Student Name

Mathematics Stage 5 Paths

NON-LINEAR RELATIONSHIPS C

Book 3 Graph hyperbolas and describe their features and transformations

Version: 260207

Report mistakes & suggest improvements: https://MrDingMaths.com

CONTENTS

The Hyperbola 2

Hyperbola Transformations 5

OVERVIEW

SYLLABUS CONTENT

MA5-NLI-P-01 interprets and compares non-linear relationships and their transformations, both algebraically and graphically

Graph hyperbolas and describe their features and transformations

- Use graphing applications to graph, compare and describe hyperbolic relationships of the form $y = \frac{k}{x}$ for integer values of k
- Use graphing applications to graph and describe a variety of hyperbolas, including where the equation is given in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x - b}$ for integer values of $k,\ b$ and c

THE HYPERBOLA

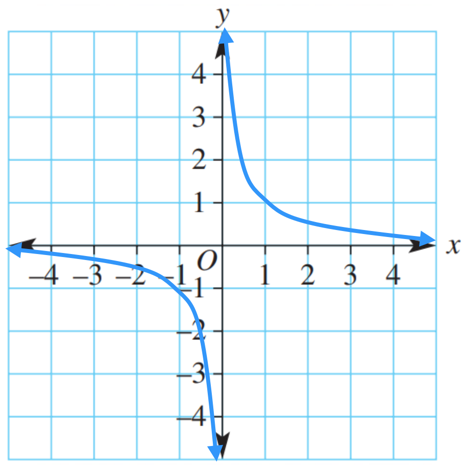
RECIPROCAL RELATIONSHIPS

- **Reciprocal relationships** are in the form $y = \frac{k}{x}$.
  Their graph will not be a line; **non-linear.**
- We can graph reciprocal relationships using a table of values.

Example Graph reciprocal relationships using a table of values.

Draw the graph of $y = \frac{1}{x}$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	Und	2	1	$\frac{1}{2}$



What happens to the value of y as $x \to \infty$?

y approaches …… from the ……….. side. $y \to$ ……

What happens to the value of y as $x \to -\infty$?

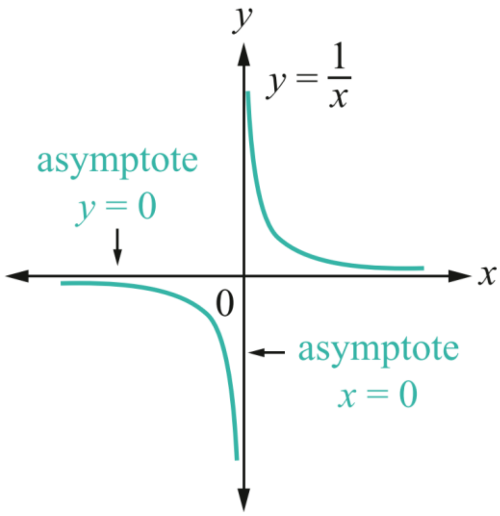
y approaches …… from the ……….. side. $y \to$ ……

Why does y not exist at $x = 0$?

At $x = 0$, $y =$ ……….., which is not possible.

THE HYPERBOLA

- A **reciprocal** relationship $y = \frac{k}{x}$ has a graph in the shape of a **hyperbola**.
- Hyperbolas have:
  - Two **asymptotes**: lines where the graph approaches but never reaches.
  - No x or y-intercepts.
  - Two symmetrical halves.



1 Plot $y = \frac{2}{x}$ and $y = \frac{3}{x}$ on the same axis.

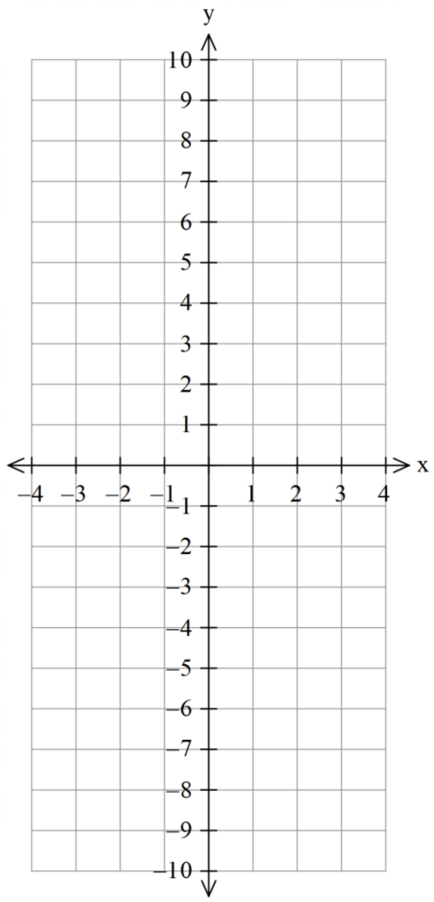
$y = \frac{2}{x}$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y							

Quadrants:

Behaviour as $x \to \infty$:

Behaviour as $x \to -\infty$:



$y = \frac{3}{x}$

x	-2	-1	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	2
y							

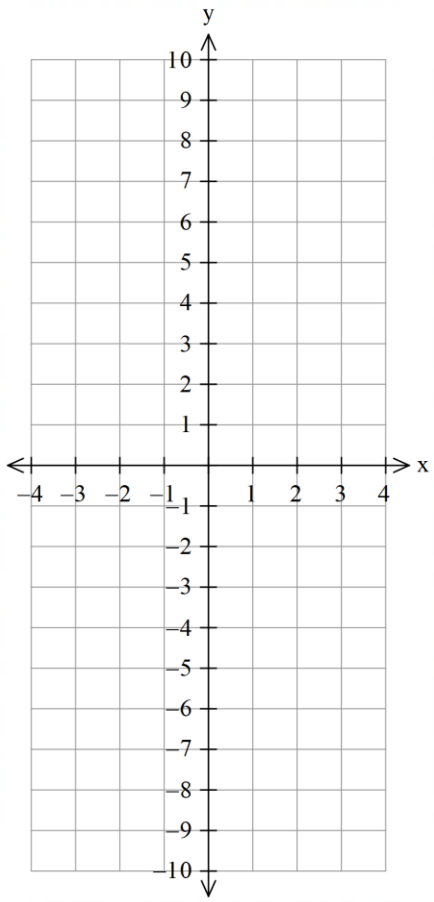
Quadrants:

Behaviour as $x \to \infty$:

Behaviour as $x \to -\infty$:

2 Plot these graphs on the same axis.

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$y = \frac{2}{x}$							
$y = -\frac{2}{x}$							



Describe their differences in terms of:

Quadrants:

Behaviour as $x \to \infty$:

Behaviour as $x \to -\infty$:

SKETCHING HYPERBOLAS

Investigation Equation and the graph of hyperbola.

On Desmos https://www.desmos.com/calculator

a Sketch these graphs

$y = \frac{1}{x}$

$y = \frac{2}{x}$

$y = \frac{3}{x}$

$y = \frac{4}{x}$

How does increasing the value of k affect the hyperbola?

....

....

....

b Sketch these graphs.

$y = \frac{-1}{x}$

$y = \frac{-2}{x}$

$y = \frac{-3}{x}$

$y = \frac{-4}{x}$

How do negative values of k affect the hyperbola?

....

....

....

c

$y = \frac{1}{x} + 1$

$y = \frac{1}{x} + 2$

$y = \frac{1}{x} + 3$

$y = \frac{1}{x} - 3$

$y = \frac{1}{x} - 4$

How does changing the constant term affect the hyperbola?

....

....

....

d

$y = \frac{1}{x+1}$

$y = \frac{1}{x+2}$

$y = \frac{1}{x+3}$

$y = \frac{1}{x-3}$

$y = \frac{1}{x-4}$

How does the constant term in the denominator affect the hyperbola?

....

....

....

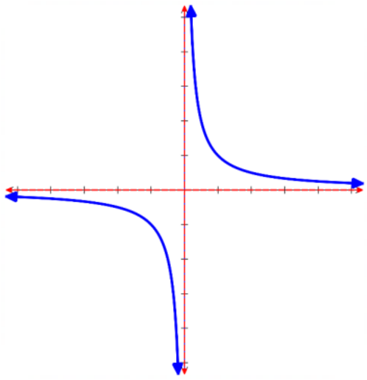
Hyperbola Transformations

Consider the equation of a hyperbola $\boldsymbol{y = \frac{k}{x-b} + c}$, where k is an integer.

The parameters k, b, c are numbers that affect the graph in predictable ways.

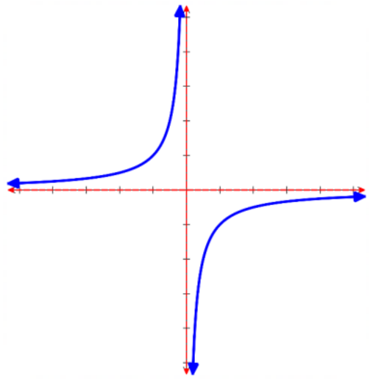
k is positive, the hyperbola is in the ……….. and ……….. quadrants.

$y = \frac{1}{x}$



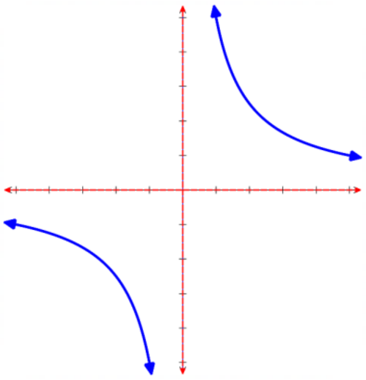
k is negative, the hyperbola is in the ……….. and ……….. quadrants.

$y = -\frac{1}{x}$

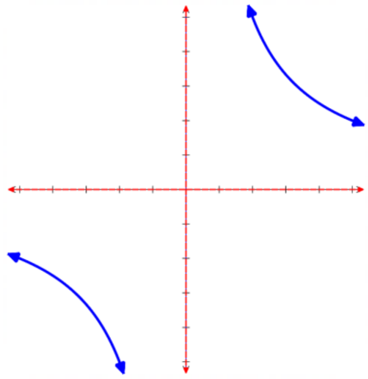


As $|k|$ increases, the hyperbola gets *dilated*; it stretches away from the axes.

$y = \frac{5}{x}$

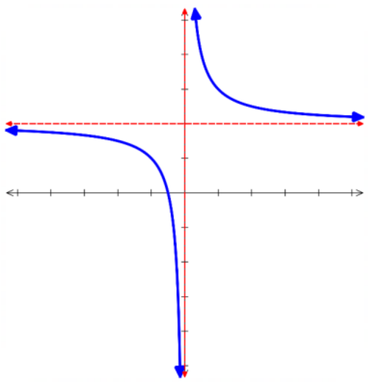


$y = \frac{10}{x}$

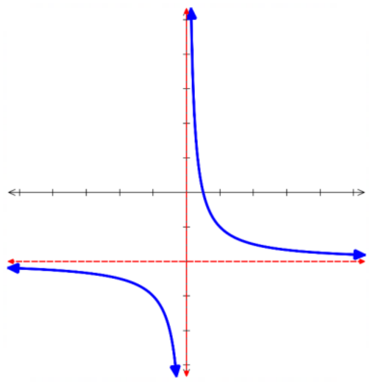


c shifts the curve ………………… The value of c is the ………….. asymptote.

$y = \frac{1}{x} + 2$

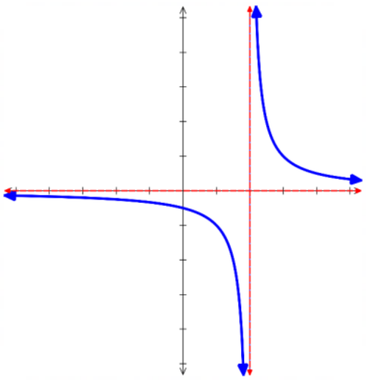


$y = \frac{1}{x} - 2$

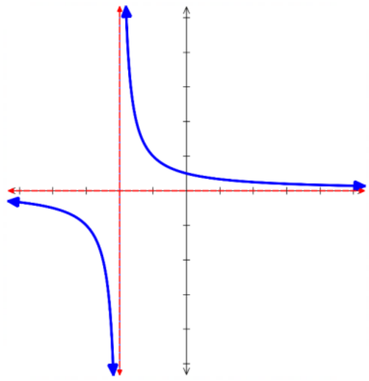


b shifts the curve ………………… The value of b is the ………….. asymptote.

$y = \frac{1}{x-2}$



$y = \frac{1}{x+2}$



SKETCHING HYPERBOLAS

Hyperbola Transformations

Dilates hyperbola

$$y = \frac{k}{x-b} + c$$

Horizontal translation
$x = b$ is horizontal asymptote

Vertical translation
$y = c$ is horizontal asymptote

- To sketch hyperbolas:

1. Find y-intercept, if any, by letting $x = 0$.
2. Find x-intercept, if any, by letting $y = 0$.
3. Find vertical asymptote at $x = b$ and horizontal asymptote at $y = c$,

Example Sketch hyperbolas, showing intercepts and asymptotes.

Worked Example	Your Turn
$y = \frac{1}{x+1} + 2$	$y = \frac{1}{x-3} + 2$

Worked Example

1. y-int: Let $x = 0$,

$$y = \frac{1}{1} + 2 = 3$$

2. x-int: Let $y = 0$

$$0 = \frac{1}{x+1} + 2$$

$$-2 = \frac{1}{x+1}$$

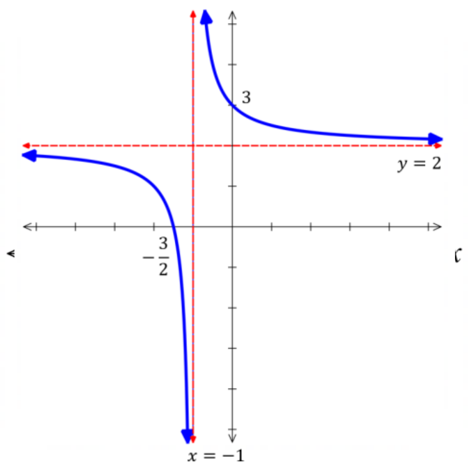
$$-2x - 2 = 1$$

$$-2x = 3$$

$$x = -\frac{3}{2}$$

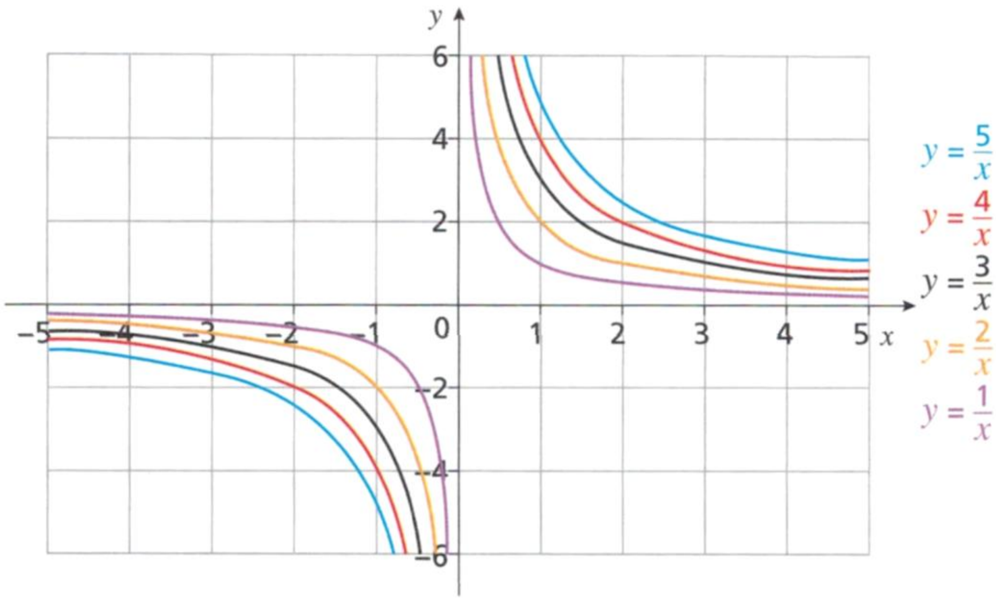
3. Asymptotes: $x = -1$

$y = 2$



FOUNDATION

1 Here are the graphs of $y = \frac{1}{x}$, $y = \frac{2}{x}$, $y = \frac{3}{x}$, $y = \frac{4}{x}$ and $y = \frac{5}{x}$



What's the same and what's different about the graphs?

Same:

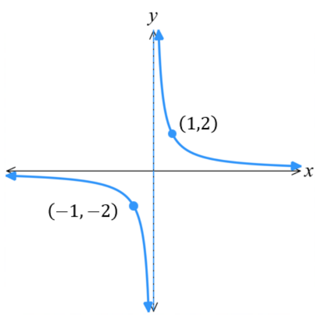
Asymptotes at $x = 0$ and $y = 0$, in quadrants 1 and 3.

Different:

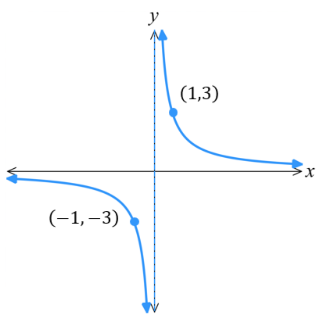
For $y = \frac{k}{x}$, the larger the value of k, the further away from the axis.

2 Sketch these hyperbolas, labelling the point where $x = 1$ and $x = -1$.

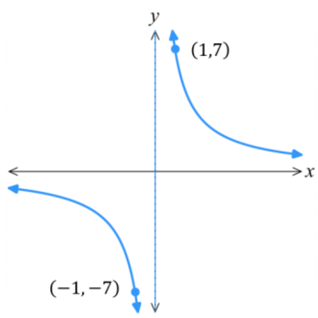
a $y = \frac{2}{x}$



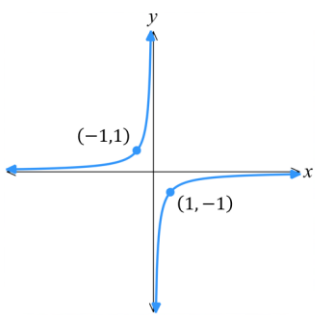
b $y = \frac{3}{x}$



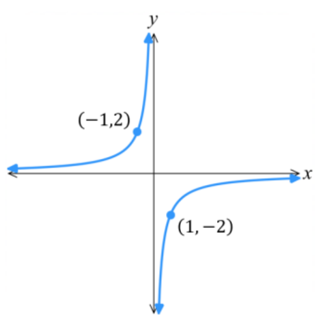
c $y = \frac{7}{x}$



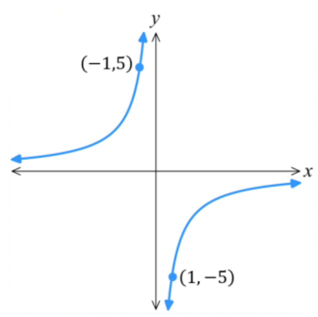
d $y = -\frac{1}{x}$



e $y = -\frac{2}{x}$



f $y = -\frac{5}{x}$



3 Choose the word *left, right, up* or *down* to complete each sentence.

a Compared with $y=\frac{1}{x}$, $y=\frac{1}{x-3}$ is translated right by 3

b Compared with $y=\frac{1}{x}$, $y=\frac{1}{x+2}$ is translated ....

Left 2

c Compared with $y=\frac{1}{x}$, $y=\frac{1}{x}+3$ is translated ....

Up 3

d Compared with $y=\frac{1}{x}$, $y=\frac{1}{x}-6$ is translated ....

Down 6

e Compared with $y=\frac{1}{x}$, $y=\frac{1}{x+3}-2$ is translated ....

Left 3, down 2

f Compared with $y=\frac{1}{x}$, $y=\frac{1}{x+5}+7$ is translated ....

Left 5, up 7

g Compared with $y=\frac{1}{x}$, $y=\frac{1}{x-5}+3$ is translated ....

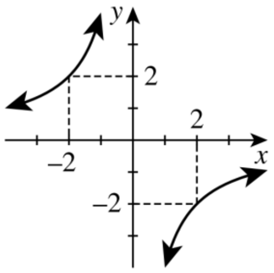
Right 5, up 3

h Compared with $y=\frac{1}{x}$, $y=\frac{1}{x+1}-9$ is translated ....

Left 1, down 9

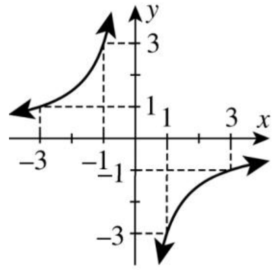
4 These hyperbolas have equations in the form $y=\frac{k}{x}$.
Determine the value of k and hence the equation of the hyperbola.

a



$y=-\frac{4}{x}$

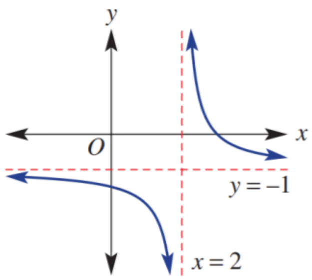
b



$y=-\frac{3}{x}$

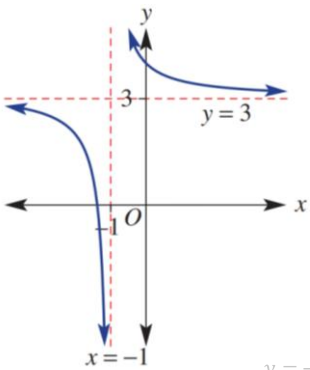
5 Determine the equation of these hyperbolas in the form $y=\frac{1}{x-b}+c$

a



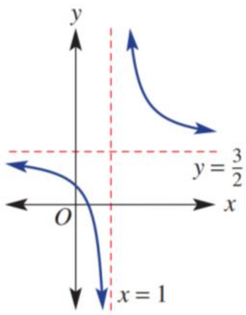
$y=\frac{1}{x-2}-1$

b



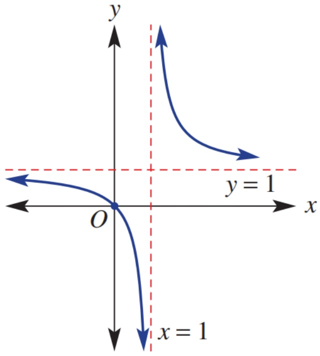
$y=\frac{1}{x+1}+3$

c



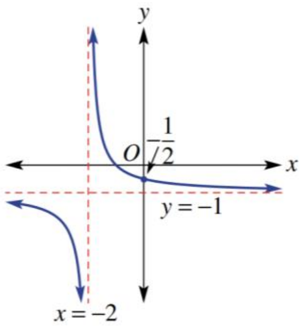
$y=\frac{1}{x-1}+\frac{3}{2}$

d



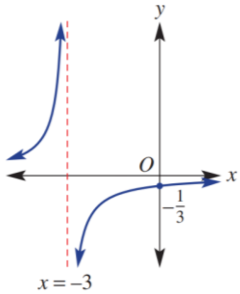
$y=\frac{1}{x-1}+1$

e



$y=\frac{1}{x+2}-1$

f

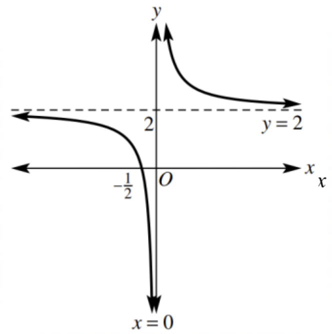


$y=-\frac{1}{x+3}$

6 Sketch these graphs on the same axis to show the transformation.
Clearly label the asymptotes and intercepts.

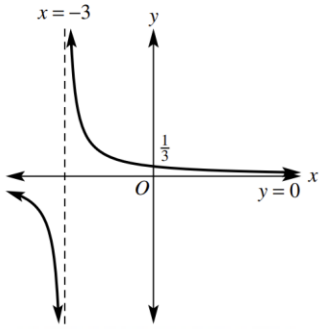
a

$$y = \frac{1}{x} + 2$$



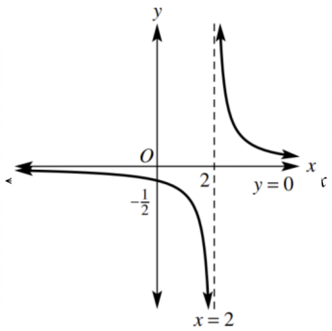
b

$$y = \frac{1}{x+3}$$



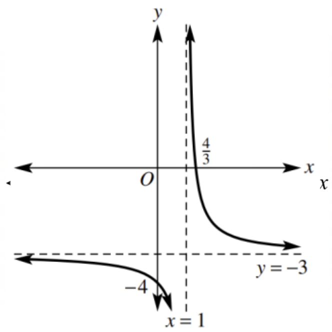
c

$$y = \frac{1}{x-2}$$



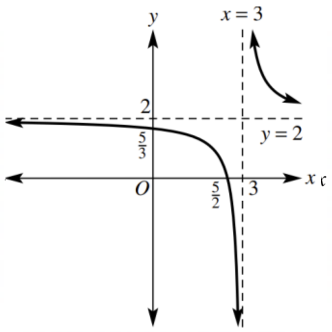
d

$$y = \frac{1}{x-1} - 3$$



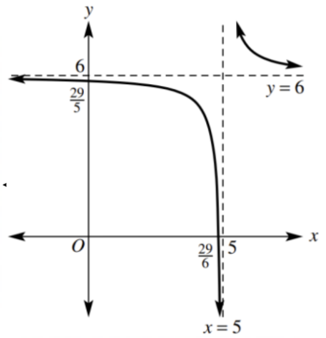
e

$$y = \frac{1}{x-3} + 2$$



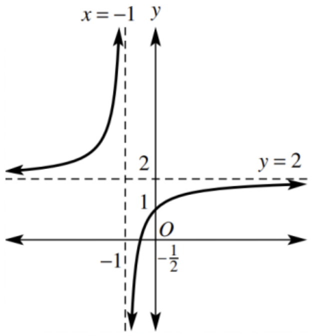
f

$$y = \frac{1}{x-5} + 6$$



g

$$y = \frac{-1}{x+1} + 2$$



h

$$y = \frac{-2}{x-3} - 2$$