Student Name

Mathematics Stage 5 Paths

NON-LINEAR RELATIONSHIPS C

Book 1 Graph parabolas and describe their features and transformations

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-NLI-P-01 interprets and compares non-linear relationships and their transformations, both algebraically and graphically

Graph parabolas and describe their features and transformations

- Use graphing applications to compare parabolas of the form $y = kx^2$, $y = kx^2 + c$, $y = k(x-b)^2$ and $y = k(x-b)^2 + c$, and describe their features and transformations
- Find x- and y-intercepts algebraically, where appropriate, for the graph of $y = ax^2 + bx + c$, given a, b and c
- Determine the equation of the axis of symmetry of a parabola using either the formula $x = -\frac{b}{2a}$ or the midpoint of the x-intercepts
- Find the coordinates of a parabola's vertex using a variety of methods
- Graph quadratic relationships of the form $y = ax^2 + bx + c$ by identifying and applying features of parabolas and their equations without graphing software

REVIEW OF PRIOR KNOWLEDGE

1 Solve these quadratic equations.

a $x^2 + 3x = 0$	**b** $x^2 + \frac{1}{3}x = 0$	**c** $-5x^2 + 15x = 0$
d $x^2 - 4 = 0$	**e** $x^2 - 12x + 32 = 0$	**f** $x^2 - 12x + 36 = 0$
g $x^2 - 12x + 20 = 0$	**h** $20 = 9x - x^2$	**i** $3x^2 - 4x - 15 = 0$
j $6x^2 + 7x - 20 = 0$	**k** $6x^2 + 2x - 4 = 0$	**l** $17x = 12 - 5x^2$

2 $|x|$ is called the **magnitude of x.** It is the distance from zero; i.e., ignore the negative.

a $\|3\| = 3$	**b** $\|-3\| = 3$	**c** $\|5\|$	**d** $\|-5\|$
e $\|17\|$	**f** $\|-21\|$	**g** $\|-0.6\|$	**h** $\|0.5\|$

3 Simplify these surds.

a $\sqrt{12}$	**b** $\sqrt{54}$	**c** $3\sqrt{8}$
d $\frac{\sqrt{48}}{2}$	**e** $\frac{5\sqrt{40}}{6}$	**f** $\frac{3+\sqrt{90}}{6}$

4 Solve these quadratic expressions by completing the square.

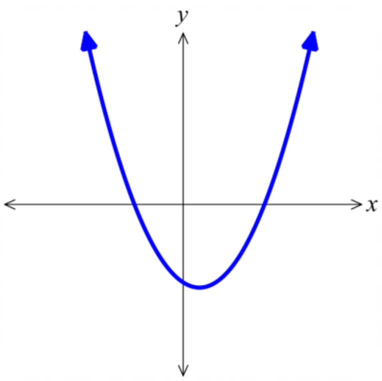
a $x^2 + 8x + 4 = 0$	**b** $x^2 - 10x - 3 = 0$	**c** $x^2 + 12x - 18 = 0$

SKETCHING QUADRATIC EQUATIONS BY FACTORISATION

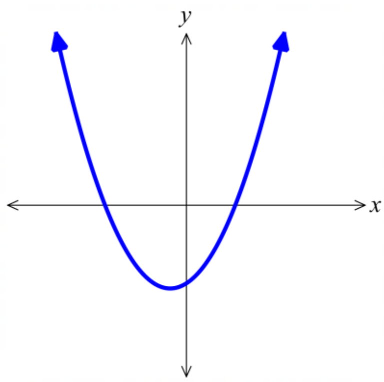
General Form of a Parabola	Factorised form of a Parabola
$y = ax^2 + bx + c$ Concavity y-intercept	$y = k(x - \alpha)(x - \beta)$ Concavity x-intercepts

Example Identify intercepts of a parabola from the equation.

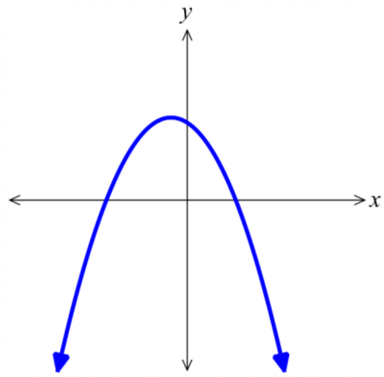
a $y = (x + 3)(x - 5)$



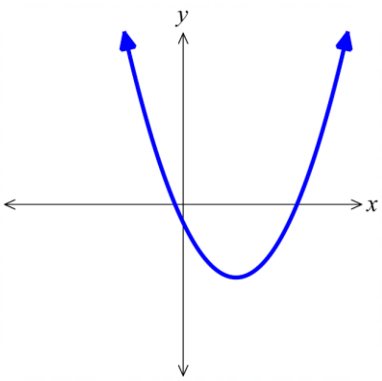
b $y = (x - 3)(x + 5)$



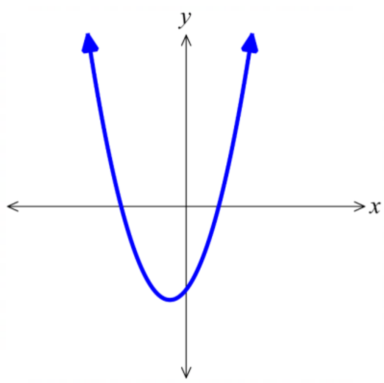
c $y = -(x - 3)(x + 5)$



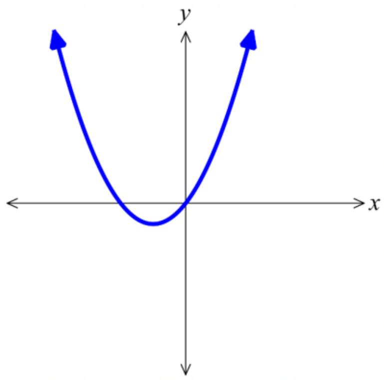
d $y = \left(x + \frac{1}{2}\right)(x - 7)$



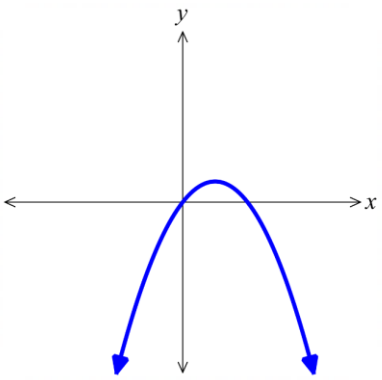
e $y = (2x + 8)(x - 2)$



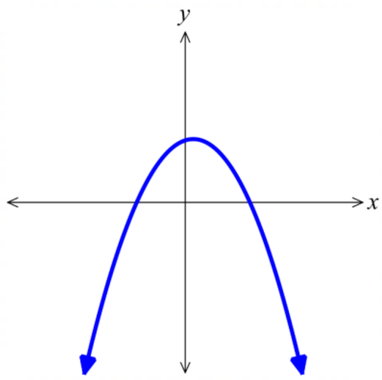
f $y = x(x + 4)$



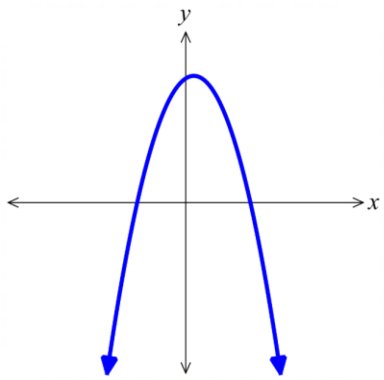
g $y = x(4 - x)$



h $y = (x + 3)(4 - x)$



i $y = (2x + 6)(4 - x)$



SKETCHING PARABOLAS BY FACTORISATION

0. Factorise $y = ax^2 + bx + c$ using PSF, perfect squares or difference of two squares.

- To sketch $y = k(x-\alpha)(x-\beta)$:
  1. Determine the **concavity**. $k > 0$, concave up U $k < 0$ concave down ∩.
  2. Find the **y-intercept**, $x = 0$.
  3. Find the **x-intercepts**, if any. $y = 0$.
  4. Determine the **vertex**.
     Use the mean of the x-intercepts to find the x-coordinate,
     then substitute into the equation to find the y −coordinate.

Example Sketch parabolas from factored form.

Sketch $y = x^2 - 6x + 5$

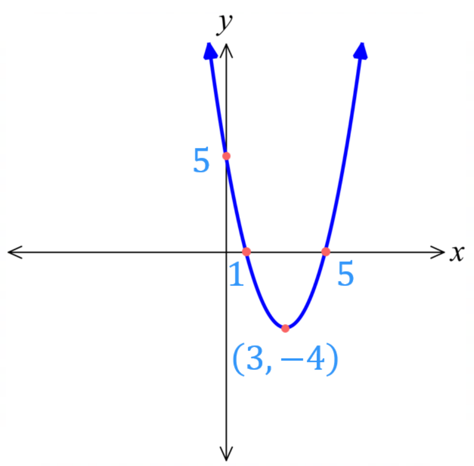
0. Factorise: $y = (x-5)(x-1)$

1. Concavity: $k > 0$, concave up

2. y-int: Let $x = 0$
$y = 5$

3. x-int: Let $y = 0$
$0 = (x-5)(x-1)$
$x = 5$ or 1

4. Vertex: Avg of x-intercepts: $\frac{1+5}{2} = 3$
Sub $x = 3$ into equation
$y = 3^2 - 6(3) + 5$
$y = -4$



Sketch $y = x^2 + 4x - 12$

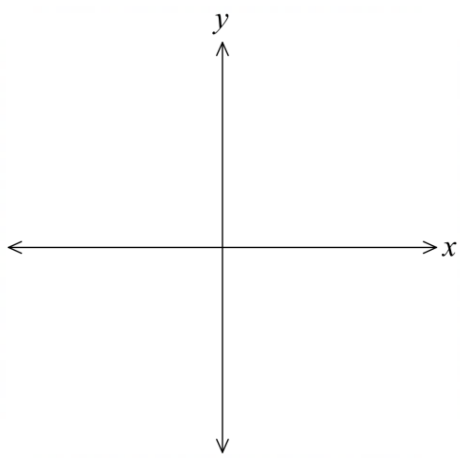
0. Factorise:

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



a $y = -x^2 + 4x$

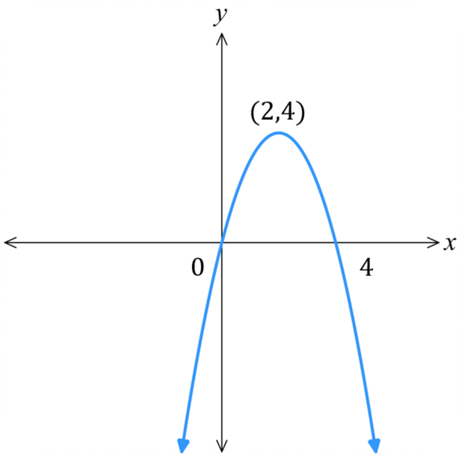
0. Factorise:

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



b $y = 2x^2 + 7x + 5$

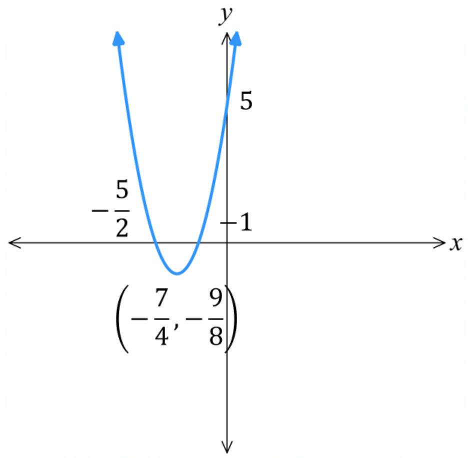
0. Factorise:

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



1 Substitute $y = 0$ then use the null factor law to find the x-intercepts of these quadratics.

a $y = (x+1)(x-2)$

$x = -1, 2$

b $y = (x-3)(x-4)$

$x = 3, 4$

c $y = (x+1)(x+5)$

$x = -1, -5$

d $y = x(x-3)$

$x = 0, 3$

e $y = 2x(x-5)$

$x = 0, 5$

f $y = -3x(x+2)$

$x = 0, -2$

g $y = \left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)$

$x = -\sqrt{5}, \sqrt{5}$

h $y = \left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)$

$x = -\sqrt{7}, \sqrt{7}$

i $y = \left(x+2\sqrt{2}\right)\left(x-2\sqrt{2}\right)$

$x = -2\sqrt{2}, 2\sqrt{2}$

2 Factorise and use the null factor law to find the x-intercepts for these quadratics.

a $y = x^2 + 3x + 2$

$x = -2, -1$

b $y = x^2 + 2x - 8$

$x = -4, 2$

c $y = x^2 - 8x + 16$

$x = 4$

d $y = x^2 - 4x$

$x = 0, 4$

e $y = x^2 - 6x$

$x = 0, 6$

f $y = 2x^2 + 10x$

$x = 0, -5$

g $y = x^2 - 9$

$x = -3, 3$

h $y = x^2 - 25$

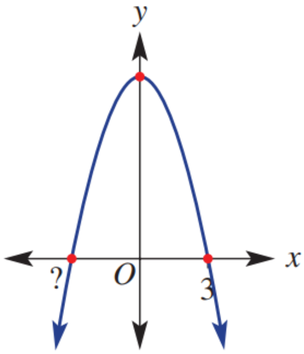
$x = -5, 5$

i $y = x^2 - 10$

$x = -\sqrt{10}, \sqrt{10}$

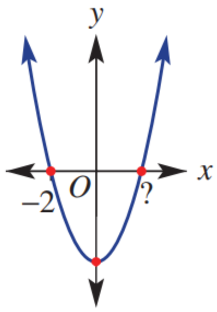
3 Using symmetry, state the missing coordinate in these quadratic graphs.

a



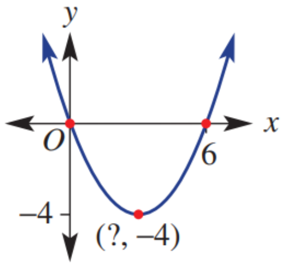
-3

b



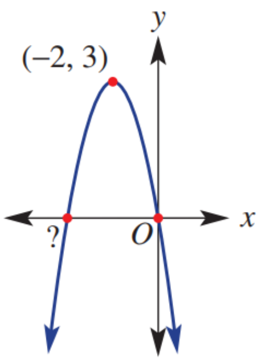
2

c



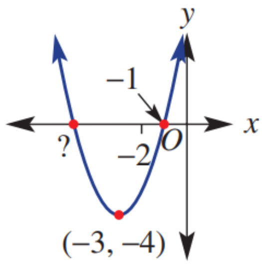
3

d



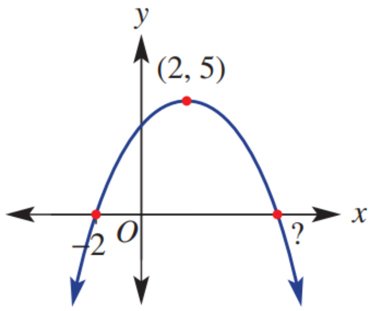
-4

e



-5

f



6

4 Sketch $y = (x-2)(x-8)$:

a Determine the concavity: ....

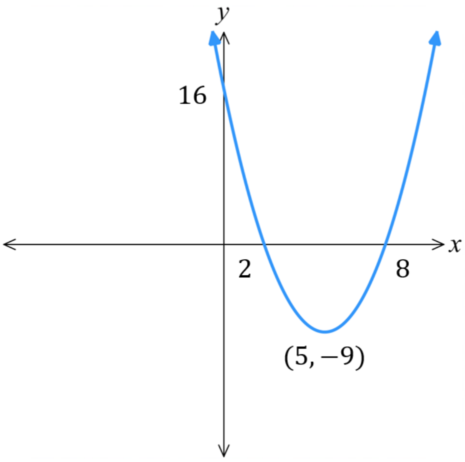
b Determine the y-intercept: ....

c Determine the x-intercepts: ....

d Determine the coordinates of the vertex:

....

....



5 Sketch $y = -x(x+2)$:

a Determine the concavity: ....

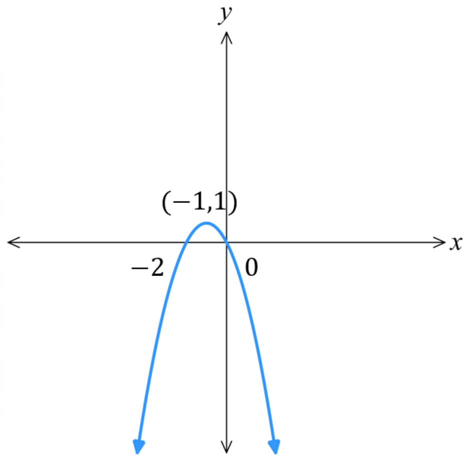
b Determine the y-intercept: ....

c Determine the x-intercepts: ....

d Determine the coordinates of the vertex:

....

....



6 Sketch $y = (x+1)^2$:

a Determine the concavity: ....

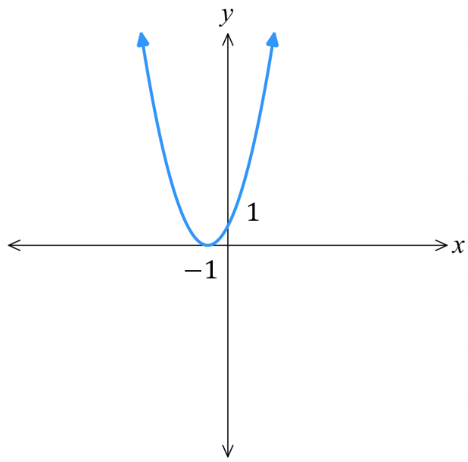
b Determine the y-intercept: ....

c Determine the x-intercepts: ....

d Determine the coordinates of the vertex:

....

....



7 Sketch $y = x^2 - 2x - 48$:

a Factorise the equation: ....

b Determine the concavity: ....

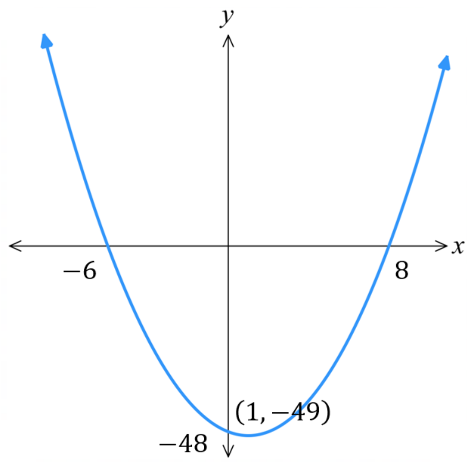
c Determine the y-intercept: ....

d Determine the x-intercepts: ....

e Determine the coordinates of the vertex:

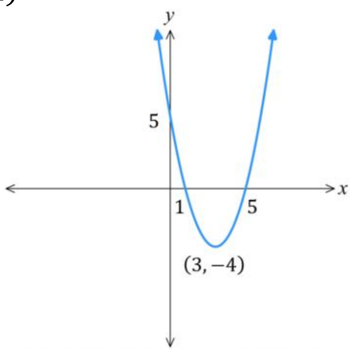
....

....

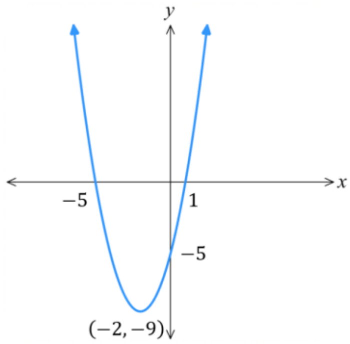


8 Sketch these parabolas. Clearly label the vertex and intercepts.

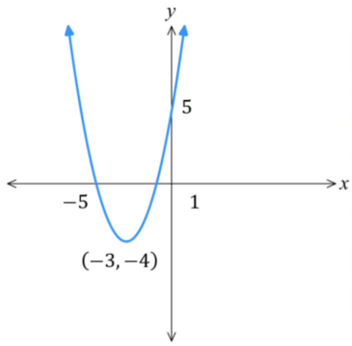
a $y = (x-5)(x-1)$



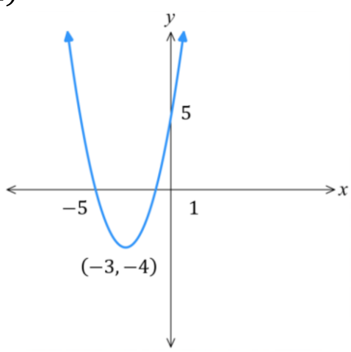
b $y = (x+5)(x-1)$



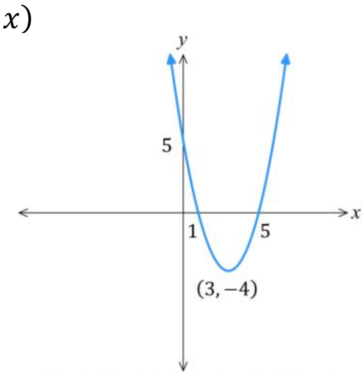
c $y = (x+5)(x+1)$



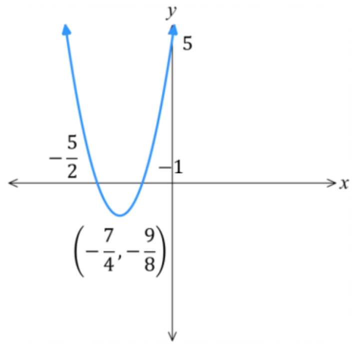
d $y = (5+x)(x+1)$



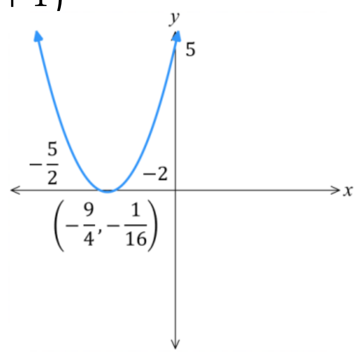
e $y = (5-x)(1-x)$



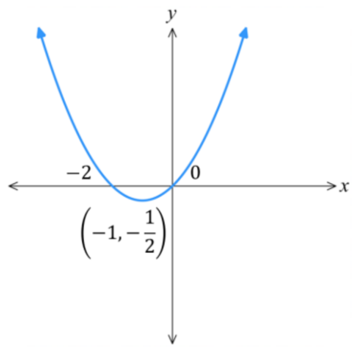
f $y = (2x+5)(x+1)$



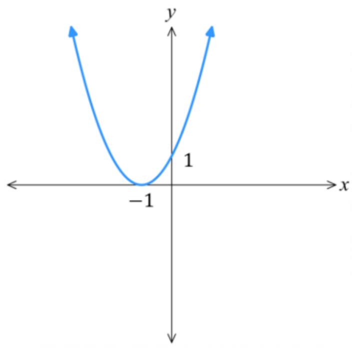
g $y = (2x+5)\left(\frac{1}{2}x+1\right)$



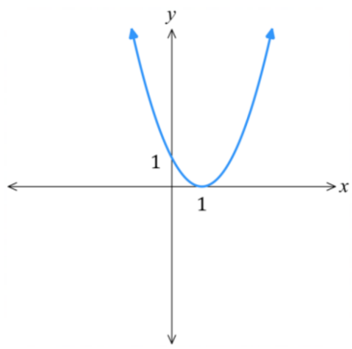
h $y = x\left(\frac{1}{2}x+1\right)$



i $y = (x+1)^2$

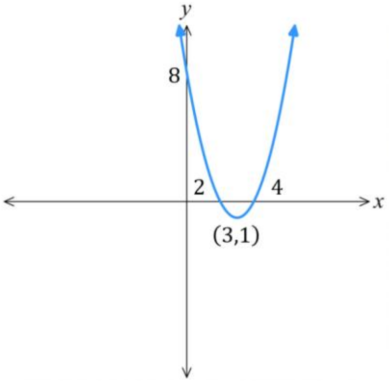


j $y = (x-1)^2$

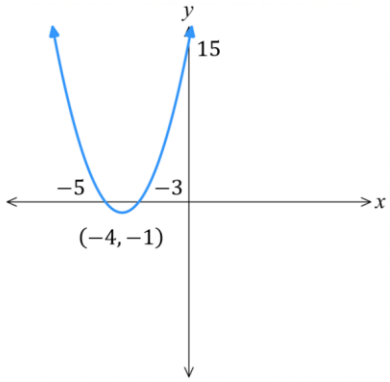


9 Sketch these parabolas by factorising first. Show the vertex and all intercepts.

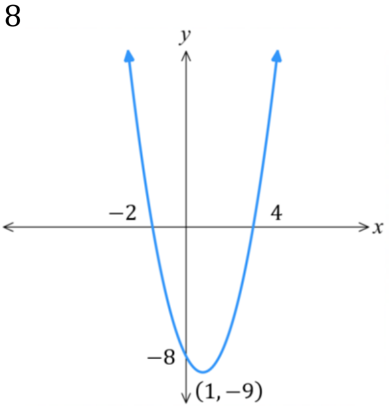
a $y = x^2 - 6x + 8$



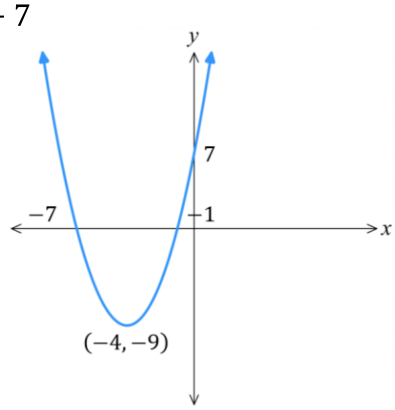
b $y = x^2 + 8x + 15$



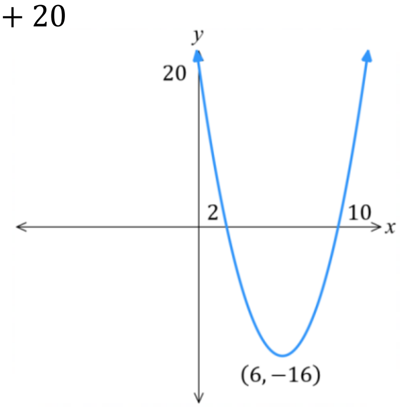
c $y = x^2 - 2x - 8$



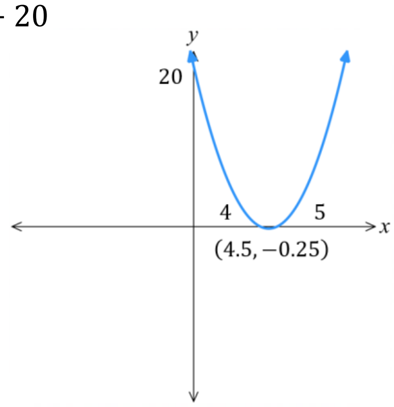
d $y = x^2 + 8x + 7$



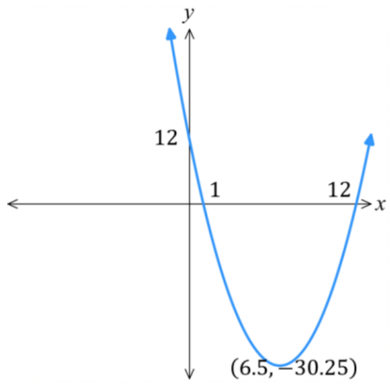
e $y = x^2 - 12x + 20$



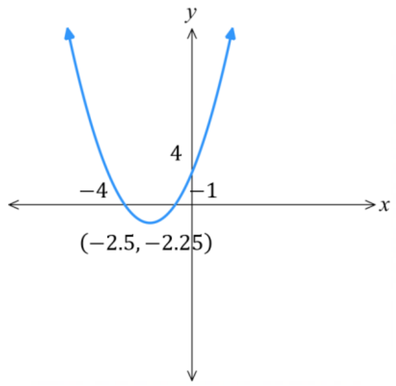
f $y = x^2 - 9x + 20$



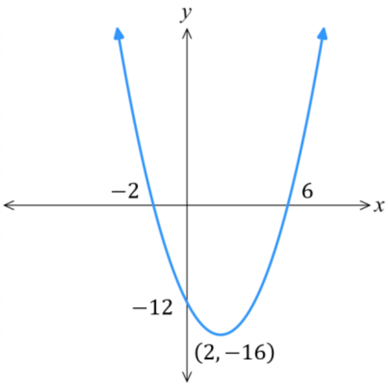
g $y = x^2 - 13x + 12$



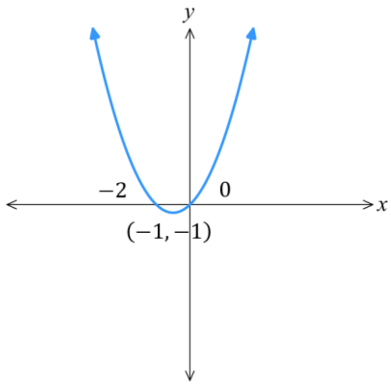
h $y = x^2 + 5x + 4$



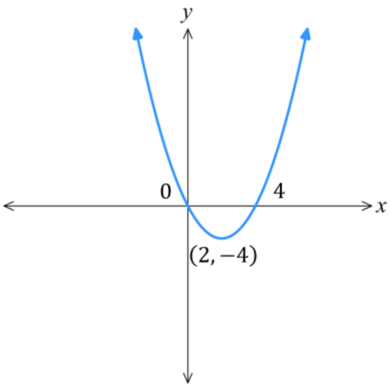
i $y = x^2 - 4x - 12$



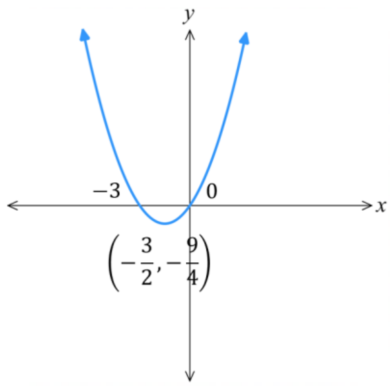
j $y = x^2 + 2x$



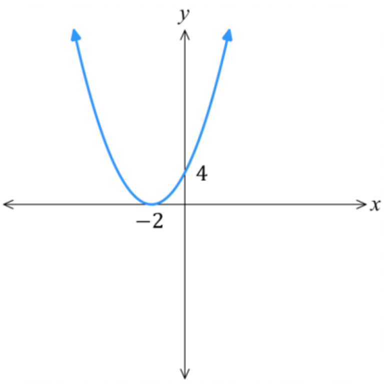
k $y = x^2 - 4x$



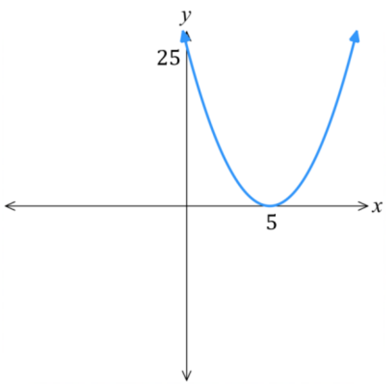
l $y = x^2 + 3x$



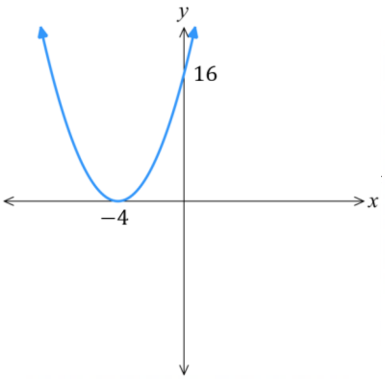
m $y = x^2 + 4x + 4$



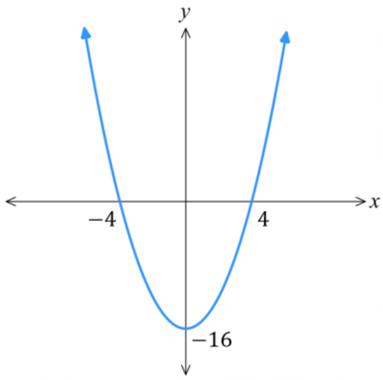
n $y = x^2 - 10x + 25$



o $y = x^2 + 8x + 16$

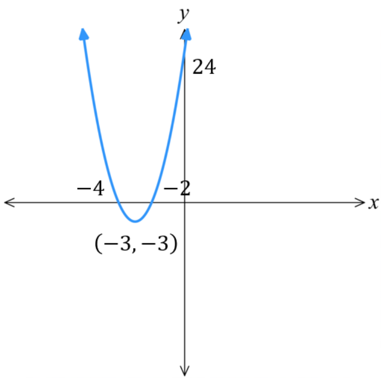


p $y = x^2 - 16$

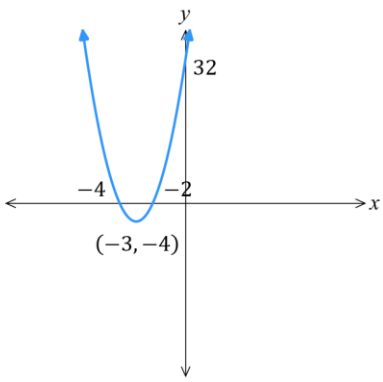


10 Sketch these parabolas. Show the vertex and all intercepts.

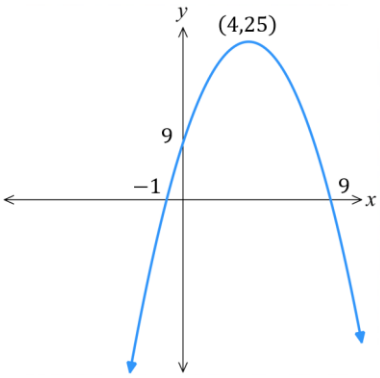
a $y = 3x^2 + 18x + 24$



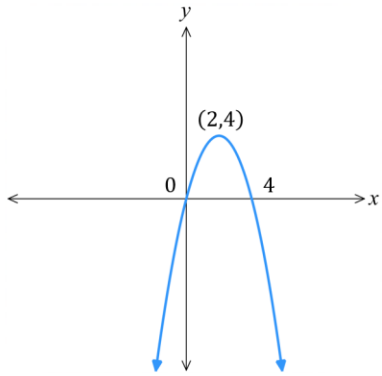
b $y = 4x^2 + 24x + 32$



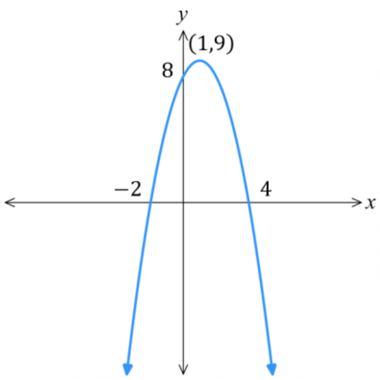
c $y = -x^2 + 8x + 9$



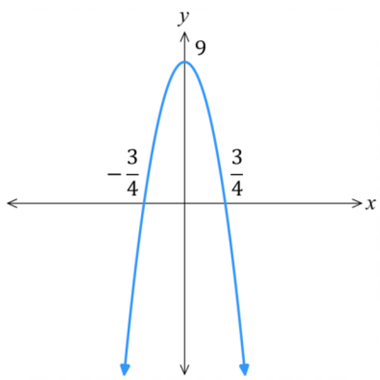
d $y = 4x - x^2$



e $y = -x^2 + 2x + 8$



f $y = 9 - 16x^2$



11 With reference to the null factor law, explain why $y = (x - 3)(x - 5)$ and $y = 2(x - 3)(x - 5)$ both have the same x-intercepts.

For both equations, null factor law says either $x - 3 = 0$ or $x - 5 = 0$

PARABOLA TRANSFORMATIONS

Investigation Equation and the graph of a parabola.

On Desmos https://www.desmos.com/calculator

a Sketch these graphs

$y = x^2$
$y = 2x^2$
$y = 3x^2$
$y = 4x^2$

How does increasing the coefficient of x^2 change the parabola?

....
....
....

b Sketch these graphs.

$y = x^2$
$y = \frac{1}{2}x^2$
$y = \frac{1}{3}x^2$
$y = \frac{1}{4}x^2$

How does decreasing the coefficient of x^2 change the parabola?

....
....
....

c Sketch these graphs.

$y = x^2$
$y = -x^2$
$y = -\frac{1}{2}x^2$
$y = -2x^2$

How does a negative coefficient of x^2 change the parabola?

....
....
....

d Sketch these graphs.

$y = x^2$
$y = x^2 + 1$
$y = x^2 + 2$
$y = x^2 + 3$
$y = x^2 - 1$
$y = x^2 - 2$

How does the constant term change the parabola?

....
....
....

e Sketch these graphs.

$y = (x - 1)^2$
$y = (x - 2)^2$
$y = (x + 2)^2$
$y = (x + 3)^2$

How does the term inside the square change the parabola?

....
....
....

Parabola Transformations

For a parabola $y = k(x-b)^2 + c$

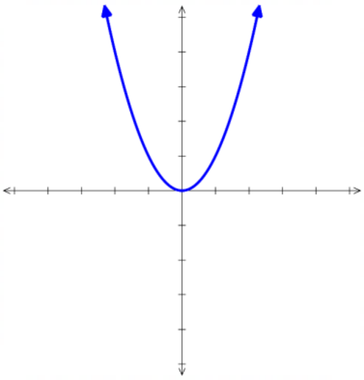
The variables b, c, k are numbers that affect the graph in predictable ways.

These special variables are called **parameters**.

$k > 0$, the parabola is concave ....,

with a .... turning point.

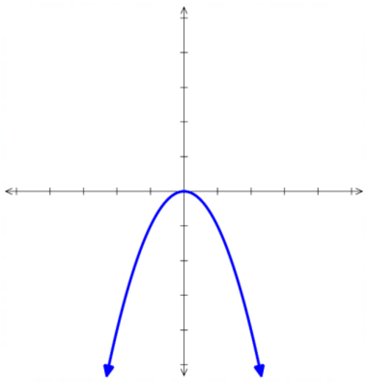
$y = 1x^2$



$k < 0$, the parabola is concave ....,

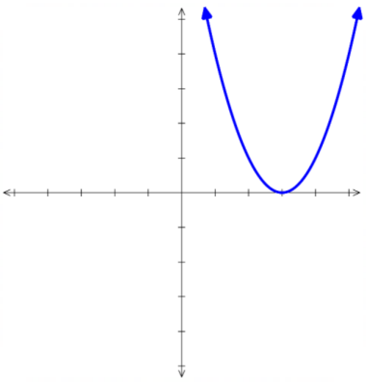
with a .... turning point.

$y = -1x^2$



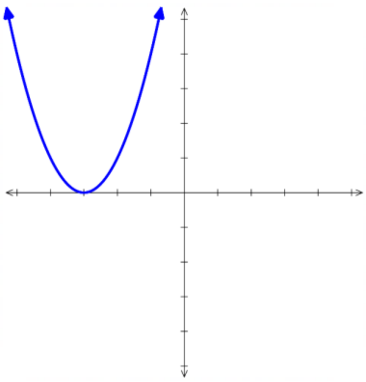
$b > 0$, the parabola is shifted ....

$y = (x-3)^2$



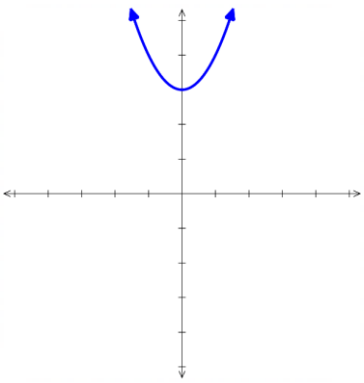
$b < 0$, the parabola is shifted ....

$y = (x+3)^2$



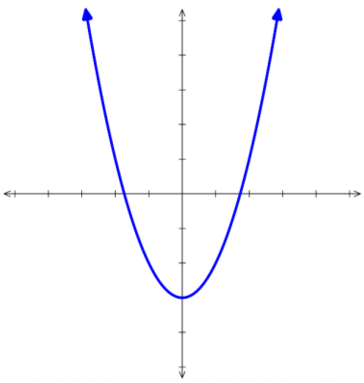
$c > 0$, the parabola is shifted ....

$y = x^2 + 3$



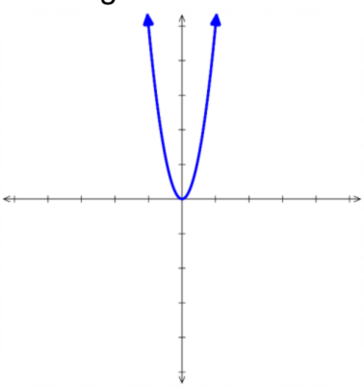
$c < 0$, the parabola is shifted ....

$y = x^2 - 3$



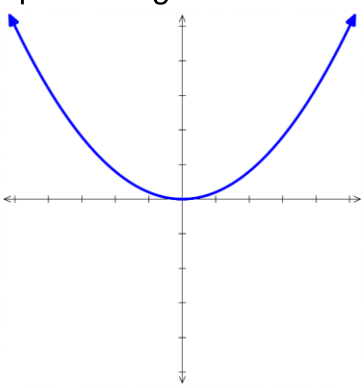
$|k| > 1$, the parabola gets narrower.

$y = 5x^2$



$0 < |k| < 1$ the parabola gets wider.

$y = \frac{1}{5}x^2$



VERTEX FORM OF A PARABOLA

Vertex Form of a Parabola
(b, c) is the vertex, or turning point, of the parabola.

Horizontal translation

$$y = k(x-b)^2 + c$$

Concavity

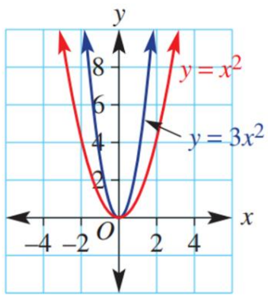
Vertical translation

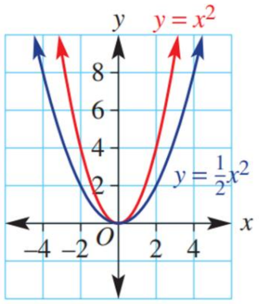
Example Identify k, b, c of the general form of the parabola.

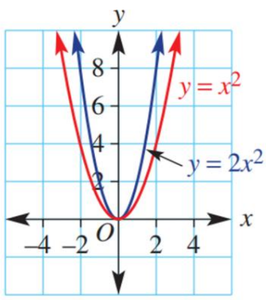
For each equation of a parabola, identify the value of $k, b,$ and c

Equation	k	b	c	Turning Point (Vertex
$y = 3(x-4)^2 + 2$	3	4	2	$(4,2)$
$y = 3(x-4)^2 - 2$	3	4	-2	$(4,-2)$
$y = -3(x-4)^2 - 2$	-3	4	-2	$(4,-2)$
$y = -3(x+4)^2 - 2$	-3	-4	-2	$(-4,-2)$
$y = -3(x+4)^2 + 2$	-3	-4	2	$(-4,2)$
$y = \frac{1}{3}(x+4)^2 + 2$	$\frac{1}{3}$	-4	2	$(-4,2)$
$y = \frac{(x+4)^2}{3} + 2$	$\frac{1}{3}$	-4	2	$(-4,2)$
$y = \frac{(x+4)^2 + 2}{3}$	$\frac{1}{3}$	-4	$\frac{2}{3}$	$\left(-4, \frac{2}{3}\right)$
$y = (x+4)^2 + 2$	1	-4	2	$(-4,2)$
$y = (x+4)^2$	1	-4	0	$(-4,0)$
$y = x^2$	1	0	0	$(0,0)$
$y = x^2 + 5$	1	0	5	$(0,5)$
$y = \frac{x^2}{5}$	$\frac{1}{5}$	0	0	$(0,0)$
$y = -9(x+1)^2 - 5$	-9	-1	-5	$(-1,-5)$

1 Consider these parabolas of the form $y = kx^2$





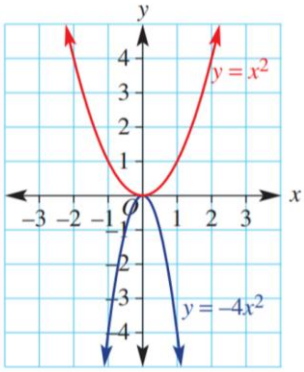


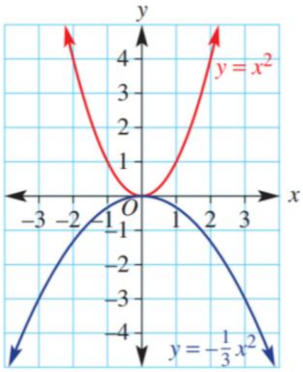
For $y = kx^2$, when $k > 1$, the parabola becomes n……………………

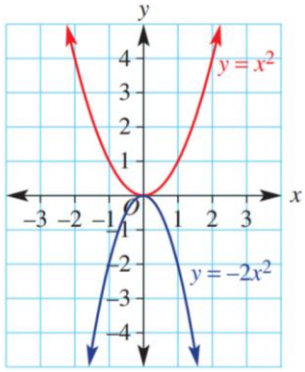
when $0 < k < 1$, the parabola becomes w……………………

narrower, wider

2 Consider these parabolas of the form $y = kx^2$







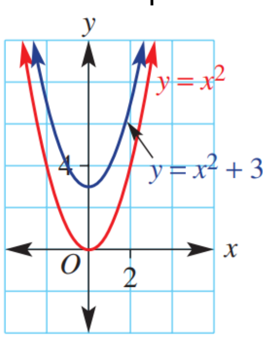
For $y = kx^2$, when $k < 0$, the parabola is r………………… across the x-axis.

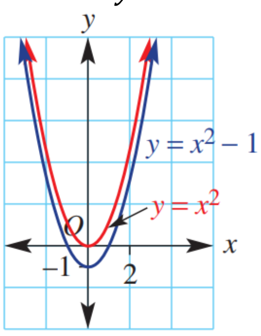
when $|k| > 1$, the parabola becomes n……………………

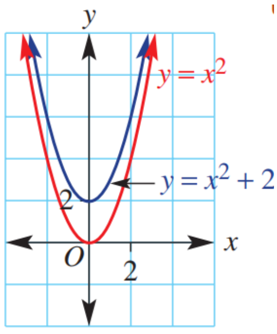
when $0 < |k| < 1$, the parabola becomes w……………………

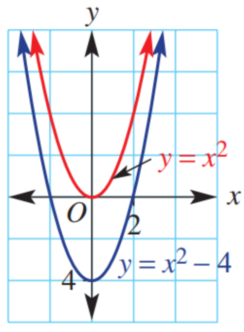
reflected, narrower, wider

3 Consider the parabolas of the form $y = x + c^2$







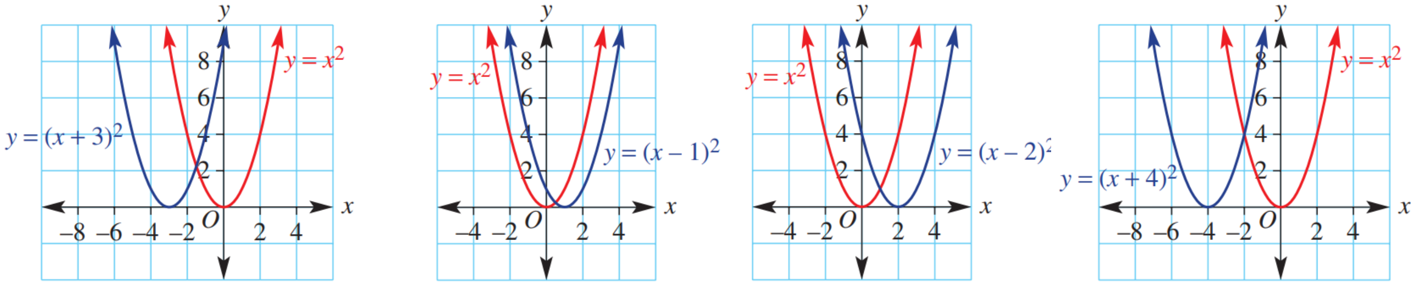


For $y = x^2 + c$, when $c > 0$, the parabola translates u………………

when $c < 0$, the parabola translates d………………

up, down

4 Consider the parabolas of the form $(x-b)^2$:



For $y=(x-b)^2$, when $b>0$, $y=(x-b)^2$, the parabola translates to the l...........

when $b<0$, $y=(x+b)^2$, the parabola translates to the r...........

left, right

5 Here are eight parabolas of the form $y=ax^2$:

A $y=6x^2$	**B** $y=-7x^2$	**C** $y=4x^2$	**D** $y=\frac{1}{9}x^2$
E $y=\frac{x^2}{7}$	**F** $y=0.3x^2$	**G** $y=-4.8x^2$	**H** $y=-0.5x^2$

a Which equation would give a parabola that is concave up and the narrowest? ....

b Which equation would give a parabola that is concave down and the widest? ....

A, H

6 Identify the translation and complete these sentences.

a Compared with $y=x^2$, $y=x^2+3$ is translated up by 3

b Compared with $y=x^2$, $y=x^2-6$ is translated ....

Down 6

c Compared with $y=x^2$, $y=(x-3)^2$ is translated ....

Right 3

d Compared with $y=x^2$, $y=(x+1)^2$ is translated ....

Left 1

e Compared with $y=x^2$, $y=(x+3)^2-2$ is translated left by ...... and down by ......

Left 3, down 2

f Compared with $y=x^2$, $y=(x-5)^2-3$ is translated ....

Right 5, down 3

g Compared with $y=x^2$, $y=(x-2)^2+3$ is translated ....

Right 2, up 3

h Compared with $y=x^2$, $y=(x+4)^2+5$ is translated ....

Left 4, up 5

7 Write the coordinates of the turning point for the parabolas formed by these equations.

a $y = x^2$ (0,0)	**b** $y = -x^2$ (0,0)	**c** $y = x^2 + 3$ (0,3)
d $y = -x^2 - 3$ $(0,-3)$	**e** $y = -x^2 + 7$ (0,7)	**f** $y = (x-2)^2$ (2,0)
g $y = (x+5)^2$ $(-5,0)$	**h** $y = 2x^2$ (0,0)	**i** $y = -\frac{1}{3}x^2$ (0,0)

8 Write the coordinates of the turning point for these parabolas.

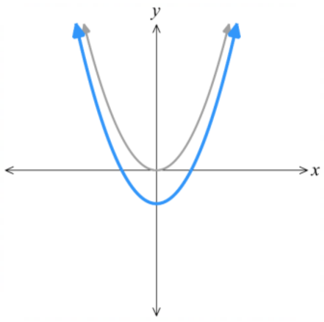
a $y = (x+3)^2 + 1$ $(-3,1)$	**b** $y = (x+2)^2 - 4$ $(-2,-4)$	**c** $y = (x-1)^2 + 3$ (1,3)
d $y = (x-4)^2 - 2$ $(4,-2)$	**e** $y = (x-2)^2 + 2$ (2,2)	**f** $y = -(x-3)^2 + 3$ (3,3)
g $y = -(x+1)^2 + 4$ $(-1,4)$	**h** $y = -(x-2)^2 - 5$ $(2,-5)$	**i** $y = -(x-4)^2 - 10$ $(4,-10)$

9 Write the equation for these parabolas, when compared to $y = x^2$.

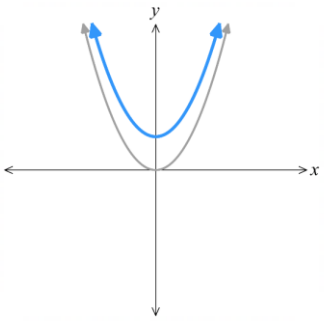
a reflected across x-axis $y = -x^2$	**b** translated 2 units left $y = (x+2)^2$	**c** translated 5 units down $y = x^2 - 5$
d translated 4 units up $y = x^2 + 4$	**e** translated 1 unit right $y = (x-1)^2$	**f** reflected across x-axis and translated 2 units up $y = -x^2 + 2$
g reflected across x-axis translated 3 units left $y = -(x+3)^2$	**h** translated 5 units left and 3 units down $y = (x+5)^2 - 3$	**i** translated 6 units right and 1 unit up $y = (x-6)^2 + 1$

10 Sketch these parabolas. The graph of $y = x^2$ is given. Show the turning point.

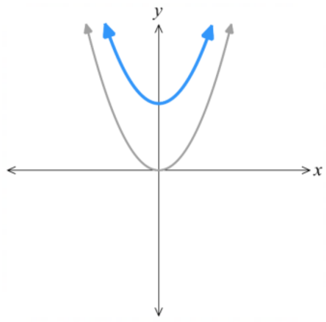
a $y = x^2 - 1$



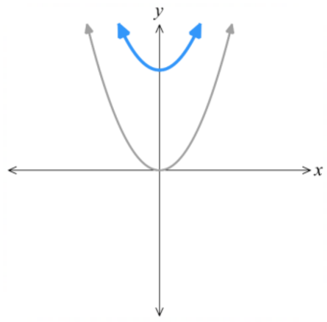
b $y = x^2 + 1$



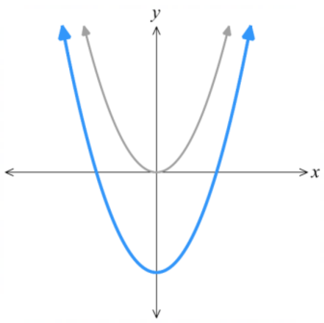
c $y = x^2 + 2$



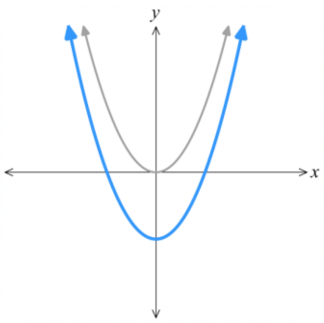
d $y = x^2 + 3$



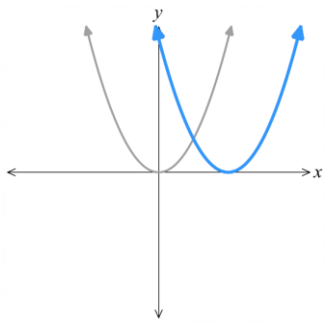
e $y = x^2 - 3$



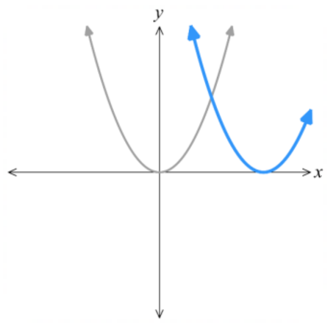
f $y = x^2 - 2$



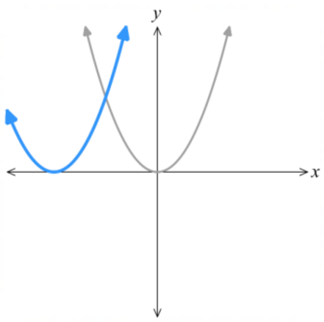
g $y = (x - 2)^2$



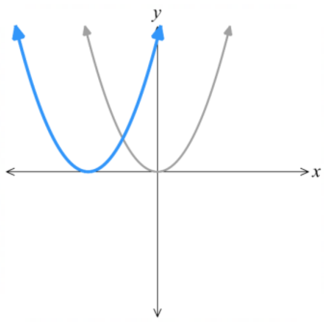
h $y = (x - 3)^2$



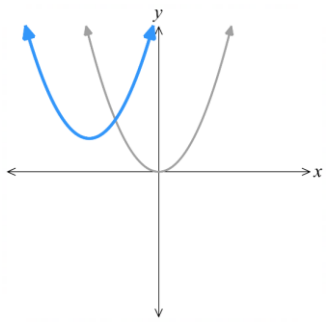
i $y = (x + 3)^2$



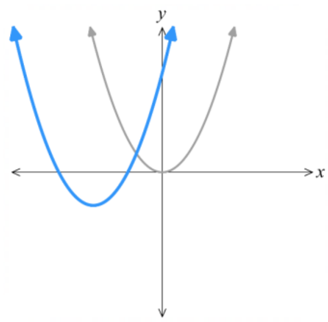
j $y = (x + 2)^2$



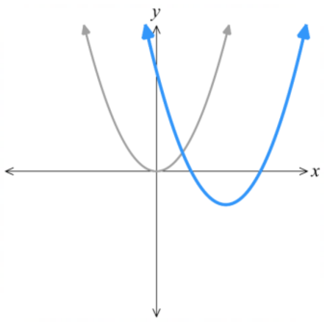
k $y = (x + 2)^2 + 1$



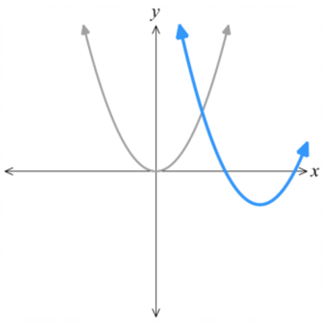
l $y = (x + 2)^2 - 1$



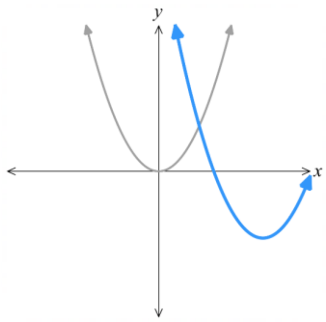
m $y = (x - 2)^2 - 1$



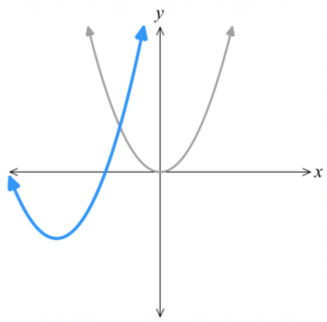
n $y = (x - 3)^2 - 1$



o $y = (x - 3)^2 - 2$

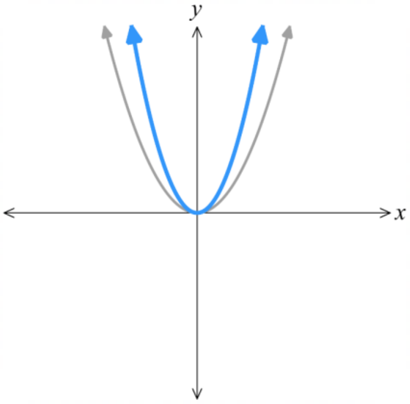


p $y = (x + 3)^2 - 2$

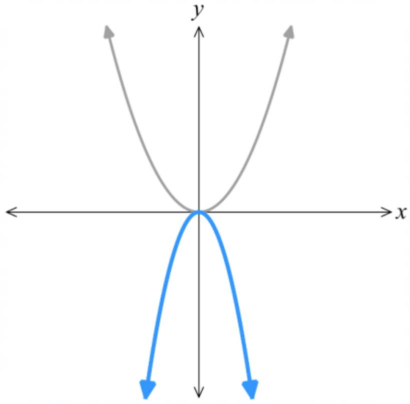


11 Sketch these parabolas. The graph of $y = x^2$ is given. Show the turning point.

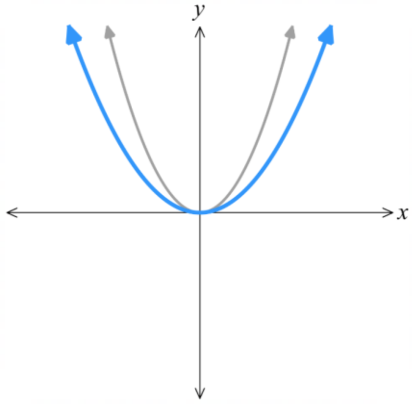
a $y = 2x^2$



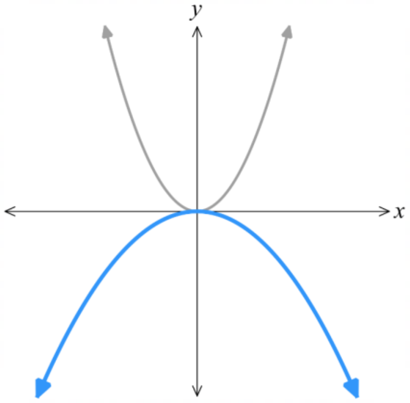
b $y = -3x^2$



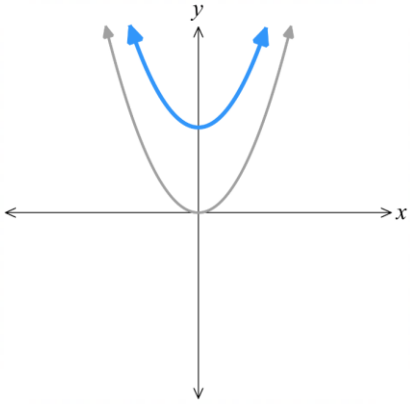
c $y = \frac{1}{2}x^2$



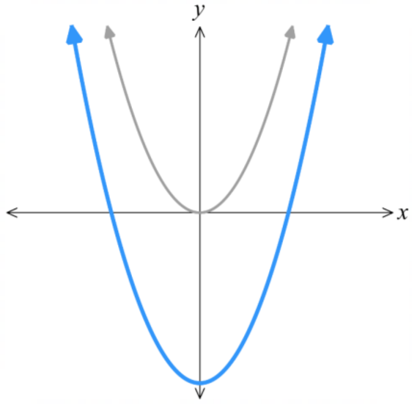
d $y = -\frac{1}{3}x^2$



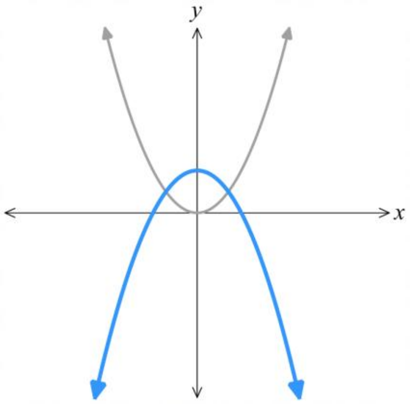
e $y = x^2 + 2$



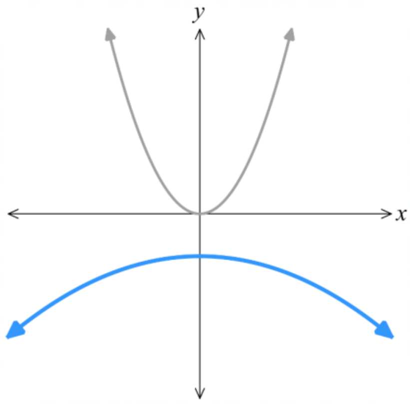
f $y = x^2 - 4$



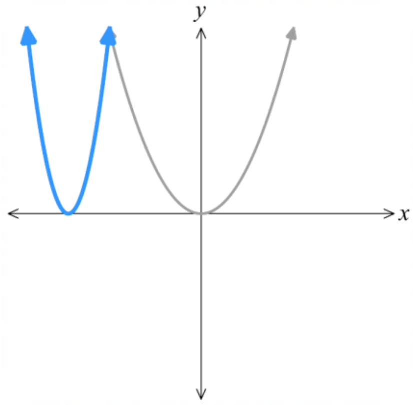
g $y = -x^2 + 1$



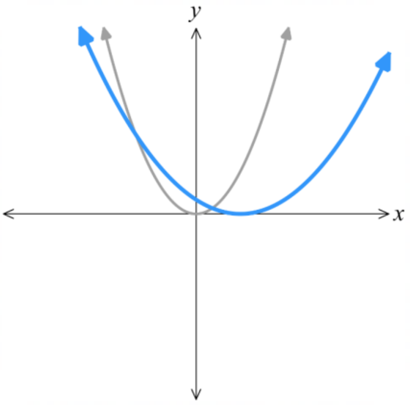
h $y = -0.1x^2 - 1$



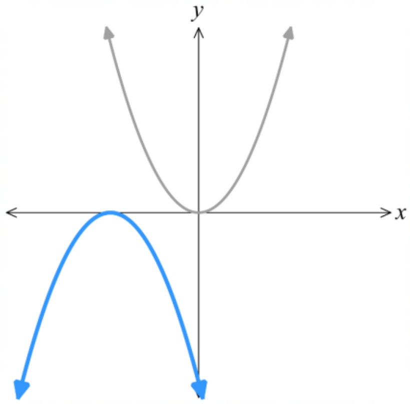
i $y = 5(x + 3)^2$



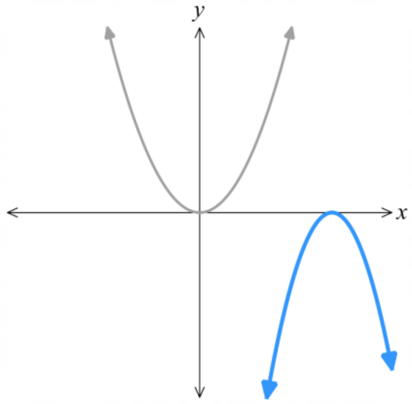
j $y = \frac{(x-1)^2}{3}$



k $y = -(x + 2)^2$



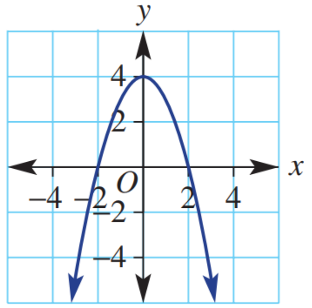
l $y = -2(x - 3)^2$



12 Match each equation to a graph.

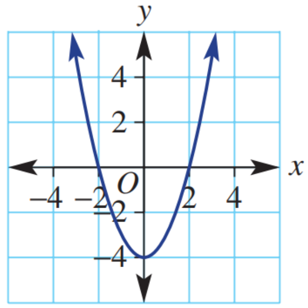
$y = x^2$	$y = (x + 2)^2$	$y = x^2 - 4$	$y = -x^2 - 3$
$y = (x - 2)^2$	$y = x^2 + 4$	$y = 4 - x^2$	$y = (x + 3)^2$
$y = (x - 5)^2$	$y = x^2 + 3$	$y = -(x + 3)^2$	$y = -x^2 + 3$

a



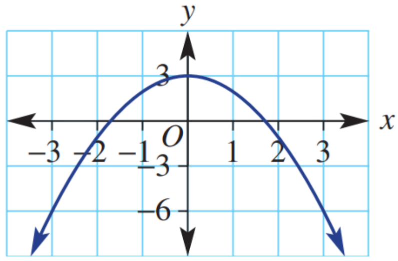
$y = x^2 + 4$

b



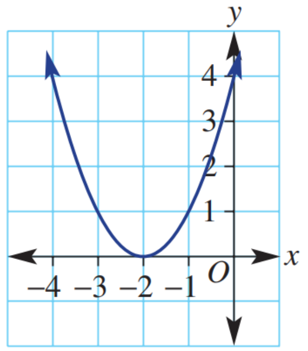
$y = x^2 - 4$

c



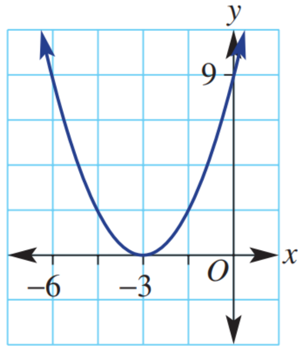
$y = -x^2 + 3$

d



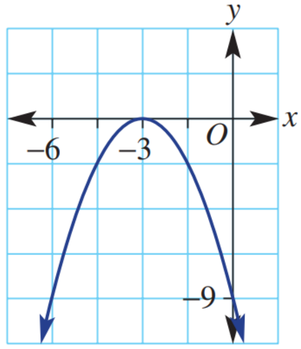
$y = (x + 2)^2$

e



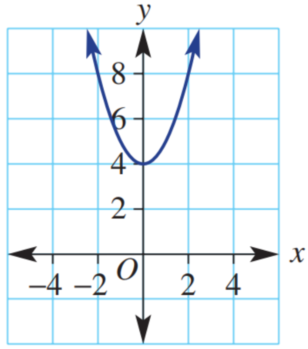
$y = (x + 3)^2$

f



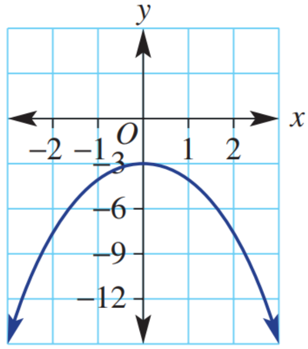
$y = -(x + 3)^2$

g



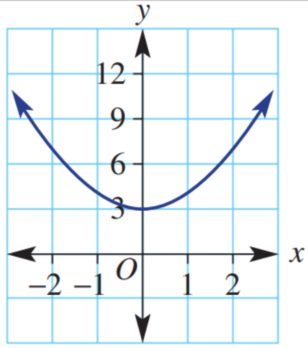
$y = x^2 + 4$

h



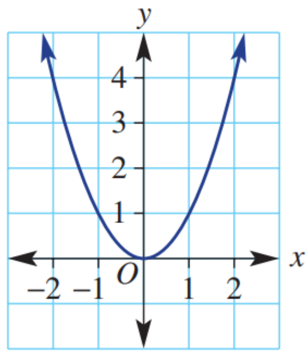
$y = -x^2 - 3$

i



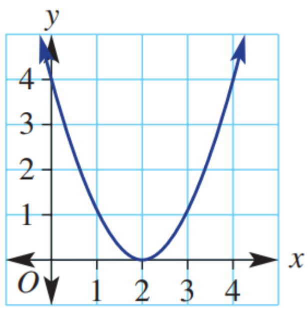
$y = x^2 + 3$

j



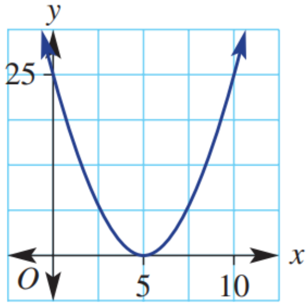
$y - x^2$

k



$y = (x - 2)^2$

l

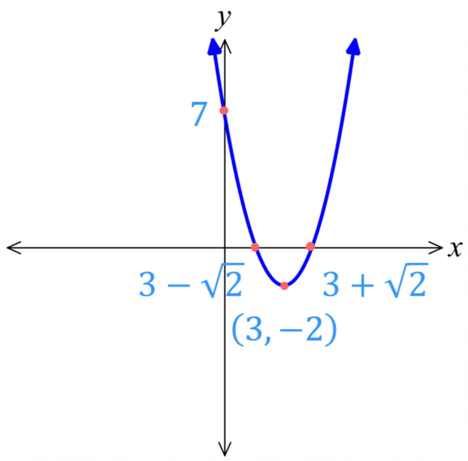
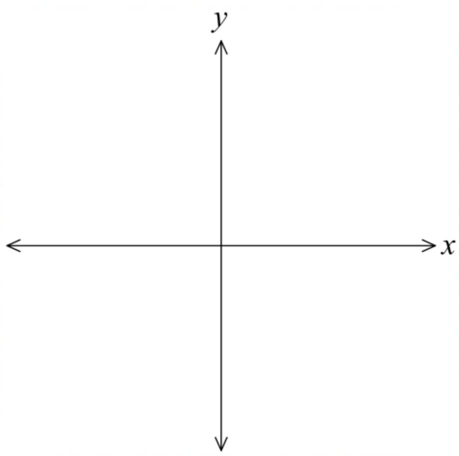


$y = (x - 5)^2$

SKETCHING QUADRATIC EQUATIONS FROM VERTEX FORM

- To sketch $y = k(x-b)^2 + c$:
  1. Determine the **concavity**.
     $k > 0$, concave up U $k < 0$ concave down ∩.
  2. Find the **y-intercept**.
     Substitute $x = 0$.
  3. Find the **x-intercepts**, if any.
     Substitute $y = 0$.
  4. Determine the **vertex**.
     Using (b, c).

Example Sketch parabolas from vertex form.

Worked Example	Your Turn
Sketch $y = (x-3)^2 - 2$	Sketch $y = (x+3)^2 - 1$
1. Concavity: $k > 0$, concave up.	1. Concavity:
2. y-int: Let $x = 0$ $y = (0-3)^2 - 2 = 7$	2. y-int:
3. x-int: Let $y = 0$ $0 = (x-3)^2 - 2$ $2 = (x-3)^2$ $\pm\sqrt{2} = x - 3$ $3 \pm \sqrt{2} = x$ x-ints at $3 + \sqrt{2}$ and $3 - \sqrt{2}$	3. x-int:
4. Vertex: $(3, -2)$	4. Vertex:
	

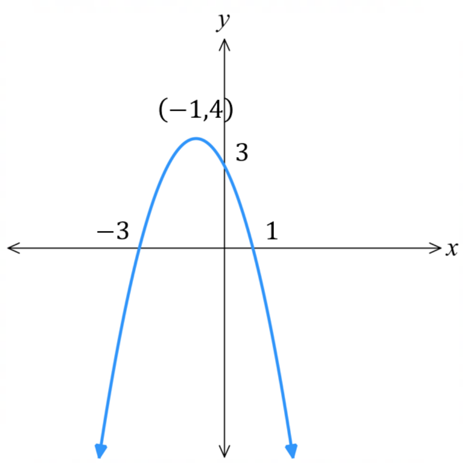
a $y = -(x+1)^2 + 4$

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



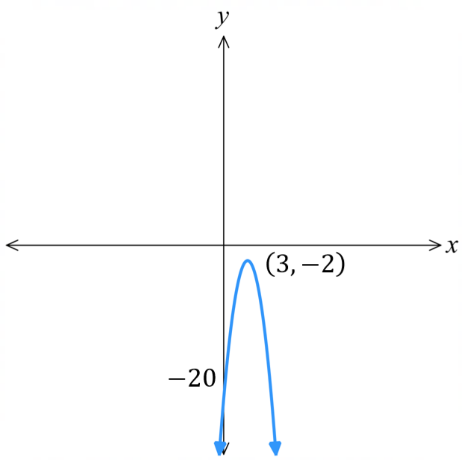
b Sketch $y = -2(x-3)^2 - 2$

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



1 By substituting $x = 0$, find the y-intercept of these graphs.

a $y = x^2 + 3$ — 3

b $y = -x^2 - 3$ — -3

c $y = 2x^2 - 3$ — -3

d $y = -6x^2 - 1$ — -1

e $y = (x - 1)^2$ — 1

f $y = (x + 2)^2$ — 4

g $y = -(x + 4)^2$ — -16

h $y = -(x - 5)^2$ — -25

i $y = -2(x + 1)^2$ — -2

j $y = (x + 1)^2 + 1$ — 2

k $y = (x - 3)^2 - 3$ — 6

l $y = -(x + 7)^2 - 14$ — -63

2 By substituting $y = 0$, find the x-intercepts of these graphs.

a $y = (x + 4)^2$ — -4

b $y = -(x - 5)^2$ — 5

c $y = -2(x + 1)^2$ — -1

d $y = x^2 + 3$ — none

e $y = -x^2 - 3$ — none

f $y = 2x^2 - 3$ — $\pm\sqrt{\frac{3}{2}}$

g $y = -6x^2 - 1$ — none

h $y = (x - 1)^2 - 3$ — $1 \pm \sqrt{3}$

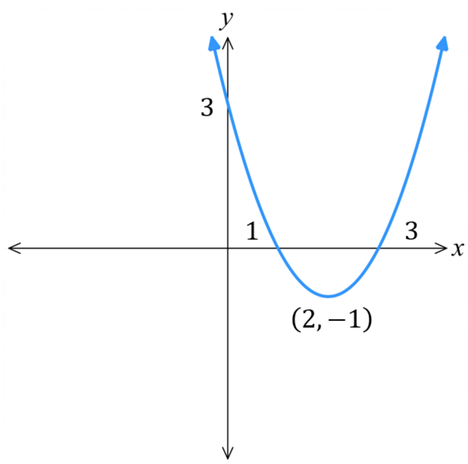
i $y = -(x + 2)^2 + 3$ — $-2 \pm \sqrt{3}$

j $y = (x + 1)^2 + 1$ — none

k $y = (x - 3)^2 - 3$ — $3 \pm \sqrt{3}$

l $y = -(x + 7)^2 - 14$ — none

3 The equation of a parabola is $y = (x-2)^2 - 1$



a Determine the concavity: ....

b Determine the y-intercept: ....

c Determine the x-intercepts:

....

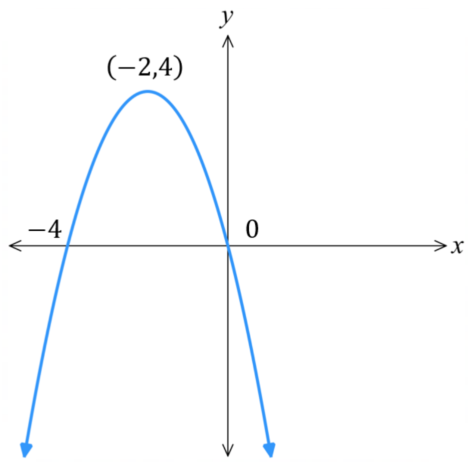
....

....

d Determine the coordinates of the vertex:

....

4 The equation of a parabola is $y = -(x+2)^2 + 4$



a Determine the concavity: ....

b Determine the y-intercept: ....

c Determine the x-intercepts:

....

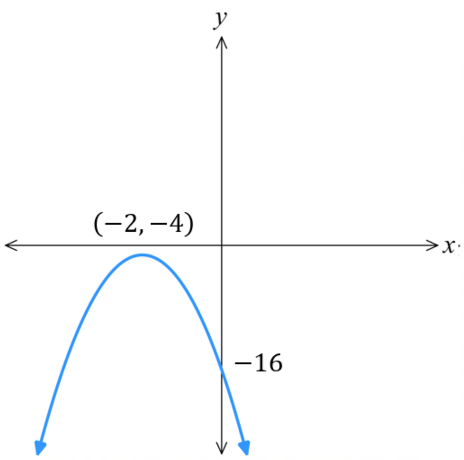
....

....

d Determine the coordinates of the vertex:

....

5 The equation of a parabola is $y = -3(x+2)^2 - 4$



a Determine the concavity: ....

b Determine the y-intercept: ....

c Determine the x-intercepts:

....

....

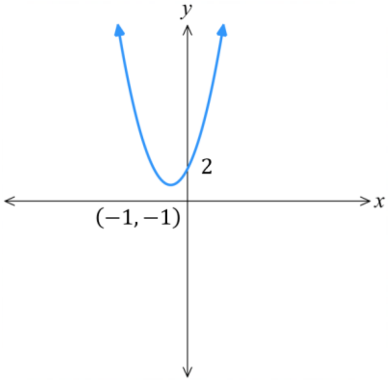
....

d Determine the coordinates of the vertex:

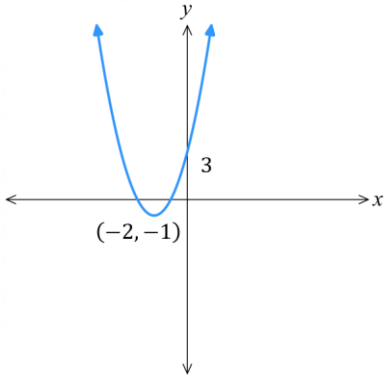
....

6 Sketch these graphs. Clearly show the turning point and intercepts.

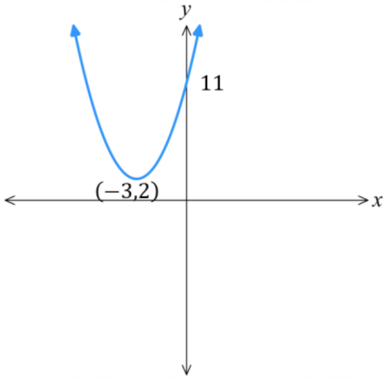
a $y = (x+1)^2 + 1$



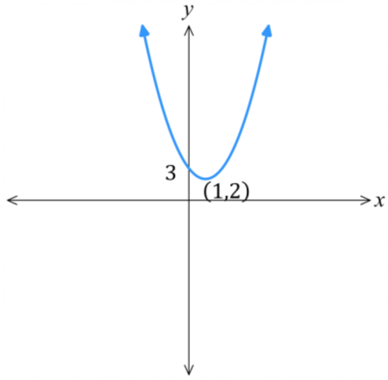
b $y = (x+2)^2 - 1$



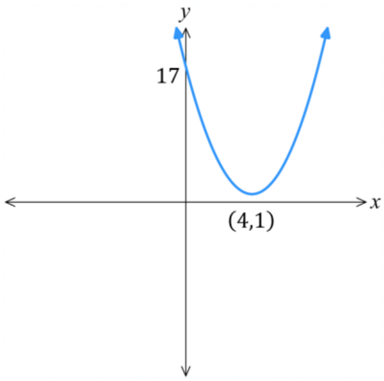
c $y = (x+3)^2 + 2$



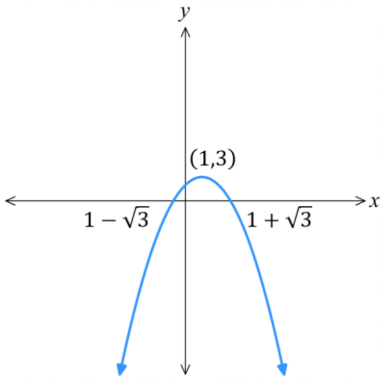
d $y = (x-1)^2 + 2$



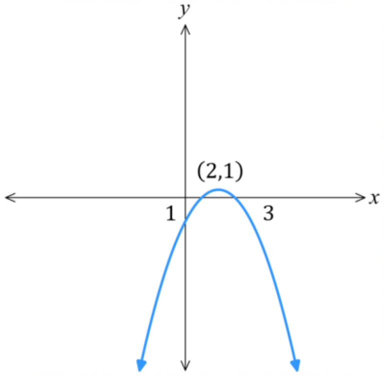
e $y = (x-4)^2 + 1$



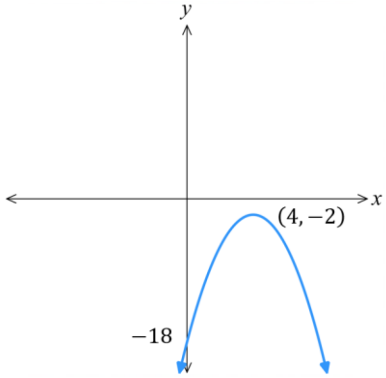
f $y = -(x-1)^2 + 3$



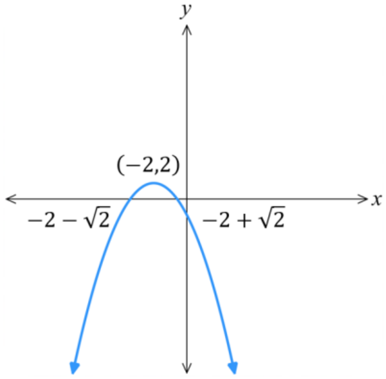
g $y = -(x-2)^2 + 1$



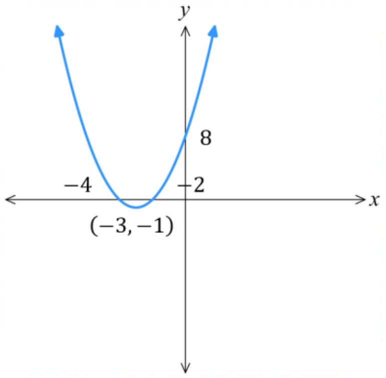
h $y = -(x-4)^2 - 2$



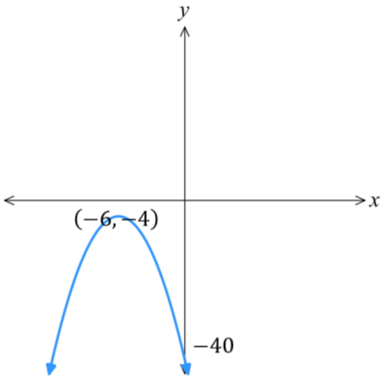
i $y = -(x+2)^2 + 2$



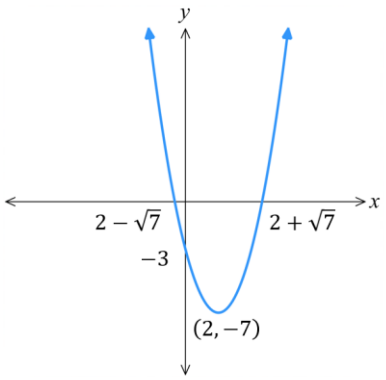
j $y = (x+3)^2 - 1$



k $y = -(x+6)^2 - 4$

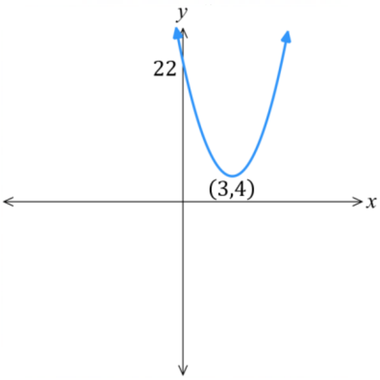


l $y = (x-2)^2 - 7$

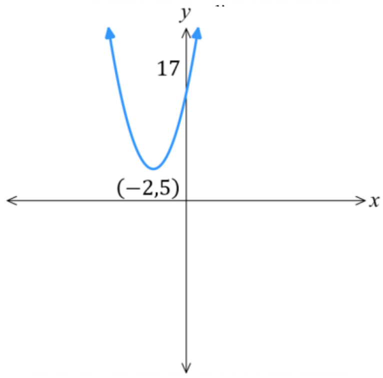


7 Sketch these parabolas. Clearly show the turning points and intercepts.

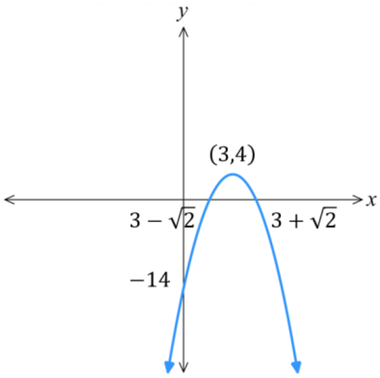
a $y = 2(x-3)^2 + 4$



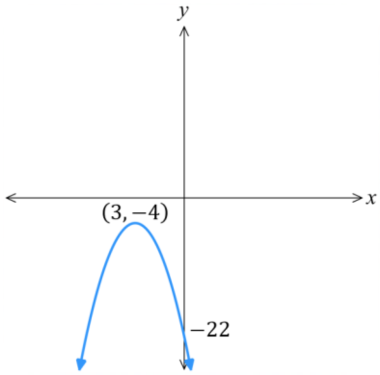
b $y = 3(x+2)^2 + 5$



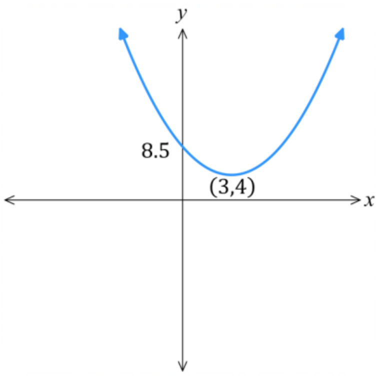
c $y = -2(x-3)^2 + 4$



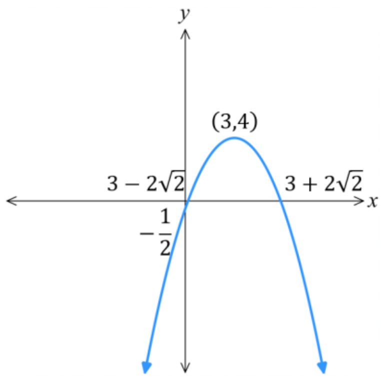
d $y = -2(x+3)^2 - 4$



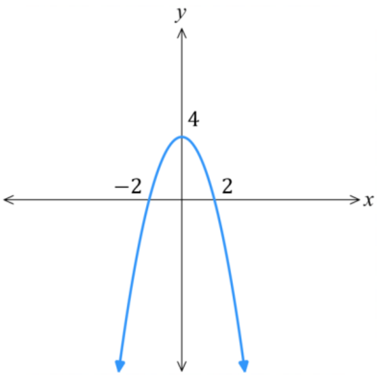
e $y = \frac{1}{2}(x-3)^2 + 4$



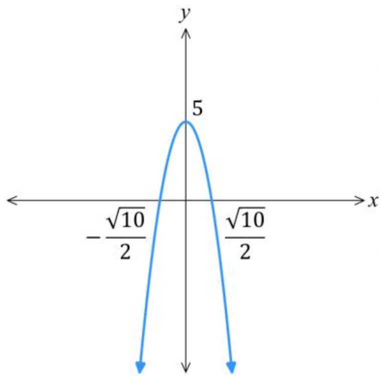
f $y = -\frac{1}{2}(x-3)^2 + 4$



g $y = 4 - x^2$



h $y = 5 - 2x^2$



SKETCHING QUADRATIC EQUATIONS BY CTS

- When factorisation isn't possible, **complete the square** to write a quadratic in vertex form.
  0. Factorise by CTS: halve the coefficient of x, square it, then add subtract this value.
  1. Concavity
  2. y-int
  3. x-int
  4. Vertex

Example Sketch parabolas by completing the square.

Sketch $y = x^2 - 4x - 5$

0. Factorise by CTS

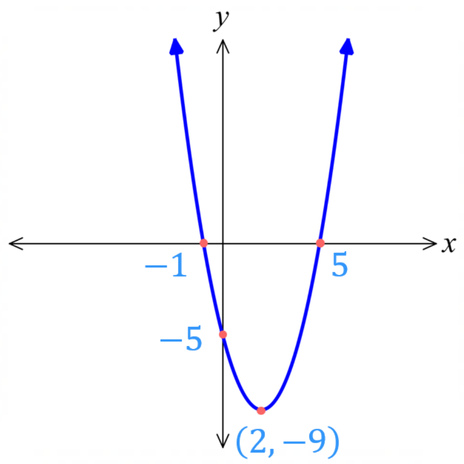
$$y = x^2 - 4x + 4 - 4 - 5$$
$$y = (x^2 - 4x + 4) - 4 - 5$$
$$y = (x - 2)^2 - 9$$

1. Concavity $k > 0$, concave up.

2. y-int: Let $x = 0$,
$y = (0 - 2)^2 - 9 = -5$

3. x-int: Let $y = 0$
$(x - 2)^2 - 9 = 0$
$(x - 2)^2 = 9$
$x - 2 = \pm 3$
$x = 2 \pm 3$
$x = 5$ or -1

4. Vertex: $(2, -9)$



Sketch $y = x^2 + 6x + 15$

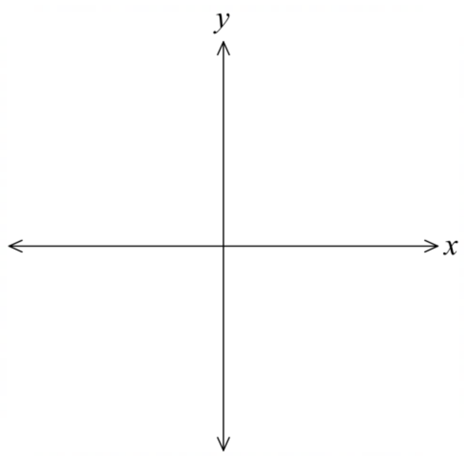
0. Factorise by CTS

1. Concavity

2. y-int:

3. x-int:

4. Vertex:



a $y = x^2 - 4x - 1$

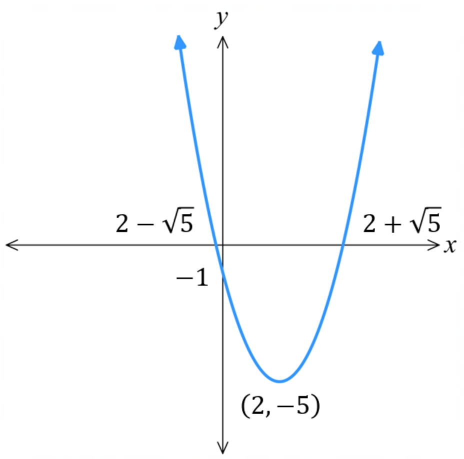
0. Factorise by CTS

1. Concavity

2. y-int:

3. x-int:

4. Vertex:



b $y = x^2 - 8x - 11$

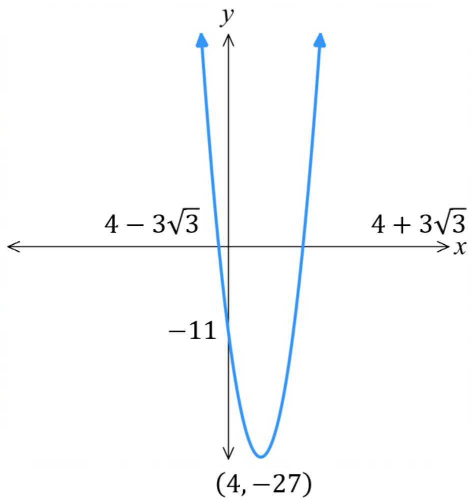
0. Factorise by CTS

1. Concavity

2. y-int:

3. x-int:

4. Vertex:

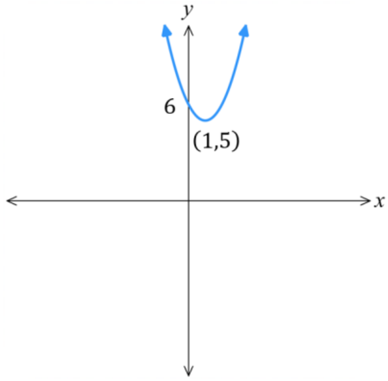


1 Sketch these parabolas by completing the square first. Show the vertex and all intercepts.

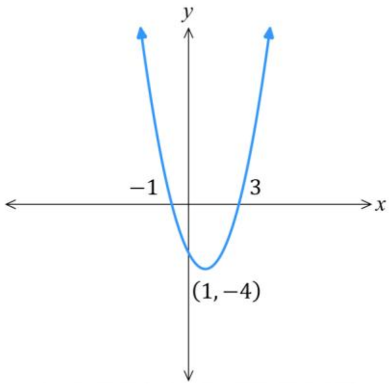
a $y = x^2 - 2x + 6$

$y = x^2 - 2x + 1 - 1 + 6$

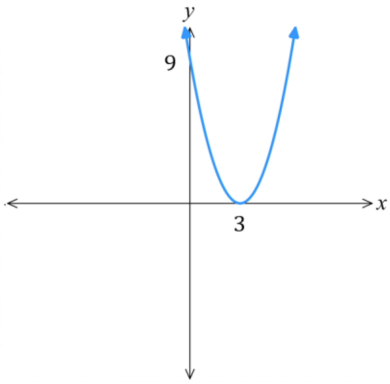
$y = (x - 1)^2 + 5$



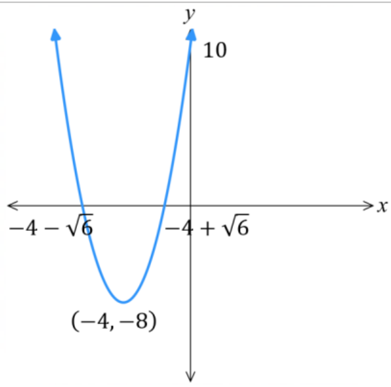
b $y = x^2 - 2x - 3$



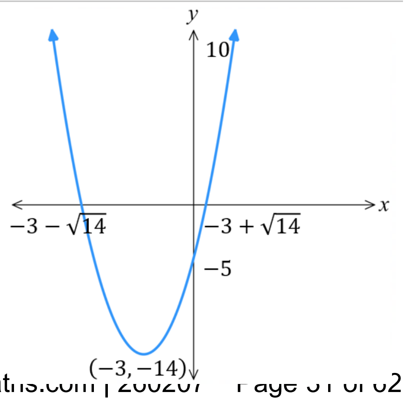
c $y = x^2 - 6x + 9$



d $y = x^2 + 8x + 10$



e $y = x^2 + 6x - 5$



2 Sketch these parabolas by taking out a common factor, then CTS. Show vertex and intercepts.

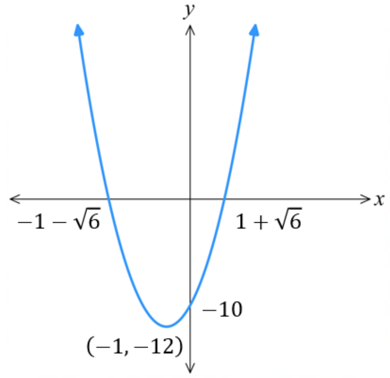
a $y = 2x^2 + 4x - 10$

$y = 2(x^2 + 2x - 5)$

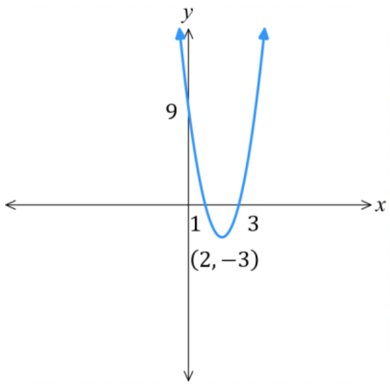
$y = 2(x^2 + 2x + 1 - 1 - 5)$

$y = 2((x + 1)^2 - 6)$

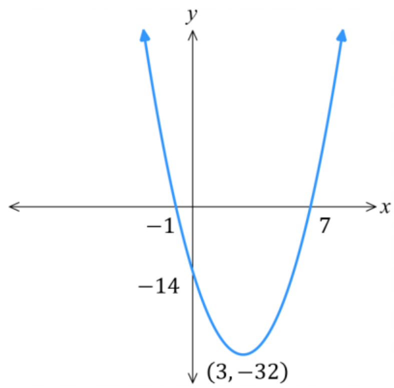
$y = 2(x + 1)^2 - 12$



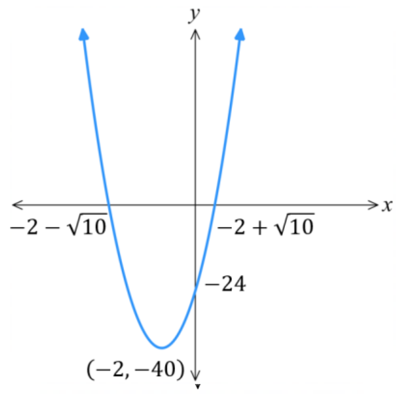
b $y = 3x^2 - 12x + 9$



c $y = 2x^2 - 12x - 14$



d $y = 4x^2 + 16x - 24$



3 Sketch these parabolas by taking out the negative, then completing the square. Show the vertex and all intercepts.

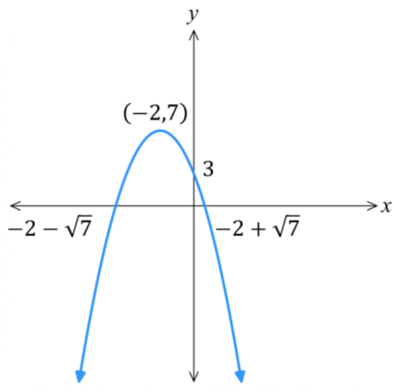
a $y = -x^2 - 4x + 3$

$y = -(x^2 + 4x - 3)$

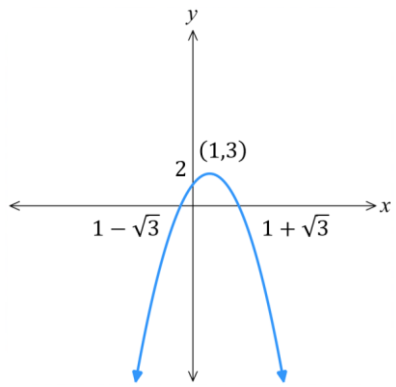
$y = -(x^2 + 4x + 4 - 4 - 3)$

$y = -((x + 2)^2 - 7)$

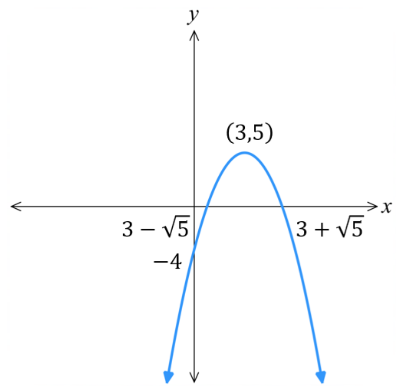
$y = -(x + 2)^2 + 7$



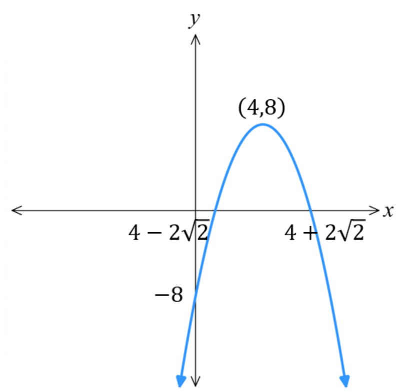
b $y = -x^2 + 2x + 2$



c $y = -x^2 + 6x - 4$



d $y = -x^2 + 8x - 8$



4 Sketch these parabolas by making the quadratic monic.

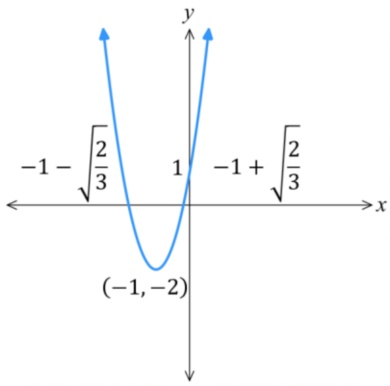
a $y = 3x^2 + 6x + 1$

$y = 3\left(x^2 + 2x + \frac{1}{3}\right)$

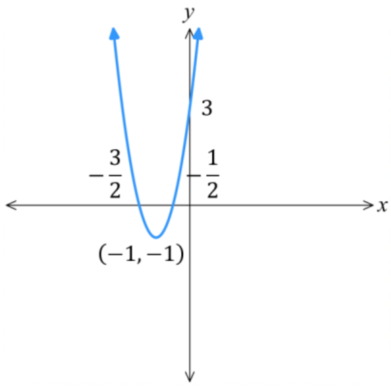
$y = 3\left(x^2 + 2x + 1 - 1 + \frac{1}{3}\right)$

$y = 3\left((x+1)^2 - \frac{2}{3}\right)$

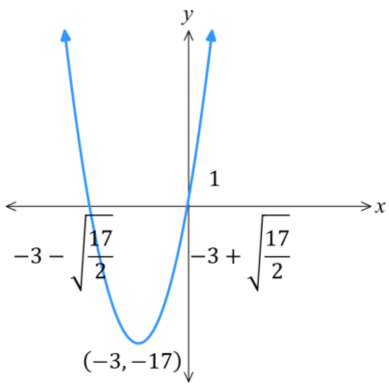
$y = 3(x+1)^2 - 2$



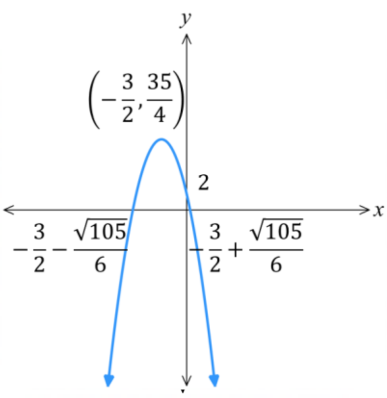
b $y = 4x^2 + 8x + 3$



c $y = 2x^2 + 12x + 1$

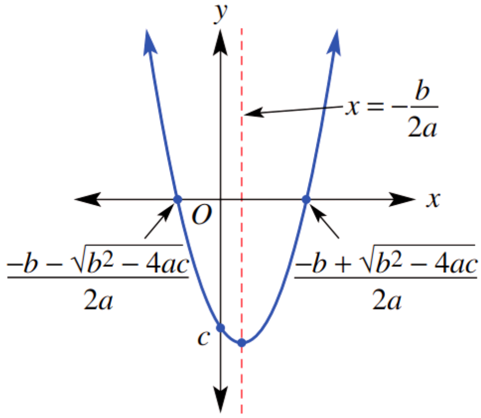


d $y = -3x^2 - 9x + 2$



SKETCHING QUADRATIC EQUATIONS BY THE FORMULA

- When factorisation isn't possible, use the quadratic formula to find the x-intercepts and vertex.
  1. Concavity.
  2. y-int.
  3. x-ints using $x = \frac{-b \pm \sqrt{b^2 - ac}}{2a}$
  4. Vertex using $x = -\frac{b}{2a}$,
     then substitute value of x into the equation to find y.



Example Sketch parabolas by using the quadratic formula.

Sketch $y = -x^2 + 6x + 1$

1. Concavity: $k < 0$, concave down.

2. y-int: Let $x = 0$,
$y = 1$

3. x-int:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(1)}}{2(-1)}$$

$$= \frac{-6 \pm \sqrt{40}}{-2}$$

$$= \frac{-6 \pm 2\sqrt{10}}{-2}$$

$$= 3 \pm \sqrt{10}$$

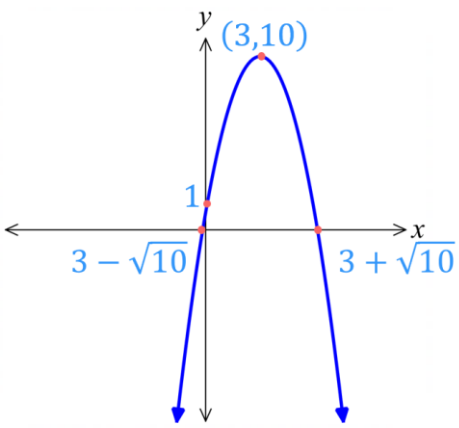
4. Vertex:

$$x = \frac{-6}{2(-1)} = 3$$

$$y = -(3)^2 + 6(3) + 1$$

$$= 10$$

Vertex at $(3,10)$



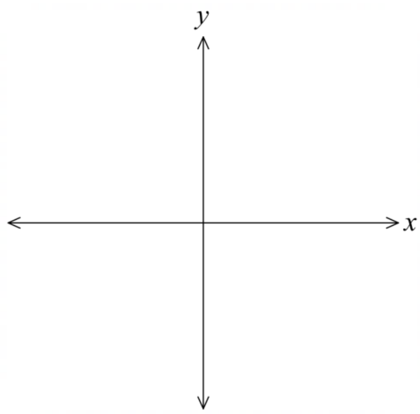
Sketch $y = x^2 + 4x + 1$

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



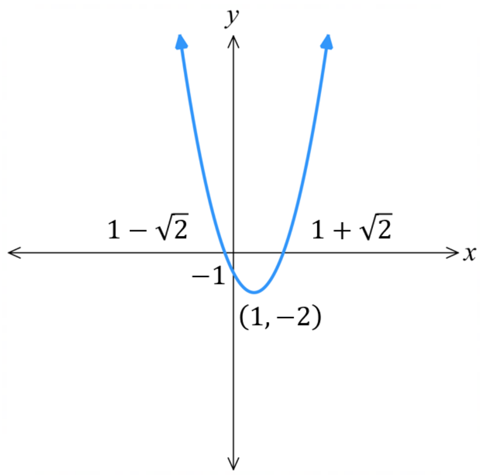
a $y = x^2 - 2x - 1$

1. Concavity:

2. y-int:

3. x-int:

4. Vertex:



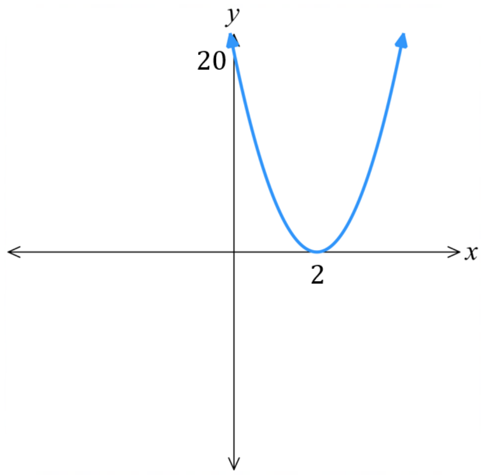
b $y = 5x^2 - 20x + 20$

1. Concavity:

2. y-int:

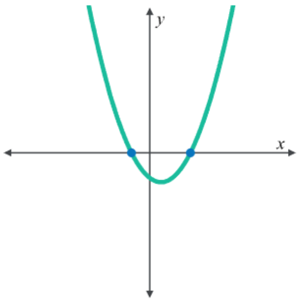
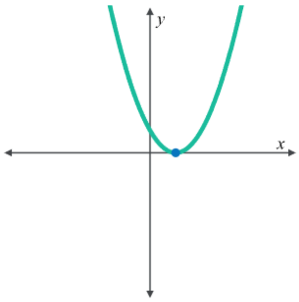
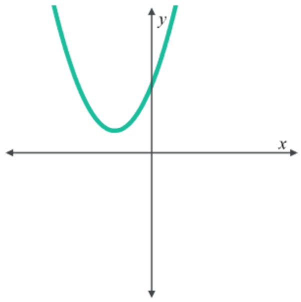
3. x-int:

4. Vertex:



THE DISCRIMINANT

- The **discriminant** $\Delta = b^2 - 4ac$ is the expression inside the square root of the quadratic formula.
- The value of the discriminant tells **how many solutions**, or x-intercepts, a quadratic has.

$\boldsymbol{\Delta > 0}$	$\boldsymbol{\Delta = 0}$	$\boldsymbol{\Delta < 0}$
2 solutions	1 solution	No solutions
		

- Check the discriminant first, it tells you if x-intercepts exist.

Example Sketch parabolas using the discriminant.

Sketch $y = 3x^2 - 6x + 4$

1. Concavity: $k > 0$, concave up.

2. y-int: Let $x = 0$,
$y = 4$

3. x-int:

$\Delta = b^2 - 4ac$
$= (-6)^2 - 4(3)(4)$
$= -12$

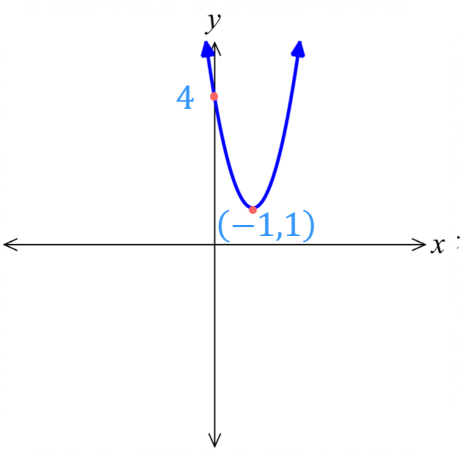
No x intercepts.

4. Vertex:

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1$$

$y = 3(1)^2 - 6(1) + 4 = 1$

Vertex at $(1,1)$



How did we know there were no x-intercepts?

The value of the discriminant was ……………

What would have happened if you tried to use the quadratic formula?

You would be square rooting a ………………

How did finding the discriminant save us time?

We did not need to attempt to find the …… intercepts.

How did we find the vertex?

We found the x-coordinate using the formula …………, then substituted into the ………………… to find y.

Sketch these parabolas by checking the discriminant first.

$y = x^2 - 4x + 5$

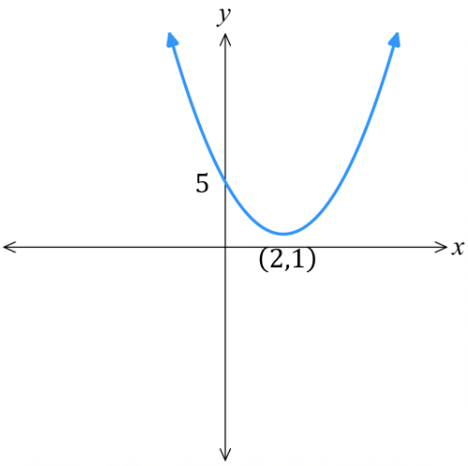
1. Concavity:

2. y-int:

3. x-int:

$\Delta = b^2 - 4ac$

4. Vertex:



$y = -4x^2 - 13x - 53$

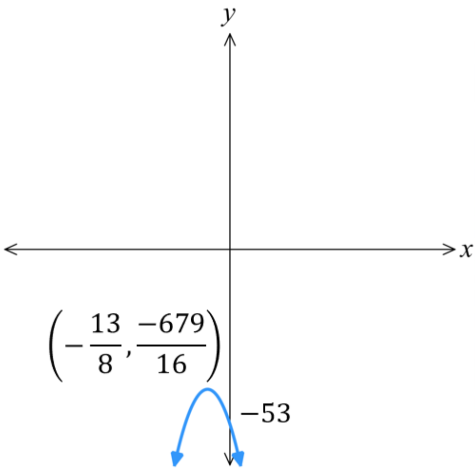
1. Concavity:

2. y-int:

3. x-int:

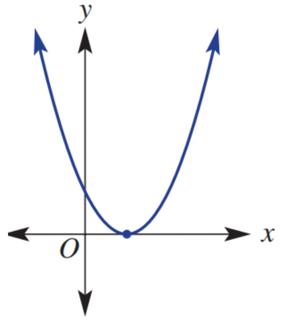
$\Delta = b^2 - 4ac$

4. Vertex:



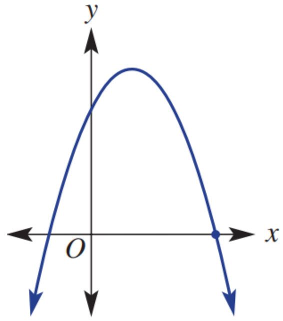
1 For these graphs, state whether the discriminant would be zero, positive, or negative.

a



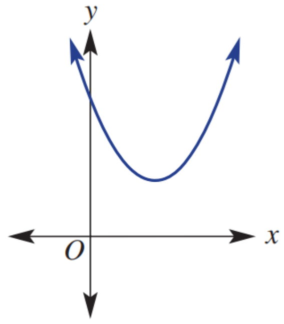
$\Delta = 0$

b



$\Delta > 0$

c



$\Delta < 0$

2 Find the discriminant of these graphs.

a $y = x^2 + 2x + 3$

$\Delta = b^2 - 4ac$

$= \dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots$

-8

b $y = -2x^2 - 5x + 8$

89

c $y = 5x^2 - 7$

140

3 Use $x = -\dfrac{b}{2a}$ to determine the x-coordinate of the vertex.

a $y = x^2 + 4x - 1$

$x = -\dfrac{b}{2a}$

$= \dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots$

$x = -2$

b $y = -x^2 - 3x + 4$

$x = -\dfrac{3}{2}$

c $y = 4x^2 - 3x$

$x = \dfrac{3}{8}$

4 A parabola has the equation $y = x^2 - 2x - 1$

a Determine the concavity: $\dots\dots\dots\dots\dots\dots$

b Determine the y- intercept: $\dots\dots\dots\dots\dots\dots$

c Determine the x- intercepts:

$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

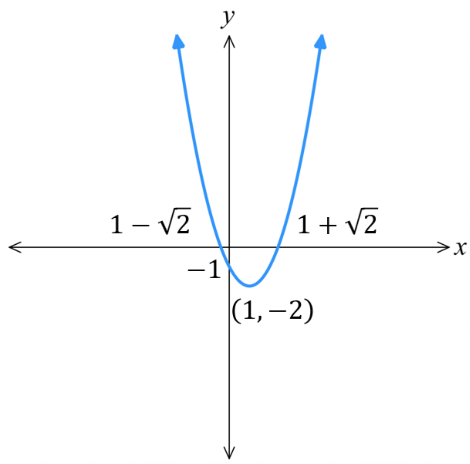
$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

d Determine the coordinates of the vertex:

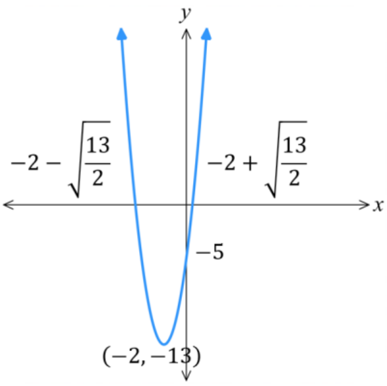
$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

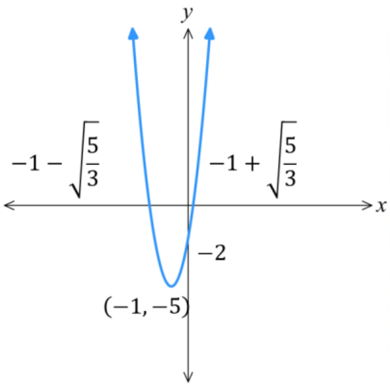


5 Sketch graphs of these quadratics. Give roots in exact surd form.

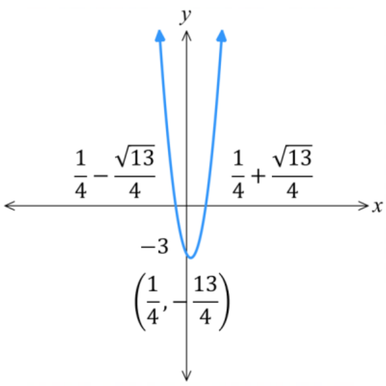
a $y = 2x^2 + 8x - 5$



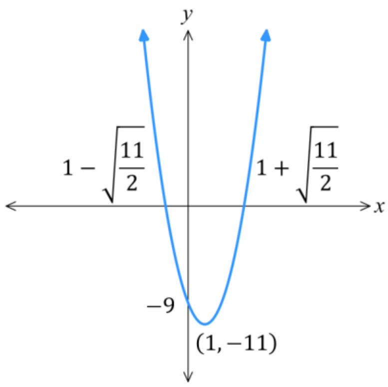
b $y = 3x^2 + 6x - 2$



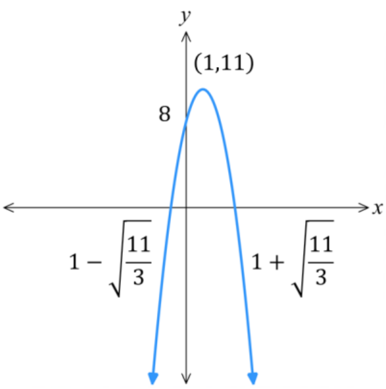
c $y = 4x^2 - 2x - 3$



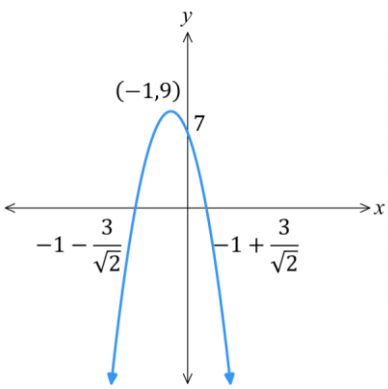
d $y = 2x^2 - 4x - 9$



e $y = -3x^2 + 6x + 8$



f $y = -2x^2 - 4x + 7$



6 Use the quadratic formula to find the roots α and β for each equation $ax^2 + bx + c = 0$.

a $x^2 - 6x + 1 = 0$

$x = 3 - 2\sqrt{2},\ 3 + 2\sqrt{2}$

b $x^2 - 2x - 4 = 0$

$x = 1 - \sqrt{5},\ 1 + \sqrt{5}$

c $-3x^2 + 10x - 5 = 0$

$x = \frac{5}{3} - \frac{\sqrt{10}}{3},\quad \frac{5}{3} + \frac{\sqrt{10}}{3}$

For each pair of roots above, find $\alpha + \beta$ and $\alpha \times \beta$

$\alpha + \beta =$

6

$\alpha \times \beta =$

1

$\alpha + \beta =$

2

$\alpha \times \beta =$

-4

$\alpha + \beta =$

$\frac{10}{3}$

$\alpha \times \beta =$

$\frac{5}{3}$

What do you notice about the sums and products of roots and the coefficients of the equation?

The sum of roots is negative the coefficient of x when the equation is monic.

The product of roots is the constant term when the equation is monic.

8

a Express $y = x^2 - 6x + 10$ in vertex form.

$y = (x - 3)^2 + 1$

b Using your answer to part **a**, explain why the parabola is always above the x-axis.

The value of y cannot be negative, since $(x - 3)^2 \geq 0$

c Express $y = -x^2 + 2x - 4$ in vertex form.

$y = -(x - 1)^2 - 3$

d Georgie says that $y = -x^2 + 2x - 4$ is always above the x-axis. Explain why she is wrong.

It is always below the x-axis.

7 Consider the general quadratic equation $y = ax^2 + bx + c$
Match each statement to one of the graphs.

a ……… $\Delta = 0$ and $a > 0$.

D

b ……… $\Delta = 0$ and $a < 0$.

B

c ……… $\Delta > 0$ and $a > 0$.

A

d ……… $\Delta > 0$ and $a < 0$.

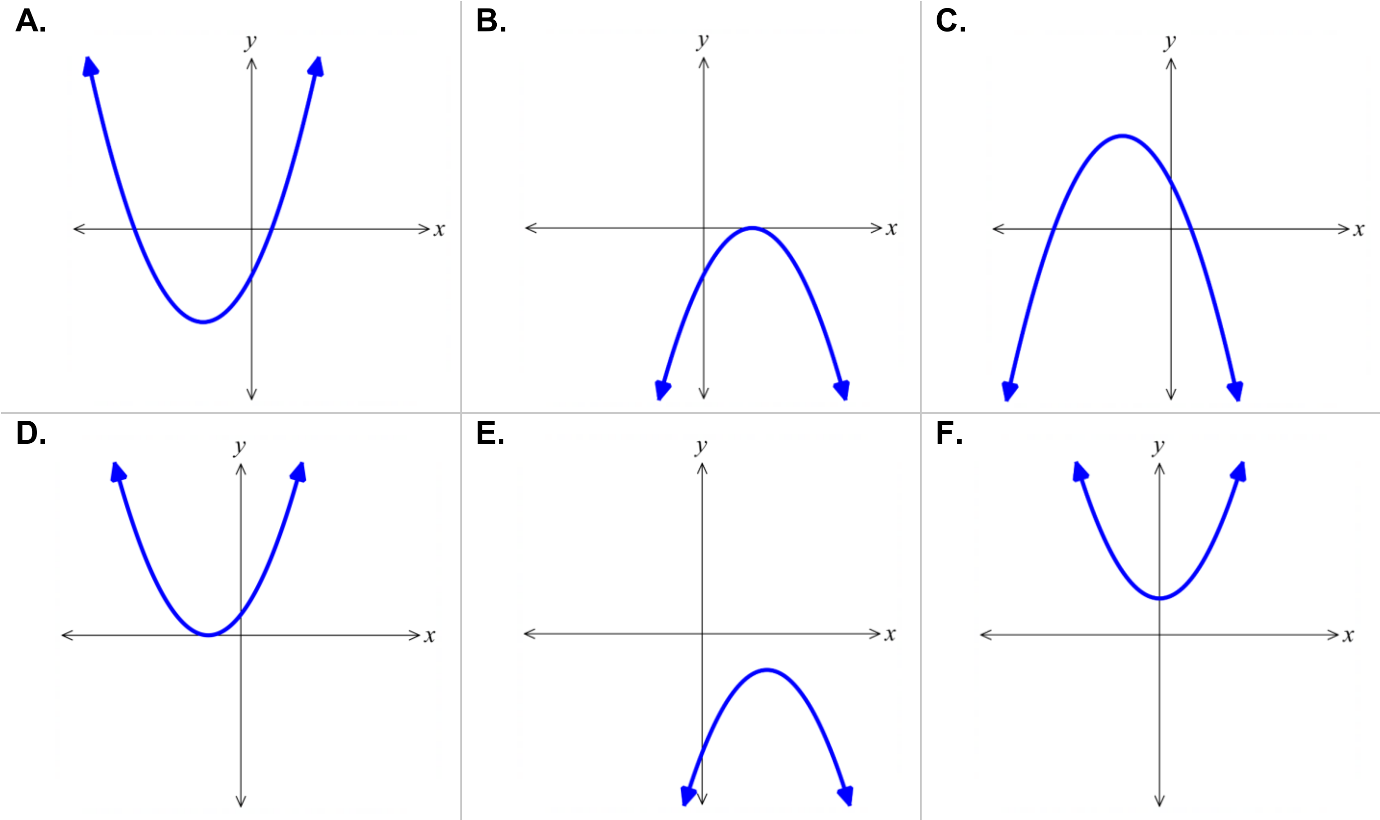
C

e ……… $\Delta < 0$ and $a > 0$.

F

f ……… $\Delta < 0$ and $a < 0$.

E



12 Explain why $y = x^2 - 2x + 1$ has only one x-intercept.

The discriminant is 0.

13 Explain why $y = x^2 + 2$ has no x-intercepts.

The discriminant $\Delta = -8$ is negative.

8 Consider the general quadratic equation $y = ax^2 + bx + c$

Match each statement to one or more of the graphs.

a .... The axis of symmetry is $x = 3$. A

b .... The coefficient of x^2 is less than one and positive. C

c .... The axis of symmetry is $x = k$, where $k < 0$. E, F

d .... The discriminant is negative and the coefficient of x^2 is positive. H

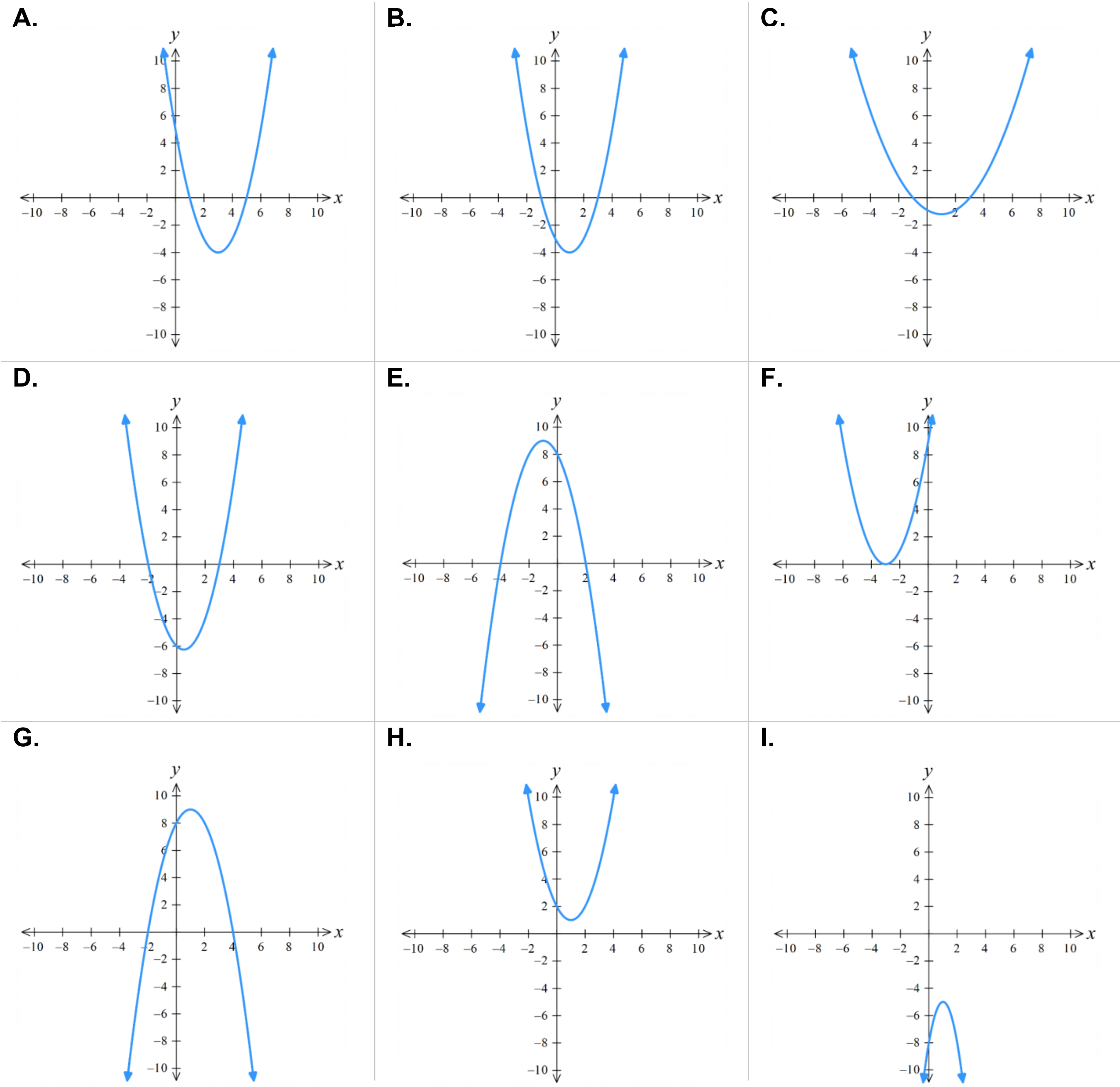
e .... The value of $-\frac{b}{2a} = 1$. B, C, G, H, I

f .... The constant term is -8. I

g .... The points $(0,8)$ and $(2,8)$ lie on the curve. G

h .... The discriminant is zero. F

i .... When factorised to $y = (x - \alpha)(x - \beta)$, $\alpha + \beta$ is odd. D, F



9

a Show that the discriminant of $x^2 - (p+1)x + p - 1 = 0$ is $\Delta = (p-1)^2 + 4$

b Hence explain why the parabola will always have two intercepts.

The discriminant $\Delta = (p-1)^2 + 4$ is always positive, so always 2 solutions and hence 2 x-intercepts.

10 If $y = px^2 + (p+1)x + \frac{p}{4}$, find the values of p such that the parabola is always below the x-axis.

$$\Delta = (p+1)^2 - 4p\left(\frac{p}{4}\right)$$
$$= p^2 + 2p + 1 - p^2$$
$$= 2p + 1$$
Let $\Delta < 0$
$$\therefore p < -\frac{1}{2}$$

11 Show that for $y = x^2 + 2mx + m^2 + n^2$, the graph is always touching or above the x-axis for all real values of m and n.

$$\Delta = 4m^2 - 4(m^2 + n^2)$$
$$= -4n^2$$
$-4n^2$ cannot be positive, so either $\Delta = 0$; graph touching x-axis, or $\Delta < 0$; graph is always above the x-axis and never touches it (always above since a>0)

12 Show that $y = (p-3)x^2 + 6x - (p+3)$ will always have rational roots for all real values of p.

$$\Delta = 36 + 4(p-3)(p+3)$$
$$= 46 + 4(p^2 - 9)$$
$$= 4p^2$$
Since the discriminant is always a perfect square, roots will always be rational.

FINDING THE EQUATION OF A PARABOLA FROM ITS GRAPH

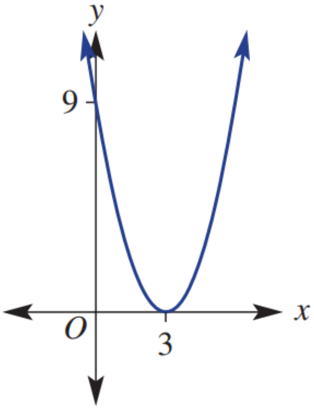
FOUNDATION

1 Match each equation to a graph.

$y = 2x^2$ $\quad$ $y = x^2 - 6$ $\quad$ $y = (x + 2)^2$ $\quad$ $y = -5x^2$

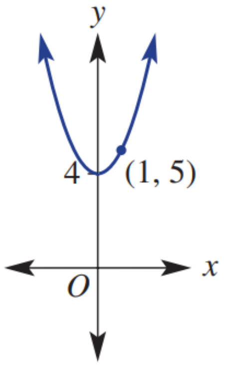
$y = (x - 3)^2$ $\quad$ $y = \frac{1}{4}x^2$ $\quad$ $y = x^2 + 4$ $\quad$ $y = (x - 2)^2 + 3$

a



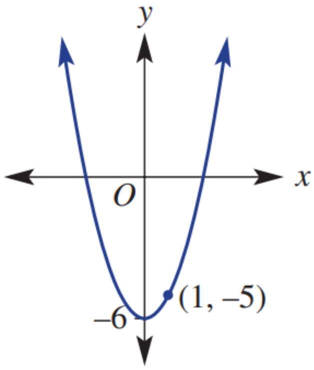
$y = (x - 3)^2$

b



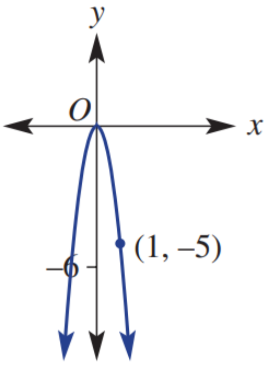
$y = x^2 + 4$

c



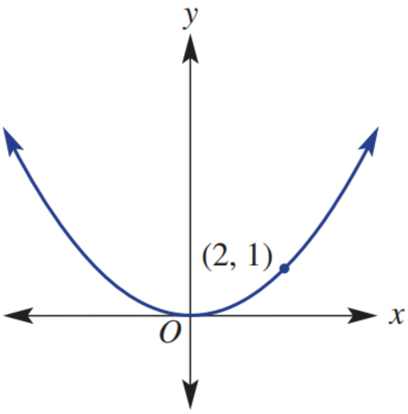
$y = x^2 - 6$

d



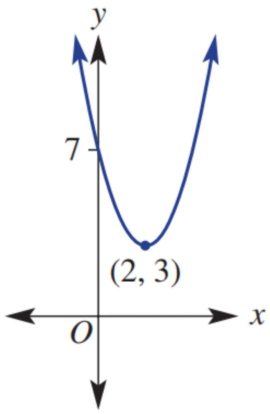
$y = -5x^2$

e



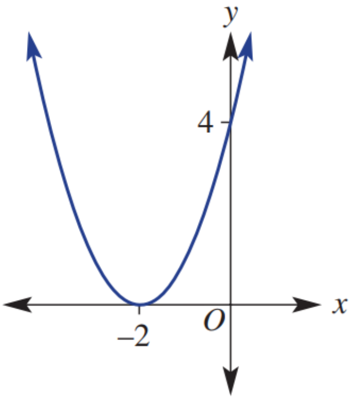
$y = \frac{1}{4}x^2$

f



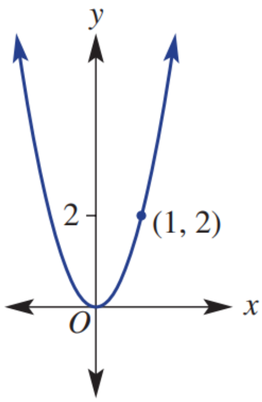
$y = (x - 2)^2 + 3$

g



$y = (x + 2)^2$

h

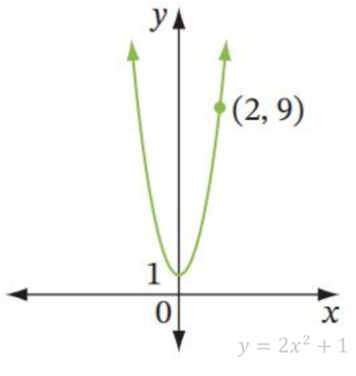


$y = 2x^2$

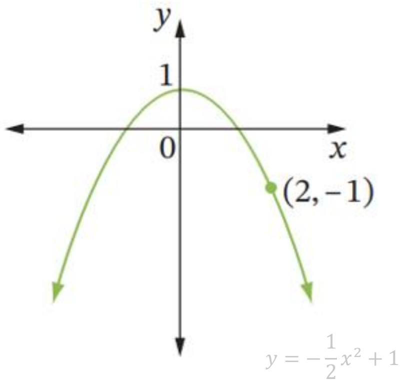
2 Match these parabolas to their equation.

$y = 5x^2$	$y = 2x^2 + 1$	$y = \frac{1}{2}x^2 - 1$
$y = \frac{1}{5}x^2$	$y = 2x^2 - 1$	$y = -5x^2$
$y = -\frac{1}{2}x^2 + 1$	$y = -\frac{1}{5}x^2$	$y = -2x^2 - 1$
$y = \frac{1}{2}x^2 + 1$	$y = -2x^2 + 1$	$y = -\frac{1}{2}x^2 - 1$

a



b



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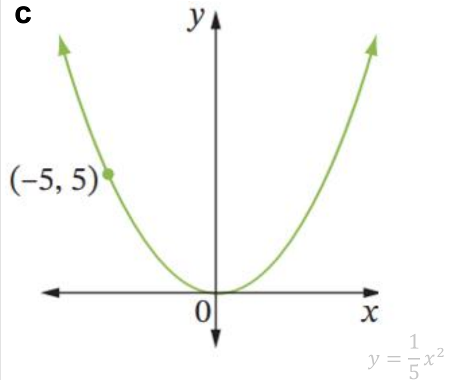

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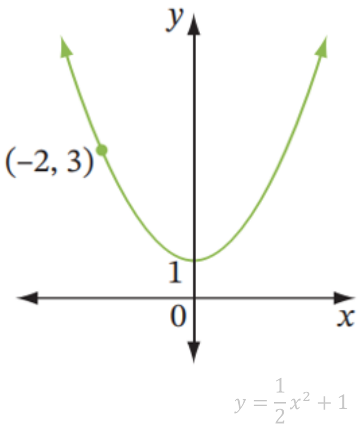
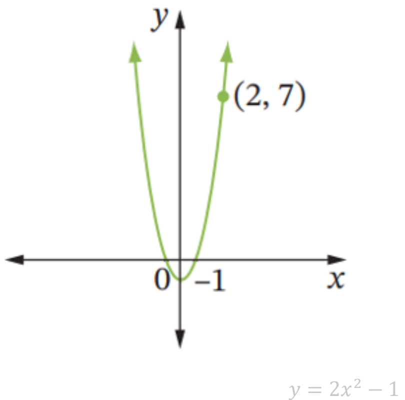


e



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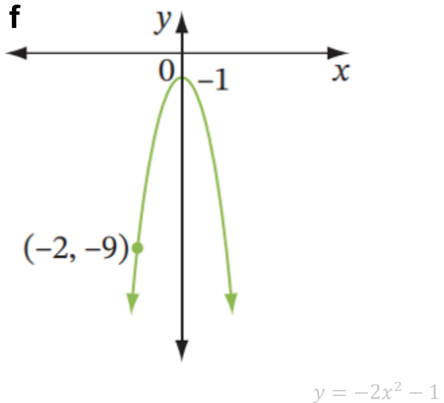
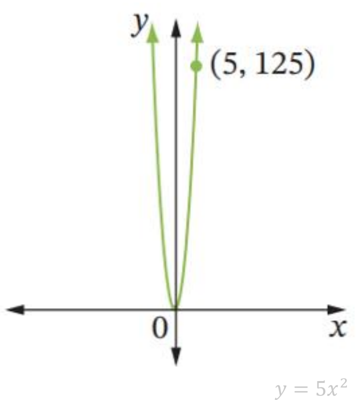


g



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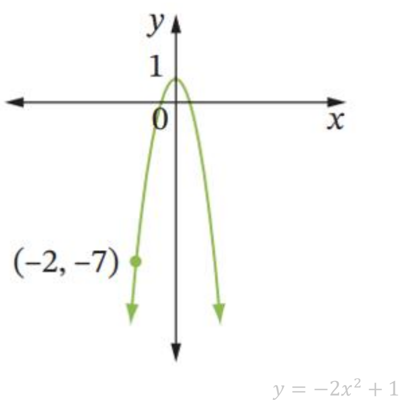
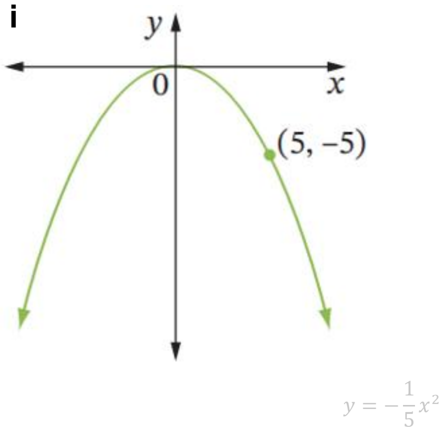


i



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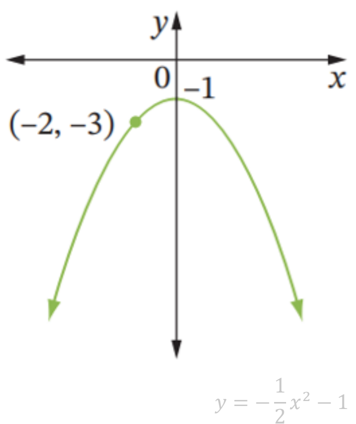
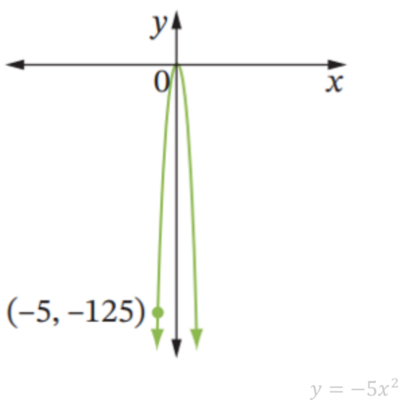
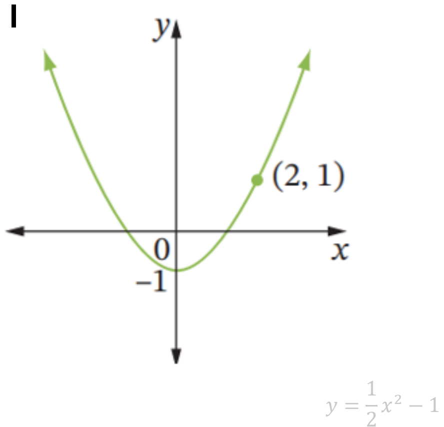


k

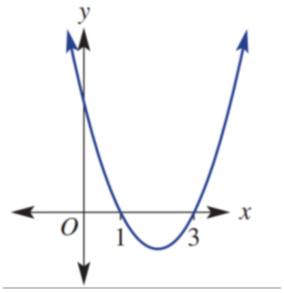


l



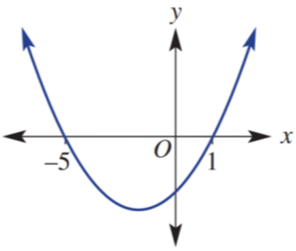
3 These equations are of the form $y = (x + \alpha)(x + \beta)$.
Use the x-intercepts to find the values of α and β, then find the coordinates of the turning point.

a



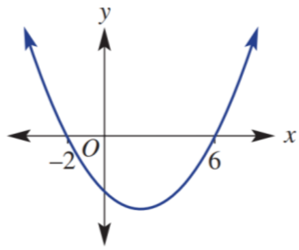
$y = (x - 1)(x - 3)$, turning point $(2, -1)$

b



$y = (x + 5)(x - 1)$, turning point $(-2 - 9)$

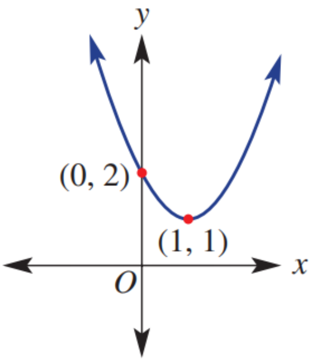
c



$y = (x + 2)(x - 6)$, turning point $(2, -16)$

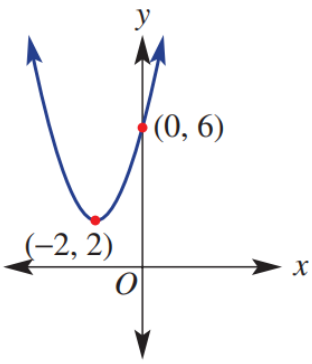
4 Write the equation for each of these graphs in the form $y = (x - b)^2 + c$.

a



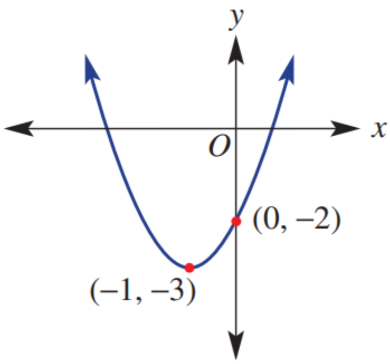
$y = (x - 1)^2 + 1$

b



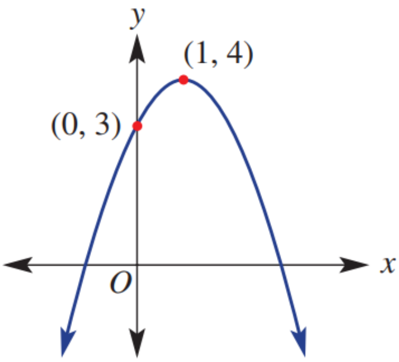
$y = (x + 2)^2 + 2$

c



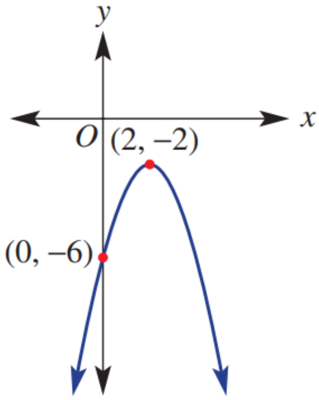
$y = (x + 1)^2 - 3$

d



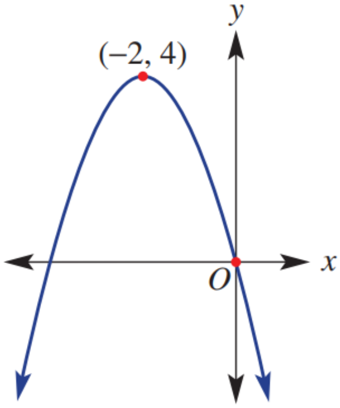
$y = -(x - 2)^2 + 4$

e



$y = -(x - 2)^2 - 2$

f



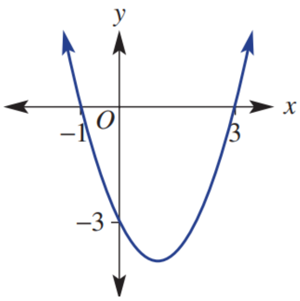
$y = -(x + 2)^2 + 4$

5 When given the x-intercepts of a parabola, we can find the equation using these steps:

1. Write the equation $y = k(x - \alpha)(x - \beta)$.
2. Substitute x-intercepts α and β.
3. Substitute a third point on the parabola (x_1, y_1) into the equation.
4. Solve for k.

Find the equation of these parabolas.

a



$y = k(x - \alpha)(x - \beta)$

$y = k(x + 1)(x - 3)$

Sub $(0, -3)$

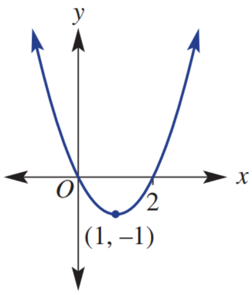
$-3 = k(0 + 1)(0 - 3)$

$-3 = -3k$

$k = 1$

$\therefore y = (x + 1)(x - 3)$

b



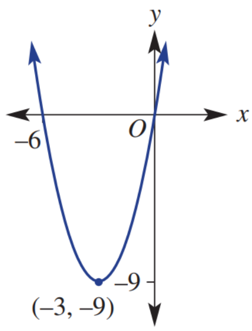
$y = k(x - \alpha)(x - \beta)$

$y = k(x + 0)(x - 2)$

Sub $(1, -1)$

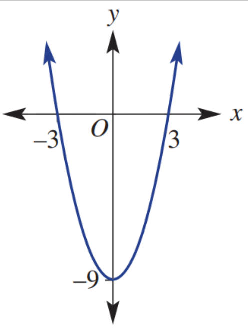
$y = x(x - 2)$

c



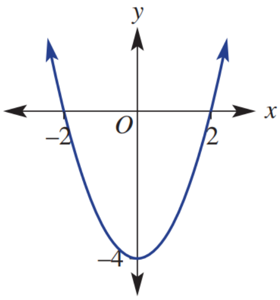
$y = x(x + 6)$

d



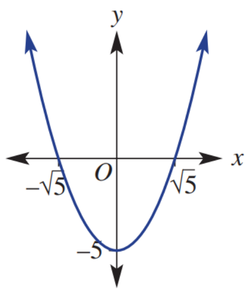
$y = (x + 3)(x - 3)$

e



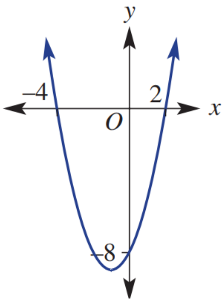
$y = (x + 2)(x - 2)$

f



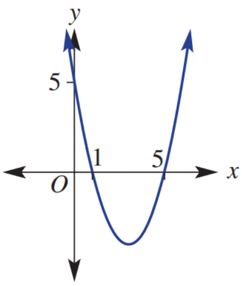
$y = (x + \sqrt{5})(x - \sqrt{5})$

g



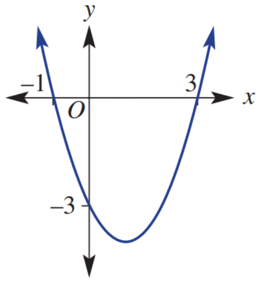
$y = 2(x+4)(x-2)$

h



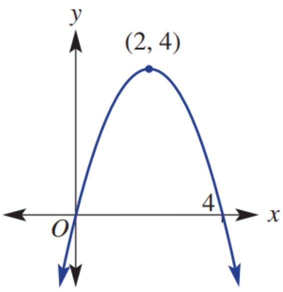
$y = 3(x-1)(x-5)$

i



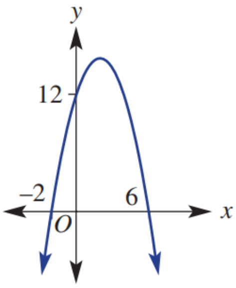
$y = 2(x+1)(x-3)$

j



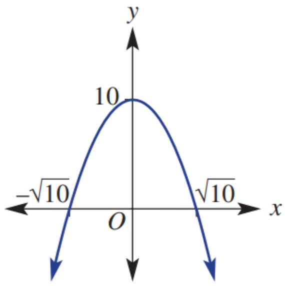
$y = -x(x-4)$

k



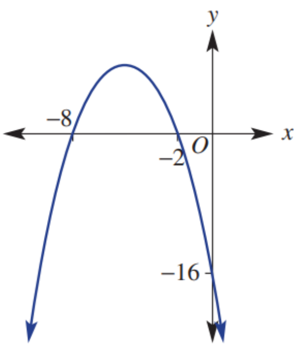
$y = -\frac{1}{2}(x+2)(x-6)$

l



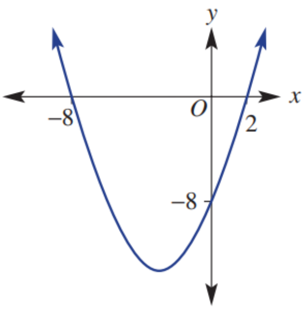
$y = -(x-\sqrt{10})(x+\sqrt{10})$

m



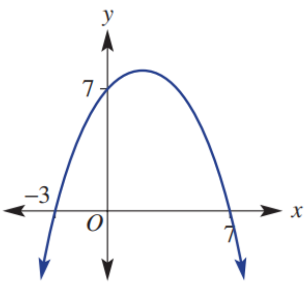
$y = -(x+8)(x+2)$

n



$y = \frac{1}{2}(x-2)(x+8)$

o



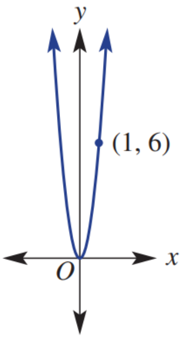
$y = -\frac{1}{3}(x+3)(x-7)$

13 When given the vertex of a parabola, we can determine the equation by:

1. Write the equation $y = k(x-b)^2 + c$.
2. Substitute the vertex (a, b).
3. Substitute a second point on the parabola (x_1, y_1).
4. Solve for k.

Determine the equation of these parabolas in the form $y = k(x-b)^2 + c$

a



$y = k(x-b)^2 + c$

Sub (0,0) $y = k(x-0)^2 + 0$

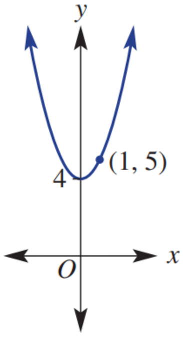
$y = kx^2$

Sub (1,6) $6 = k(1)^2$

$6 = k$

$\therefore y = 6x^2$

b



$y = k(x-b)^2 + c$

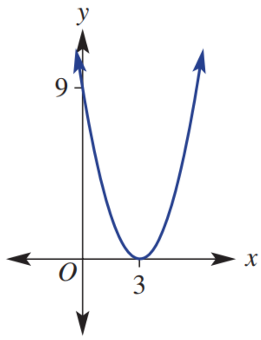
Sub (0,4) $y = k(x-0)^2 + 4$

$y = kx^2 + 4$

Sub (1,5)

$y = x^2 + 4$

c

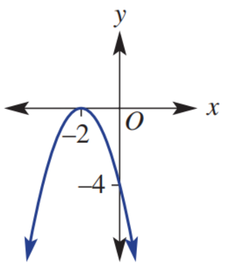


$y = k(x-b)^2 + c$

Sub (3,0)

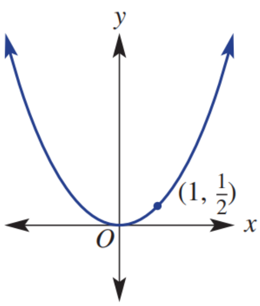
$y = (x-3)^2$

d



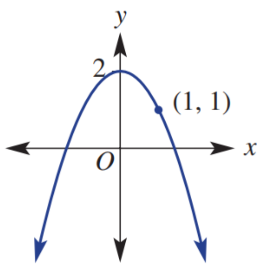
$y = -(x+2)^2$

e



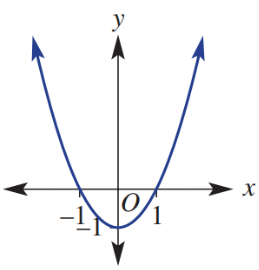
$y = \frac{1}{2}x^2$

f



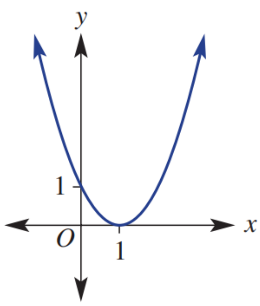
$y = -x^2 + 2$

g



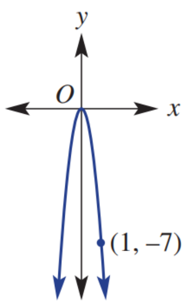
$y = x^2 - 1$

h



$y = (x-1)^2$

i



$y = -7x^2$

6 If the graph of $y = k(x + 2)(x - 4)$ passes through the point $(2, 16)$, determine the value of k using substitution, and hence the coordinates of the turning point for this parabola.

$16 = k(2+2)(2-4), \therefore k = -2$

Turning point is halfway between $x = -2$ and $x = 4$, $\therefore x = 1$

Substitute $x = 1$ into $y = -2(x+2)(x-4)$; $y = 18$

Turning point at $(1,18)$

14 $y = k(x+4)^2 + 2$ is the form of a quadratic with vertex $(-4,2)$.
Find the equation of such a quadratic if:

a The quadratic is monic.

$y = (x+4)^2 + 2$

b The coefficient of x^2 is 3.

$y = 3(x+4)^2 + 2$

c The y-intercept is 5.2

$y = \frac{1}{5}(x+4)^2 + 2$

d The curve passes through the origin.

$y = -\frac{1}{8}(x+4)^2 + 2$

9 Use the quadratic formula to find the roots and hence write the function in factored form

$$y = (x-a)(x-b)$$

a $y = x^2 - 6x + 4$

$(x + \sqrt{5} - 3)(x - \sqrt{5} - 3)$

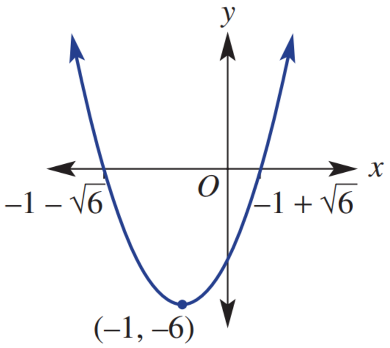
b $y = x^2 + 2x - 1$

$(x + \sqrt{2} + 1)(x - \sqrt{2} + 1)$

c $y = -x^2 + 3x - 1$

$\left(-x + \frac{\sqrt{5}+3}{2}\right)\left(x + \frac{\sqrt{5}-3}{2}\right)$

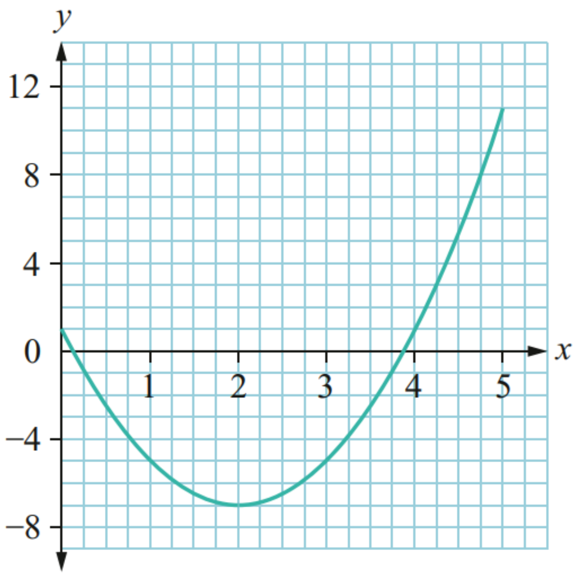
10 Find the equation in the form $y = ax^2 + bx + c$ that matches this graph.



$y = (x+1)^2 - 6$ or

$y = (x+1+\sqrt{6})(x+1-\sqrt{6})$

15 Which of these equations matches the parabola shown? Explain your reasoning.



A. $y = -2x^2 - 4x - 7$

B. $y = -2x^2 - 4x + 1$

C. $y = 2x^2 - 8x + 1$

D. $y = 2x^2 - 8x + 4$

Must be C or D as parabola is concave up.

y-intecept is 1, so constant term is 1.

Answer is C

16 $y = k(x-2)(x-8)$ is the form of a quadratic with x-intercepts at $x = 2$ and $x = 8$.
Find the equation of such a quadratic if:

a The coefficient of x^2 is 3

$k = 3$

b The y-intercept is -16

$k = -1$

c The vertex is $(5, -12)$

$-12 = k(5-2)(5-8)$

$k = \frac{4}{3}$

d The curve passes through $(1, -20)$

$-20 = k(1-2)(1-8)$

$k = -\frac{20}{7}$

17 A parabola has x-intercepts at $x = -1$ and $x = 6$. The y-intercept is 12. Find the equation.

$$y = k(x+1)(x-6)$$
$$12 = k(0+1)(0-6)$$
$$k = -2$$
$$\therefore y = -2(x+1)(x-6)$$

18 A parabola has x-intercepts at $x = -3$ and $x = 3$. The y-intercept is $-\frac{9}{2}$. Find the equation.

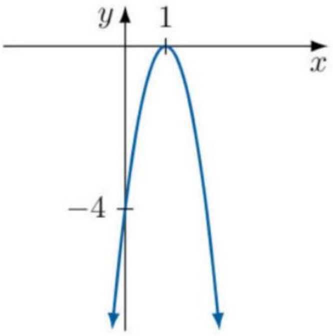
$$y = k(x+3)(x-3)$$
$$-\frac{9}{2} = k(0+3)(0-3)$$
$$k = \frac{1}{2}$$
$$\therefore y = \frac{1}{2}(x+3)(x-3)$$

19 A parabola has vertex $(-2,1)$ and passes through $(-5,28)$. Find the equation of the parabola.

$$y = k(x+2)^2 + 1$$
$$28 = k(-5+2)^2 + 1$$
$$k = 3$$
$$\therefore y = 3(x+2)^2 + 1$$

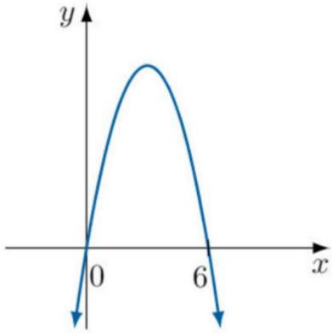
20 Find the equation of each parabola. You can answer in any appropriate form.

a



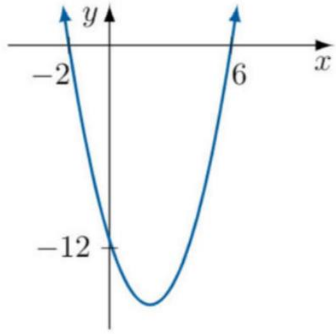
$y = -4(x-1)^2$

b



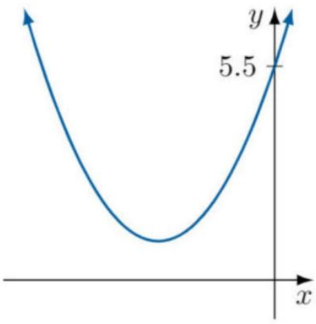
$y = -x(x-6)$

c



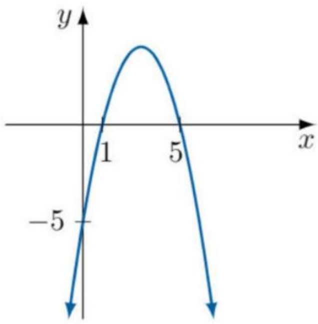
$y = (x+2)(x-6)$

d



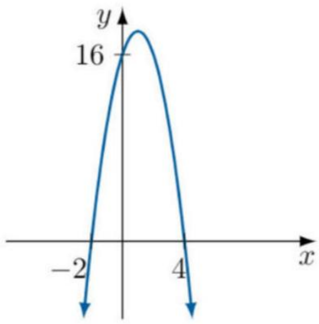
$y = \frac{1}{2}(x+3)^2 + 1$

e



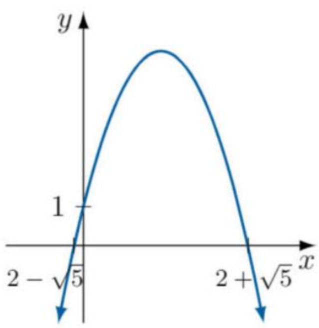
$y = -(x-1)(x-5)$

f



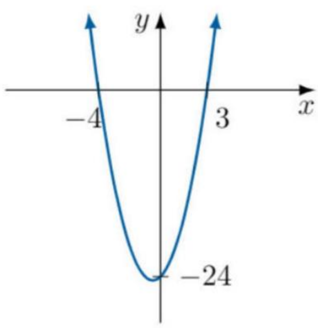
$y = -2(x+2)(x-4)$

g



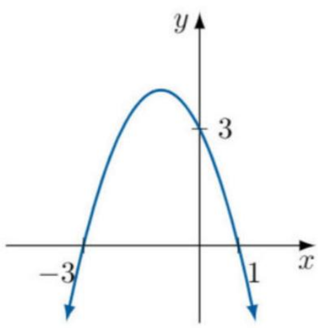
$y = -(x-2)^2 + 5$ or
$y = -(x-2-\sqrt{5})(x-2+\sqrt{5})$

h



$y = 2(x-3)(x-4)$

i



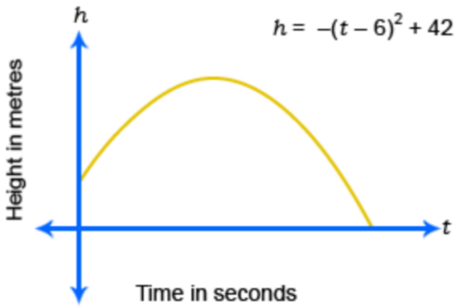
$y = -(x+1)^2 + 4$

APPLYING PARABOLAS TO SOLVE PROBLEMS

In this section, you will use your knowledge of quadratic modelling and sketching parabolas to solve application problems.

FOUNDATION

1 An arrow was shot into the air. The height, h metres, of the arrow above the ground at any time, t seconds, is given by the formula $h = -(t - 6)^2 + 42$. The included sketch may be useful.



a Using the vertex of this parabola, what was the maximum height reached by the arrow and at what time was this?

42 metres at 6 seconds

b From what height was the arrow initially fired? (Hint: 'Initially' means 'at the start', so t = 0)

6m

c How high above the ground was the arrow after 2 seconds?

26 m

d How high above the ground was the arrow after 10 seconds?

26 m

e How do the answers to (c) and (d) relate to the answers for (a)?

6 is the midpoint of 2 and 10. The height of the arrow is the same because a parabola is symmetrical.

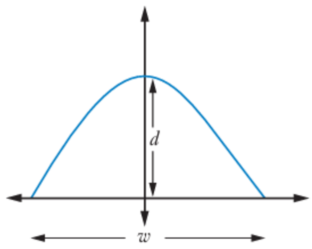
f How long will it take the arrow to hit the ground? (Hint: what height is the ground?)

Let $h = 0$ and solve; $t = -0.48$ s and 12.48 s. The arrow will hit the ground at 12.48 seconds.

g Another arrow's height can be determined using the formula $h = -a(t - n)^2 + p$, where a, n and p are unknown numbers.
Find the equation of its path if the arrow was initially fired from a height of 1 metre and it reached a maximum height of 28 m after 9 seconds.

$h = -\frac{1}{3}(t - 9)^2 + 28$

2 A satellite dish is in the shape of a parabola with equation $y = -3x^2 + 6$.
All dimensions are in metres.



a Find the depth of the dish.

y-intercept at 6 metres

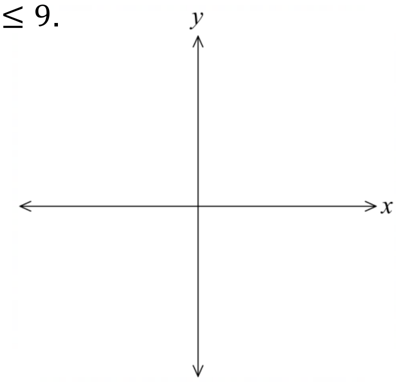
b Find the width of the dish. Answer to 1 d.p,

Let $y = 0$, $x = \pm\sqrt{2}$

So, the distance is $2\sqrt{2} \approx 2.8$ m

3 A carpenter programs a wood lathe to carve out a bowl according to the formula $d = \frac{1}{3}x^2 - 27$

a Sketch the graph for $-9 \leq x \leq 9$.



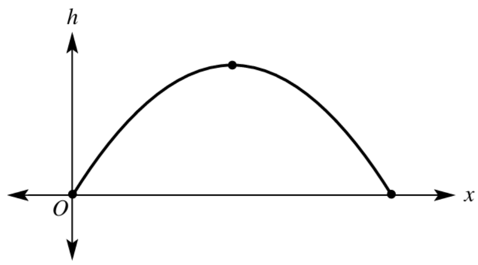
b What is the breadth of the bowl?

18 cm

c What is the maximum depth of the bowl?

27 cm

4 The equation for the arch of a bridge is $h = -\frac{1}{500}(x - 100)^2 + 20$,
where h is the height above the base in metres and x is distance from the left part of the bridge.



a Determine the coordinate of the turning point of the parabola.

$(100, 20)$

b Determining the x-intercepts of the parabola.

$x = 0$ and $x = 200$

c Determine the span (horizontal length) of the arch.

200 m

d What is the maximum height of the arch?

20 m

5 A 20 cm piece of wire is bent to form a rectangle. Let x cm be the breadth of the rectangle.

a Using the perimeter, explain why the length of the rectangle can be written as $10 - x$.

The perimeter is $x + x + l + l = 20$

$2x + 2l = 20$

$2l = 20 - 2x$

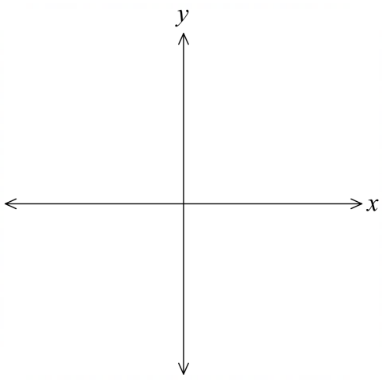
$l = 10 - x$

b Write an equation for the area of the rectangle A cm^2 in terms of x.

$A = lw$

$= x(10 - x)$

c Sketch the graph of A vs x for suitable values of x.



d Use the graph to determine the maximum area that can be formed.

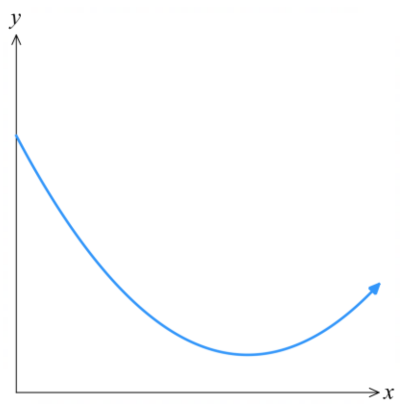
25 cm^2

e What will be the dimensions of the rectangle to achieve its maximum area?

$A = 25, x = 5$, therefore $l = 5$

6 A small toy glider is released from a cliff. It follows a parabolic path as shown.
The lowest point of the glider is 11 metres above the ground, 8 metres from the cliff.

a Write an equation in the form $y = x^2 + bx + c$ to model the glider's path.



Min turning point at (8,11)

The question tells us it is a monic quadratic. $y = (x - 8)^2 + 11$

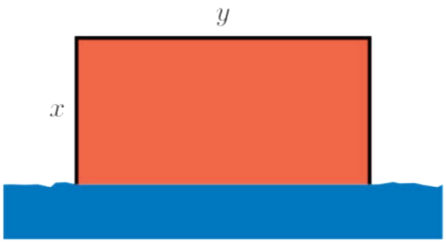
By expanding and equating coefficients. $b = -16$, $c = 75$

$\therefore y = x^2 - 16x + 75$

b Hence, or otherwise, from what height was the glider released?

$at\ x = 0, y = 75.$ 75 m

7 A farmer has 400 metres of fencing to construct a rectangular field.
One of the sides is a river, so the farmer only needs to fence three sides of the rectangle.
The farmer wants to use all the fencing available to make the largest possible area.



a Show that we can model this situation as $2x + y = 400$

The amount of fencing required is $x + y + x$, and we have 400 metres,
so $2x + y = 400$

b Show that the area of the field is $A = 400x - 2x^2$

$$A = xy$$
$$= x(400 - 2x)$$
$$= 400x - 2x^2$$

c By completing the square of the above expression, or otherwise, find the maximum area of the field that the farmer can construct.

By CTS, we get $A = -2(x - 100)^2 + 20000$, so the maximum turning point is at $(100,20000)$

Therefore, maximum area would be 20000 m^2

Alternatively,

$$-\frac{b}{2a} = 100$$

The maximum turning point is at $x = 100$, substitute into $A = 400 - 2x^2$

$\therefore$Maximum area is $A = 20000$ m^2

8 **2013 HSC Standard 2 Band 6**

Leanne wants to build a rectangular vegetable garden in her backyard. She has 20 metres of fencing and will use a wall as one side of the garden. The plan for her garden is shown, where x metres is the width of her garden.

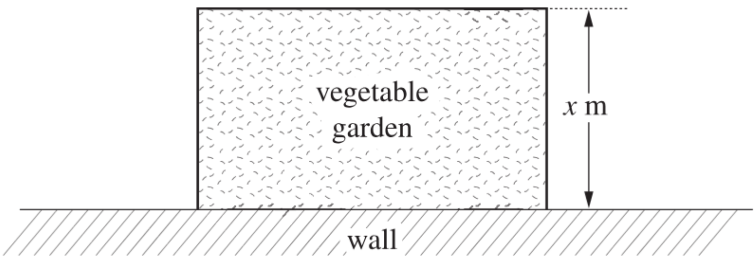
Which equation gives the area of her vegetable garden?

A. $A = 10x - x^2$

B. $A = 10x - 2x^2$

C. $A = 20x - x^2$

D. $A = 20x - 2x^2$



9 Let $C = 2x^2 - 12x + 25$ represent the cost of manufacturing products in a day, where x is the number of machines operating and $C(x)$ is in tens of thousands of dollars.

a Find the cost of production if one machine is operating.

$150 000

b If the cost of production is $90 000, how many machines are operating?

Solve $9 = 2x^2 - 12x + 25$

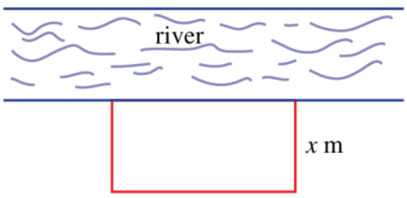
2 or 4 machines

c Find how many machines should be operated to minimise the cost of production and hence find the minimum cost of production.

By either completing the square or $x = -\frac{b}{2a}$, we can determine the minimum value is at $x = 3$.

3 machines, at a minimum production cost of $70,000

10 A farmer has 100 m of fencing to form a rectangular paddock with a river on one side (that does not require fencing), as shown.
What will be the dimensions of the paddock to achieve its maximum area?



Let x be width and y be length.

$2x + y = 100$ (from perimeter) and $A = xy$

$y = 100 - 2x$, substitute into area equation

$A = x(100 - 2x)$

$= 2x(50 - x)$

Therefore, maximum of this parabola is at $x = 25$

Substitute $x = 25$ into $y = 100 - 2x$ to get $y = 50$

Breadth: 25 m, Length: 50 m

11 Revenue for a burger truck can be modelled by $R = np$, where n is the number of burgers sold and p is the price of a burger.
A burger truck normally sells 60 burgers a day for \$10 per burger.
The owner is advised that they should increase the price of the burger.
However, for every 50 cents extra that the burger costs, the number of daily sales will drop by 2.
The owner decides to increase the price in 50-cent increments.
Let x be the number of 50-cent price increases.

a Given that revenue is $\text{Price} \times \text{Sales}$, show that this scenario can be modelled using

$$R = (30 - x)(20 + x)$$

Revenue is $R = \text{Price} \times \text{Number of Sales}$

After x lots of 50c increases, price would be $10 + \frac{x}{2}$ and number sold is $60 - 2x$

So, $R = \left(10 + \frac{x}{2}\right)(60 - 2x)$

$= \frac{1}{2}(20 + x) \times 2(30 - x)$

$= (20 + x)(30 - x)$

b Find the price that the owner should charge per burger to maximise the daily revenue, and state what the maximum daily revenue is.

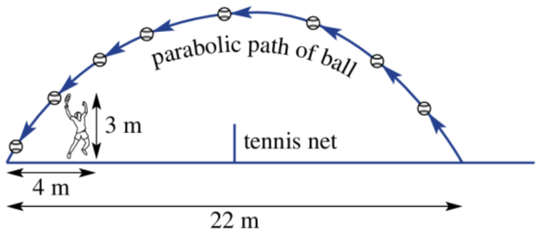
x-intercepts of the equation are at -20 and 30. Maximum turning point is at the midpoint, 5.

Therefore, 5 50c increases; the owner should charge \$12.50 per burger.

The revenue is $R = (20 + 5)(30 - 5) = \$625$

12 A professional tennis player returns a ball from ground level. It was hit in such a way that it will cover a horizontal distance of 22 m and land just inside the opposite end of the court.
The opponent is standing 4 m from the baseline and he can hit any ball less than 3 m high.
What is the lowest max height the ball must reach to fly over the opponent and win the point?

(Hint: Find the equation of the parabola using the x-intercepts and the point $(4,3)$)



Let h be height and x be distance from left

Using intercepts at $x = 0$ and $x = 22$ and the formula $y = k(x - \alpha)(x - \beta)$:

$\therefore y = kx(x - 22)$, substitute $(4,3)$ into equation and solve to get $k = -\frac{1}{24}$

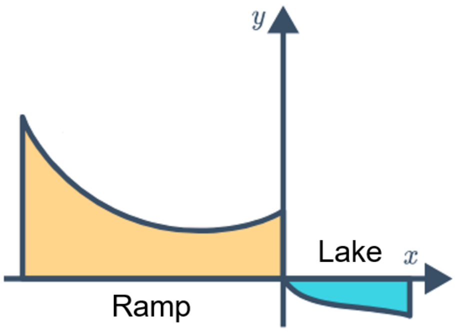
$\therefore y = -\frac{x}{24}(x - 22)$

The turning point is at $x = 11$, substitute into equation to find y:

$y = -\frac{11}{24}(11 - 22) = \frac{121}{24}$

$\therefore$ Max height must be at least $5\frac{1}{24}$ metres

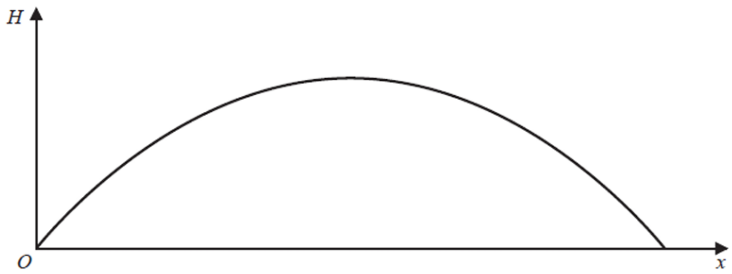
13 Over the summer, Mr Ding and his daredevil maths teacher friends want to build a bike ramp to launch themselves into a lake. He decided that the shape of the ramp would be parabolic.
The right end of the ramp should have a height of 5 metres.
The lowest point of the ramp should be 2 metres above the ground, 3 metres from the right end.
If the left end of the ramp is 7 metres from the right end, from what height should the ramp start?



The equation of the ramp is $y = \frac{1}{3}(x + 3)^2 + 2$

The ramp should start from a height of 7.33 metres.

14 The graph shows the trajectory of an AFL ball kicked by a Year 11 student.
The ball reaches a maximum height of 12 metres and hits the ground 40 metres from where it was kicked. As the ball is falling, it just passes over the head of a Year 7 girl who is 134 cm tall.
How far away was the year 7 girl standing from where the ball was kicked?
Answer in metres rounded to 2 decimals.



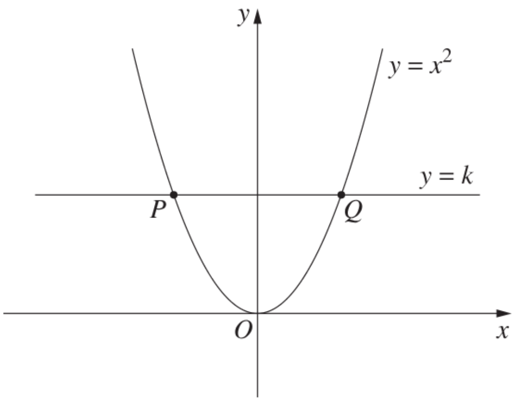
Equation of the parabola is $y = -\frac{3x}{100}(x - 40)$

Using quadratic formula, solve for when $y = 134$. Y7 girl was standing 38.85 metres away

15 **2023 HSC Advanced Band 6**

The graph $y = x^2$ meets the line $y = k$, where $k > 0$, at points P and Q as shown.

The length of interval PQ is L.



Let a be a positive number. The graph $y = \frac{x^2}{a^2}$ meets the line $y = k$ at points S and T.

What is the length of ST? (Hint: find an expression for L, then find an expression for ST.)

A. $\frac{L}{a}$

B. $\frac{L}{a^2}$

C. aL

D. a^2L

Intersection of $y = x^2$ and $x = k$ is at $x = \pm\sqrt{k}$

Therefore, length of PQ is $L = 2\sqrt{k}$

Intersection of $y = \frac{x^2}{a^2}$ and $y = k$ is at $x = \pm a\sqrt{k}$

Therefore, length of $ST = 2a\sqrt{k} = aL$

Answer is C