Student Name

Mathematics Stage 5 Core

NON-LINEAR RELATIONSHIPS B

Book 2 Graph and examine exponential relationships

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

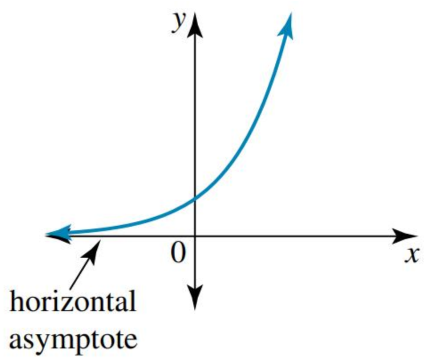
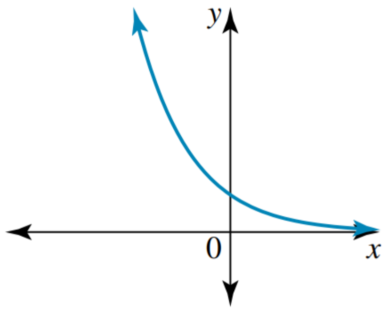
MA5-NLI-C-02 identifies and compares features of parabolas and exponential curves in various contexts

Graph and examine exponential relationships

- Graph and compare exponential curves of the form $y = a^x$ using graphing applications
- Describe features of exponential curves including the y-intercept, asymptote and the nature of the curve for very large and very small values of x

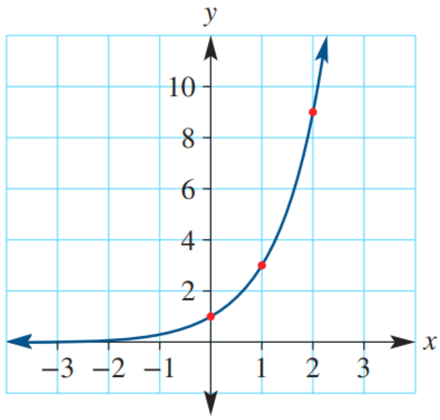
PROPERTIES OF THE EXPONENTIAL CURVE

- An **exponential relationship** $y = a^x$ produces **exponential curve** graphs.
- Exponential curves have these features:
  - An **asymptote**: a line the graph approaches but never reaches.
  - The **nature** of the graph (**growth** or **decay**) depends on the coefficient of x.
  - Consecutive y values change by a **common ratio**, which is the base number a.

$y = a^x$	$y = a^{-x}$
Increasing; exponential growth	Decreasing; exponential decay
	

Example Identify key features of an exponential curve.

Identify the features of this exponential curve.



y – intercept: $(0,1)$

Asymptote: $y = 0$

Nature: Increasing; exponential growth

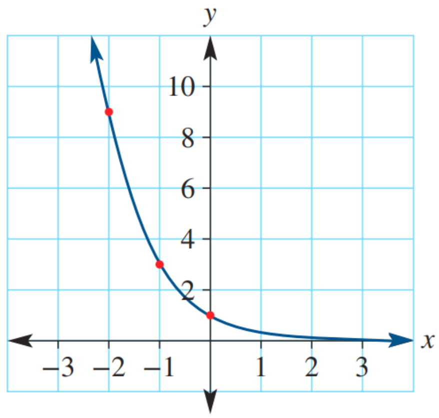
What happens when $x \to \infty$?

y is very large, $y \to \infty$

What happens when $x \to -\infty$?

y is very small, $y \to 0$

Identify the features of this exponential curve.



y – intercept:

$(0,1)$

Asymptote:

$y = 0$

Nature:

Decreasing, exponential decay

What happens when $x \to \infty$?

$y \to 0$

What happens when $x \to -\infty$?

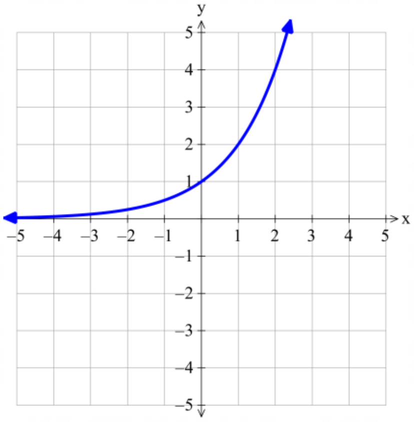
$y \to \infty$

Key ideas

1. Exponential curves have ………………, which are values it approaches but never reaches.
2. Exponential curves can either represent g………….. or d……………

1 Consider these exponential curves.

a $y = 2^x$



y-intercept:

$(0,1)$

Asymptote:

$y = 0$

Nature:

Increasing; exponential growth

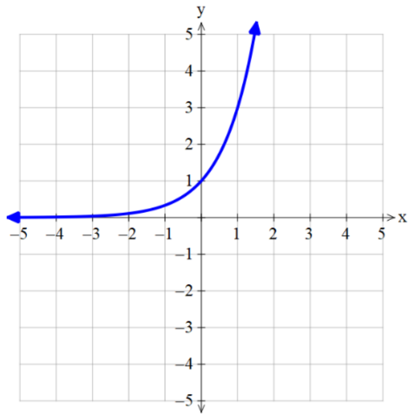
What happens when $x \to \infty$?

$y \to \infty$

What happens when $x \to -\infty$?

$y \to 0$

b $y = 3^x$



y-intercept:

$(0,1)$

Asymptote:

$y = 0$

Nature:

Increasing; exponential growth

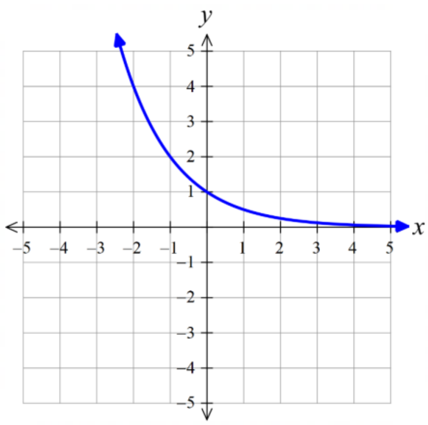
What happens when $x \to \infty$?

$y \to \infty$

What happens when $x \to -\infty$?

$y \to 0$

c $y = \left(\frac{1}{2}\right)^x$



y-intercept:

$(0,1)$

Asymptote:

$y = 0$

Nature:

decreasing; exponential decay

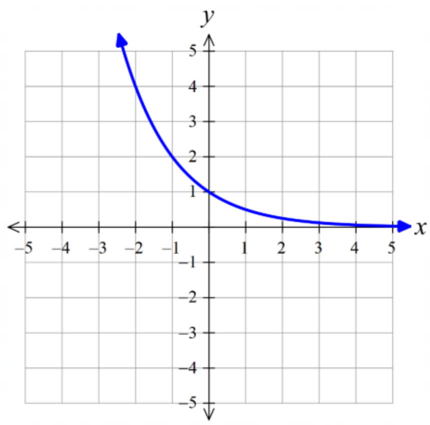
What happens when $x \to \infty$?

$y \to 0$

What happens when $x \to -\infty$?

$y \to \infty$

d $y = 2^{-x}$



y-intercept:

$(0,1)$

Asymptote:

$y = 0$

Nature:

decreasing; exponential decay

What happens when $x \to \infty$?

$y \to 0$

What happens when $x \to -\infty$?

$y \to \infty$

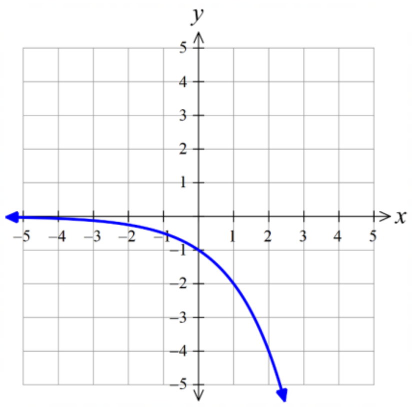
2 What do you notice about part **c** and **d** above?

....

They are the same.

3 Consider these exponential curves.

a $y = -2^x$



y-intercept:

$(0,-1)$

Asymptote:

$y = 0$

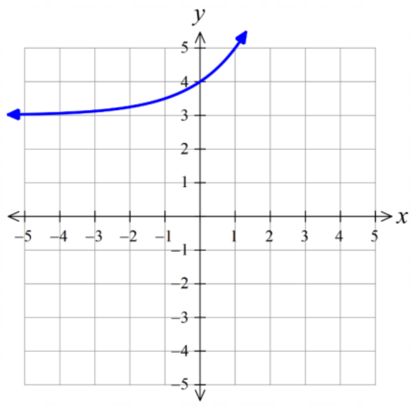
What happens when $x \to \infty$?

$y \to -\infty$

What happens when $x \to -\infty$?

$y \to 0$

b $y = 2^x + 3$



y-intercept:

$(0,4)$

Asymptote:

$y = 3$

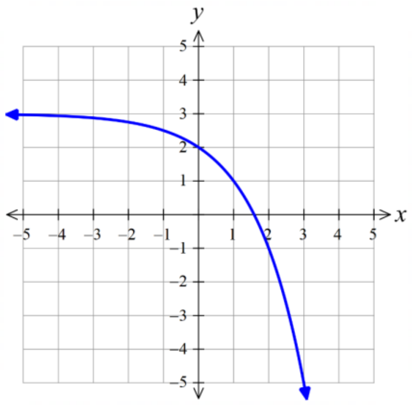
What happens when $x \to \infty$?

$y \to \infty$

What happens when $x \to -\infty$?

$y \to 3$

c $y = -2^x + 3$



y-intercept:

$(0,2)$

Asymptote:

$y = 3$

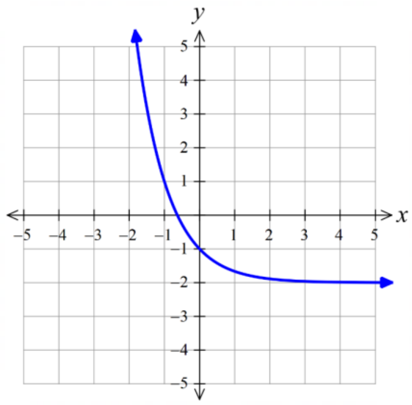
What happens when $x \to \infty$?

$y \to -\infty$

What happens when $x \to -\infty$?

$y \to 3$

d $y = \left(\frac{1}{3}\right)^x - 2$



y-intercept:

$(0,-1)$

Asymptote:

$y = -2$

What happens when $x \to \infty$?

$y \to -2$

What happens when $x \to -\infty$?

$y \to -\infty$

EXPONENTIAL GRAPH TRANSFORMATIONS

Investigation Equation and the graph of an exponential curve.

https://www.desmos.com/calculator

a Sketch these graphs

$y = 2^x$

$y = 3^x$

$y = 4^x$

$y = 5^x$

How does increasing the base value $a > 1$ affect the exponential curve?

....

....

....

b Sketch these graphs.

$y = 2^x$

$y = 0.8^x$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{3}\right)^x$

How does $0 < a < 1$ affect the exponential curve?

....

....

....

c

$y = 2^x$

$y = -2^x$

$y = \left(\frac{1}{2}\right)^x$

$y = -\left(\frac{1}{2}\right)^x$

How is $y = -a^x$ different to $y = a^x$?

....

....

....

d

$y = 2^x$

$y = 2(2)^x$

$y = 3(2)^x$

$y = -3(2)^x$

$y = -4(2)^x$

How does the factor in front of the exponential affect the curve?

....

....

e

$y = \left(\frac{1}{2}\right)^x$

$y = 2\left(\frac{1}{2}\right)^x$

$y = -3\left(\frac{1}{2}\right)^x$

How does the factor in front of the exponential affect the curve?

....

....

f

$y = 2^{-x}$

$y = \left(\frac{1}{2}\right)^x$

Why is $y = 2^{-x}$ the same as $y = \left(\frac{1}{2}\right)^x$?

....

....

....

Exponential Curve Transformations

https://www.desmos.com/calculator/63kvbi5cwn

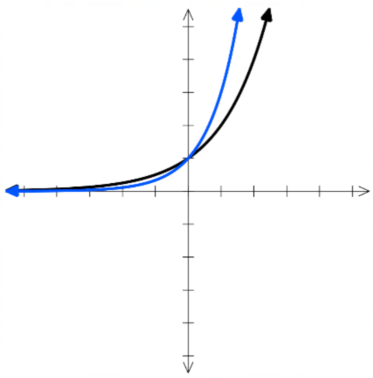
Consider the equation of an exponential curve $\boldsymbol{y = k(a)^x}$, where $a > 0$

The parameters a, k are numbers that affect the graph in predictable ways.

$a > 1$, we call this exponential ………………

a tells you the common r…………

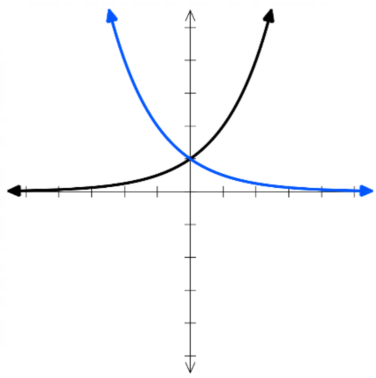
$y = 3^x$



$0 < a < 1$, we call this exponential …………

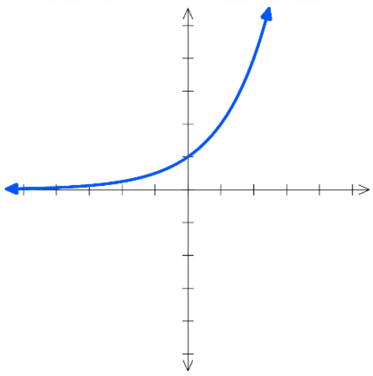
a tells you the common r…………

$y = \left(\frac{1}{2}\right)^x$



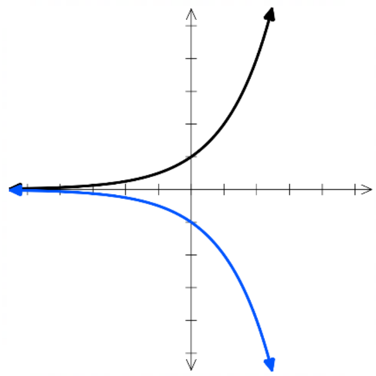
$k > 0$

$y = 1(2^x)$



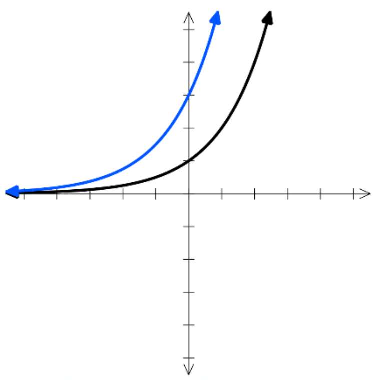
$k < 0$, reflect the graph across x-axis

$y = -1\,(2^x)$



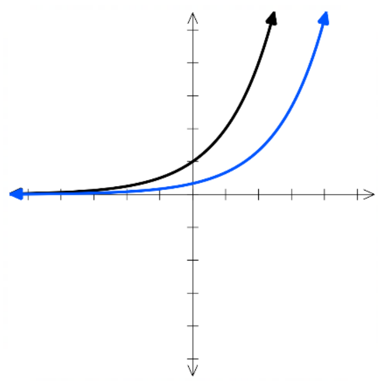
$|k| > 1$, the curve changes more quickly.
The value of k is the …… -intercept.

$y = 3(2^x)$

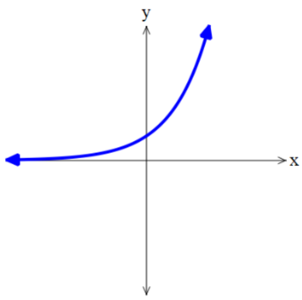
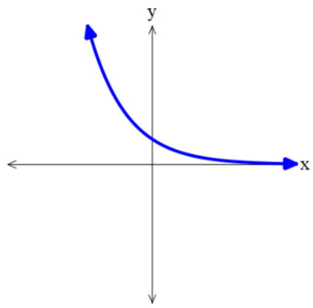
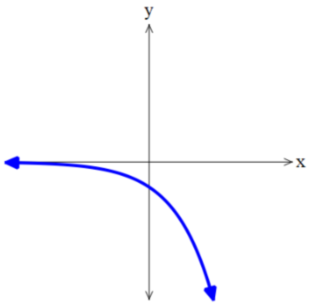
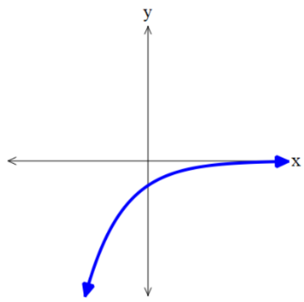


$0 < |k| < 1$ the curve changes more slowly.
The value of k is the …… -intercept.

$y = \frac{1}{3}(2^x)$



Exponential Curve Transformations	
Common ratio $a > 1,$ exponential growth $0 < a < 1,$ exponential decay $y = k(a)^x$ Factor; affects steepness	$a > 1,$ exponential decay $0 < a < 1,$ exponential growth $y = k(a)^{-x}$

$y = 2^x$	$y = 2^{-x}$ or $y = \left(\frac{1}{2}\right)^x$	$y = -2^x$	$y = -2^{-x}$ or $y = -\left(\frac{1}{2}\right)^x$
			

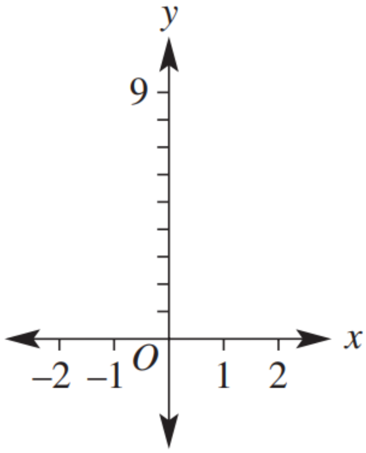
1 Plot $y = 2^x$ and $y = 3^x$ on the same axis.

$y = 2^x$

x	-2	-1	0	1	2
y					

$y = 3^x$

x	-2	-1	0	1	2
y					



Describe their similarities and differences.

Similarities: ………………………………………………………………………………………

Differences: ………………………………………………………………………………………

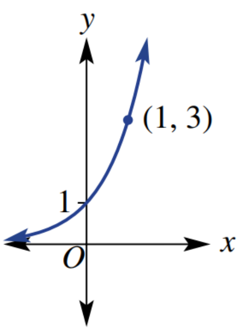
Similar: both exponential growth, same y-intercept at $(0,1)$., same asymptote at $y = 0$.

Different: $y = 3^x$ increases faster.

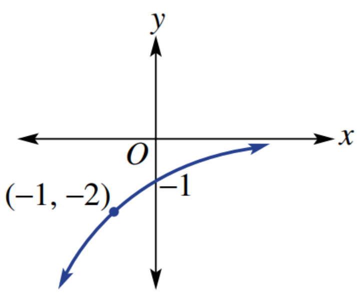
2 Match each equation to its graph. Check by substituting the given point into the equation.

A $y = -x - 2$	**B** $y = 3^x$	**C** $y = 3^{-x}$
D $y = -2^x$	**E** $y = x$	**F** $y = -2^{-x}$

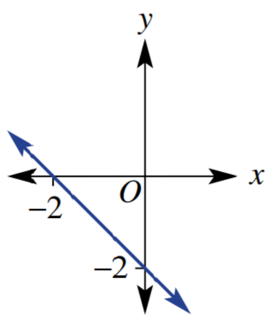
a



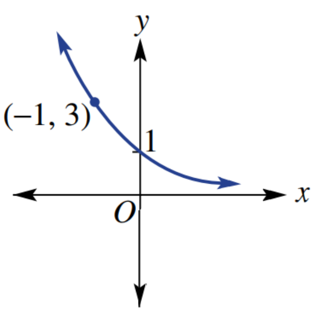
b



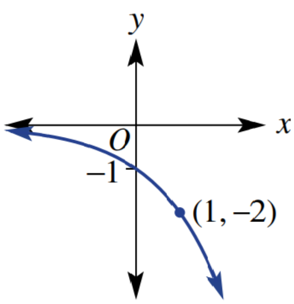
c



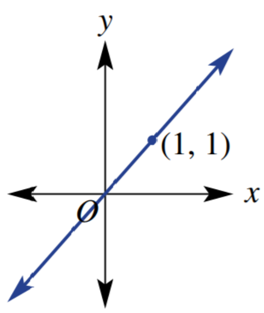
d



e



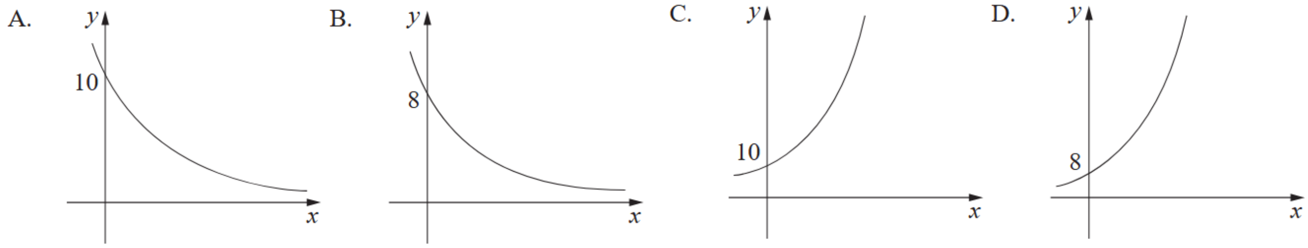
f



A – c, B – a, C – d, D – e, E – f, F - b

3 **2021 HSC Standard 2 Band 5**

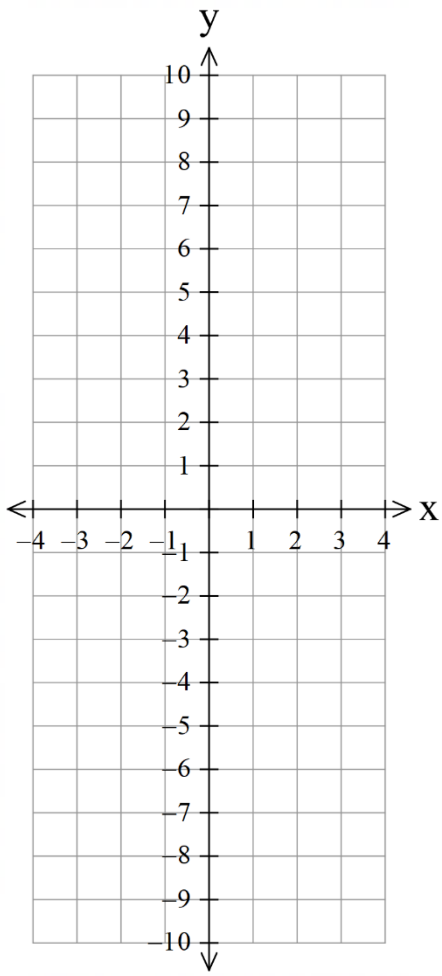
Which graph best represents the graph of $y = 10(0.8)^x$?



A

4 Plot these graphs on the same axis.

x	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = -2^x$							
$y = 2^{-x}$							



5

a Show the difference between 2^{-2} and -2^2

$2^{-2} = \frac{1}{4}, \quad -2^2 = -4$

b Write these with negative powers:

$\frac{1}{2}$ | $\frac{1}{3^2}$ | $\frac{1}{5^3}$

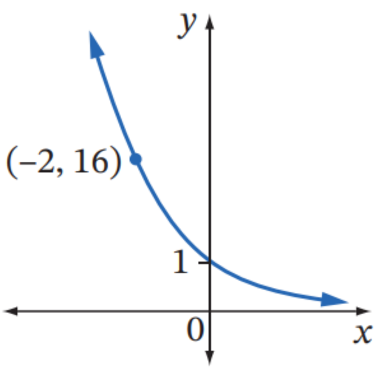
2^{-1} 3^{-2} 5^3

c Evaluate these expressions.

-3^2 | -2^3 | $(-2)^3$ | $(-2)^{-3}$ | 3^{-2}

-9 -8 -8 $-\frac{1}{8}$ $\frac{1}{9}$

6 Find an equation in the form $y = a^x$ for this graph:



$y = 4^{-x}$

7 Consider the exponential curves $y = 2^{-x}$ and $y = \left(\frac{1}{2}\right)^x$

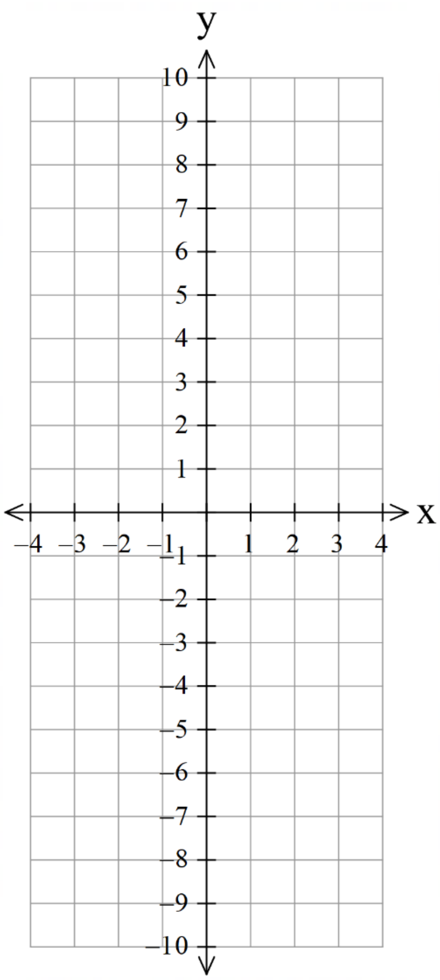
a Sketch each graph on the same axis.

$y = 2^{-x}$

x	-3	-2	-1	0	1	2	3
y							

$y = \left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
y							



b What do you notice?

They are the same graph

c Rewrite these equations in the form $y = a^x$, where $0 < a < 1$.

i $y = 3^{-x}$

$y = \left(\frac{1}{3}\right)^x$

ii $y = 5^{-x}$

$y = \left(\frac{1}{5}\right)^x$

iii $y = 10^{-x}$

$y = \left(\frac{1}{10}\right)^x$

d Write these rules in the form $y = a^{-x}$, where $a > 1$.

i $y = \left(\frac{1}{4}\right)^x$

$y = 4^{-x}$

ii $y = \left(\frac{1}{7}\right)^x$

$y = 7^{-x}$

iii $y = \left(\frac{1}{11}\right)^x$

$y = 11^{-x}$

e Show, using index laws, that this is always true: $\left(\frac{1}{a}\right)^x = a^{-x}$

$\left(\frac{1}{a}\right)^x = (a^{-1})^x = a^{-x}$