

Student Name

Mathematics Stage 5 Paths

NON-LINEAR RELATIONSHIPS C

Book 6 Graph and compare polynomial curves and
describe their features and transformations

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

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OVERVIEW

SYLLABUS CONTENT

MA5-NLI-P-01 interprets and compares non-linear relationships and their transformations, both algebraically and graphically

Graph and compare polynomial curves and describe their features and transformations

- Use graphing applications to graph and compare features of cubic equations of the form $y = ax^3 + c$, where a and c are integers
- Use graphing applications to graph a variety of equations of the form $y = kx^n$, where n is an integer and $n \geq 2$, and describe the effect on the shape of the curve where n is an odd or an even number
- Use graphing applications to graph curves of the form $y = kx^n + c$ and $y = k(x - b)^n$ where n is an integer and $n \geq 2$, and describe the transformations from $y = kx^n$

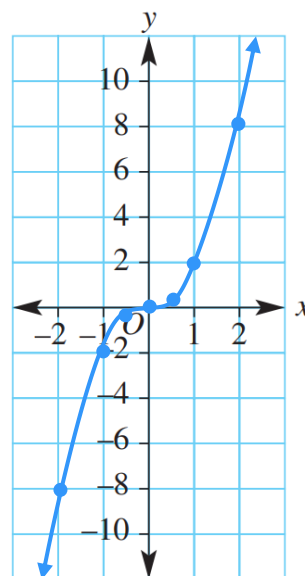
CUBIC CURVE TRANSFORMATIONS

- A **cubic** function has an equation where x has a **highest power of 3**.
The most basic form is $y = kx^3$
- The graph has a **point of inflection**, where the concavity changes.

Example Graph cubic relationships using a table of values.

Draw the graph of $y = x^3$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	-8	-1	$-\frac{1}{8}$	0	$\frac{1}{8}$	1	8



Where is the point of inflection?

The point of inflection (POI) is where the graph changes

In this example, the POI is at

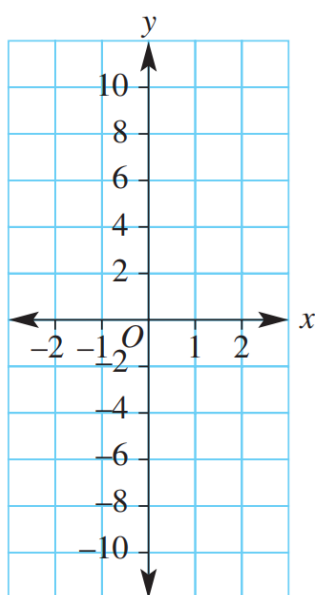
How is the gradient is changing?

Before the POI, the curve is at a rate,

After the POI, the curve is at a rate.

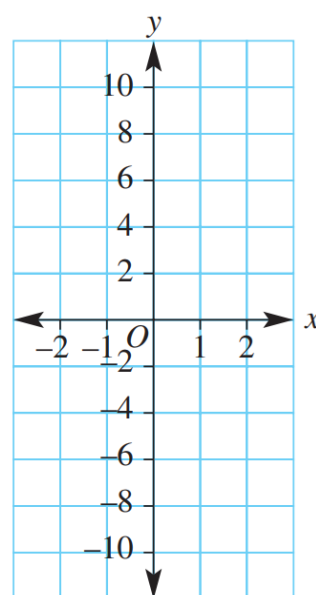
a Draw the graph of $y = x^3 - 2$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y							



b Draw the graph of $y = -x^3 + 1$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y							



Investigation Equation and the graph of a cubic curve.

On Desmos <https://www.desmos.com/calculator>

a Sketch these graphs

$$y = x^3$$

$$y = 2x^3$$

$$y = 3x^3$$

$$y = 4x^3$$

How does increasing the coefficient of x^3 affect the cubic curve?

.....

.....

.....

b Sketch these graphs.

$$y = x^3$$

$$y = \frac{1}{2}x^3$$

$$y = \frac{1}{3}x^3$$

$$y = \frac{1}{4}x^3$$

How does decreasing the coefficient of x^3 affect the cubic curve?

.....

.....

.....

c Sketch these graphs.

$$y = x^3$$

$$y = -x^3$$

$$y = -\frac{1}{2}x^3$$

$$y = -2x^3$$

How does a negative coefficient of x^3 affect the cubic curve?

.....

.....

.....

d Sketch these graphs.

$$y = x^3$$

$$y = x^3 + 1$$

$$y = x^3 + 2$$

$$y = x^3 + 3$$

$$y = x^3 - 1$$

$$y = x^3 - 2$$

How does the constant term change the cubic curve?

.....

.....

.....

e Sketch these graphs.

$$y = (x - 1)^3$$

$$y = (x - 2)^3$$

$$y = (x + 2)^3$$

$$y = (x + 3)^3$$

How does the constant term inside the cube change the cubic curve?

.....

.....

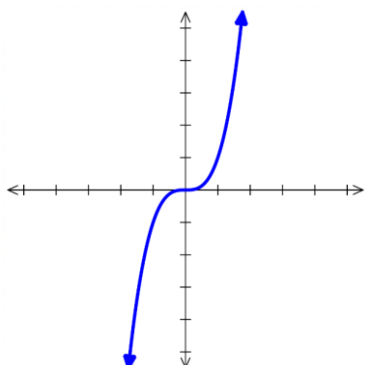
Cubic Curve Transformations

For a cubic curve $y = k(x - b)^3 + c$

The parameters b, c, k affect the graph in predictable ways.

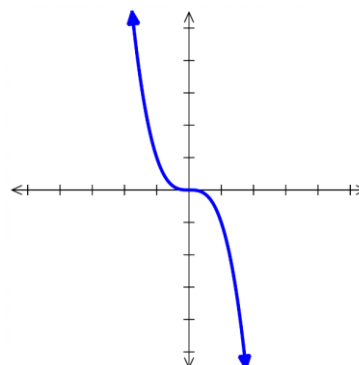
$k > 0$, the curve is increasing.

$$y = 1x^3$$



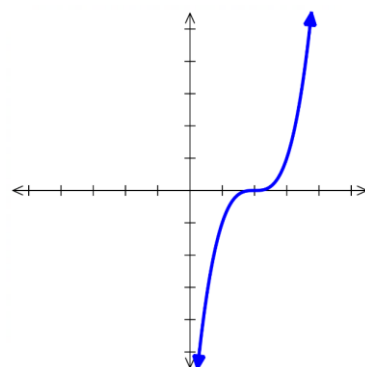
$k < 0$, the curve is decreasing

$$y = -1x^3$$



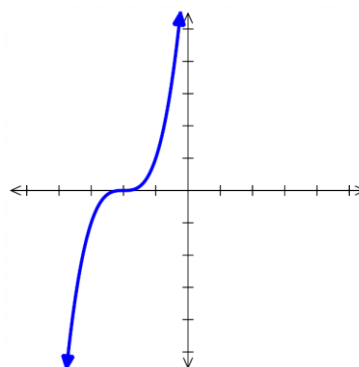
$b > 0$, the curve is shifted

$$y = (x - 2)^3$$



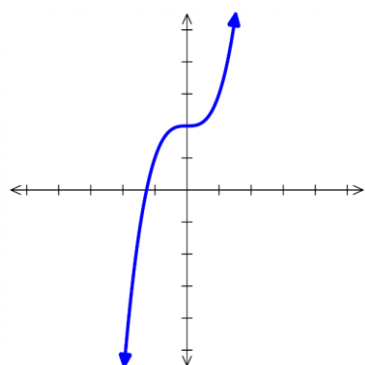
$b < 0$, the curve is shifted

$$y = (x + 2)^3$$



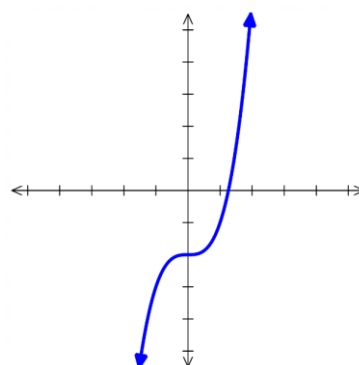
$c > 0$, the curve is shifted

$$y = x^3 + 2$$



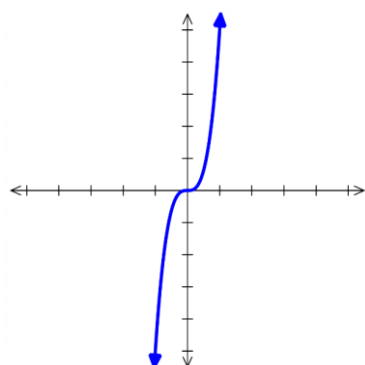
$c < 0$, the curve is shifted

$$y = x^3 - 2$$



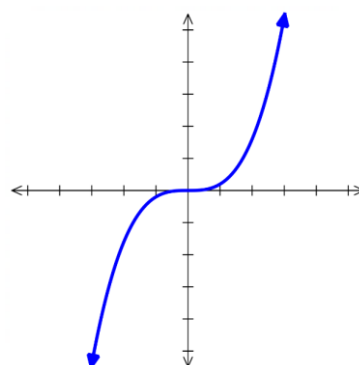
$|k| > 1$, the curve gets narrower.

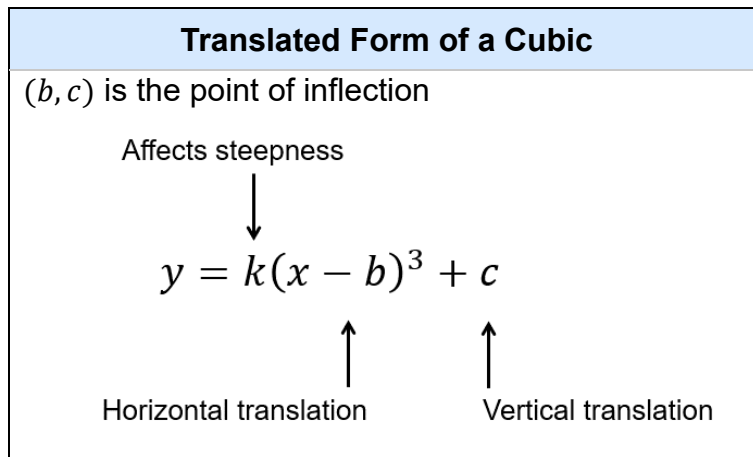
$$y = 5x^3$$



$0 < |k| < 1$ the curve gets wider.

$$y = \frac{1}{5}x^3$$





- To sketch a cubic curve:
 - Determine the behaviour of curve by substituting $x \rightarrow \infty$.
 - Find y -intercept by letting $x = 0$.
 - Find x -intercepts by letting $y = 0$.
 - Identify the point of inflection (b, c) .

Example Sketch cubic equations from translated form.

Sketch $y = x^3 - 8$

1. When $x \rightarrow \infty, y \rightarrow \infty$

2. Let $x = 0$

$$y = 0^3 - 8$$

$$= -8$$

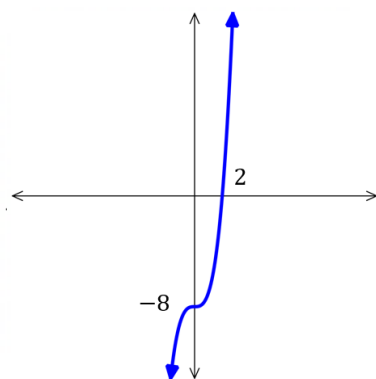
3. Let $y = 0$

$$0 = x^3 - 8$$

$$8 = x^3$$

$$2 = x$$

4. Point of inflection at $(0, -8)$



Sketch $y = -2(x - 3)^3 + 2$

1. When $x \rightarrow \infty, y \rightarrow -\infty$

2. Let $x = 0$

$$y = -2(0 - 3)^3 + 2$$

$$= 56$$

3. Let $y = 0$

$$0 = -2(x - 3)^3 + 2$$

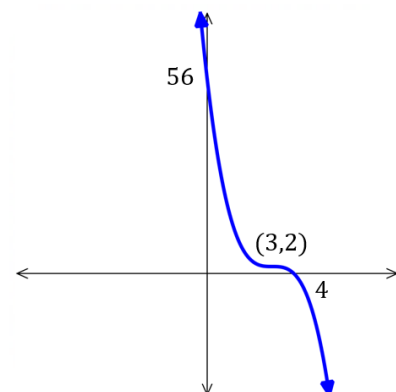
$$-2 = -2(x - 3)^3$$

$$1 = (x - 3)^3$$

$$1 = x - 3$$

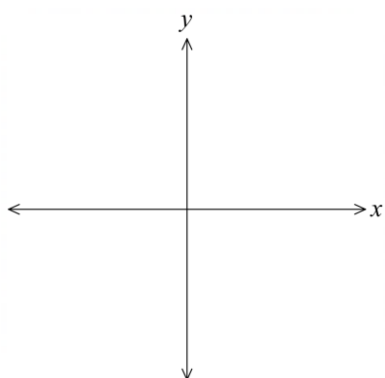
$$4 = x$$

4. Point of inflection at $(3, 2)$

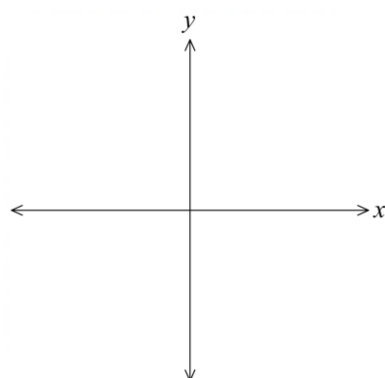


Sketch these cubic functions. Show all intercepts and the inflection point.

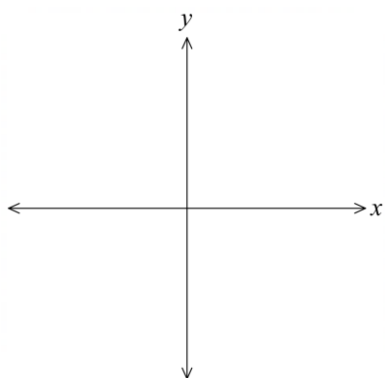
a $y = x^3 - 1$



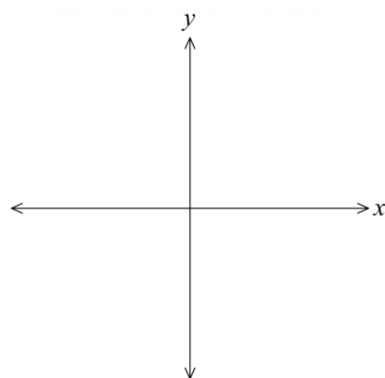
b $y = -x^3 + 8$



c $y = (x - 4)^3$



d $y = 2(x + 7)^3 - 3$



1 The graph of a cubic function is shown.

a Identify the point of inflection.

b Circle a possible equation for this cubic equation.

A $y = 2x^3$

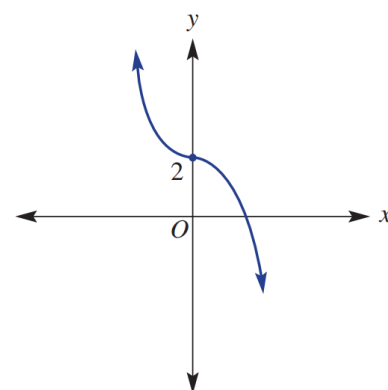
B $y = (x + 2)^3$

C $y = -2x^3$

D $y = -x^3 + 2$

E $y = x^3 + 2$

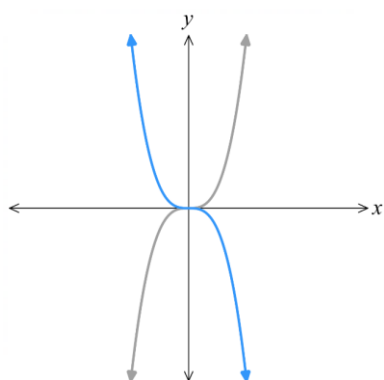
(0,2)



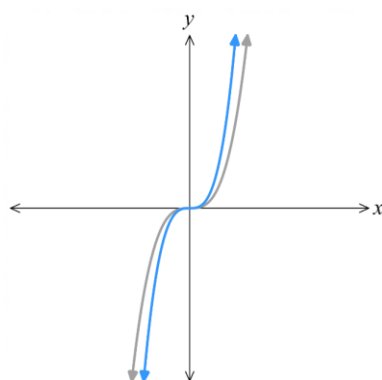
D

2 The graph of $y = x^3$ is given. Sketch these graphs on the same axis to show the transformation. Show the point of inflection.

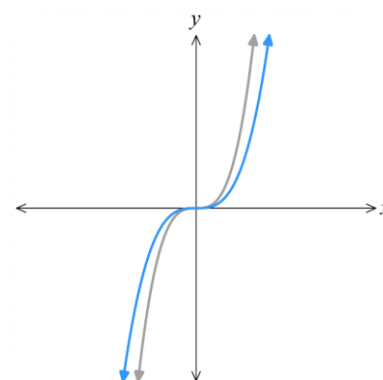
a $y = -x^3$



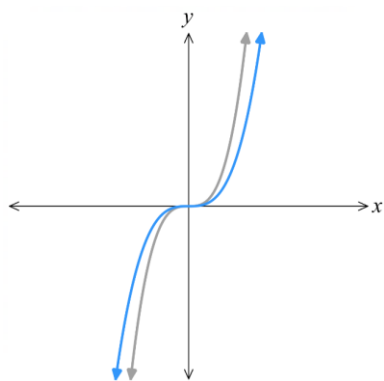
b $y = 2x^3$



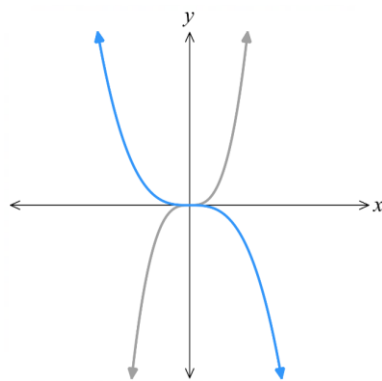
c $y = \frac{1}{2}x^3$



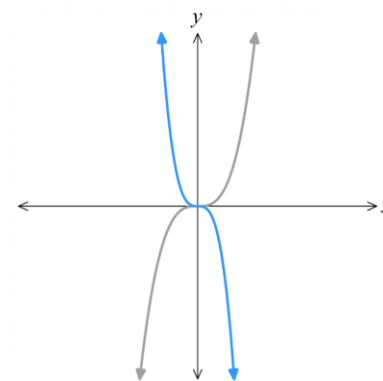
d $y = \frac{x^3}{2}$



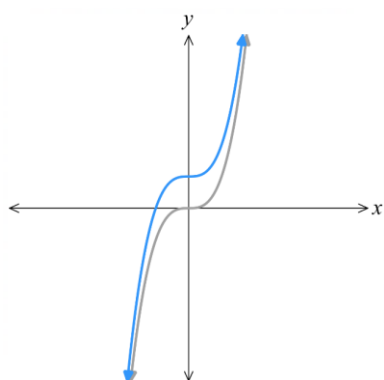
e $y = -\frac{x^3}{4}$



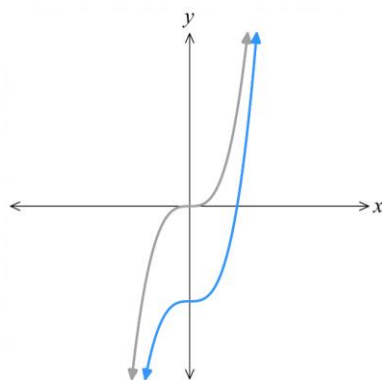
f $y = -4x^3$



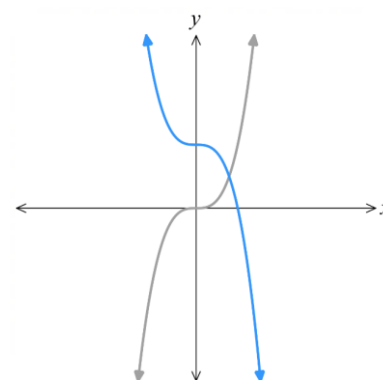
g $y = x^3 + 1$



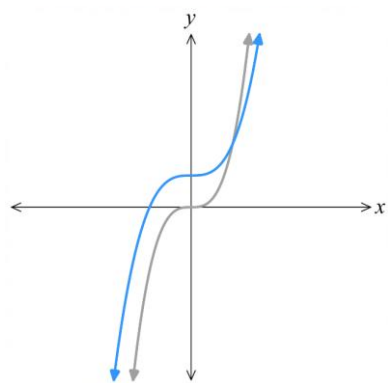
h $y = x^3 - 3$



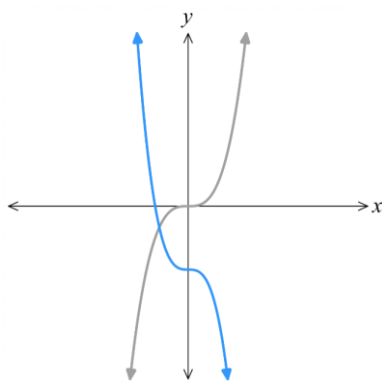
i $y = -x^3 + 2$



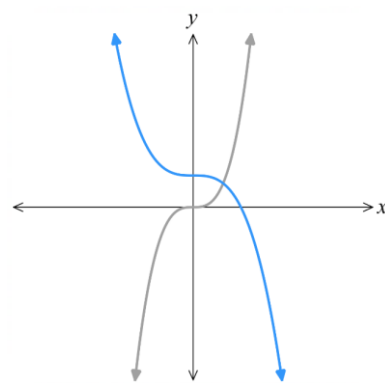
j $y = \frac{1}{2}x^3 + 1$



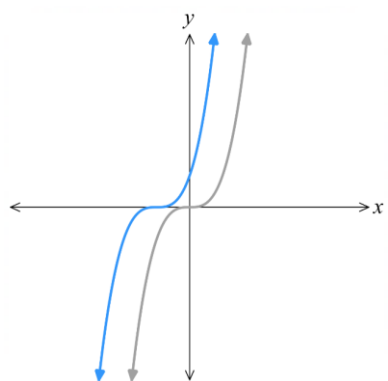
k $y = -2x^3 - 2$



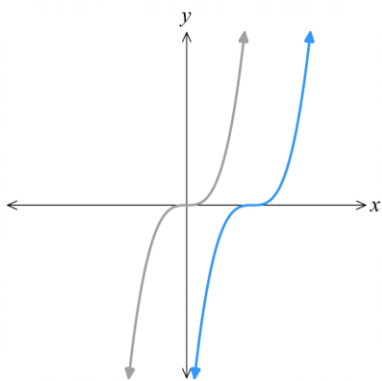
l $y = -\frac{x^3}{3} + 1$



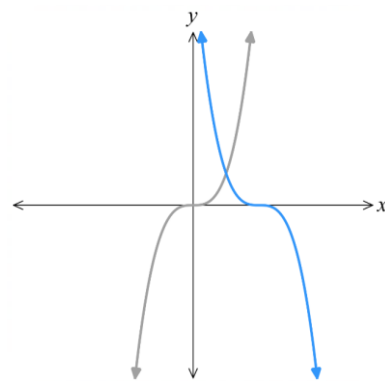
m $y = (x + 1)^3$



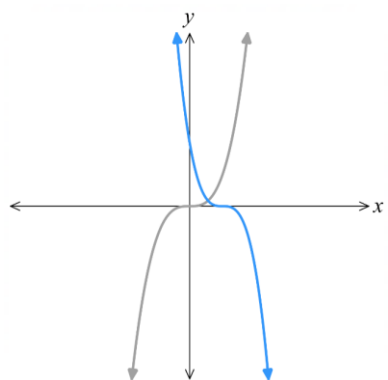
n $y = (x - 2)^3$



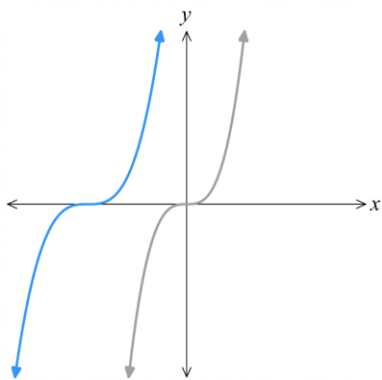
o $y = -(x - 2)^3$



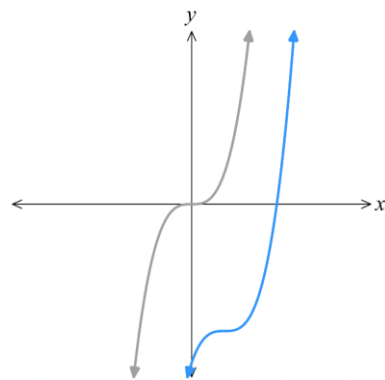
p $y = -2(x - 1)^3$



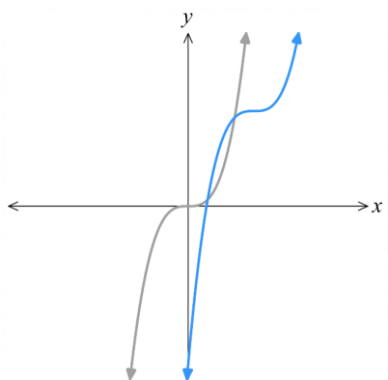
q $y = \frac{1}{2}(x + 3)^3$



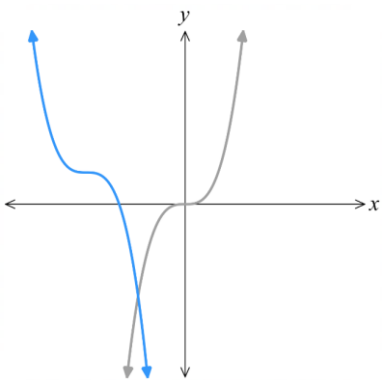
r $y = (x - 1)^3 - 4$



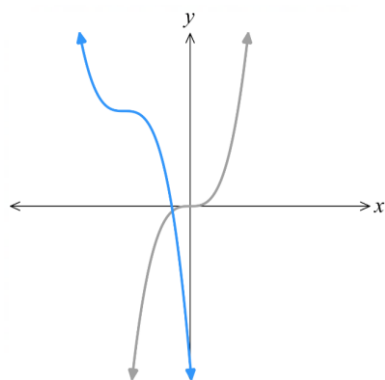
s $y = (x - 2)^3 + 3$



t $y = -(x + 3)^3 + 1$

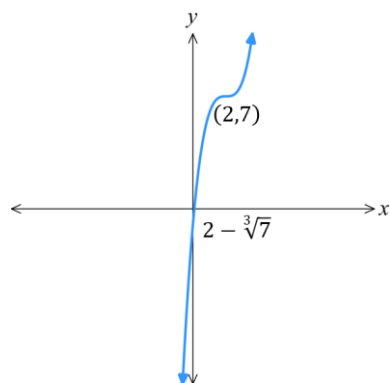


u $y = -(x + 2)^3 + 3$

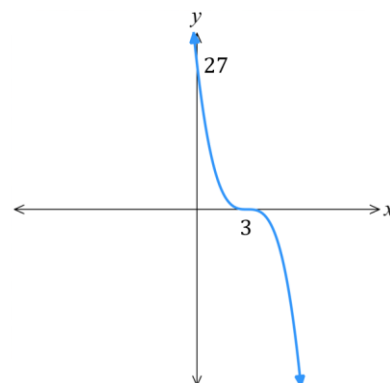


- 3** Draw a neat sketch of these equations.
Clearly label the y -intercept, x -intercept, and the point where $x = 1$.

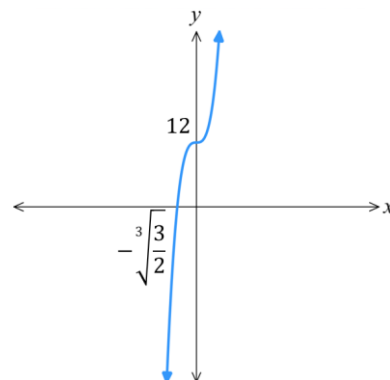
a $y = (x - 2)^3 + 7$



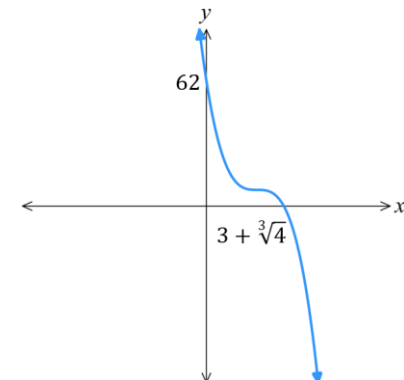
b $y = -(x + 3)^3$



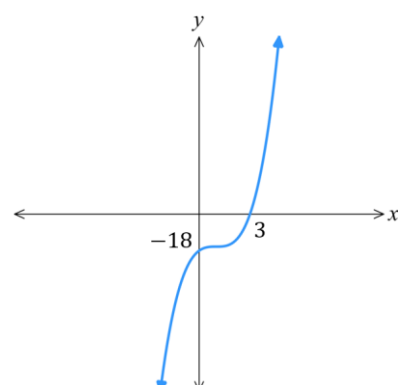
c $y = 8x^3 + 12$



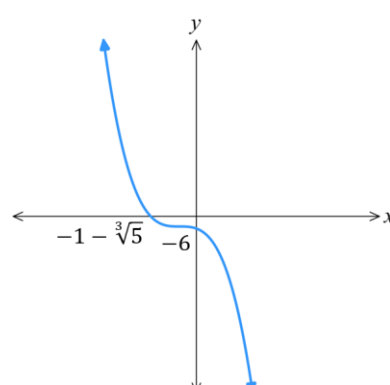
d $y = -2(x - 3)^3 + 8$



e $y = 2(x - 1)^3 - 16$



f $y = -(x + 1)^3 - 5$



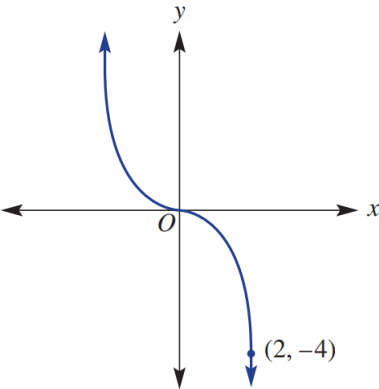
- 4 A cubic equation is in the form $y = kx^3 + c$. Find its equation if it passes through:
- a (0,5) and (2,-3) b (1,-2) and (2,33)

$y = -x^3 + 5$

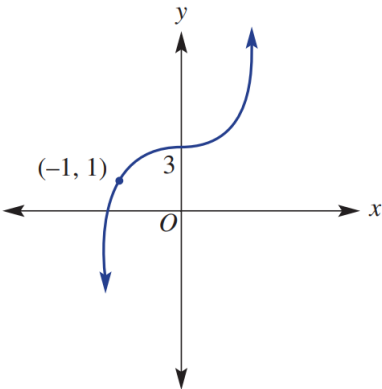
$y = 5x^3 - 7$

- 5 The following graphs have equations of the form $y = kx^3 + c$. Use the points given to find the values of k and c for each equation.

a



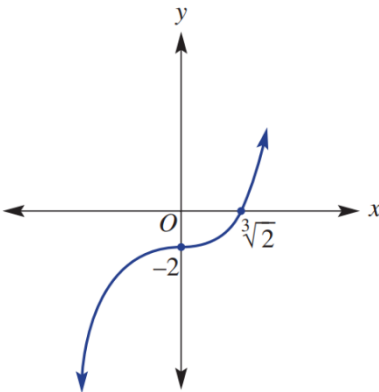
b



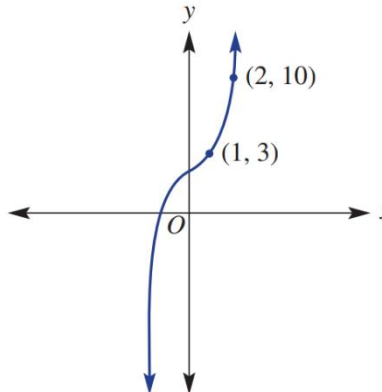
$y = -\frac{x^3}{2}$

$y = -2x^3 + 3$

c



d



$y = x^3 - 2$

$y = x^3 + 2$

POWER CURVE TRANSFORMATIONS

Power Functions

$$y = k(x - b)^n + c.$$

Investigation Equation and the graph of a power curve.

On Desmos <https://www.desmos.com/calculator>

a Sketch these graphs

$$y = x^1$$

$$y = x^2$$

$$y = x^3$$

$$y = x^4$$

$$y = x^5$$

How do odd and even powers affect the graph?

.....
.....
.....

b Sketch these graphs.

$$y = x^4$$

$$y = 2x^4$$

$$y = 3x^4$$

$$y = \frac{1}{3}x^4$$

$$y = \frac{1}{5}x^4$$

How does the factor affect the graph?

.....
.....
.....

c

$$y = x^5 + 1$$

$$y = x^5 + 3$$

$$y = x^5 - 2$$

$$y = x^5 - 4$$

How does the constant term affect the graph?

.....
.....
.....

d

$$y = (x - 1)^6$$

$$y = (x - 4)^6$$

$$y = (x + 2)^6$$

$$y = (x + 3)^6$$

How does the term with x affect the graph?

.....
.....
.....

Power Curve Transformations

For a power curve $y = k(x - b)^n + c$

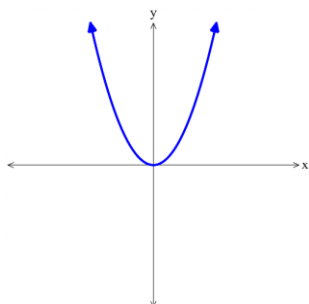
The parameters b, c, k, n affect the graph in predictable ways.

n is even, curve has a turning point.

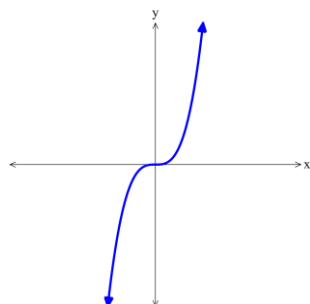
n is odd, curve has inflection point.

As n increases, the curve gets but the turning/inflection point gets

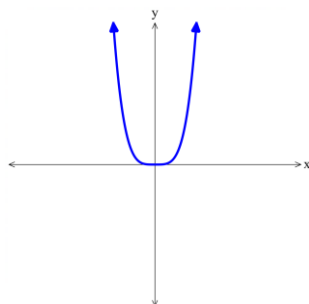
$$y = x^2$$



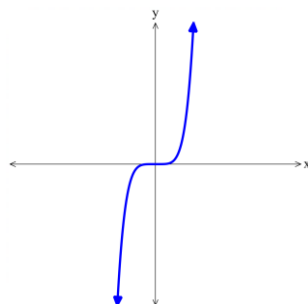
$$y = x^3$$



$$y = x^4$$

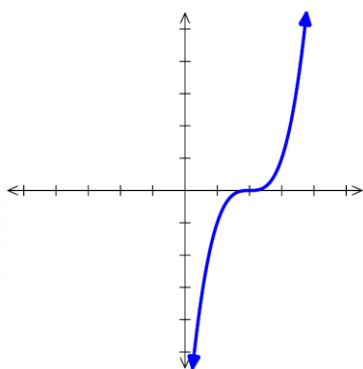


$$y = x^5$$



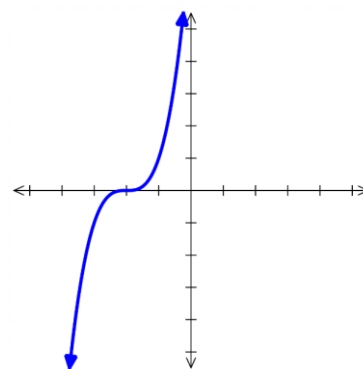
$b > 0$, the curve is shifted

$$y = (x - 2)^3$$



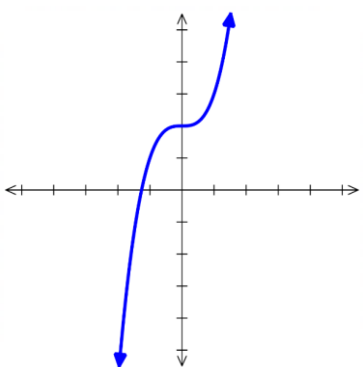
$b < 0$, the curve is shifted

$$y = (x + 2)^3$$



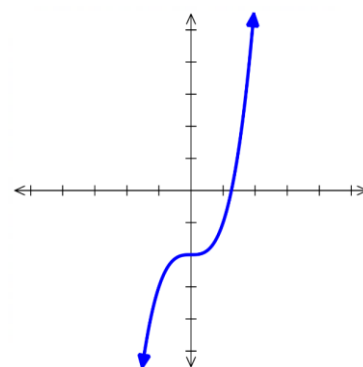
$c > 0$, the curve is shifted

$$y = x^3 + 2$$



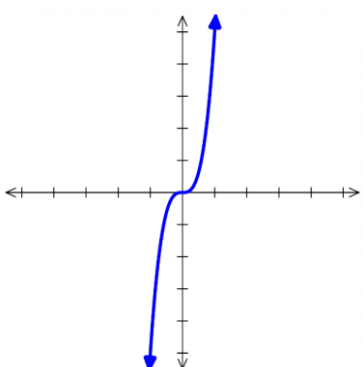
$c < 0$, the curve is shifted

$$y = x^3 - 2$$



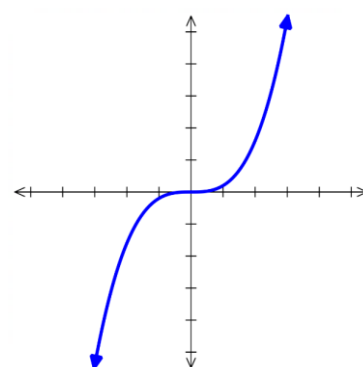
$|k| > 1$, the curve gets narrower.

$$y = 5x^3$$



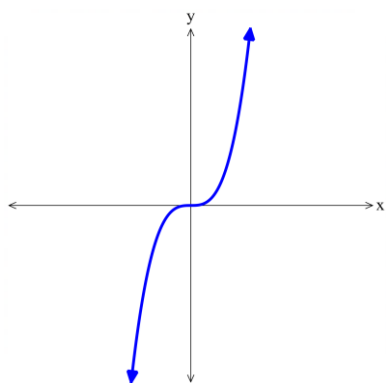
$0 < |k| < 1$ the curve gets wider.

$$y = \frac{1}{5}x^3$$



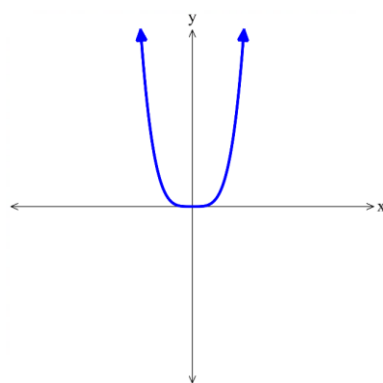
1 The graph of $y = x^n$ is given. State whether the power is even or odd.

a



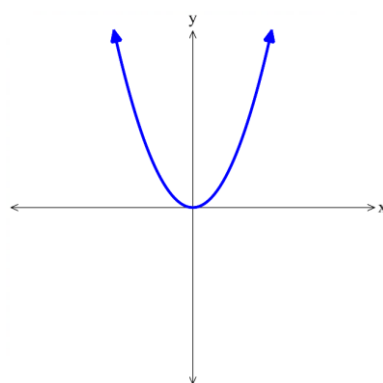
odd

b



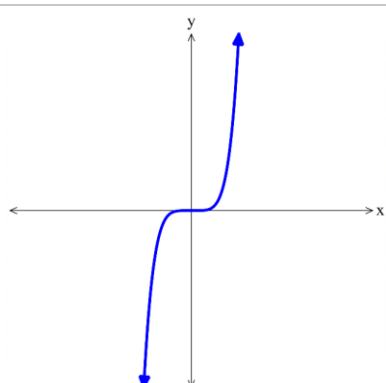
even

c



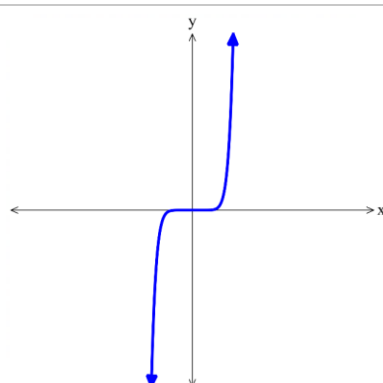
even

d



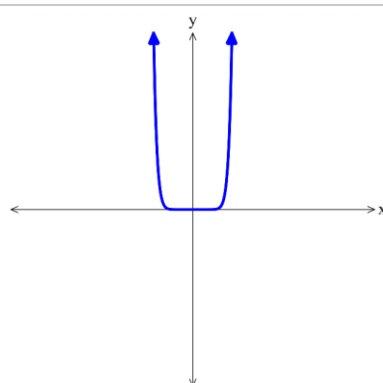
odd

e



odd

f



even

2 Match each graph its equation.

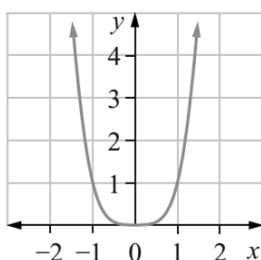
a $y = x^4$

b $y = 5x^4$

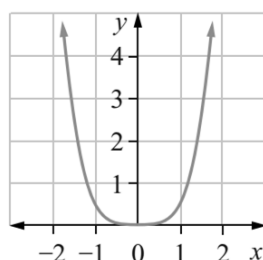
c $y = \frac{1}{2}x^4$

d $y = \frac{x^4}{6}$

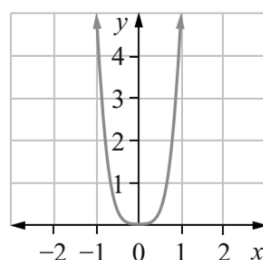
A



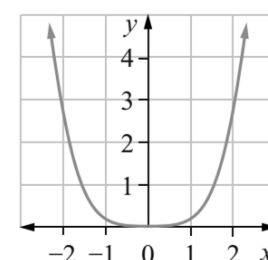
B



C



D



a - A, b - C, c - B, d - D

3 Match each graph with its equation.

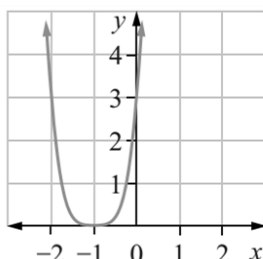
a $y = 3(x - 1)^4$

b $y = 3(x + 1)^4$

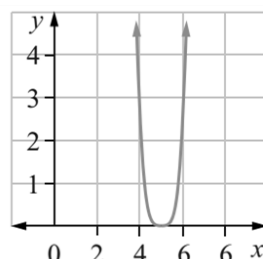
c $y = 3(x + 3)^4$

d $y = 3(x - 5)^4$

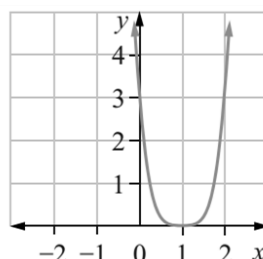
A



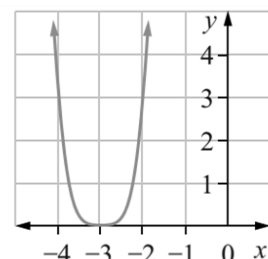
B



C



D



a - C, b - A, c - D, d - B

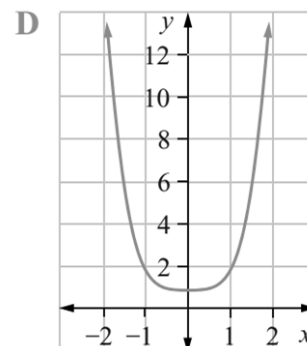
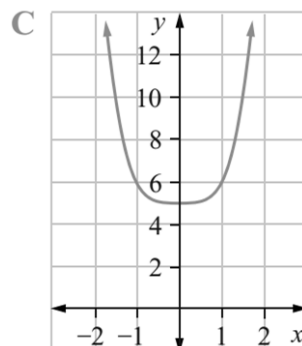
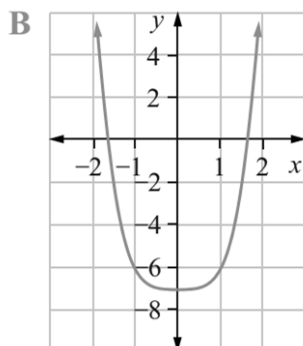
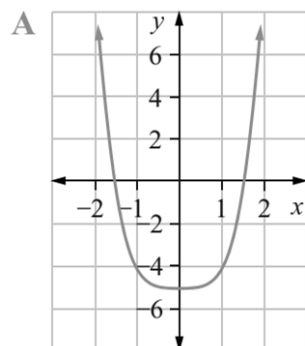
4 Match each graph with its equation.

a $y = x^4 - 5$

b $y = x^4 + 1$

c $y = x^4 - 7$

d $y = x^4 + 5$



a – A, b – D, c – B, d – C

5 Match each graph with its equation.

$y = 4x^2$

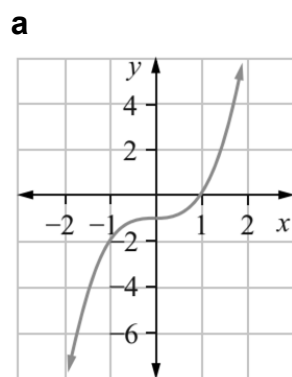
$y = -4x^2$

$y = x^3 - 1$

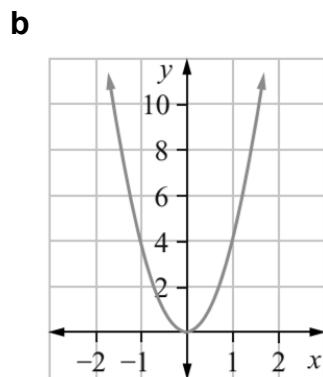
$y = -x^3 + 1$

$y = (x + 2)^6$

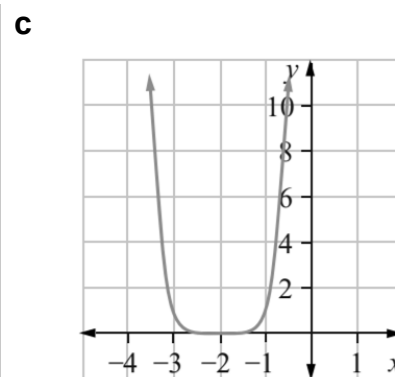
$y = -(x + 2)^6$



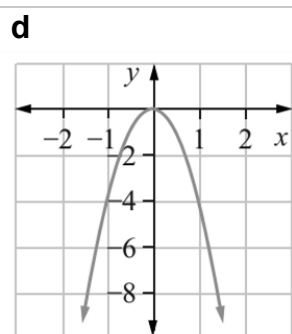
$y = x^3 - 1$



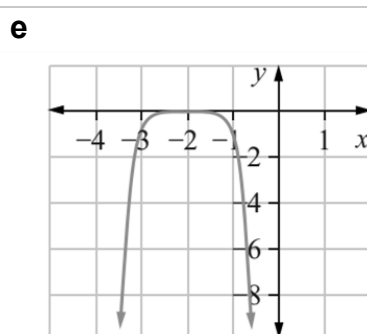
$y = 4x^2$



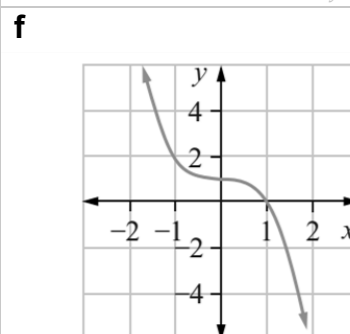
$y = (x + 2)^6$



$-4x^2$



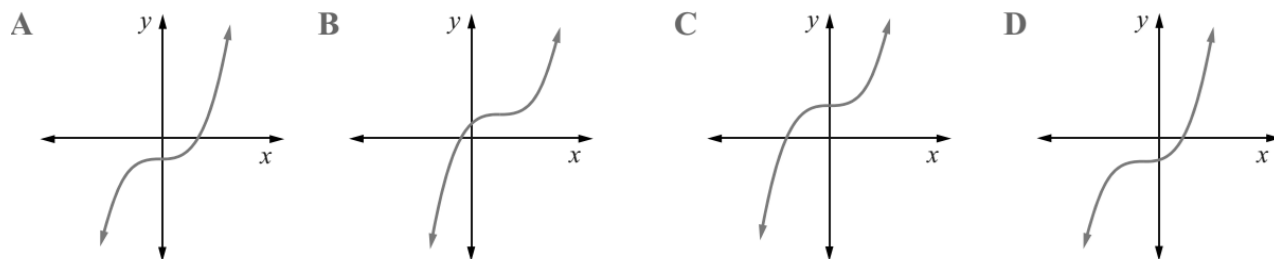
$y = -(x + 2)^6$



$y = -x^3 + 1$

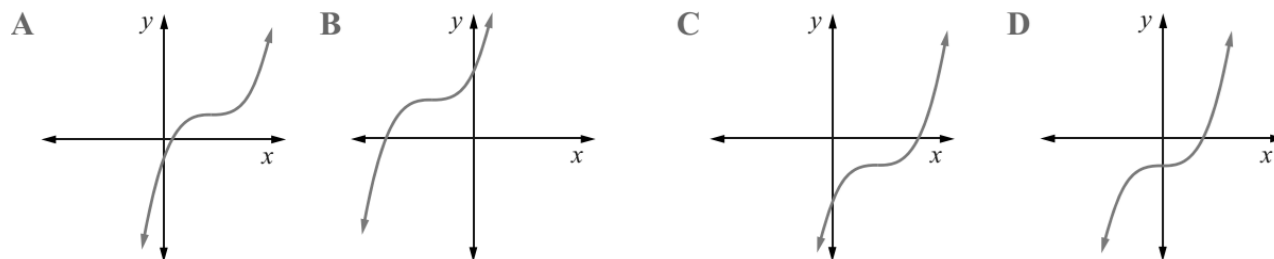
6 Which of the following could be the graph of each function?

a $y = (x + 3)^3 - 2$



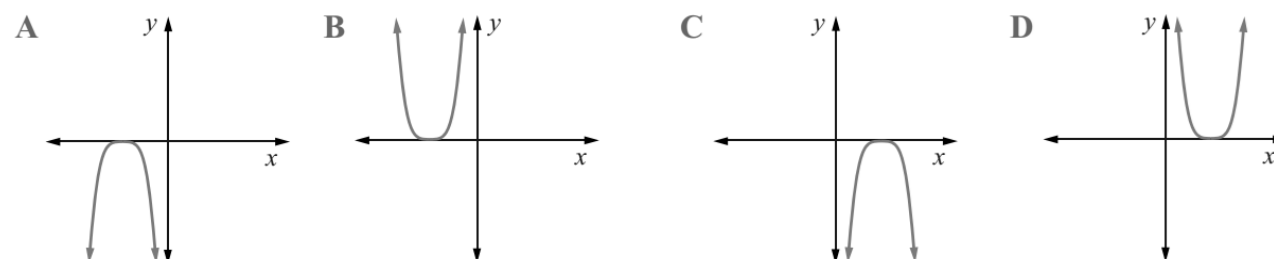
D

b $y = (x + 3)^3 + 4$



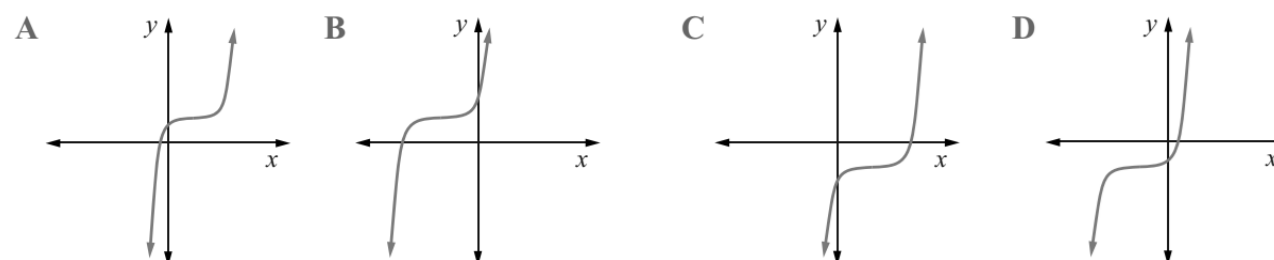
B

c $y = -(x - 3)^4$



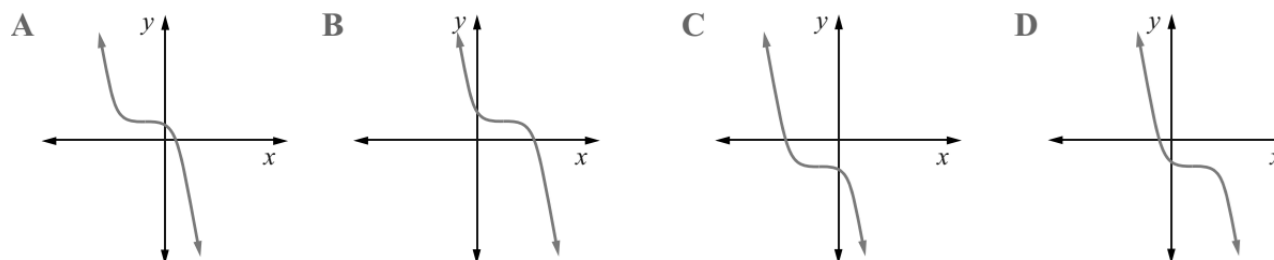
C

d $y = (x + 3)^5 - 2$



D

e $y = -(x + 2)^3 + 1$



A