Student Name

Mathematics Stage 5 Core

NON-LINEAR RELATIONSHIPS B

Book 1 Graph and examine quadratic relationships

Version: 260207

Report mistakes & suggest improvements:
https://MrDingMaths.com

CONTENTS

OVERVIEW

SYLLABUS CONTENT

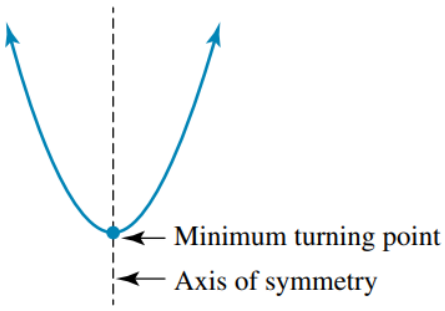
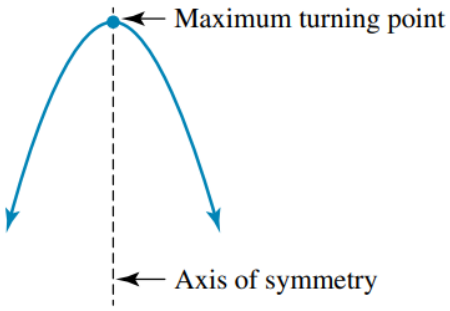
MA5-NLI-C-02 identifies and compares features of parabolas and exponential curves in various contexts

Graph and examine quadratic relationships

- Graph and compare parabolas of the form $y = kx^2$ and $y = kx^2 + c$ using graphing applications
- Identify and describe the key features of parabolas of the form $y = kx^2$ and $y = kx^2 + c$ including the vertex, x- and y-intercepts, axis of symmetry and concavity

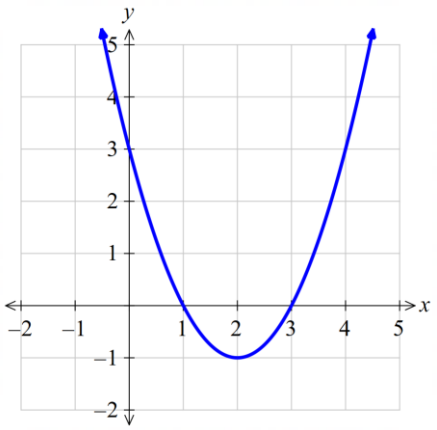
THE PARABOLA

- A **quadratic relationship** $y = ax^2 + bx + c$ produces a parabola-shaped a graph.
- Parabolas have these features:
  - **Line symmetry:** Folding the parabola along this line makes both halves match.
  - **Vertex,** or **turning point**: where the gradient changes direction.
    The vertex is always on the axis of symmetry.
  - The **nature** of the turning point (**maximum** or **minimum**) depends on coefficient of x^2

$a > 0$	$a < 0$
Concave up	Concave down
	

Example Identify key features of a parabola.

Identify the key features of this parabola:



Turning point and nature: $(2, -1)$, minimum

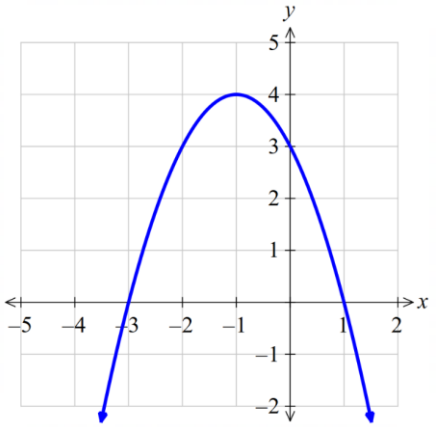
Axis of symmetry: $x = 2$

y-intercept: $(0,3)$

x-intercept: $(1,0)$ and $(3,0)$

Minimum value: $y = -1$

Identify the key features of this parabola:



Turning point and nature: $(-1,4)$ max

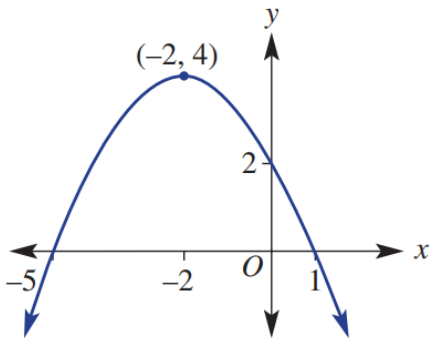
Axis of symmetry: $x = -1$

y-intercept: $(0,3)$

x-intercept: $(-3,0)$ and $(1,0)$

Maximum value: $y = 4$

1 Complete the sentences.



a The parabola has a …… turning point.

maximum

b The coordinates of the turning point are ……

(−2,4)

c The y-intercept is ……

(0,2)

d The x -intercepts are ……and ……

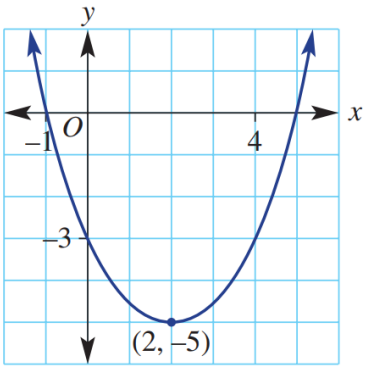
(−5,0) (1,0)

e The axis of symmetry is ……

$x = -2$

2 Determine the key features of these parabolas.

a



Vertex:

(2, −5)

Nature of turning point:

minimum

Axis of symmetry:

$x = 2$

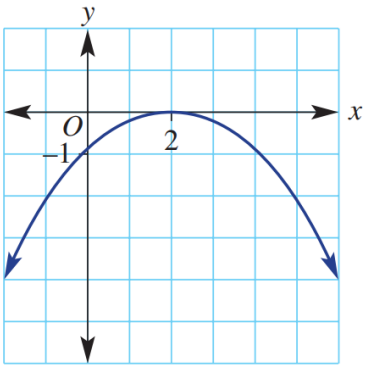
x intercepts:

(−1,0) (5,0)

y intercept:

(0, −3)

b



Vertex:

(2,0)

Nature of turning point:

maximum

Axis of symmetry:

$x = 2$

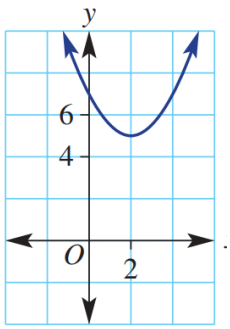
x intercepts:

(2,0)

y intercept:

(0, −1)

c



Vertex:

(2,5)

Nature of turning point:

minimum

Axis of symmetry:

$x = 2$

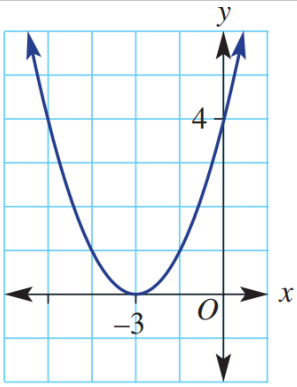
x intercepts:

none

y intercept:

(0,7)

d



Vertex:

(−3,0)

Nature of turning point:

minimum

Axis of symmetry:

$x = -3$

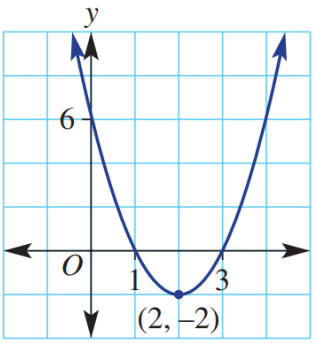
x intercepts:

(−3,0)

y intercept:

(0,4)

e



Vertex:

(2, −2)

Nature of turning point:

minimum

Axis of symmetry:

$x = 2$

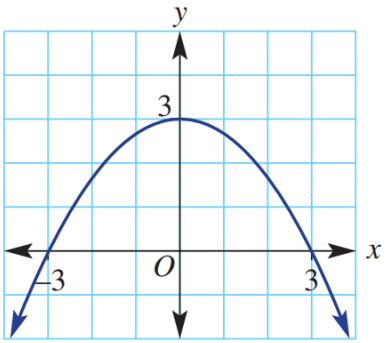
x intercepts:

(1,0) (3,0)

y intercept:

(0,6)

f



Vertex:

(0,3)

Nature of turning point:

maximum

Axis of symmetry:

$x = 0$

x intercepts:

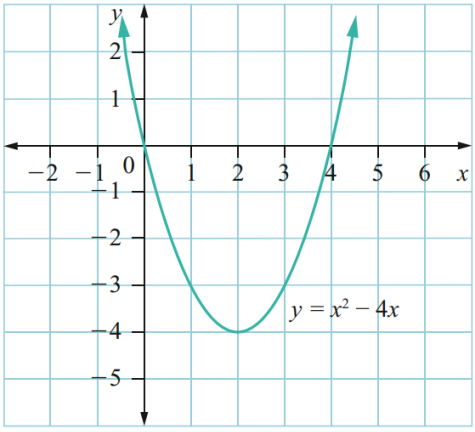
(−3,0) (3,0)

y intercept:

(0,3)

3 Determine the key features of these parabolas.

a



Vertex: $(2, -4)$

Nature of turning point: minimum

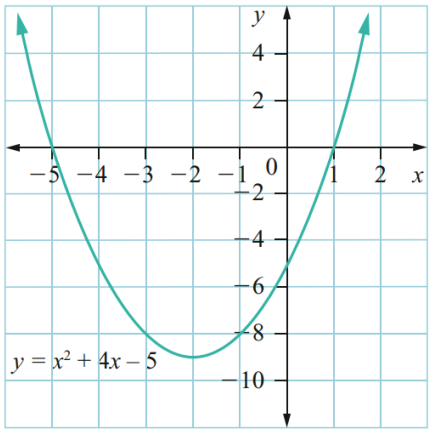
Axis of symmetry: $x = 2$

y-intercept: $(0, -4)$

x-intercepts: $(0,0)$ $(4,0)$

Maximum/minimum value: Min at $y = -4$

b



Vertex: $(-2, -9)$

Nature of turning point: minimum

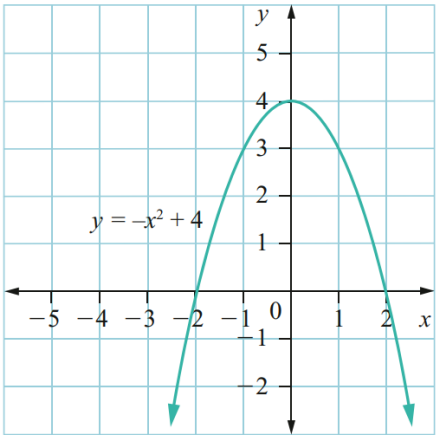
Axis of symmetry: $x = -2$

y-intercept: $(0, -5)$

x-intercepts: $(-5,0)$ $(1,0)$

Maximum/minimum value: Min at $y = -9$

c



Vertex: $(0,4)$

Nature of turning point: maximum

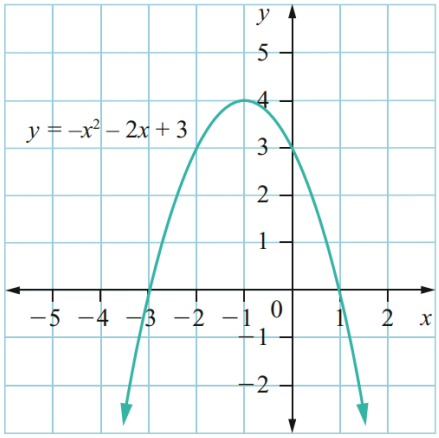
Axis of symmetry: $x = 0$

y-intercept: $(0,4)$

x-intercepts: $(-2,0)$ $(2,0)$

Maximum/minimum value: Max at $y = 4$

d



Vertex: $(-1,4)$

Nature of turning point: maximum

Axis of symmetry: $x = -1$

y-intercept: $(0,3)$

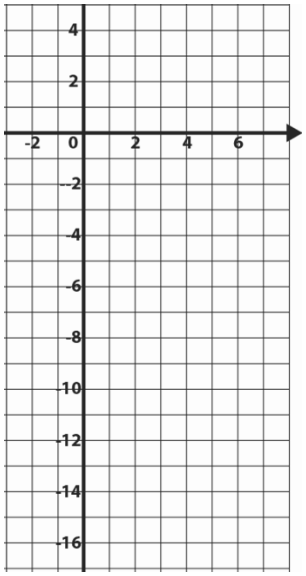
x-intercepts: $(-3,0)$ $(1,0)$

Maximum/minimum value: Max at $y = 4$

4 Sketch these parabolas from their equation.

a $y = -x^2 + 6x - 5$

x	-1	0	1	2	3	4	5
y							



Vertex:

$(3,4)$

Nature of turning point:

maximum

Axis of symmetry:

$x = 3$

y-intercept:

$(0,-5)$

x-intercepts:

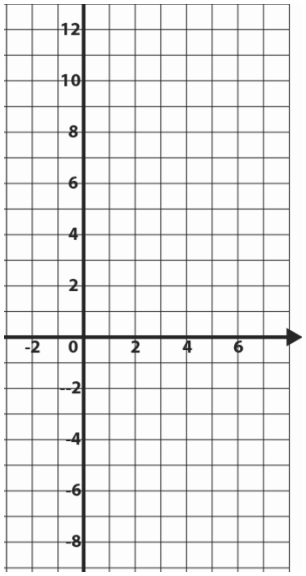
$(1,0)$ $(5,0)$

Maximum/minimum value:

Max at $y = 4$

b $y = 2x^2 - 4x - 6$

x	-2	-1	0	1	2	3	4
y							



Vertex:

$(1,-8)$

Nature of turning point:

minimum

Axis of symmetry:

$x = 1$

y-intercept:

$(0,-6)$

x-intercepts:

$(-1,0)$ $(3,0)$

Maximum/minimum value:

Min at $y = -8$

5 Rhys is drawing the graph of $y = x^2 - 5x + 3$

x	-1	0	1	2	3	4	5	6
$y = x^2 - 5x + 3$	9	3	-1	-3	-3	-1	3	9

He notices that the x-values 2 and 3 have the same y-value and thinks that the graph will look like this.

Rhys is wrong.

At what value of x will the lowest point of the graph be? How do you know?

2.5, as it is the midpoint of 2 and 3

What will the y-value be at this point?

$y = (2.5)^2 - 5(2.5) + 3 = -3.25$

PARABOLA TRANSFORMATIONS

Investigation Equation and the graph of a parabola.

https://www.desmos.com/calculator

a Sketch these graphs

$y = x^2$

$y = 2x^2$

$y = 3x^2$

$y = 4x^2$

How does increasing the coefficient of x^2 change the parabola?

.....

.....

.....

b Sketch these graphs.

$y = x^2$

$y = \frac{1}{2}x^2$

$y = \frac{1}{3}x^2$

$y = \frac{1}{4}x^2$

How does decreasing the coefficient of x^2 change the parabola?

.....

.....

.....

c Sketch these graphs.

$y = x^2$

$y = -x^2$

$y = -\frac{1}{2}x^2$

$y = -2x^2$

How does a negative coefficient of x^2 change the parabola?

.....

.....

.....

d Sketch these graphs.

$y = x^2$

$y = x^2 + 1$

$y = x^2 + 2$

$y = x^2 + 3$

$y = x^2 - 1$

$y = x^2 - 2$

How does the constant term change the parabola?

.....

.....

.....

Parabola Transformations

https://www.desmos.com/calculator/m4rrcqccef

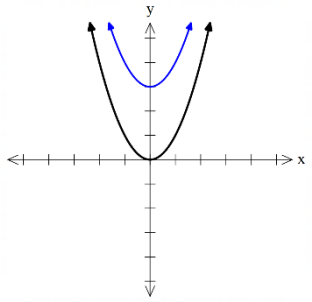
For a parabola $y = kx^2 + c$

The variables c, k are numbers that affect the graph in predictable ways.

These special variables are called **parameters**.

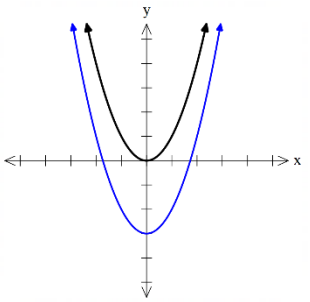
$c > 0$, the parabola is shifted ……

$y = x^2 + 3$



$c < 0$, the parabola is shifted ……

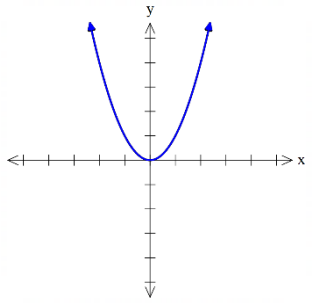
$y = x^2 - 3$



$k > 0$, the parabola is concave ……,

with a …… turning point.

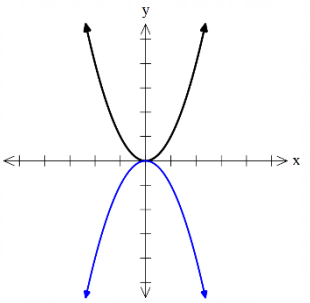
$y = 1x^2$



$k < 0$, the parabola is concave ……,

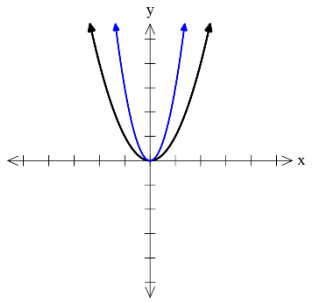
with a …… turning point.

$y = -1x^2$



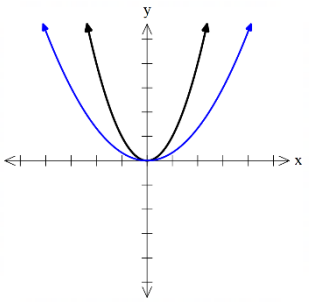
$|k| > 1$, the parabola gets narrower.

$y = 3x^2$



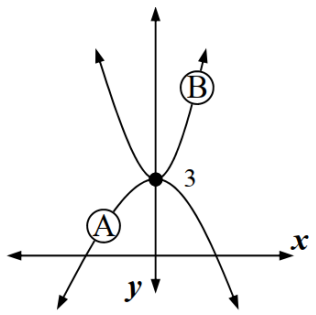
$0 < |k| < 1$ the parabola gets wider.

$y = \frac{1}{3}x^2$



1 Choose the equation that matches each equation.

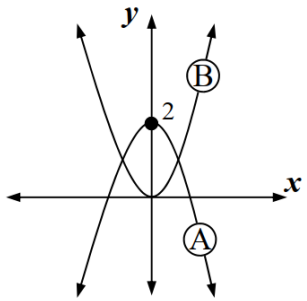
a



$y = x^2 + 3$ Ⓐ Ⓑ

$y = -\frac{1}{2}x^2 + 3$ Ⓐ Ⓑ

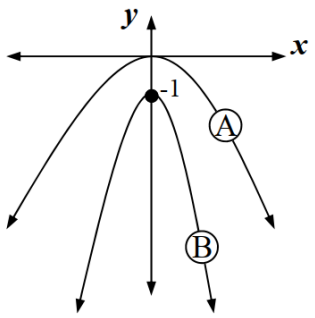
b



$y = x^2$ Ⓐ Ⓑ

$y = -x^2 + 2$ Ⓐ Ⓑ

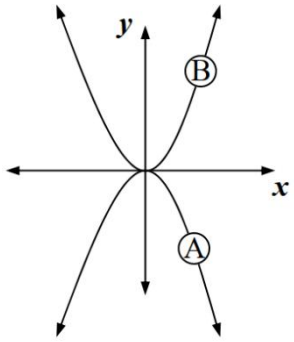
c



$y = -\frac{1}{2}x^2$ Ⓐ Ⓑ

$y = -2x^2 - 1$ Ⓐ Ⓑ

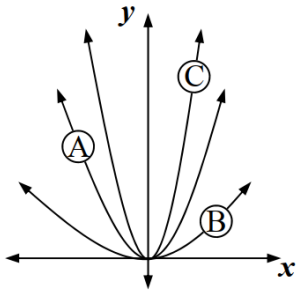
d



$y = -x^2$ Ⓐ Ⓑ

$y = x^2$ Ⓐ Ⓑ

e

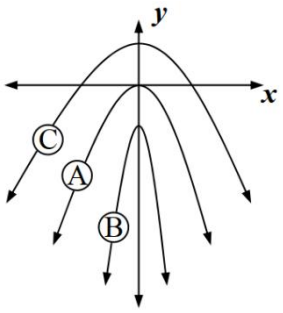


$y = x^2$ ⒶⒷⒸ

$y = 2x^2$ ⒶⒷⒸ

$y = \frac{1}{2}x^2$ ⒶⒷⒸ

f



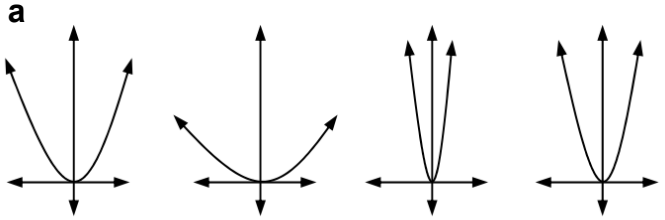
$y = -\frac{1}{2}x^2 + 3$ ⒶⒷⒸ

$y = -3x^2 - 3$ ⒶⒷⒸ

$y = -x^2$ ⒶⒷⒸ

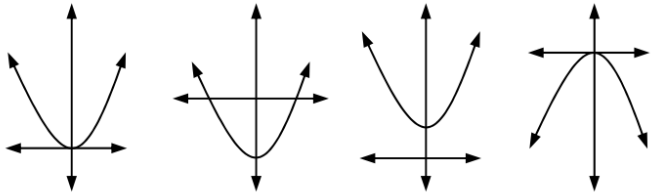
2 Match each parabola to its equation.

a



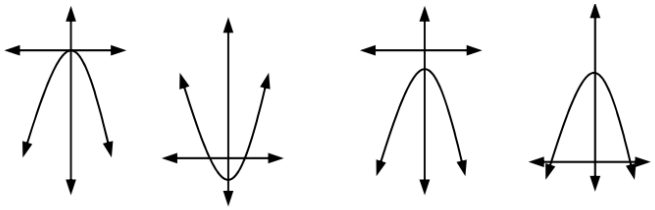
$y = 2x^2$ $y = x^2$ $y = \frac{1}{2}x^2$ $y = 3x^2$

b



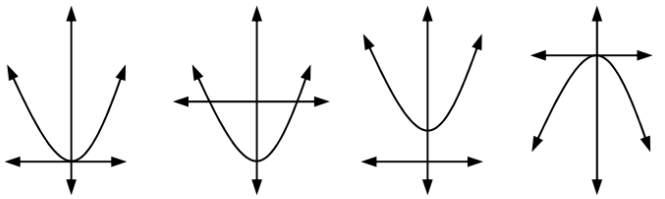
$y = x^2 + 2$ $y = -x^2$ $y = x^2$ $y = x^2 - 2$

c



$y = -x^2 - 2$ $y = -x^2 + 7$ $y = x^2 - 2$ $y = -x^2$

d



$y = x^2 + 2$ $y = -x^2$ $y = x^2$ $y = x^2 - 2$

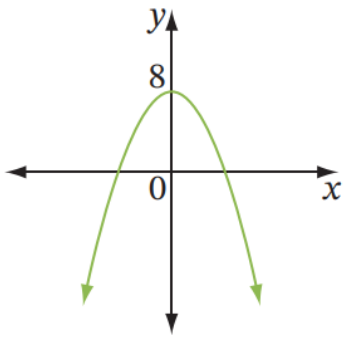
3 Match these parabolas to their equation.

$y = x^2$ $\quad y = -x^2$ $\quad y = x^2 - 8$ $\quad y = -12 - x^2$

$y = \frac{1}{2} + x^2$ $\quad y = 8 - x^2$ $\quad y = 8 + x^2$ $\quad y = -x^2 + \frac{1}{2}$

$y = x^2 - 12$ $\quad y = 12 - x^2$ $\quad y = -x^2 - 8$ $\quad y = x^2 + 12$

a



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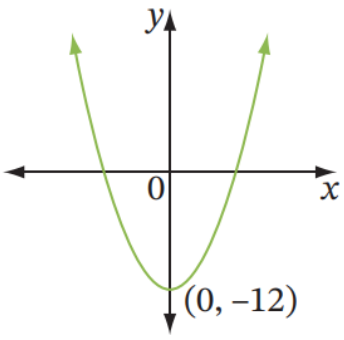

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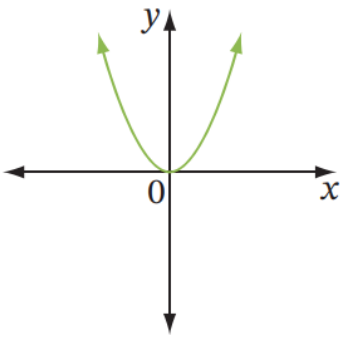

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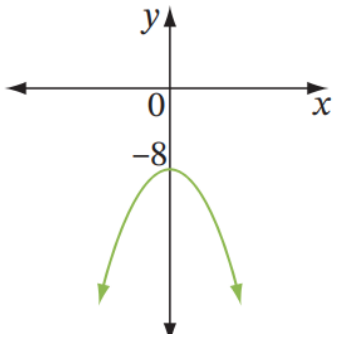
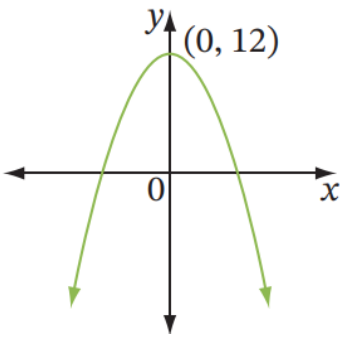


e



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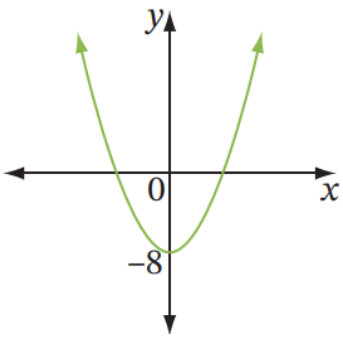
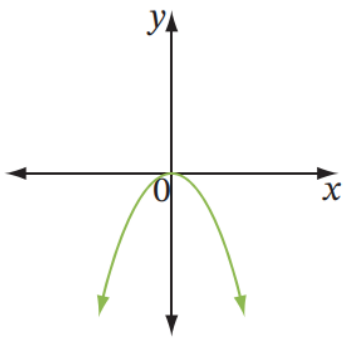


g



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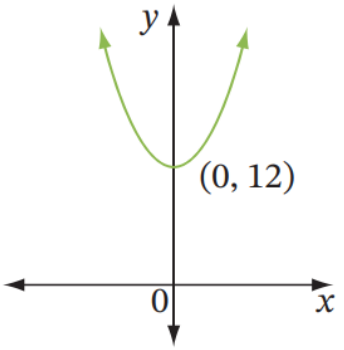

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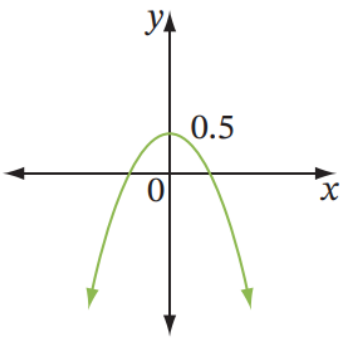

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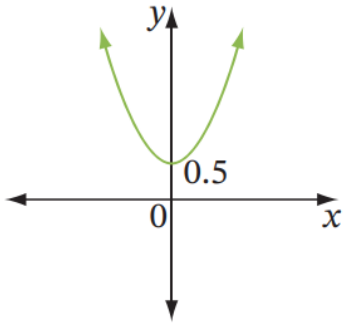

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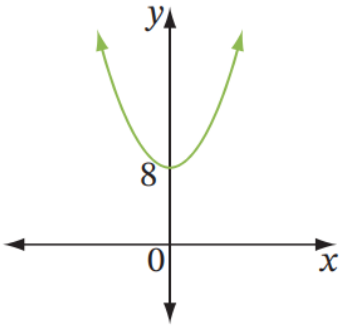

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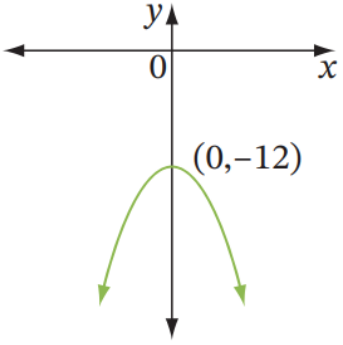

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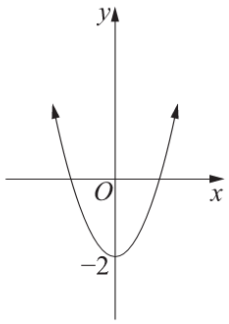
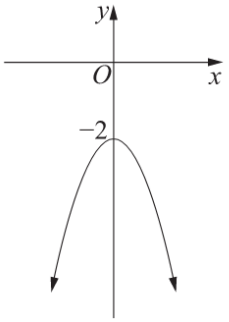
Which graph best represents the equation $y = x^2 - 2$?

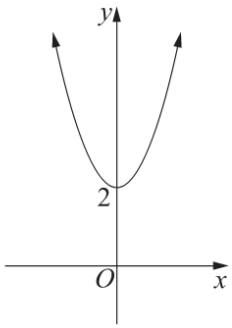
A.



B.



C.



D.