben says:
"If graph A represents direct proportion, then graph B represents inverse proportion."



Explain why he is incorrect.

.....

.....

Inverse variation is a hyperbolic curve.

2 The variables x and y are inversely proportional.

x	8	16		2.4
y	12		10	

a Express this relationship in the form $xy = k$, where k is the constant of proportionality.

$xy = 96$

b Complete the table.

6, 9.6, 40

3 The variables x and y are inversely proportional.

p	8		5	2
q	5	10		

a Express this relationship in the form $pq = k$, where k is the constant of proportionality.

$pq = 40$

b Complete the table.

4, 8, 20

4 State whether each table shows direct proportion, inverse proportion, or neither.

a

x	2	3	4
y	6	4	3

inverse

b

a	4	5	8
b	12	13	16

neither

c

p	1	5	10
q	1.6	8	16

direct