Student Name

Mathematics Stage 4

RIGHT-ANGLED TRIANGLES

Book 1 Identify and define Pythagoras' theorem

Version: 260207

Report mistakes & suggest improvements:
https://MrDingMaths.com

CONTENTS

OVERVIEW

SYLLABUS DOTPOINTS

MA4-PYT-C-01 applies Pythagoras' theorem to solve problems in various contexts

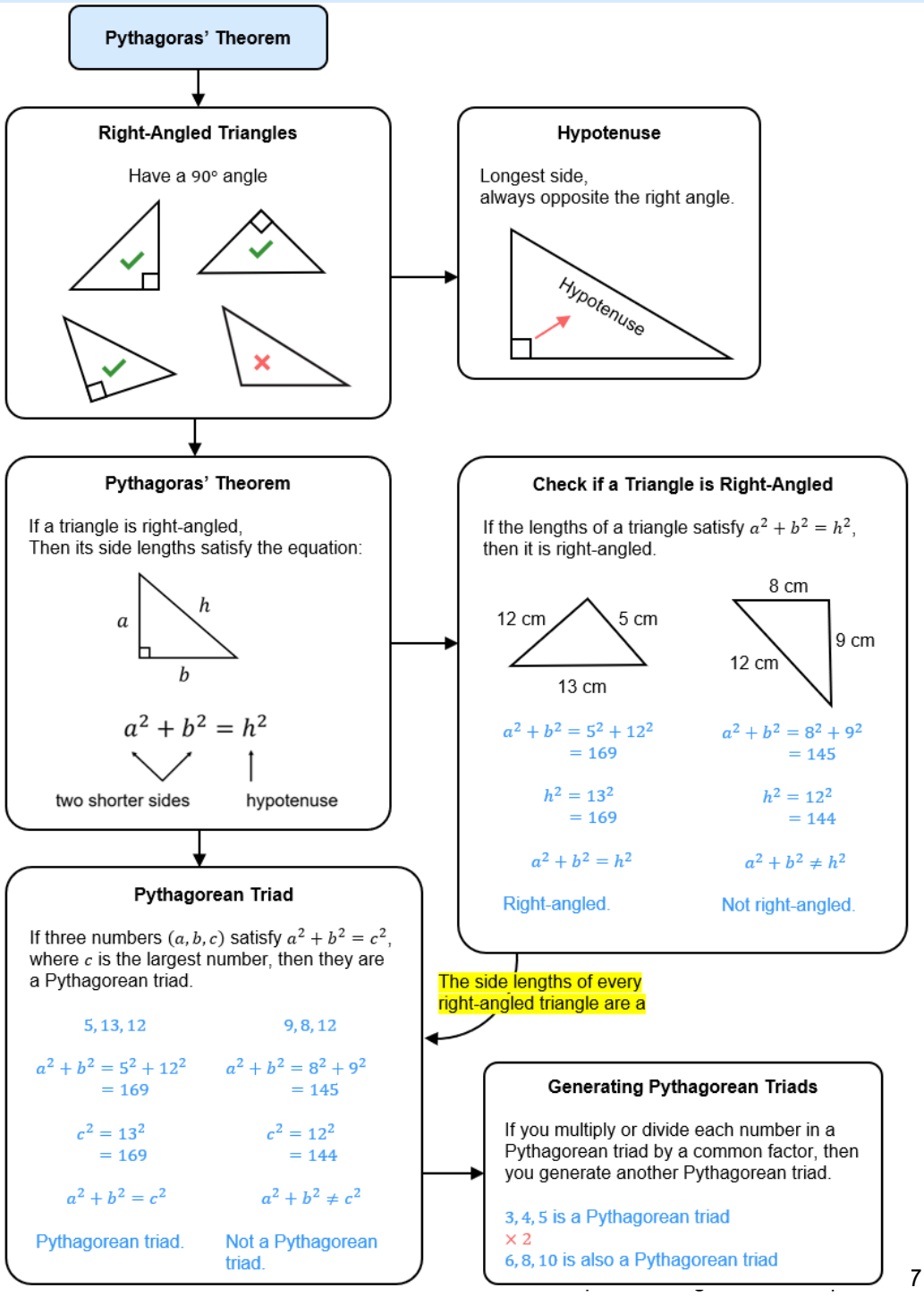
Identify and define Pythagoras' theorem

- Identify and describe the hypotenuse as the side opposite the right angle and the longest side in any right-angled triangle
- Establish the relationship between the lengths of the sides of a right-angled triangle
- Use the relationship to record and define Pythagoras' theorem both algebraically and in words

Examine problems involving Pythagoras' theorem

- Justify whether a set of 3 integers is a Pythagorean triad

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Square these numbers.

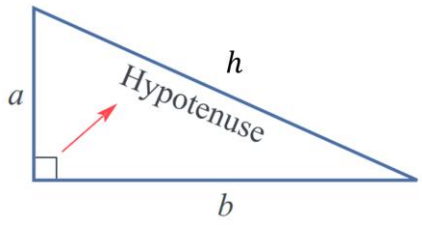
a	3^2	**b**	12^2	**c**	5^2	**d**	1.5^2

2 Evaluate these expressions.

a	$3^2 + 4^2$	**b**	$6^2 + 8^2$	**c**	$12^2 + 5^2$	**a**	$20^2 + 21^2$

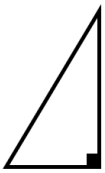
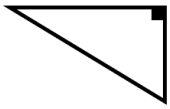
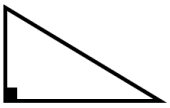
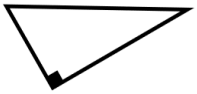
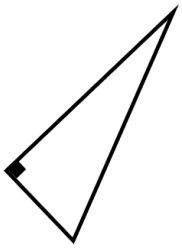
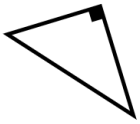
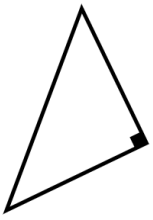
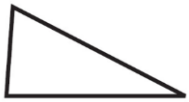
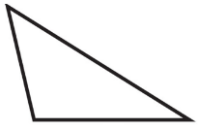
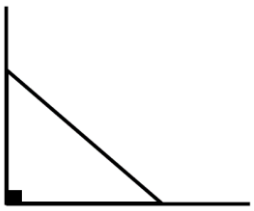
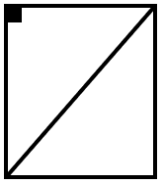
THE HYPOTENUSE

- A triangle with a **90-degree angle** is a **right-angled triangle**.
- In a right-angled triangle, we call the longest side opposite the right-angle the **hypotenuse**.



Example Identify the hypotenuse of a right-angled triangle.

Identify the hypotenuse in each diagram.

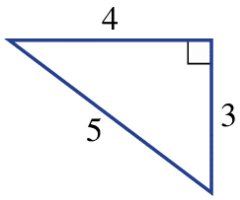
a 	**b** 	**c** 	**d** 
e 	**f** 	**g** 	**h** 
i 	**j** 	**k** 	**l** 

Key ideas

1. If a triangle has a 90° angle, then it is ……-……
2. If a triangle is right-angled, then we call the longest side opposite the right angle the ……

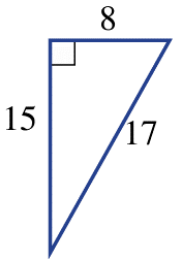
1 Identify the hypotenuse of these triangles.

a



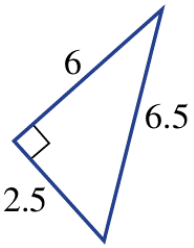
5

b



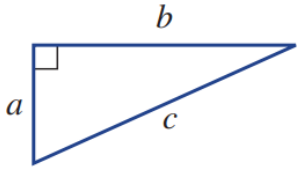
17

c



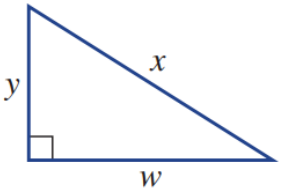
6.5

d



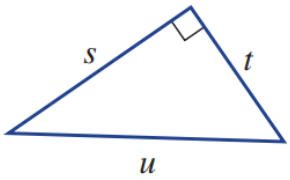
c

e



x

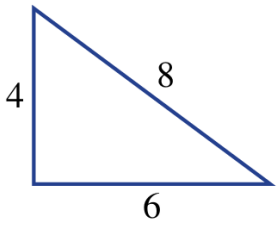
f



u

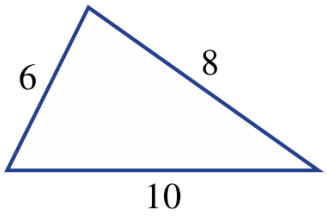
2 These triangles are right-angled. Show on the diagram where right angle would be.

a



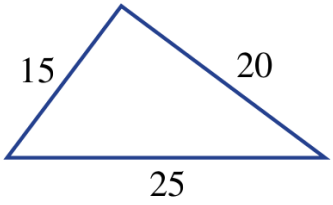
Between 4 and 6

b



Between 6 and 8

c



Between 15 and 20

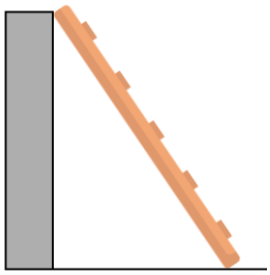
3 A ladder is leaning against a wall.

Which statement (A, B, or C) describes the hypotenuse?

A. The horizontal distance between the base of the ladder and the wall.

B. The vertical distance between the floor and the top of the ladder.

C. The length of the ladder.

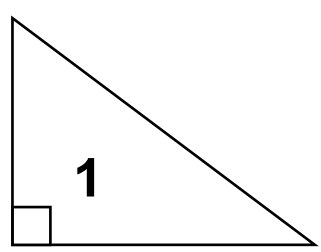


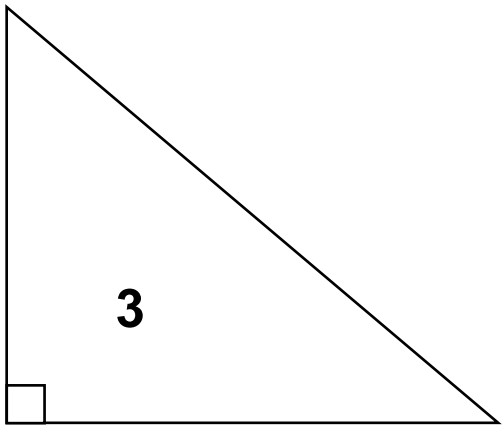
C

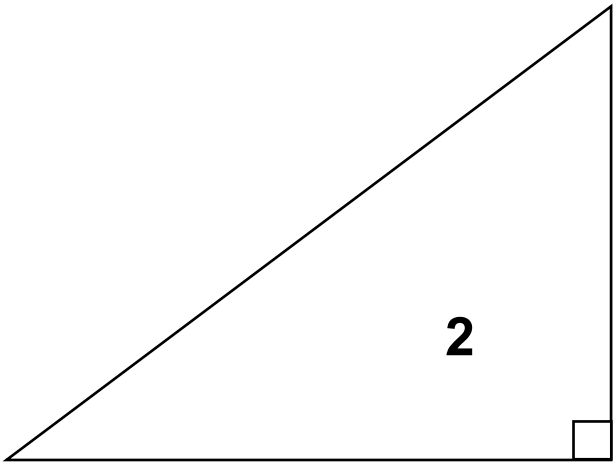
PYTHAGORAS' THEOREM

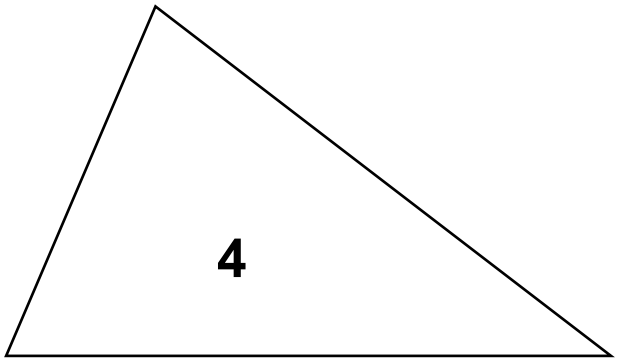
Investigation Pythagoras' Theorem.

Measure each side of these triangles and complete the table.









	Shorter sides		Longest Side				
Triangle	a	b	c	a^2	b^2	$a^2 + b^2$	c^2
1							
2							
3							
4							

For a right-angled triangle, c^2 = …… + ……

This relationship does not exist if the triangle is not …… - angled.

- Pythagoras' theorem: https://www.geogebra.org/m/bt27dew8
- Pythagoras' theorem visual proof: https://www.geogebra.org/m/t3zvxd9u

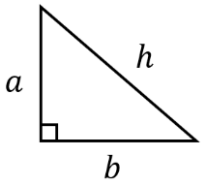
PYTHAGORAS' THEOREM

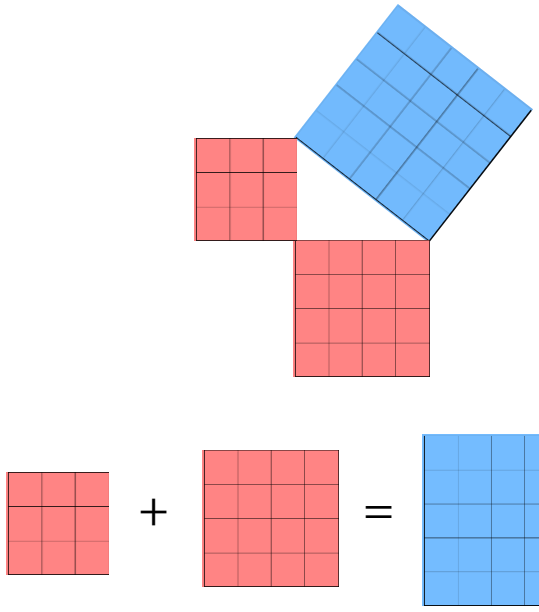
Pythagoras' Theorem

If a triangle is right-angled, then its side lengths satisfy the equation:

$$a^2 + b^2 = h^2$$

two shorter sides — hypotenuse





$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

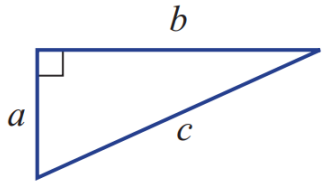
Key ideas

1. If a triangle is ……-……, its side lengths satisfy the equation: $a^2 + b^2 = h^2$, where a and b are the two …… sides, and h is the ……

1 Write an equation describing the relationship between the sides of these right-angled triangles.

$$\text{side}^2 + \text{side}^2 = \text{hypotenuse}^2$$

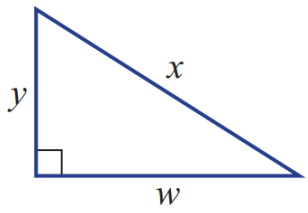
a



$\ldots\ldots^2 + \ldots\ldots^2 = \ldots\ldots^2$

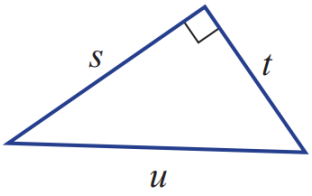
$a^2 + b^2 = c^2$

b



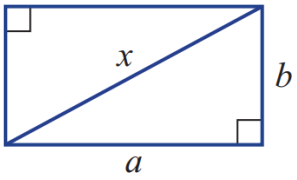
$w^2 + y^2 = x^2$

c



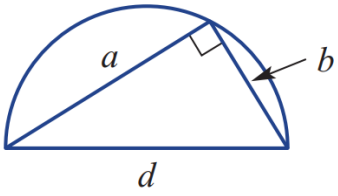
$s^2 + t^2 = u^2$

d



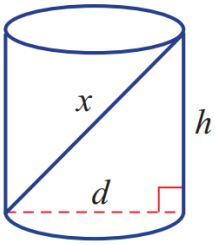
$a^2 + b^2 = x^2$

e



$a^2 + b^2 = d^2$

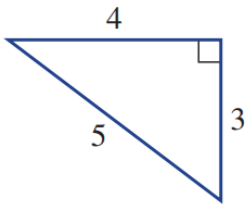
f



$d^2 + h^2 = x^2$

2 Check that $a^2 + b^2 = h^2$ for all these right-angled triangles.

a



$a^2 + b^2 = 3^2 + 4^2$

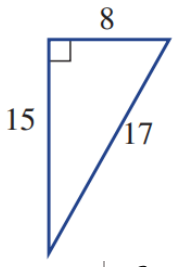
$= \ldots\ldots + \ldots\ldots$

$= \ldots\ldots$

$h^2 = 5^2$

$= \ldots\ldots$

b



$a^2 + b^2 = \ldots\ldots + \ldots\ldots$

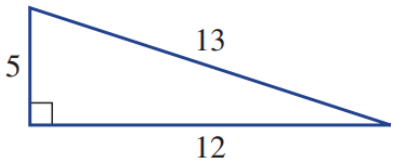
$= \ldots\ldots + \ldots\ldots$

$= \ldots\ldots$

$h^2 = \ldots\ldots$

$= \ldots\ldots$

c



$a^2 + b^2 = \ldots\ldots + \ldots\ldots$

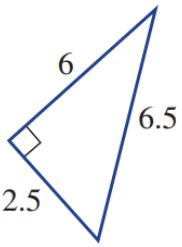
$= \ldots\ldots + \ldots\ldots$

$= \ldots\ldots$

$h^2 = \ldots\ldots$

$= \ldots\ldots$

d



$a^2 + b^2 = \ldots\ldots + \ldots\ldots$

$= \ldots\ldots + \ldots\ldots$

$= \ldots\ldots$

$h^2 = \ldots\ldots$

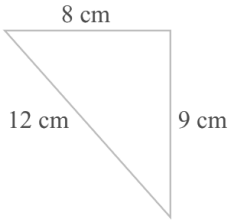
$= \ldots\ldots$

PROVE THAT A TRIANGLE IS RIGHT-ANGLED

- A triangle is **right-angled** if its side lengths satisfy $a^2 + b^2 = h^2$.
- If the triangle's side lengths do not satisfy the equation $a^2 + b^2 = h^2$, then it's not right-angled.

Example Prove whether a triangle is right-angled using Pythagoras' theorem.

Determine whether this triangle is right-angled.



$a^2 + b^2 = 8^2 + 9^2$
$= 64 + 81$
$= 145$

$h^2 = 12^2$
$= 144$

$a^2 + b^2 \neq h^2$

So, this is not a right-angled triangle.

Why can't we assume the triangle is right-angled?

The diagram does not have a ..... symbol

How can we check whether a triangle is right-angled?

We check if the side lengths satisfy ..... theorem:

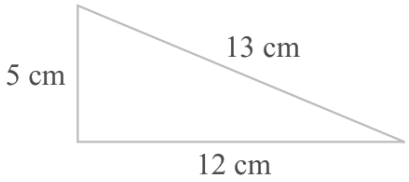
..... + ... = ...

Why is this triangle not right angled?

The side lengths .... satisfy Pythagoras' Theorem

Determine whether these triangles are right-angled.

a



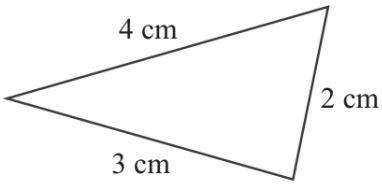
$a^2 + b^2 =$..... + ...
=..... + ...
=.....

$h^2 =$ .....
= .....

.....
.....

Right-angled

b



$a^2 + b^2 =$..... + ...
=..... + ...
=.....

$h^2 =$ .....
= .....

.....
.....

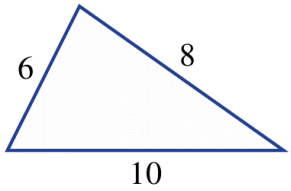
Not right-angled

Key ideas

1. If the three sides of a triangle satisfy $a^2 + b^2 = c^2$, then it is .....-.....
2. If a triangle doesn't satisfy $a^2 + b^2 = c^2$, then it's ....-.....

1 These diagrams are not drawn to scale. Prove that:

a This triangle is right-angled.



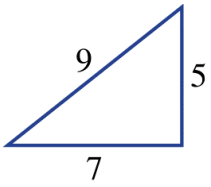
$a^2 + b^2 =$ ..... + ...
$=$ ..... + ...
$=$ .....

$h^2 =$ .....
$=$ .....

.....

$6^2 + 8^2 = 100$
$10^2 = 100$
$\therefore$ right angled, $a^2 + b^2 = h^2$

b This triangle is NOT right-angled.



$a^2 + b^2 =$ ..... + ...
$=$ ..... + ...
$=$ .....

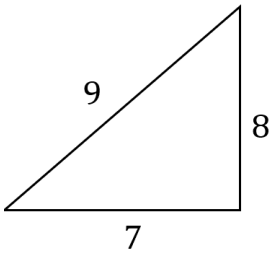
$h^2 =$ .....
$=$ .....

.....

$7^2 + 5^2 = 74$
$9^2 = 81$
$\therefore$ not right angled, $a^2 + b^2 \neq h^2$

2 Decide whether these shaded triangles are right angled.

a

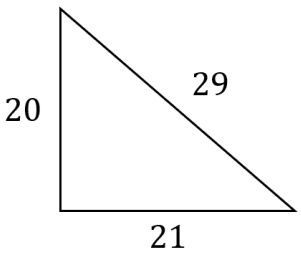


$7^2 + 8^2 = 113$ | $9^2 = 81$

.....

$\therefore$ not right angled, $a^2 + b^2 \neq h^2$

b

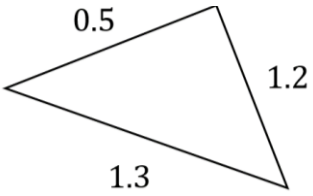


$20^2 + 21^2 = 841$ | $29^2 = 841$

.....

$\therefore$ right angled, $a^2 + b^2 = h^2$

c

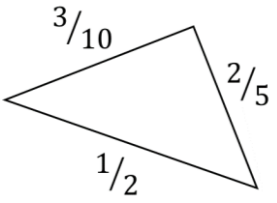


$0.5^2 + 1.2^2 = 1.69$ | $1.3^2 = 1.69$

.....

$\therefore$ right angled, $a^2 + b^2 = h^2$

d



$\left(\frac{3}{10}\right)^2 + \left(\frac{2}{5}\right)^2 = \frac{1}{4}$ | $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

.....

$\therefore$ right angled, $a^2 + b^2 = h^2$

PYTHAGOREAN TRIADS

- If three **whole numbers** (a, b, c) satisfy $a^2 + b^2 = c^2$, where c is the largest number, then they are a **Pythagorean triad**.

Example Determine whether 3 numbers are a Pythagorean triad.

Is 13, 12, 5 a Pythagorean triad? $a = 5,\ b = 12,\ c = 13$ $a^2 + b^2 = 5^2 + 12^2$ $= 169$ $c^2 = 13^2$ $= 169$ Therefore, 13, 12, 5 is a Pythagorean triad.	*What is a Pythagorean triad?* *A Pythagorean triad is ..... numbers that satisfy .....' Theorem* What should you let c be in $a^2 + b^2 = c^2$? *c should always be the ..... number in the list.*

Test whether these three numbers are Pythagorean triads.

a (8, 15, 17)

$a = \ldots\ldots, b = \ldots\ldots, c = \ldots\ldots$

$a^2 + b^2 = \ldots\ldots + \ldots\ldots$

$=$

$c^2 = \ldots\ldots\ldots$

Therefore, 8, 15, 17 is a Pythagorean triad.

b (24, 26, 10)

$a = \ldots\ldots, b = \ldots\ldots, c = \ldots\ldots$

Yes, Pythagorean triad

c (6, 4, 3)

Not Pythagorean triad

GENERATING PYTHAGOREAN TRIADS

- To generate a new Pythagorean triad, multiply or divide each number in a Pythagorean triad by the same number.

Example Generate new Pythagorean triads from an existing Pythagorean triad.

3, 4, 5 is a Pythagorean Triad.	**8, 15, 17** is a Pythagorean Triad.
Which of these are also Pythagorean triads?	Which of these are also Pythagorean triads?
❒ 6, 8, 10	❒ 9, 16, 18
❒ 4, 5, 6	❒ 16, 30, 34
❒ 9, 12, 15	❒ 24, 45, 51
❒ 6, 7, 8	❒ 6, 13, 15
Come up with one of your own:	Come up with one of your own:
……	……

Key ideas

1. If three …… numbers (a, b, c) satisfy the equation $a^2 + b^2 = c^2$, then it is called a …… ……
2. c is always the …… number.
3. If you multiply or divide each number in a Pythagorean triad by a common factor, then you generate another …… ……

1 Determine which of the following are Pythagorean triads.

a 9, 15, 12

$a = \ldots\ldots, b = \ldots\ldots, c = \ldots\ldots$

$a^2 + b^2 = \ldots\ldots + \ldots\ldots$

$=$

$c^2 = \ldots\ldots\ldots$

Therefore, ……

……

Not Pythagorean triad.

b (5, 7, 9)

$a = \ldots\ldots, b = \ldots\ldots, c = \ldots\ldots$

Not Pythagorean Triad

c (9, 40, 41)

Pythagorean Triad

d (4, 5, 9)

Not Pythagorean Triad

e (7, 24, 25)

Pythagorean Triad

f (11, 60, 61)

Pythagorean Triad

2 For each Pythagorean triad, create another four Pythagorean triads by multiplying or dividing each number by the same factor.

a (5, 12, 13)

……

……

……

……

b (16, 30, 34)

……

……

……

……

c (21, 48, 100)

……

……

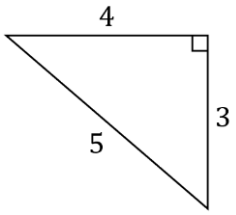
……

……

3 Here is a triangle. Prove that it is right-angled.

$a^2 + b^2 =$..... + ...
$=$..... + ...
$=$.....

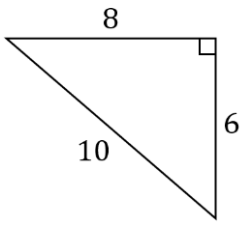
$h^2 =$ .....
$=$ .....

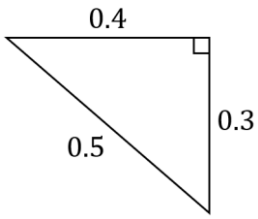


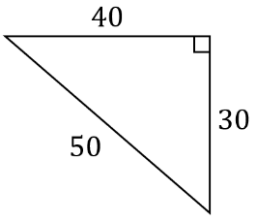
.....

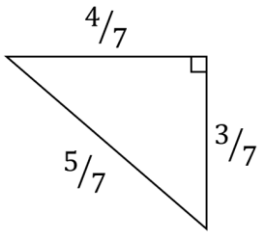
.....

Here are some more right-angled triangles.









Compare the side lengths of these triangles with the triangle above.

What's the relationship between the sides of these triangles?

.....

Each side has been multiplied or divided by the same factor.

Complete the sentence:

A right-angled triangle will stay right-angled as long as you ..... or ..... each side by the same number.

Multiply, divide

Draw three more right-angled triangles that are based on the 3, 4, 5 triangle.

4 Pythagoras developed a formula for finding Pythagorean triads (a, b, c).

If one number in the triad is a, the formulas for the other 2 numbers are:

$$b = \frac{1}{2}(a^2 - 1) \quad \text{and} \quad c = \frac{1}{2}(a^2 + 1)$$

a If $a = 5$, use the formulas to find the values of b and c.

5, 12, 13

b If $a = 7$, use the formulas to find the values of b and c.

7, 24, 25

5 $(8, 15, n)$ is a Pythagorean triad. Find the value(s) of n.

17 is the only answer.

6 $(18, 30, n)$ is a Pythagorean triad. Find the value(s) of n.

24 is the only answer.

7 $(13,85, n)$ is a Pythagorean triad. Find the value(s) of n.

84 is the only answer.