

Student Name

Mathematics Stage 5 Core

LINEAR RELATIONSHIPS A

Book 1 Find the midpoint and gradient of a line segment on the Cartesian plane

Version: 260207
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<https://MrDingMaths.com>

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OVERVIEW

SYLLABUS CONTENT

MA4-LIN-C-01 determines the midpoint, gradient and length of an interval, and graphs linear relationships, with and without digital tools

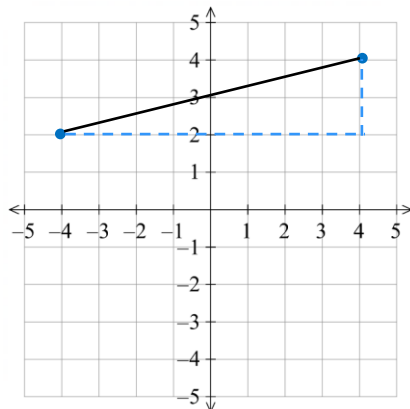
Find the midpoint and gradient of a line segment (interval) on the Cartesian plane

- Plot and join 2 points to form an interval on the Cartesian plane and use the interval as the hypotenuse of a right-angled triangle
- Apply the relationship gradient $m = \frac{\text{rise}}{\text{run}}$ to find the gradient/slope of the interval joining the 2 points
- Distinguish between intervals with positive and negative gradients from a diagram
- Explain why horizontal intervals have a gradient of 0 and vertical intervals have undefined gradients using the gradient relationship
- Determine the midpoint of horizontal and vertical intervals on the Cartesian plane
- Apply the process for calculating the mean to find the midpoint, M of the interval joining 2 points on the Cartesian plane
- Use graphing applications to find the midpoint and gradient/slope of an interval

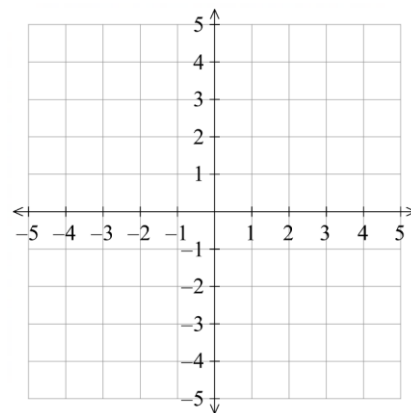
REVIEW OF PRIOR KNOWLEDGE

- 1 Plot these points on the Cartesian plane and join them to form an interval.
Show how a right-angled triangle can be made.

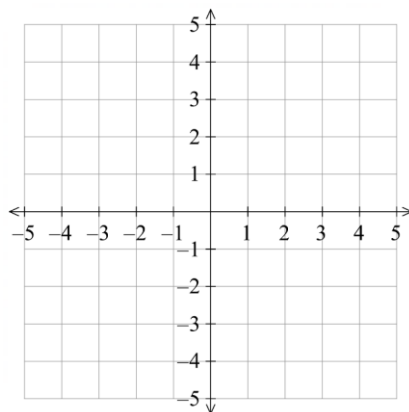
a $(-4, 2)$ and $(4, 4)$



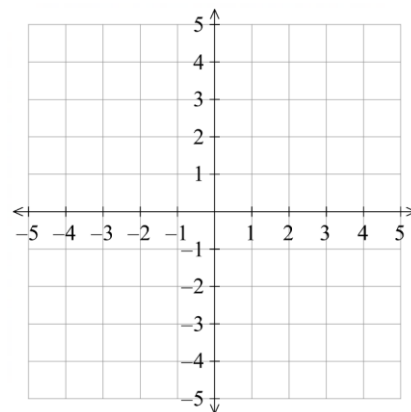
b $(0, 2)$ and $(2, 0)$



c $(2, 3)$ and $(5, -4)$.



d $(-4, 3)$ and $(1, -2)$



- 2 Find the mean of these two numbers.

a 22 and 38

$$\frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

b 26 and 6

$$\frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

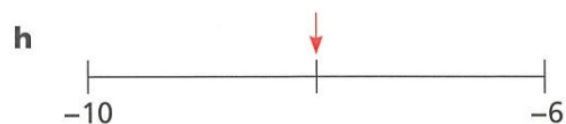
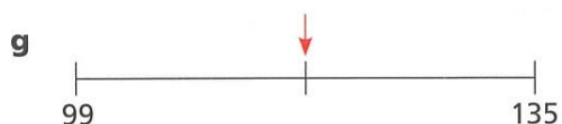
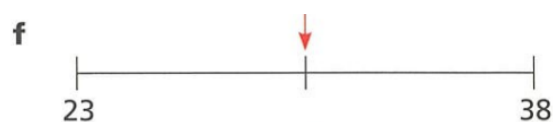
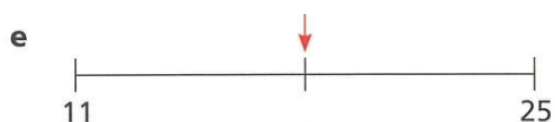
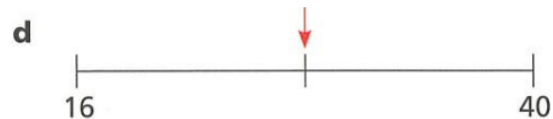
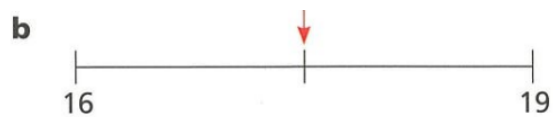
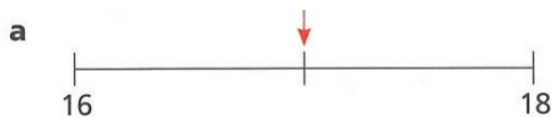
c 5 and 33

d -6 and -12

e -7 and 21

f 6 and -12

3 Find the midpoint of these numbers.



4 Find the midpoint of these two numbers using the mean.

a 3 and 5

b 4 and 6

c 4 and 7

d 3 and 7

e 3 and 9

f 3.5 and 9

g 7 and 18

h 7 and 17

i 7 and 16.5

j 7 and 15

k 0.7 and 1.5

l -7 and -15

m -7 and -16

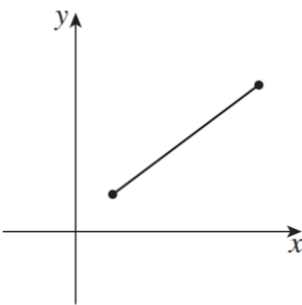
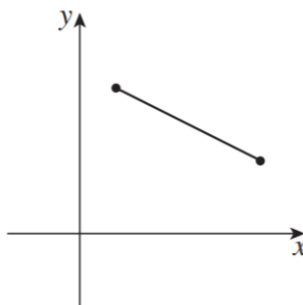
n -6 and -16

o -6 and 16

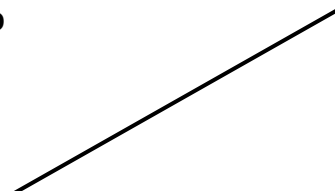
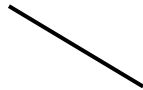
p 6 and -16

GRADIENT

- **Gradient** measures a line's **steepness**.

Positive Gradient	Negative Gradient
Line slopes up from left to right.	Line slopes down from left to right.
	

Example Identify positive and negative gradients.

a  Positive Negative	b  Positive Negative	c  Positive Negative
d  Positive Negative	e  Positive Negative	f  Positive Negative
g  Positive Negative	h  Positive Negative	i  Positive Negative
j  Positive Negative	k  Positive Negative	l  Positive Negative

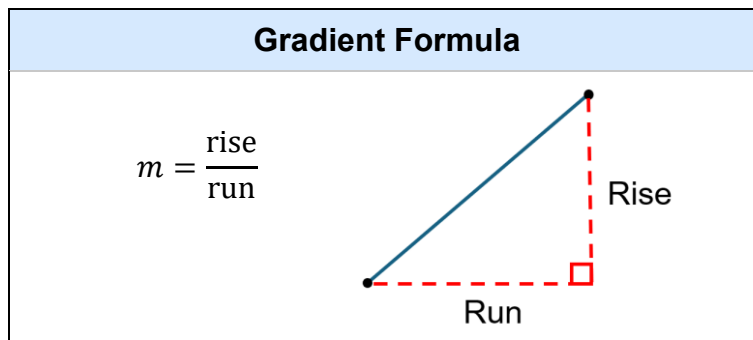
Key ideas

1. If a line goes upwards from to, the gradient is positive.
2. If a line goes downwards from left to right, the gradient is

GRADIENT FORMULA

🌐 Gradient: <https://www.geogebra.org/m/jjqsdqxr>

- Gradient, m , is the ratio of **rise** to **run**.
- Gradient shows **how far up or down for each unit across**.



- Find the **rise**: vertical change between two points, from left to right.
 - Positive if line slopes upwards from left to right.
 - Negative if line slopes downwards from left to right.
- Find the **run**: horizontal change between two points, moving from left to right.
- Calculate gradient $m = \frac{\text{rise}}{\text{run}}$.
- Simplify the fraction, if you can.

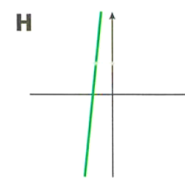
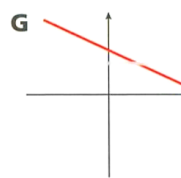
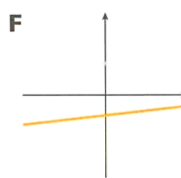
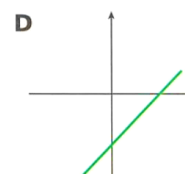
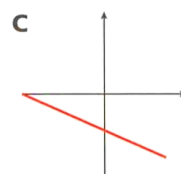
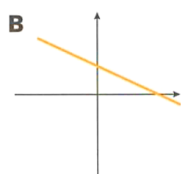
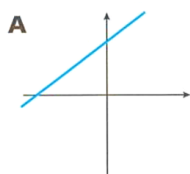
Example Calculate gradient.

a Rise: Run: $m = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square}$	b Rise: Run: $m = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square}$	c Rise: Run: $m = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square}$
d Rise: Run: $m = \frac{\text{rise}}{\text{run}}$ =	e 	f

Key ideas

- Gradient is a r..... that describes the steepness of a line by relating its and
- The gradient is a number that represents how much a line moves vertically for every 1 unit, moving from to

- 1 Consider these lines. Circle the graphs with a negative gradient.



B, C, E, G

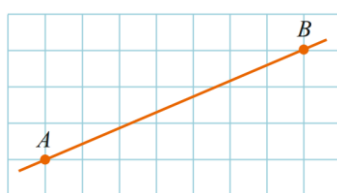
2

- a Determine the gradient of the interval AB

Rise:

Run:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\boxed{}}{\boxed{}}$$



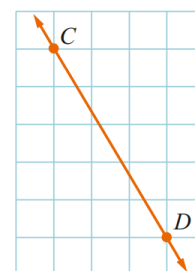
$\frac{3}{7}$

- b Determine the gradient of the interval CD

Rise:

Run:

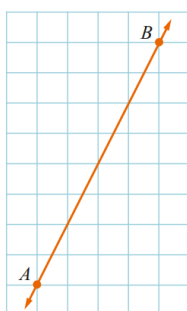
$$m = \frac{\text{rise}}{\text{run}} = \frac{\boxed{}}{\boxed{}}$$



$-\frac{5}{3}$

- 3 Determine whether the gradient is positive or negative and then find the gradient.

a

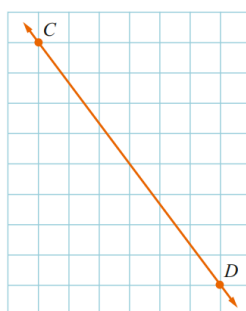


Positive | Negative

$$m = \frac{\text{rise}}{\text{run}} =$$

$\frac{8}{4} = 2$

b



Positive | Negative

$$m =$$

$-\frac{8}{6} = -\frac{4}{3}$

c

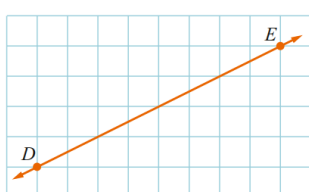


Positive | Negative

$$m =$$

$-\frac{8}{2} = -4$

d

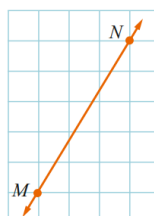


Positive | Negative

$$m =$$

$\frac{4}{8} = \frac{1}{2}$

e

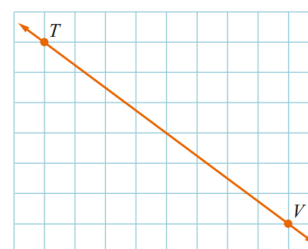


Positive | Negative

$$m =$$

$\frac{5}{3}$

f

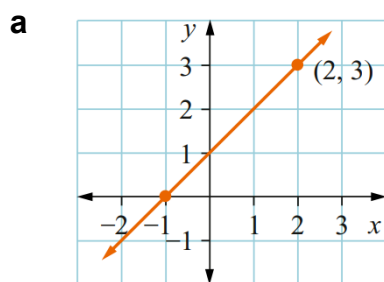


Positive | Negative

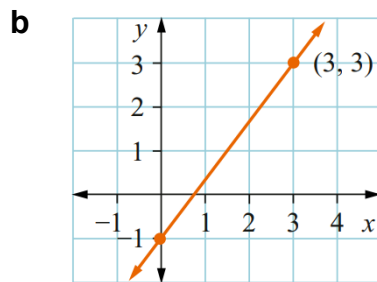
$$m =$$

$-\frac{6}{8} = -\frac{3}{4}$

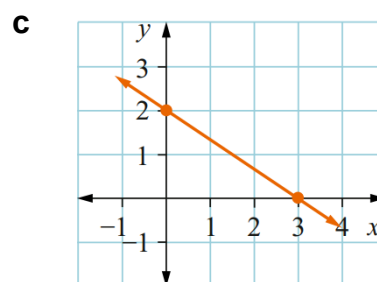
4 Find the gradient of these lines. Remember to check positive or negative.



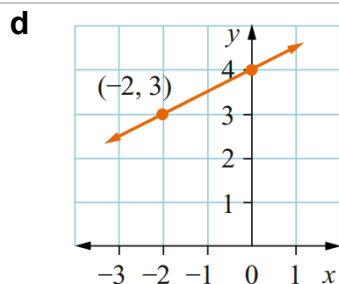
$$m = 1$$



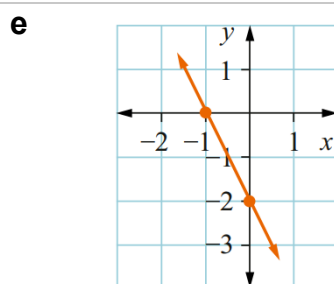
$$m = \frac{4}{3}$$



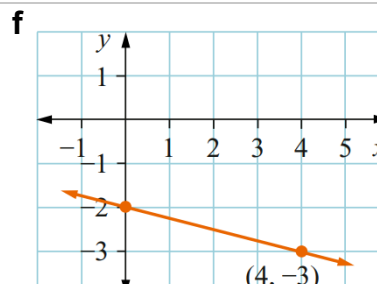
$$m = -\frac{2}{3}$$



$$m = \frac{1}{2}$$



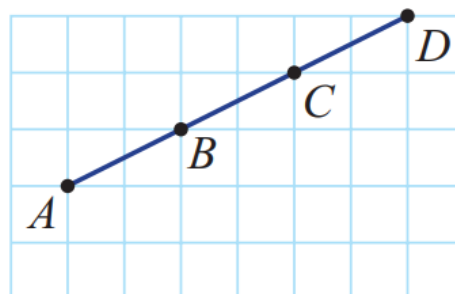
$$m = -2$$



$$m = -\frac{1}{4}$$

5 Calculate the gradient of these line segments.

Interval	Rise	Run	Gradient
AB	1	2	$\frac{1}{2}$
AC	2	4	$\frac{1}{2}$
AD	3	6	$\frac{1}{2}$
BC	1	2	$\frac{1}{2}$
BD	2	4	$\frac{1}{2}$
CD	1	2	$\frac{1}{2}$



What do you notice about the gradient between points on the same line?

They are the same

Does it matter which two points you pick on a line to calculate gradient?

No

6 Tom climbs a rocky slope that rises 12 m for each 6 metres across.
His friend Jonathan climbs a nearby grassy slope that rises 23 m for each 12 m across.
Which slope is steeper?

$$\text{Tom: } \frac{12}{6} = 2, \text{ Johnathan: } \frac{23}{12} < 2$$

Tom's slope

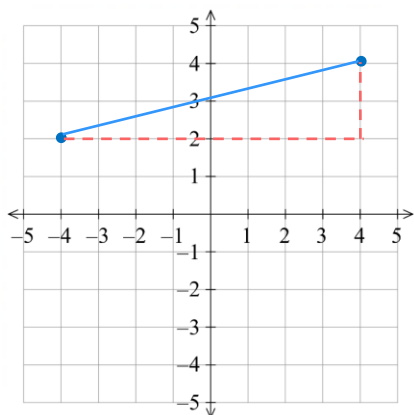
GRADIENT FROM COORDINATES

1. Rise
2. Run
3. $m = \frac{\text{Rise}}{\text{Run}}$

4. Simplify, if possible.

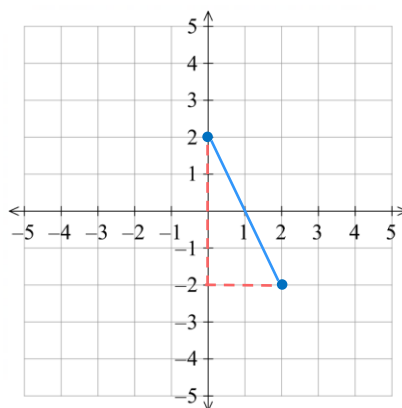
Example Calculate gradient from coordinates.

$(-4, 2)$ and $(4, 4)$



1. Rise = 2
2. Run = 8
3. $m = \frac{\text{Rise}}{\text{Run}}$
4. $= \frac{2}{8} = \frac{1}{4}$

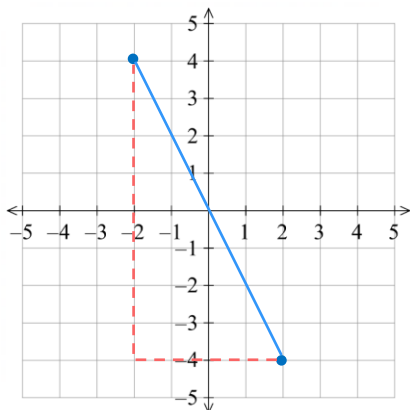
$(0, 2)$ and $(2, -2)$



1. Rise = -4
2. Run = 2
3. $m = \frac{\text{Rise}}{\text{Run}}$
4. $= \frac{-4}{2} = -2$

Calculate the gradient of these intervals.

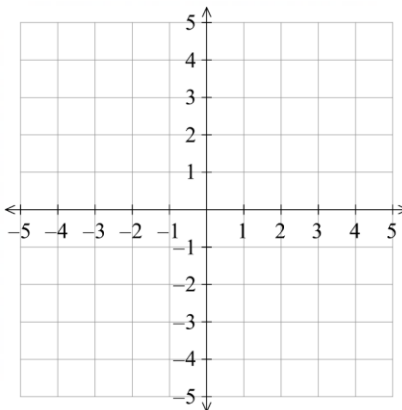
a $(-2, 4)$ and $(2, -4)$.



Rise =
Run =
 $m = \frac{\text{Rise}}{\text{Run}}$
=

-2

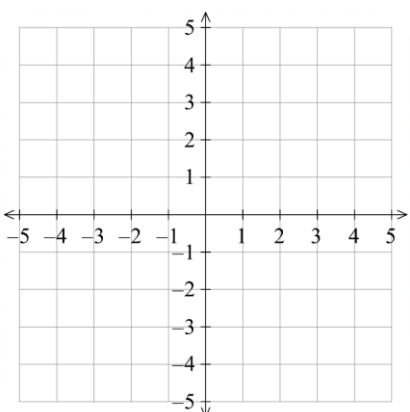
b $(0, 5)$ and $(5, 0)$



Rise =
Run =
 $m =$
=

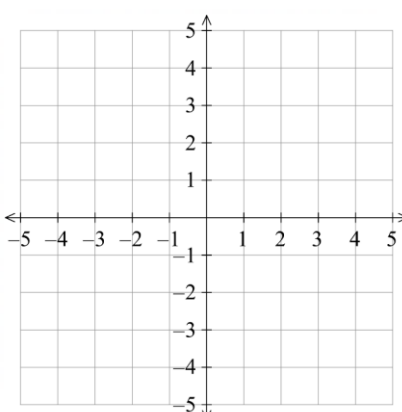
-1

c $(-3, 2)$ and $(4, 2)$



0

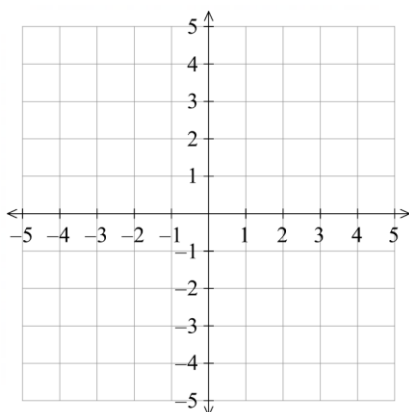
d $(-2, 0)$ and $(4, 0)$



0

1 Plot these points on the Cartesian plane and calculate the gradient of the interval.

a $A(2, 3)$ and $B(3, 5)$

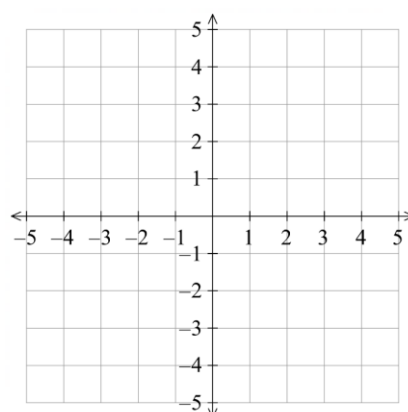


Rise:

Run:

$m = \dots\dots\dots$

b $A(0, 3)$ and $B(3, 0)$



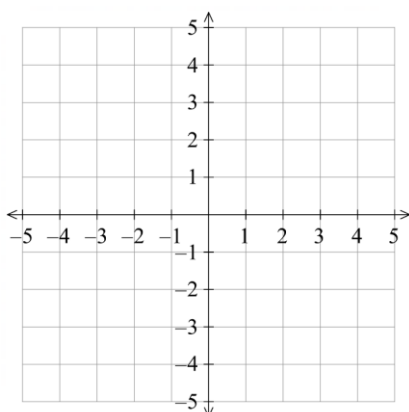
Rise:

Run:

$m = \dots\dots\dots$

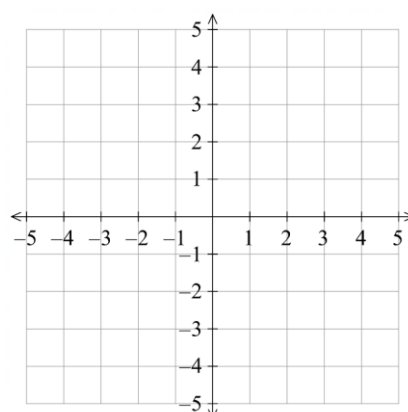
2

c $A(-1, 0)$ and $B(0, -3)$



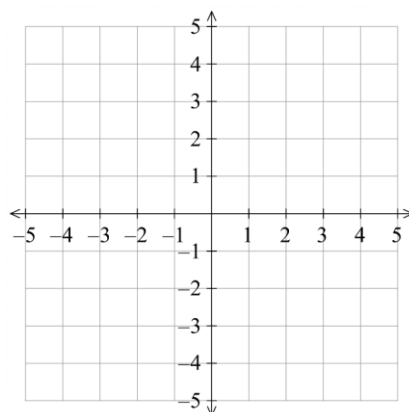
-3

d $A(-2, 5)$ and $B(0, 2)$



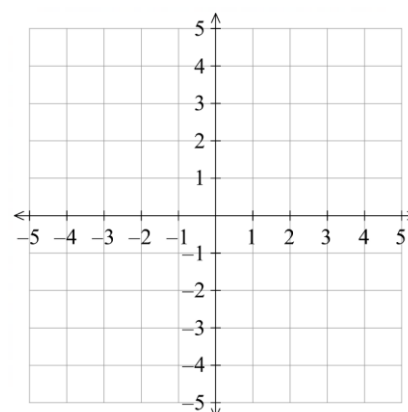
$\frac{3}{2}$

e $C(-2, 4)$ and $D(1, -2)$



-2

f $C(3, -4)$ and $D(1, 1)$



$\frac{5}{2}$

- 2 Charlotte and Naomi are arguing about a gradient of $\frac{1}{2}$



This means the line rises 1 unit for every 2 units run.



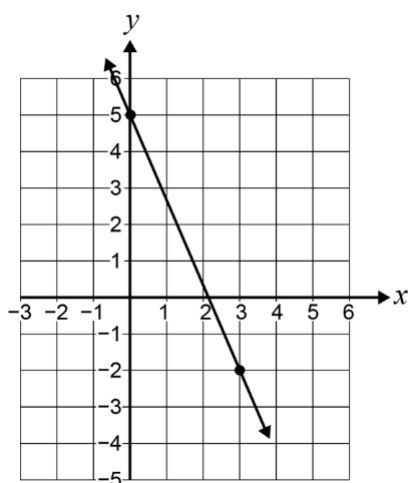
This means the line rises half a unit for every 1 unit run.

Explain why they are both correct.

Both represent the same ratio of $\frac{1}{2}$

3 HSC Standard 2 Sample Question Band 4

Leo drew a straight line through the points (0, 5) and (3, -2).
What is the gradient of the line?

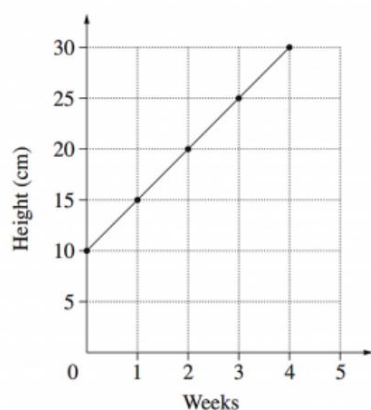


$$m = -\frac{7}{3}$$

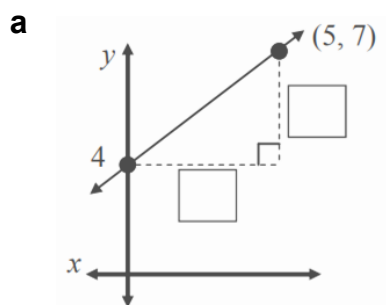
4 2004 HSC Standard 2 Band 3

What is the gradient of the line?

The axis scales are different. **Do not count the squares.**

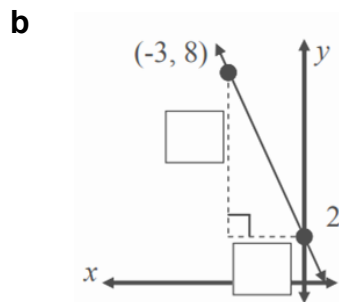


5 Calculate the gradient of these lines. There is no grid so you must use the coordinates.



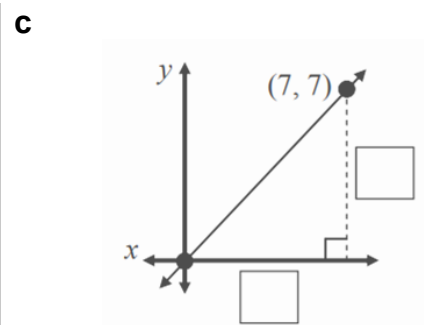
$m =$

$\frac{3}{5}$



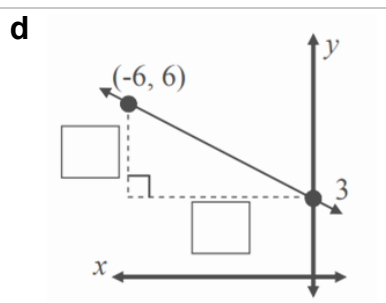
$m =$

-2



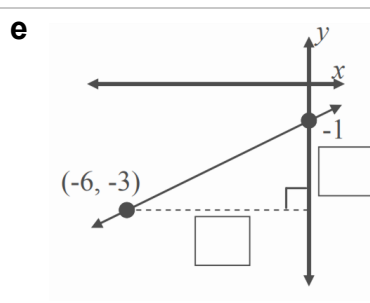
$m =$

1



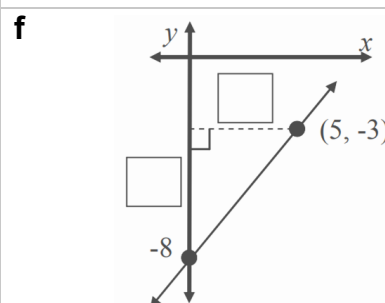
$m =$

$-\frac{1}{2}$



$m =$

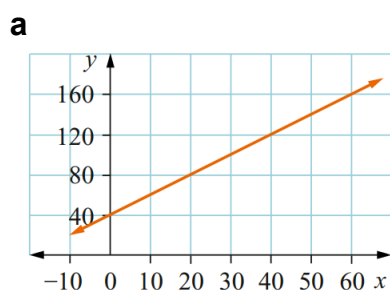
$\frac{1}{3}$



$m =$

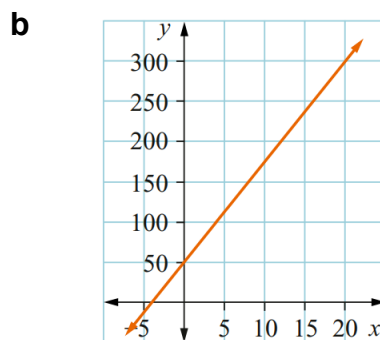
1

6 Calculate the gradient of these lines. The axis scales mean you cannot count the squares.



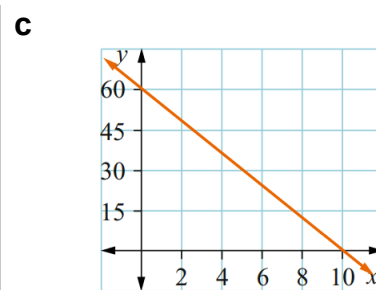
$m =$

2



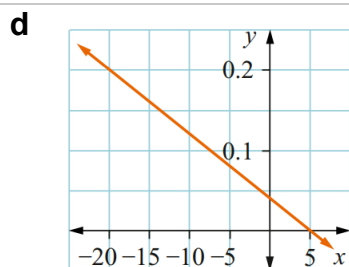
$m =$

12.5



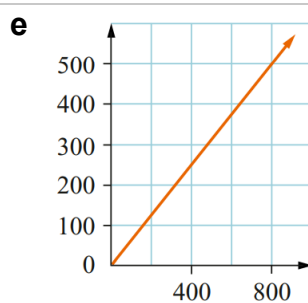
$m =$

-6



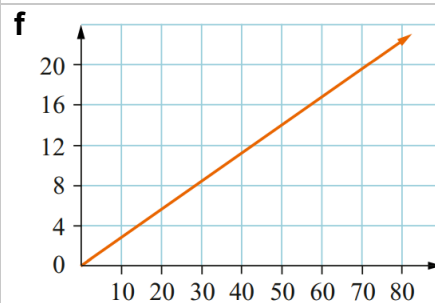
$m =$

-0.008



$m =$

$\frac{5}{8}$

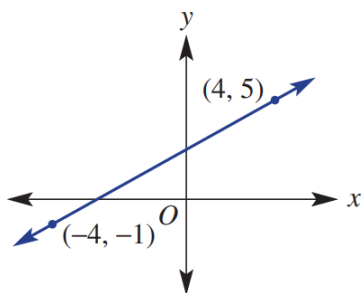


$m =$

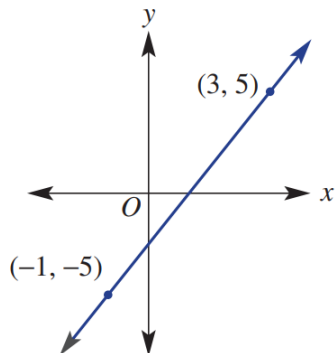
$\frac{2}{7}$

7 Find the gradient of these lines.

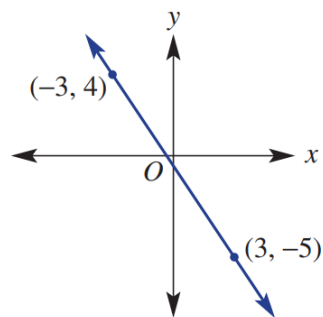
a



b



c

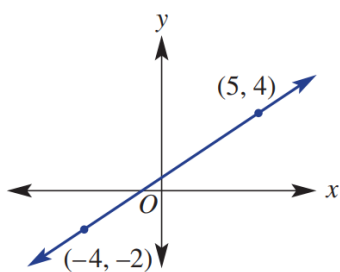


$\frac{3}{4}$

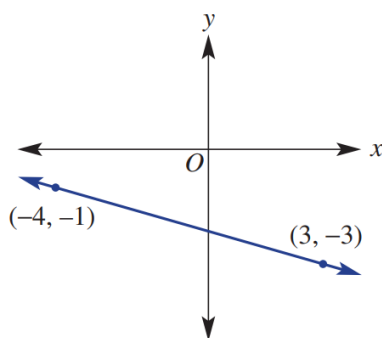
$\frac{5}{2}$

$-\frac{3}{2}$

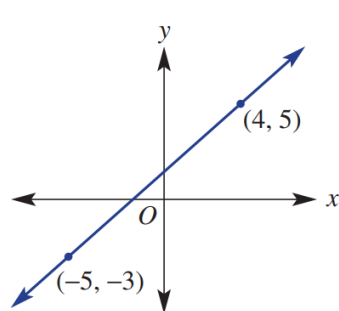
d



e



f



$\frac{2}{3}$

$-\frac{2}{7}$

$\frac{8}{9}$

g

$(-3, 1)$ and $(4, -2)$

h

$(-2, -5)$ and $(1, 7)$

i

$(5, -2)$ and $(-4, 6)$

$-\frac{3}{7}$

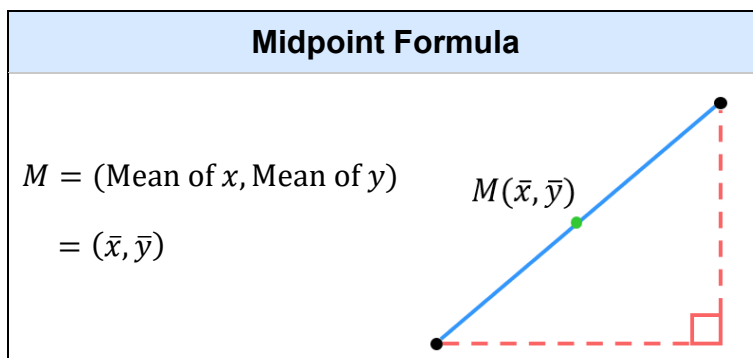
4

$-\frac{8}{9}$

MIDPOINT

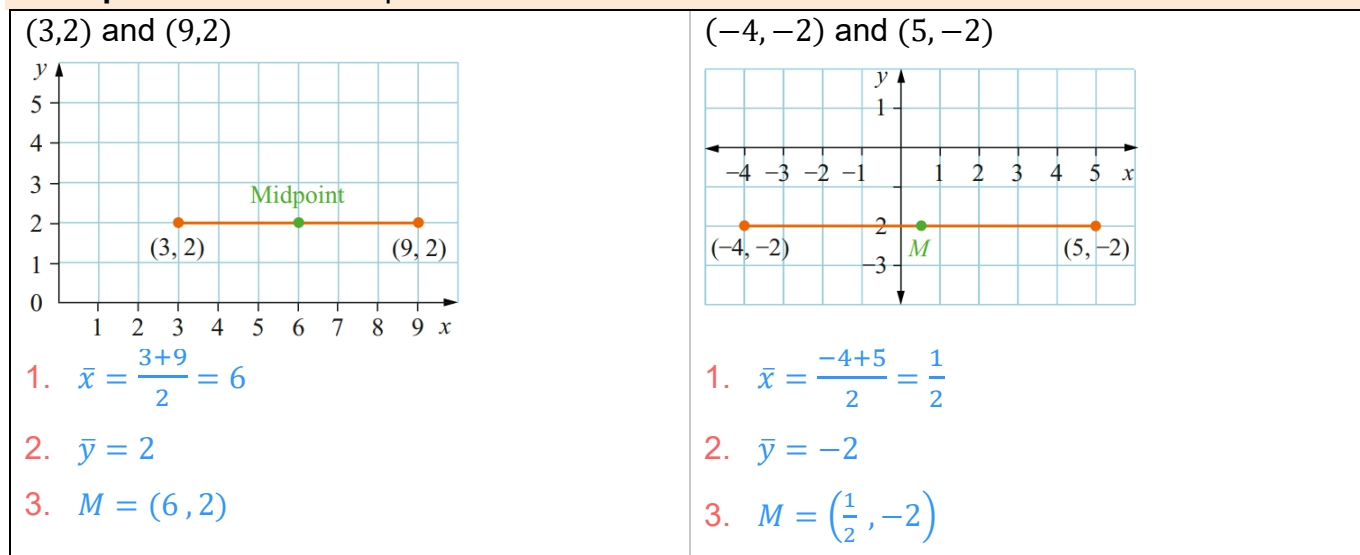
Midpoint: <https://www.geogebra.org/m/uepqnx7k>

- **Midpoint** is halfway between two endpoints.
- To find the midpoint, calculate the average, or mean, of the endpoints: $\text{mean} = \frac{a+b}{2}$



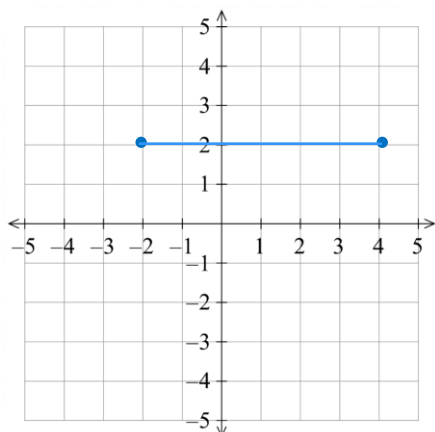
1. Find x midpoint, $\bar{x}$, the mean of the x -coordinate endpoints.
2. Find y midpoint, $\bar{y}$, the mean of the y -coordinate endpoints.
3. The midpoint is $M = (\bar{x}, \bar{y})$

Example Calculate midpoint of a horizontal interval.



Calculate midpoint of these intervals.

a (-2,2) and (4,2)



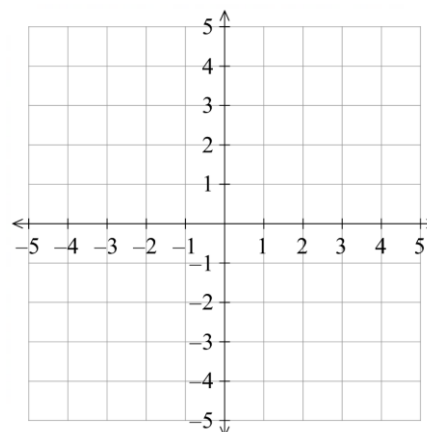
$\bar{x} = \dots\dots\dots$

$\bar{y} = \dots\dots\dots$

$M = (\quad , \quad)$

(1.2)

b (1,-4) and (4,-4)



$\bar{x} = \dots\dots\dots$

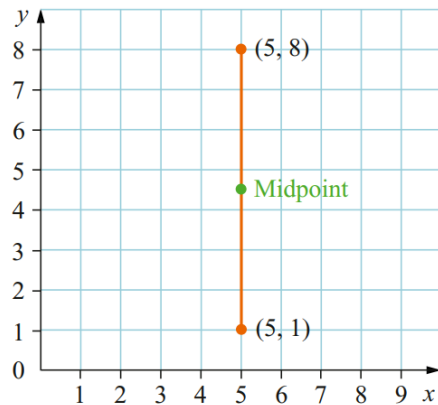
$\bar{y} = \dots\dots\dots$

$M = (\quad , \quad)$

(2.5, -4)

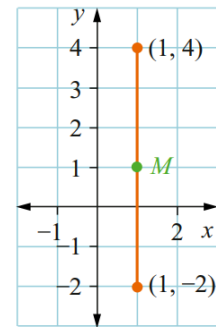
Example Calculate midpoint of a vertical interval.

(5,1) and (5,8)



1. $\bar{x} = 5$
2. $\bar{y} = \frac{1+8}{2} = 4.5$
3. $M = (5, 4.5)$

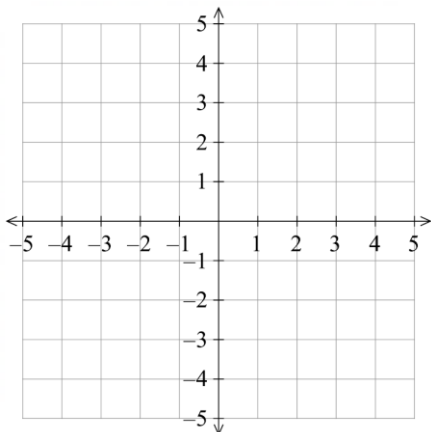
(1,-2) and (1,4)



1. $\bar{x} = 1$
2. $\bar{y} = \frac{-2+4}{2} = 1$
3. $M = (1, 1)$

Calculate midpoint of these intervals.

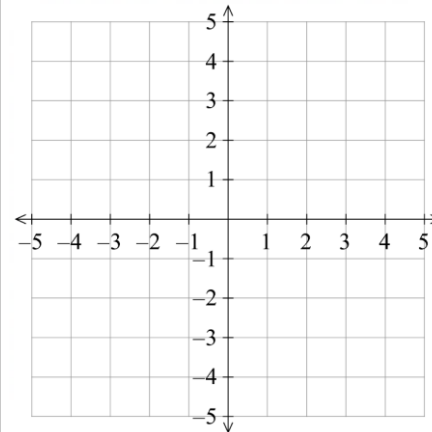
a (1,5) and (1,-1)



$\bar{x} = \dots\dots\dots$
 $\bar{y} = \dots\dots\dots$
 $M = (\quad , \quad)$

(1,2)

b (0,3) and (0,-3)

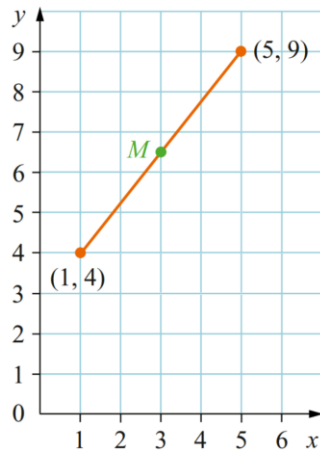


$\bar{x} = \dots\dots\dots$
 $\bar{y} = \dots\dots\dots$
 $M = (\quad , \quad)$

(0,0)

Example Calculate midpoint of an interval.

(1,4) and (5,9)

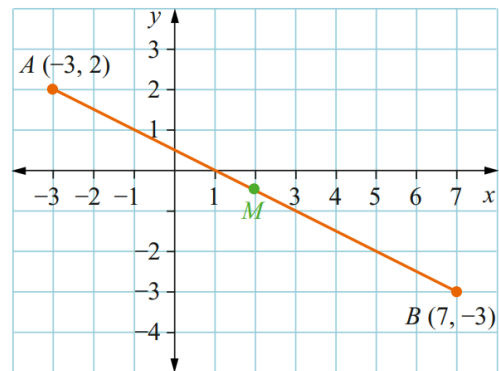


$$1. \quad \bar{x} = \frac{1+5}{2} = 3$$

$$2. \quad \bar{y} = \frac{4+9}{2} = 6.5$$

$$3. \quad M = (3, 6.5)$$

(-3,2) and (7,-3)



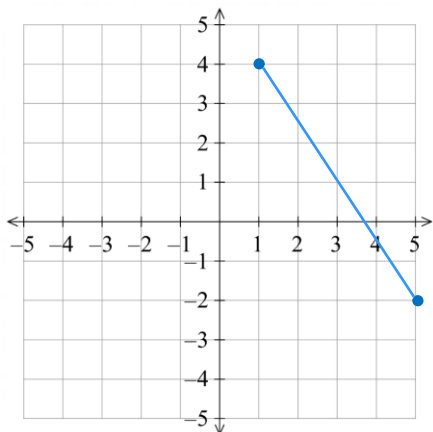
$$1. \quad \bar{x} = \frac{-3+7}{2} = 2$$

$$2. \quad \bar{y} = \frac{2+(-3)}{2} = -1.5$$

$$3. \quad M = (2, -1.5)$$

Calculate midpoint of these intervals.

a (5,-2) and (1,4)



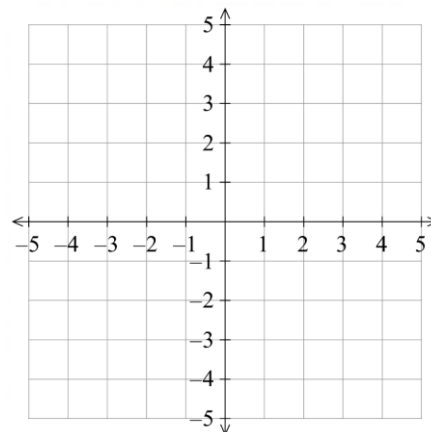
$$\bar{x} = \dots\dots\dots$$

$$\bar{y} = \dots\dots\dots$$

$$M = (\quad , \quad)$$

(3,1)

b (-2,-5) and (3,-1)



$$\bar{x} = \dots\dots\dots$$

$$\bar{y} = \dots\dots\dots$$

$$M = (\quad , \quad)$$

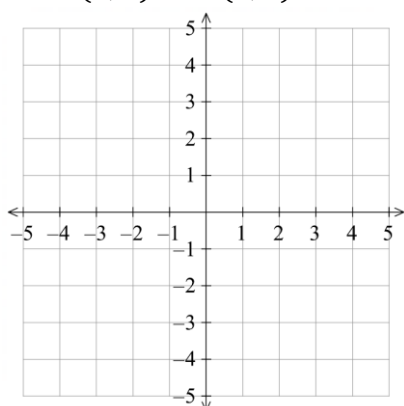
(0.5, -3)

Key ideas

1. Calculate the midpoint of a line interval by finding the of the x and y coordinates.

1 Sketch the line interval and calculate the midpoint.

a (1, 2) and (1, 4)

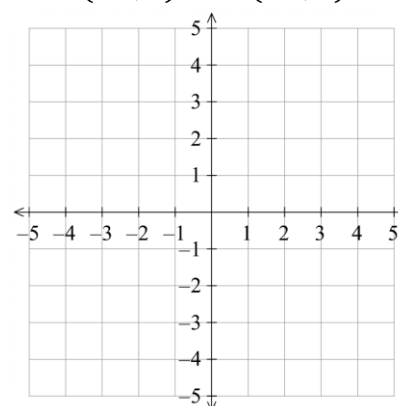


$$\bar{x} =$$

$$\bar{y} =$$

$$M = (\quad , \quad)$$

b (-2, 1) and (-2, 5)

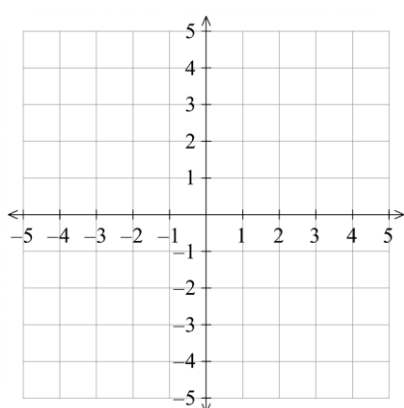


$$\bar{x} =$$

$$\bar{y} =$$

$$M = (\quad , \quad)$$

c (0, 0) and (3, 0)

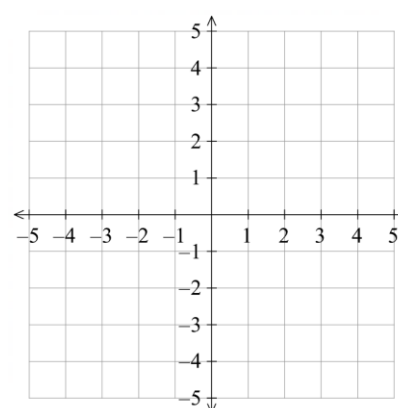


$$\bar{x} =$$

$$\bar{y} =$$

$$M = (\quad , \quad)$$

d (5, 1) and (3, 1)



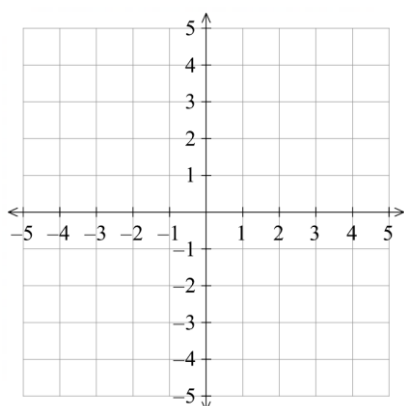
$$\bar{x} =$$

$$\bar{y} =$$

$$M = (\quad , \quad)$$

2

a (5, 2) and (1, 4)

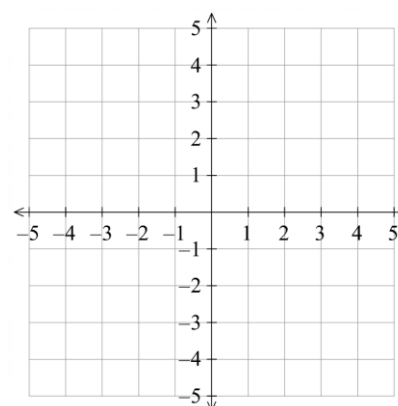


$$\bar{x} =$$

$$\bar{y} =$$

$$M = (\quad , \quad)$$

b (2, 0) and (0, 8)

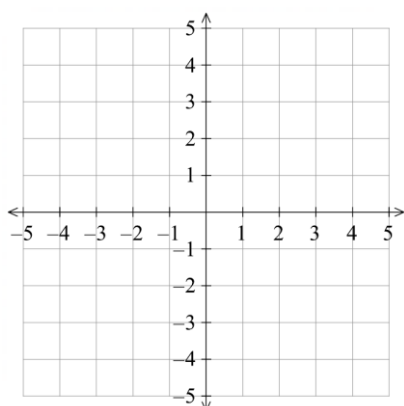


$$\bar{x} =$$

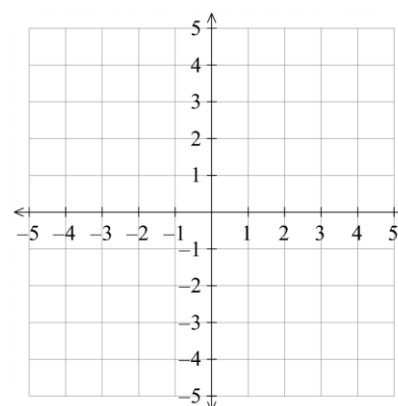
$$\bar{y} =$$

$$M = (\quad , \quad)$$

c (2, -1) and (-1, 3)

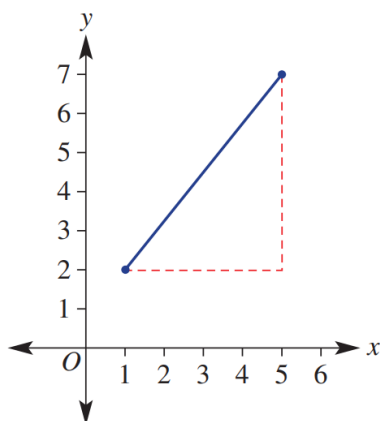


d (-4, -4) and (-1, 1)

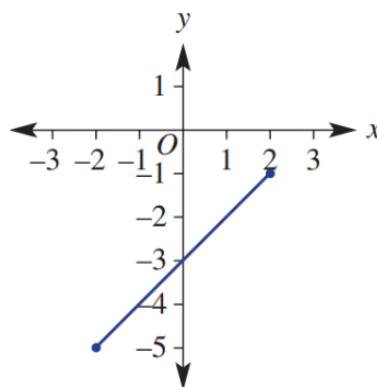


3 Calculate the midpoint of the line segment.

a

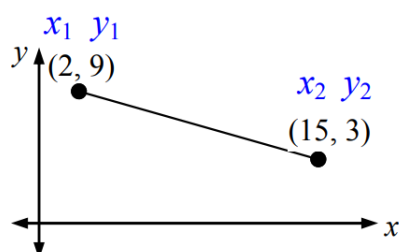


b

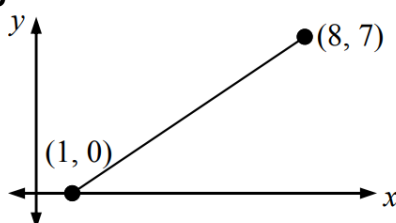


4 Find the midpoint of these line segments.

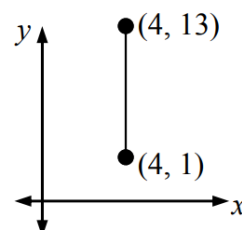
a



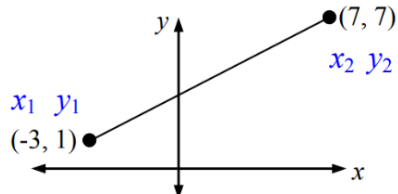
b



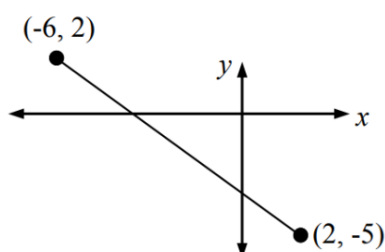
c



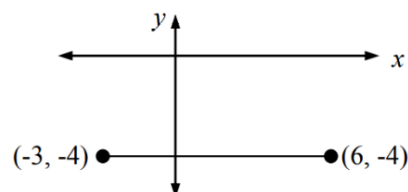
d



e



f



g

$(-3, 1)$ and $(4, -2)$

h

$(-2, -5)$ and $(1, 7)$

i

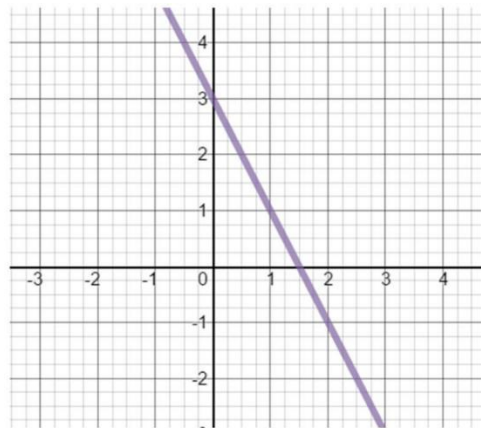
$(5, -2)$ and $(-4, 6)$

CHECK YOUR UNDERSTANDING

Gradient

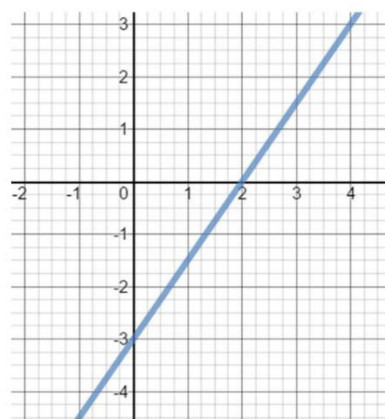
1 What is the gradient of this line?

- a -2
- b -1
- c -1.5
- d -0.5



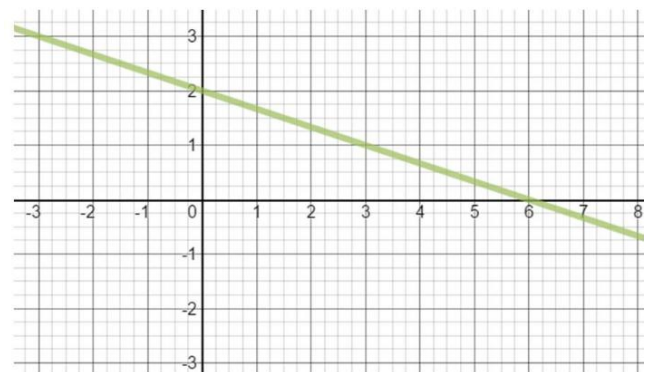
2 What is the gradient of this line?

- a 2
- b 1
- c 1.5
- d $\frac{2}{3}$



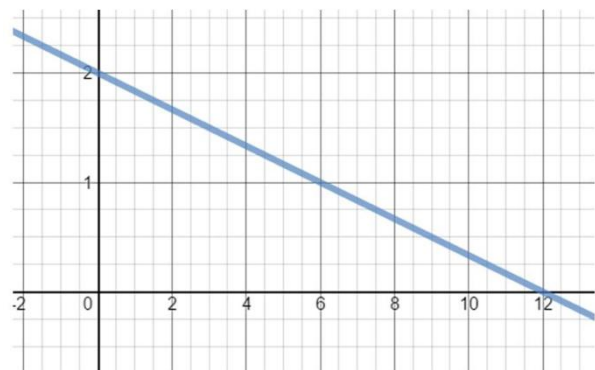
3 What is the gradient of this line?

- a -2
- b -3
- c -1.5
- d $-\frac{1}{3}$



4 What is the gradient of this line?

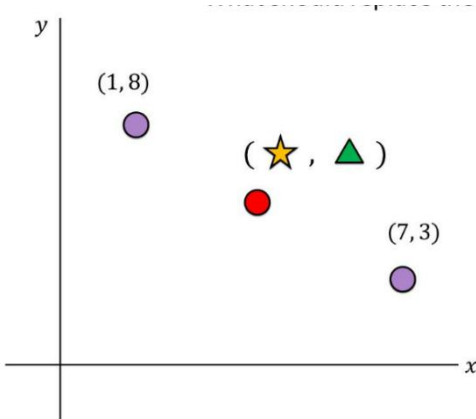
- a $-\frac{1}{3}$
- b -3
- c -6
- d $-\frac{1}{6}$



Midpoint

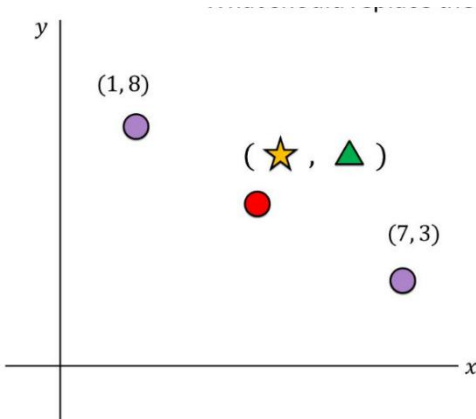
- 1 Millie is finding the midpoint.
What should replace the star?

- a 8
- b 6
- c 5
- d 5



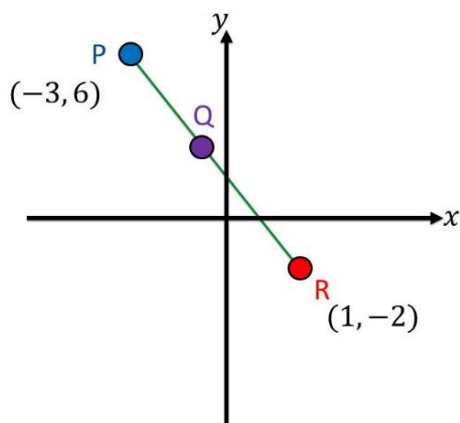
- 2 Millie is finding the midpoint.
What should replace the triangle?

- a 6
- b 5
- c 5.5
- d 11



- 3 Q is the midpoint of PR.
What are its coordinates?

- a $(-1, 3)$
- b $(-2, 4)$
- c $(-1, 2)$
- d $(-2, 2)$



- 5 What is the midpoint of $(4, 3)$ and $(1, -5)$?

- a $(2.5, -1)$
- b $(5, -2)$
- c $(2.5, -4)$
- d Impossible to tell