Student Name

Mathematics Stage 5 Paths

NON-LINEAR RELATIONSHIPS C

Book 2 Graph exponentials and describe their features and transformations

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-NLI-P-01 interprets and compares non-linear relationships and their transformations, both algebraically and graphically

Graph exponentials and describe their features and transformations

- Use graphing applications to graph exponential relationships of the form $y = k(a)^x + c$ and $y = k(a)^{-x} + c$ for integer values of k, a and c (where $a > 0$ and $a \neq 1$), and compare and describe any relevant features

EXPONENTIAL CURVE TRANSFORMATIONS

Investigation Equation and the graph of an exponential curve.

On Desmos https://www.desmos.com/calculator

a Sketch these graphs

$y = 2^x$

$y = 3^x$

$y = 4^x$

$y = 5^x$

How does increasing the base value $a > 1$ affect the exponential curve?

....

....

....

b Sketch these graphs.

$y = 2^x$

$y = 0.8^x$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{3}\right)^x$

How does $0 < a < 1$ affect the exponential curve?

....

....

....

c

$y = 2^x$

$y = -2^x$

$y = \left(\frac{1}{2}\right)^x$

$y = -\left(\frac{1}{2}\right)^x$

How is $y = -a^x$ different to $y = a^x$?

....

....

....

d

$y = 2^x$

$y = 2(2)^x$

$y = 3(2)^x$

$y = -3(2)^x$

$y = -4(2)^x$

How does the factor in front of the exponential affect the curve?

....

....

e

$y = \left(\frac{1}{2}\right)^x$

$y = 2\left(\frac{1}{2}\right)^x$

$y = -3\left(\frac{1}{2}\right)^x$

How does the factor in front of the exponential affect the curve?

....

....

f

$y = 2^{-x}$

$y = \left(\frac{1}{2}\right)^x$

Why is $y = 2^{-x}$ the same as $y = \left(\frac{1}{2}\right)^x$?

....

....

....

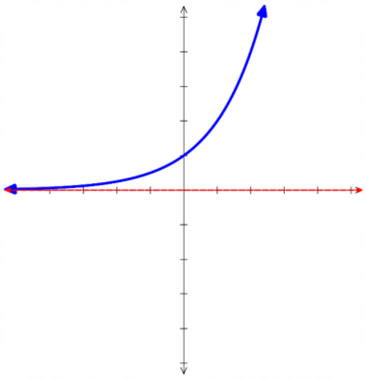
Exponential Curve Transformations

Consider the equation of an exponential curve $\boldsymbol{y = k(a)^x + c}$, where $a > 0$

The parameters a, c, k are numbers that affect the graph in predictable ways.

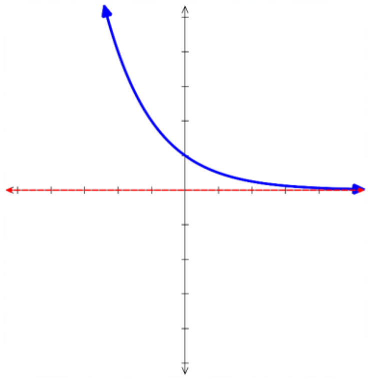
$a > 1$, we call this exponential ....

$y = 2^x$



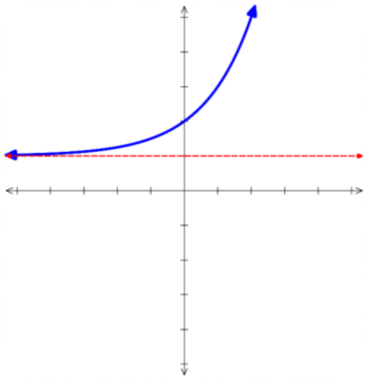
$0 < a < 1$, we call this exponential ....

$y = \left(\frac{1}{2}\right)^x$



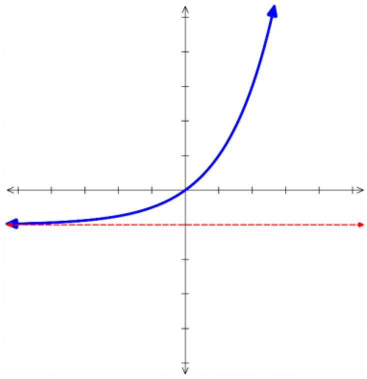
$c > 0$, the curve is shifted ....
The value of c is the ....

$y = 2^x + 1$



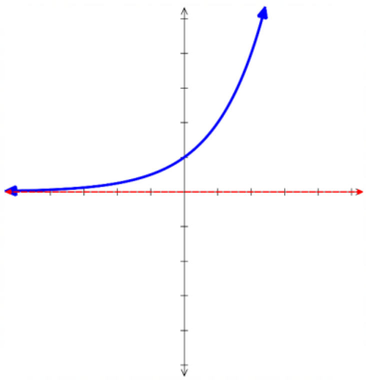
$c < 0$, the curve is shifted ....
The value of c is the ....

$y = 2^x - 1$



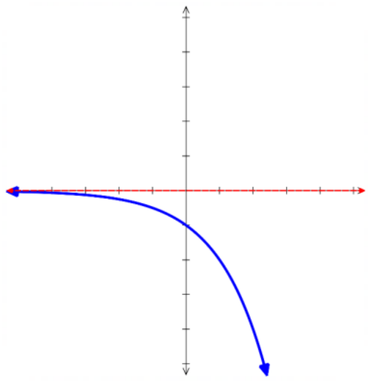
$k > 0$

$y = 1(2^x)$



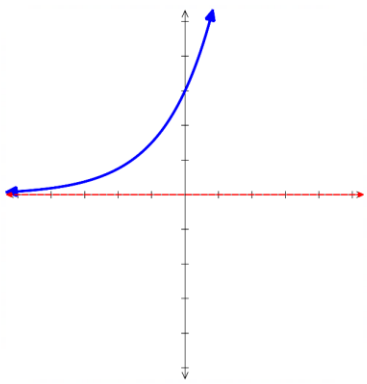
$k < 0$, reflect the graph across x-axis

$y = -1\,(2^x)$



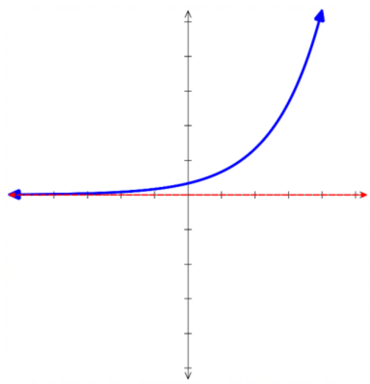
$|k| > 1$, the curve changes more quickly.

$y = 3(2^x)$



$0 < |k| < 1$ the curve changes more slowly.

$y = \frac{1}{3}(2^x)$



SKETCHING EXPONENTIAL CURVES

Exponential Curve Transformations

The common multiplier
$a > 1$, exponential growth
$0 < a < 1$, exponential decay

$$y = k(a)^x + c$$

Factor, affects steepness
$(0, k)$ is the y-intercept

Vertical translation
$y = c$ is the asymptote

The common multiplier
$a > 1$, exponential decay
$0 < a < 1$, exponential growth

$$y = k(a)^{-x} + c$$

Factor, affects steepness
$(0, k)$ is the y-intercept

Vertical translation
$y = c$ is the asymptote

- To sketch exponential curves:

1. Find y-intercept by letting $x = 0$.
2. Find asymptote: $y = c$.
3. Plot at least one other point, such as at $x = 1$.

Example Sketch exponential curves.

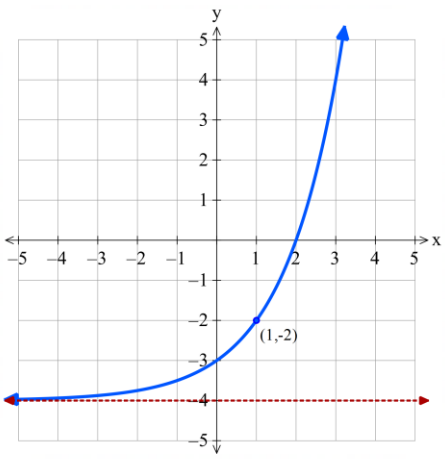
Worked Example

$y = 2^x - 4$

1. y-int: Let $x = 0$,
$y = 2^0 - 4 = -3$

2. Asymptote: $y = -4$

3. When $x = 1$: $y = 2^1 - 4$
$y = -2$



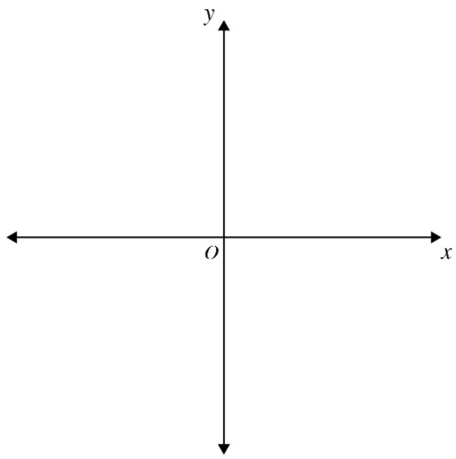
Your Turn

$y = 3^{-x} - 5$

1. y-int:

2. Asymptote:

3. When $x = 1$:



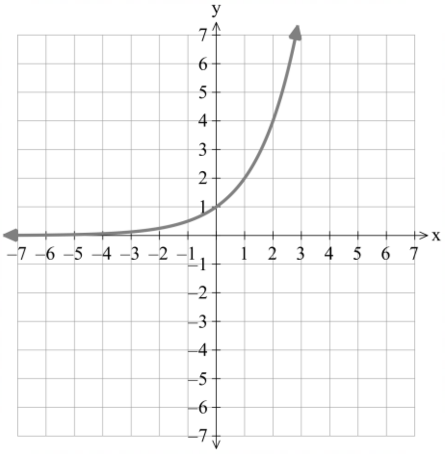
1 The graph of $y = 2^x$ is given. Sketch these graphs on the same axis to show the transformation. Clearly label the y-intercept, asymptote, and the point where $x = 1$.

a $y = 2^x + 1$

y-int:

Asymptote:

When $x = 1$:

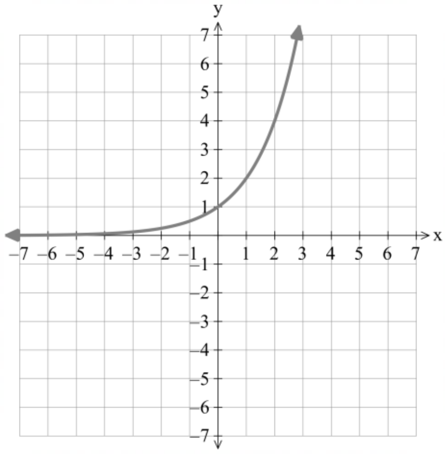


b $y = 2^x - 3$

y-int:

Asymptote:

When $x = 1$:

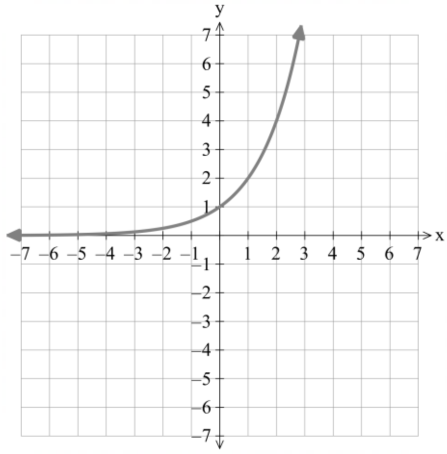


a $y = 2^x - 5$

y-int:

Asymptote:

When $x = 1$:

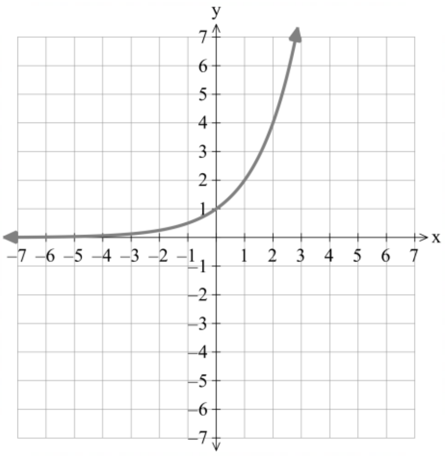


c $y = 3^x$

y-int:

Asymptote:

When $x = 1$:

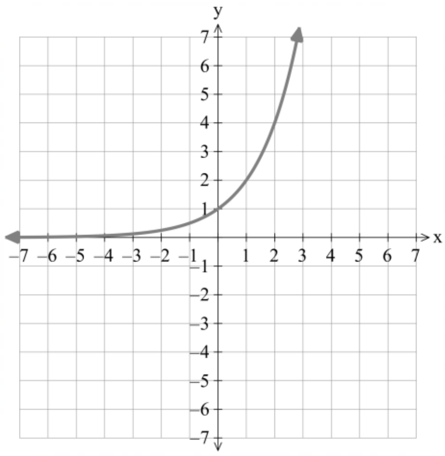


d $y = 4^x$

y-int:

Asymptote:

When $x = 1$:

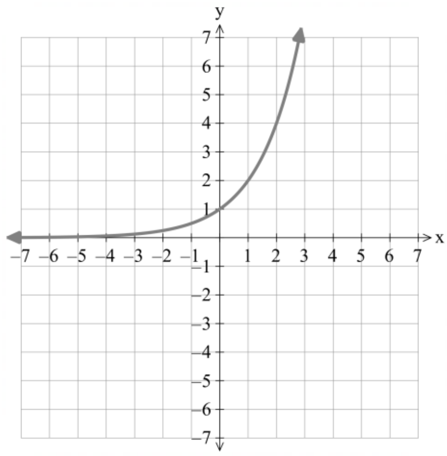


b $y = 5^x$

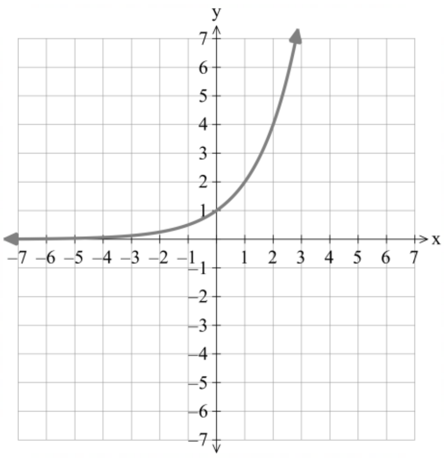
y-int:

Asymptote:

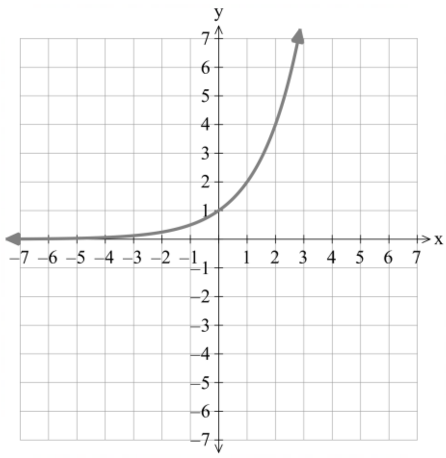
When $x = 1$:



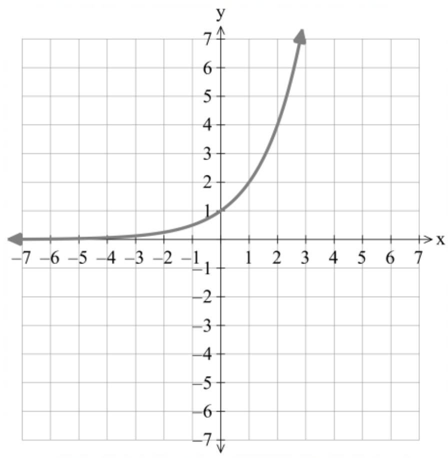
e $y = -2^x$



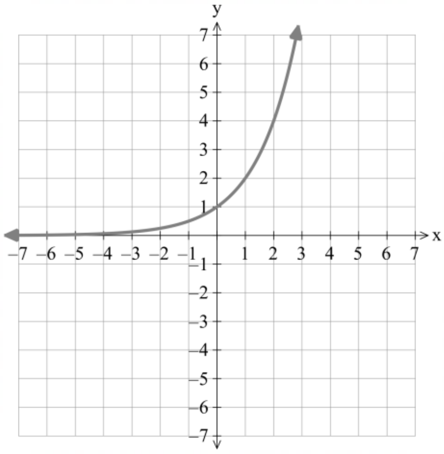
f $y = -2^x + 1$



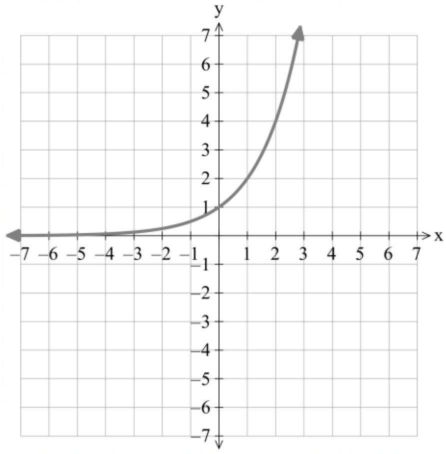
c $y = -2^x - 3$



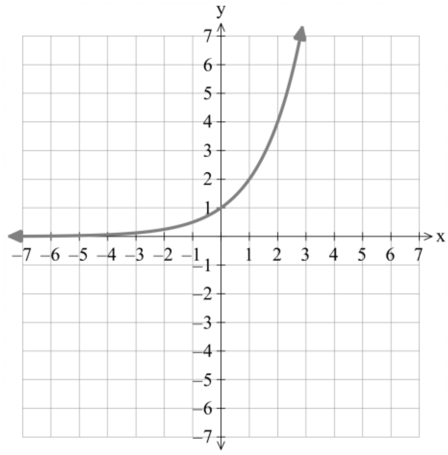
g $y = 3(2^x)$



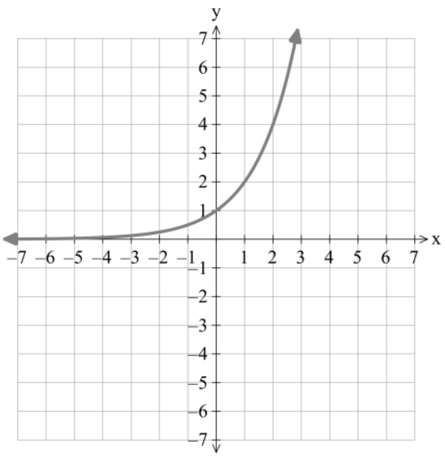
h $y = -3(2^x)$



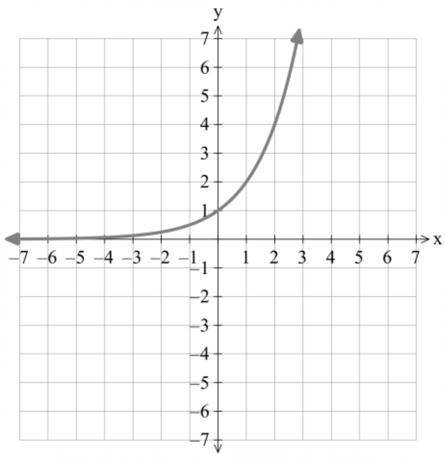
d $y = 0.5(2^x)$



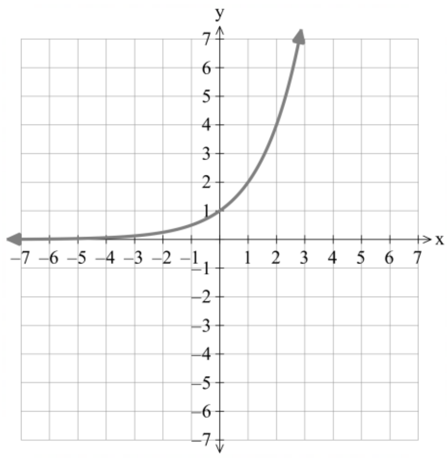
i $y = 2^{-x}$



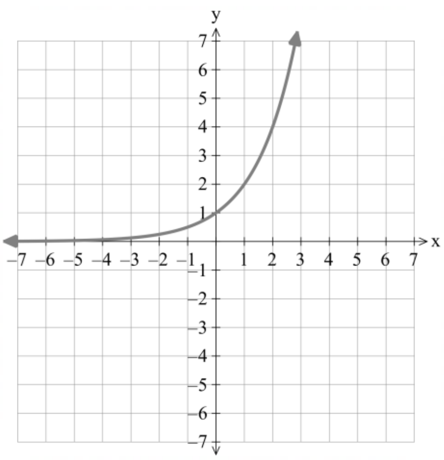
j $y = 2^{-x} + 7$



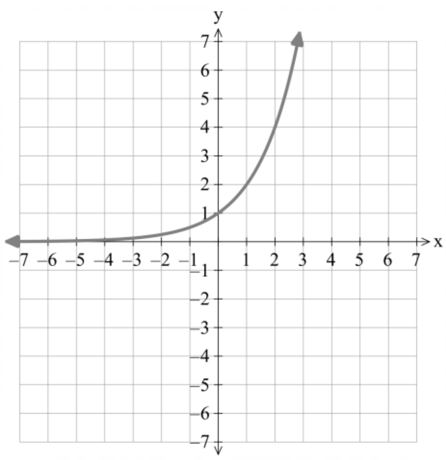
e $y = -(2^{-x})$



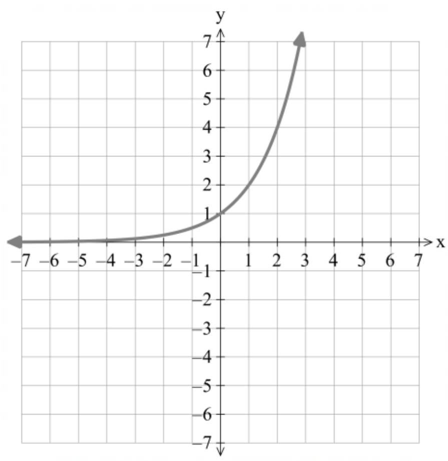
k $y = \left(\frac{1}{2}\right)^x$



l $y = \left(\frac{1}{3}\right)^x$

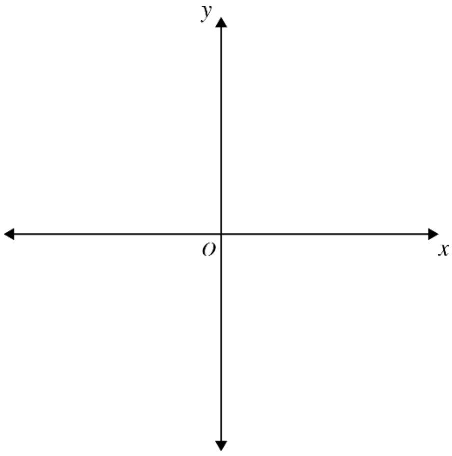


f $y = \left(\frac{1}{4}\right)^x$

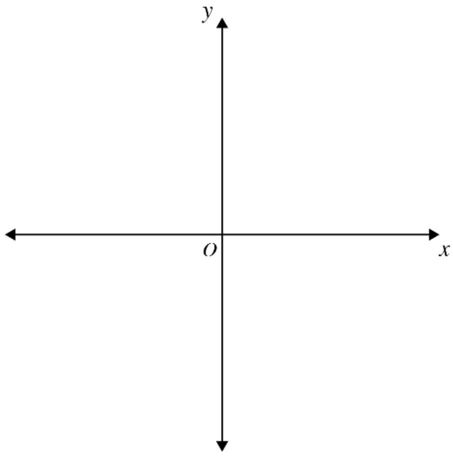


2 Sketch these exponential equations.
Clearly label the y-intercept, asymptote, and the point where $x = 1$.

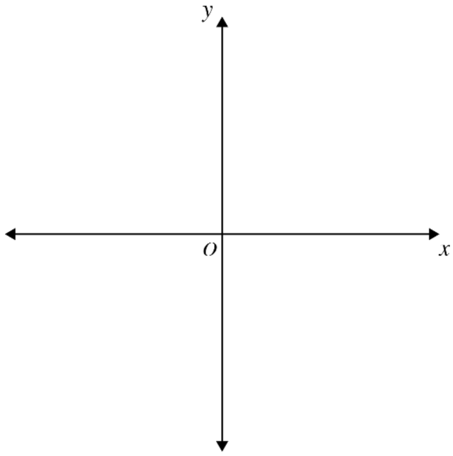
a $y = 3^x + 2$



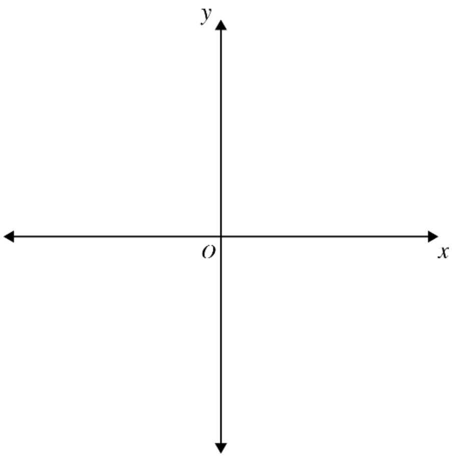
b $y = 2^x - 1$



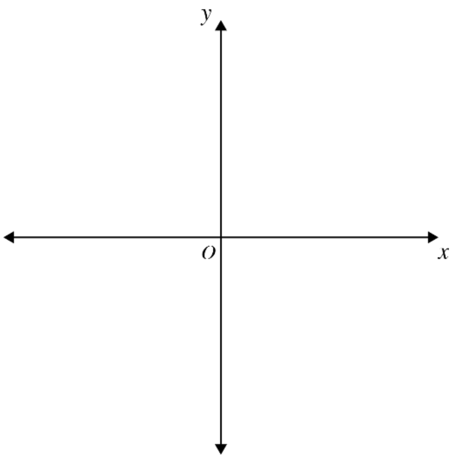
c $y = 2(3^x)$



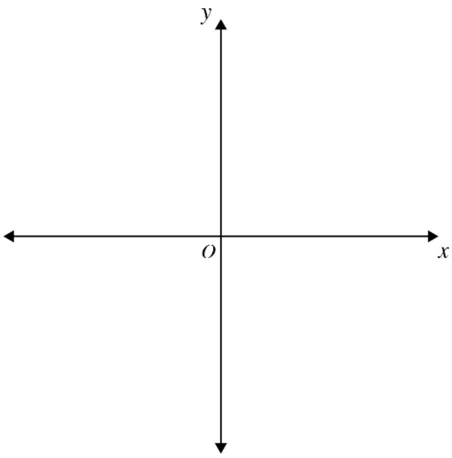
d $y = (0.25)^x$



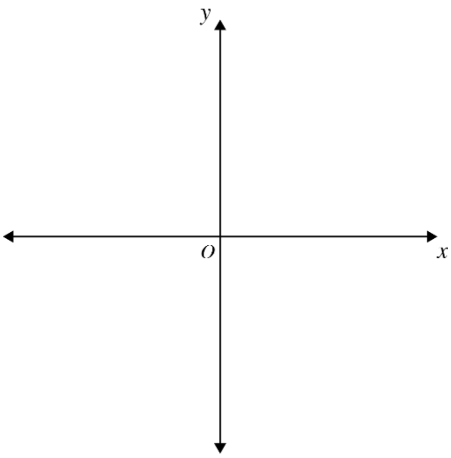
e $y = -3^x$



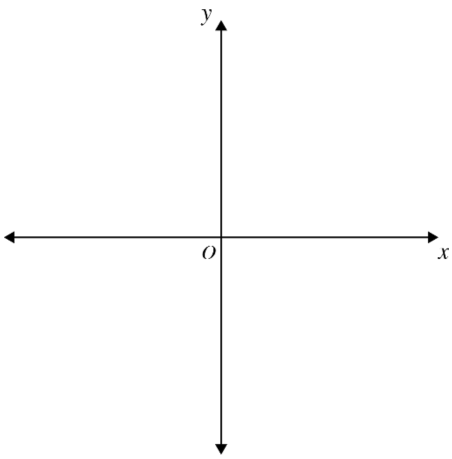
f $y = 3^{-x}$



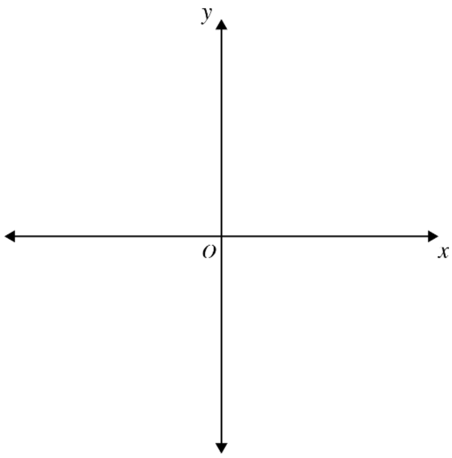
g $y = -3^{-x}$



h $y = 2^{-x} + 3$



i $y = -2(5^x)$

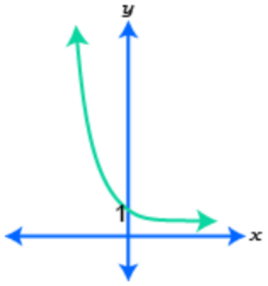


3 Match the equation to the graph.

$y = 6^x$ $\quad$ $y = -6^x$ $\quad$ $y = -6^x - 2$ $\quad$ $y = -6^x + 2$ $\quad$ $y = 6^{-x}$

$y = 6^{-x} - 2$ $\quad$ $y = 6^{-x} + 2$ $\quad$ $y = -6^{-x}$ $\quad$ $y = -6^{-x} - 2$ $\quad$ $y = 6^{-x} + 2$

a



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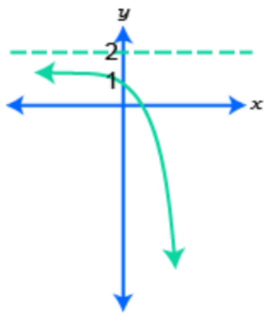

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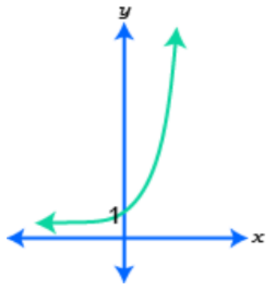

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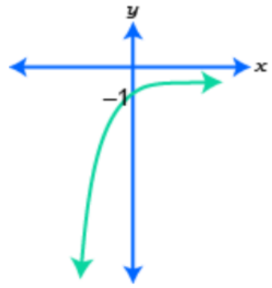

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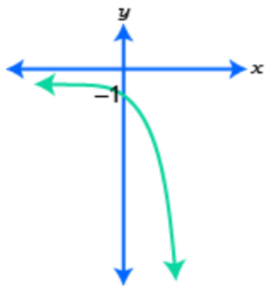

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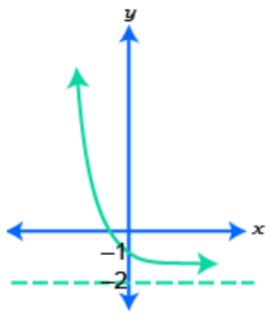

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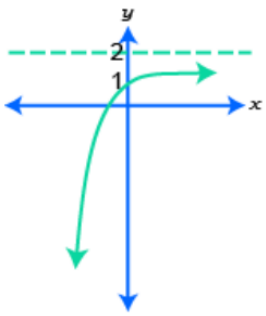

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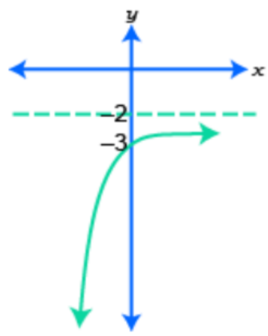

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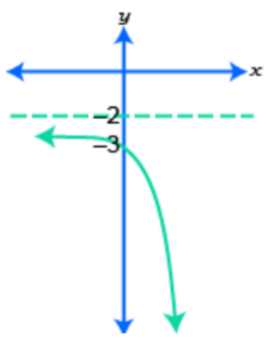

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