

Student Name

Mathematics Stage 5 Core

LINEAR RELATIONSHIPS B

Book 2 Find the equations of parallel and
perpendicular lines

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-LIN-C-02 graphs and interprets linear relationships using the gradient/slope-intercept form

Find the equations of parallel and perpendicular lines

- Explain that 2 straight lines are perpendicular if the product of their gradients is -1
- Find the equation of a straight line that is parallel or perpendicular to another given line by applying $y = mx + c$

REVIEW OF PRIOR KNOWLEDGE

1 Write the missing number in each of these calculations.

a $\frac{1}{2} \times \square = -1$

b $\frac{1}{3} \times \square = -1$

c $\square \times \frac{1}{9} = -1$

d $-11 \times \square = -1$

e $-100 \times \square = -1$

f $\square \times \frac{1}{5} = -1$

g $\frac{2}{5} \times \square = -1$

h $\square \times -\frac{3}{4} = -1$

2 Write the negative reciprocal of each number.

a 3

b 7

c $-\frac{1}{4}$

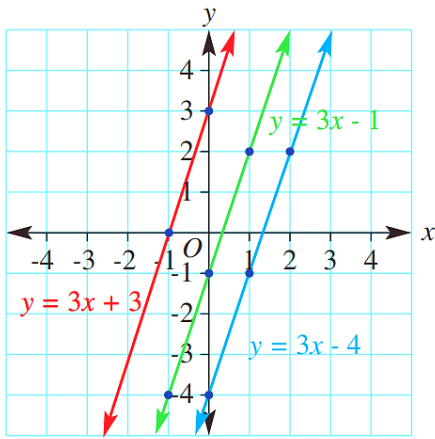
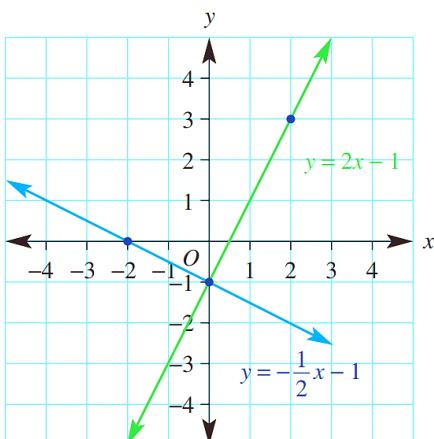
d $-\frac{1}{10}$

e $\frac{3}{2}$

f $-\frac{7}{8}$

PARALLEL AND PERPENDICULAR LINES

- Consider two lines $y = m_1x + c_1$ and $y = m_2x + c_2$
- Parallel** lines have the **same gradient**.
- Perpendicular** lines have gradients that **multiply to -1** .

Parallel Lines	Perpendicular Lines
$m_1 = m_2$ 	$m_1 \times m_2 = -1$ $m_1 = -\frac{1}{m_2}$ 

Example Decide if two lines are parallel, perpendicular, or neither.

$y = 3x - 1$ $m_1 = 3$ $y = 3x + 4$ $m_2 = 3$ The two lines are parallel $m_1 = m_2$	$y = 4x - 2$ $m_1 = 4$ $y = -\frac{1}{4}x + 3$ $m_2 = -\frac{1}{4}$ The two lines are perpendicular $m_1 \times m_2 = -1$	$y = 2x + 1$ $m_1 = 2$ $y = -2x - 5$ $m_2 = -2$ The two lines are neither parallel nor perpendicular.
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Decide whether these lines are parallel, perpendicular, or neither.

a $y = -\frac{1}{5}x + 1$
 $y = 5x + 2$

perpendicular

b $y = 2x - 1$
 $y = 2x - 3$

parallel

c $y = 7x - 2$
 $y = -7x + 2$

neither

Key ideas

- Parallel lines have the same, i.e., $m_1 = m_2$
- Perpendicular lines have gradients that are the n..... r..... of each other, i.e. $m_2 = -\frac{1}{m_1}$

1 Find the gradient the lines that are parallel to each line.

a $y = 4x + 6$

$m_{\text{par}} = 4$

b $y = -7x - 1$

c $y = -\frac{3}{4}x + 2$

d $y = \frac{8x}{7} - \frac{1}{2}$

2 Find the gradient the lines that are perpendicular to each line.

a $y = 3x - 1$

$m_{\text{perp}} = -\frac{1}{3}$

b $y = -2x + 6$

c $y = \frac{7}{8}x - \frac{2}{3}$

d $y = \frac{-4x}{9} - \frac{4}{7}$

3 Decide if these gradients would make the lines parallel, perpendicular, or neither.

	Gradient Line 1	Gradient Line 2	Parallel / Perpendicular / Neither?		Gradient Line 1	Gradient Line 2	Parallel / Perpendicular / Neither?
a	5	6		k	$-\frac{5}{8}$	-5.8	
b	5	5		l	$-\frac{5}{8}$	5.8	
c	5	-5		m	$-\frac{5}{8}$	$\frac{5}{8}$	
d	5	$-\frac{1}{5}$		n	$-\frac{5}{8}$	$-\frac{5}{8}$	
e	5	$\frac{1}{5}$		o	$-\frac{5}{8}$	$\frac{8}{5}$	
f	0.5	$\frac{1}{5}$		p	$-\frac{5}{8}$	$\frac{8}{5}$	
g	0.2	$\frac{1}{5}$		q	$-\frac{5}{8}$	1.6	
h	-0.2	$\frac{1}{5}$		r	$-\frac{5}{8}$	-0.625	
i	-5	$\frac{1}{5}$		s	1	-1	
j	5	$\frac{1}{5}$		t	1	0	

4 Decide if these pairs of lines are parallel, perpendicular, or neither.

a $y = 3x - 1$
 $y = 3x + 7$

b $y = \frac{1}{2}x - 6$
 $y = \frac{x}{2} - 4$

c $y = -\frac{2}{3}x + 1$
 $y = \frac{2}{3}x - 3$

d $y = -8x + 4$
 $y = \frac{1}{8}x - 2$

e $x - y = 4$
 $y = -x + \frac{1}{2}$

f $3x - y = 2$
 $x + 3y = 5$

5 Show that $y = 5x - 7$ is perpendicular to $5y = 10 - x$

.....
.....

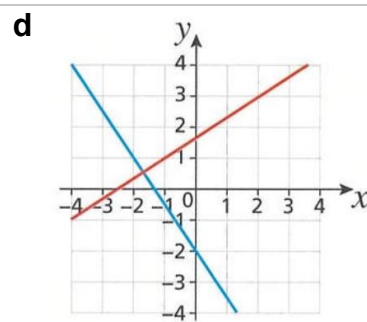
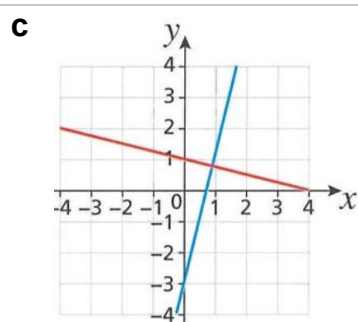
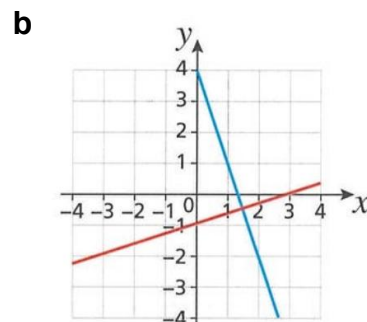
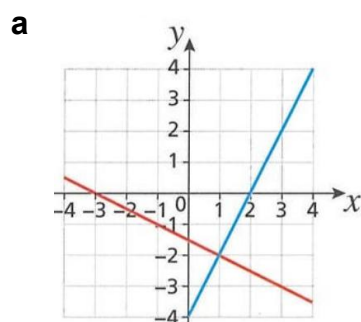
6 Beca says that $y = 8x + \sqrt{25}$ and $y = 5 + 8x$ are parallel. Marta says they are not. Who do you agree with? Explain your answer.

.....
.....

7 Write an equation of a line that is parallel to $y = \frac{5x+9}{4}$.

.....
.....

8 By finding the product of the gradients, show that each pair of lines are perpendicular.



EQUATION OF PARALLEL AND PERPENDICULAR LINES

FIND THE EQUATION OF A LINE THAT IS PARALLEL

Gradient of Parallel Line

$$m_{\text{par}} = m_{\text{original}}$$

1. m_{orig} Identify gradient of original line.
2. m_{par} Use this same gradient for the parallel line.
3. c_{par} Identify the y -intercept of the parallel line (given in the question).
4. Write the equation $y = m_{\text{par}}x + c_{\text{par}}$

Example Find the equation of a line that is parallel to another.

Find the equation of a line that is parallel to $y = 3x - 1$ and passes through (0,4)	What does m_{par} and c_{par} represent?
1. $m_{\text{orig}} = 3$	<i>The and of the parallel line.</i>
2. $m_{\text{par}} = 3$	How did we know $m_{\text{par}} = 3$?
3. $c_{\text{par}} = 4$	<i>The gradient of a parallel line is to the gradient of the original.</i>
4. $y = 3x + 4$	How did we know $c_{\text{par}} = 4$?
	<i>It was given by the</i>
	Do you need to consider the y -intercept of the original graph?

Find the equation of the line that is parallel to:

a $y = 2x + 3$ passing through (0,1)

1. $m_{\text{orig}} = \dots\dots\dots$
2. $m_{\text{par}} = \dots\dots\dots$
3. $c_{\text{par}} = \dots\dots\dots$
4. $y = \dots x + \dots$

$$y = 2x + 1$$

b $y = -x + 2$ passing through (0,5)

1.
2.
3.
4.

$$y = -x + 5$$

c $y = \frac{2x}{3} - 4$ passing through (0, -5)

$$y = \frac{2x}{3} - 5$$

d $y = -\frac{5}{3}x + 7$ passing through (0, -1.5)

$$y = -\frac{5}{3}x - 1.5$$

FIND THE EQUATION OF A LINE THAT IS PERPENDICULAR

Perpendicular Lines: <https://www.geogebra.org/m/vnr2kmmq>

Gradient of Perpendicular Line

$$m_{\text{perp}} = -\frac{1}{m_{\text{original}}}$$

1. m_{orig} Identify gradient of original line.
2. m_{perp} Calculate the perpendicular gradient. It is the negative reciprocal of the original.
3. c_{perp} Identify the y-intercept of the parallel line (given in the question).
4. Write the equation $y = m_{\text{perp}}x + c_{\text{perp}}$

Example Find the equation of a line that is perpendicular to another.

Find the equation of a line that is perpendicular to $y = 2x - 3$ and passes through $(0, -1)$	How did we know $m_{\text{perp}} = -\frac{1}{2}$?
1. $m_{\text{orig}} = 2$	The of 2 is $-\frac{1}{2}$
2. $m_{\text{perp}} = -\frac{1}{2}$	How did we know $c_{\text{perp}} = -1$?
3. $c_{\text{perp}} = -1$	It was given by the
4. $y = -\frac{1}{2}x - 1$	Do you need to consider the y-intercept of the original graph?

Find the equation of the line that is perpendicular to:

a $y = 3x - 2$ passing through $(0, 3)$

1. $m_{\text{orig}} = \dots\dots\dots$
2. $m_{\text{perp}} = \dots\dots\dots$
3. $c_{\text{perp}} = \dots\dots\dots$
4. $y = \dots x + \dots$

$$y = -\frac{1}{3}x + 3$$

b $y = -2x + 3$ passing through $(0, -4)$

$$y = \frac{1}{2}x - 4$$

c $y = \frac{3x}{2} + 9$ passing through $(0, 11)$

$$y = -\frac{2}{3}x + 11$$

d $y = x + 1$ passing through $(0, 2.8)$

$$y = -x + 2.8$$

1 Write down the equation of the line that:

a passes through (0, 2) and is parallel to another line with gradient 4.

$$m_{\text{par}} = 4, c_{\text{par}} = 2 \quad y = \dots x + \dots$$

b passes through (0, 4) and is parallel to another line with gradient 2.

.....

c passes through (0, -3) and is parallel to another line with gradient -1.

.....

d passes through (0, 3) and is perpendicular to another line with gradient 2.

.....

e passes through (0, -5) and is perpendicular to another line with gradient 3.

.....

f passes through (0, -10) and is perpendicular to another line with gradient $\frac{1}{2}$.

.....

g passes through (0, 6) and is perpendicular to another line with gradient $\frac{1}{6}$.

.....

h passes through (0, -7) and is perpendicular to another line with gradient $-\frac{1}{4}$.

.....

2 A straight line is parallel to $y = 10x + 14$ and intersects the y-axis at (0, -21).
Write the equation of the line.

.....
.....

3 Find the equation of the line that is perpendicular to $y = 8x + 9$ and passes through (0, -1)

.....
.....

4 Find the equation of the line that is perpendicular to $y = 5 - \frac{1}{4}x$ and passes through (0, 28)

.....
.....

- 5** Find the equation of the line parallel and perpendicular to each of the following that passes through the given point.

a $y = 3x - 2$ (0, 3)

Parallel:

Perpendicular:

b $y = 5x - 4$ (0, 7)

Parallel:

Perpendicular:

c $y = -2x + 3$ (0, -4)

Parallel:

Perpendicular:

d $y = -7x + 2$ $(0, -\frac{1}{2})$

Parallel:

Perpendicular:

e $y = \frac{3x}{2} - 5$ (0, -5)

Parallel:

Perpendicular:

f $y = 10 - \frac{7}{8}x$ (0, 2)

Parallel:

Perpendicular:

g $8 - x = y$ (0, 5)

Parallel:

Perpendicular:

h $y = \frac{7x}{2} + 2$ (0, -1.5)

Parallel:

Perpendicular:

i $y = -\frac{x}{19}$ (0, 5)

Parallel:

Perpendicular:

- 6** Write the equation of the line:

a parallel to $y = 4$ passing through (0,1)

b parallel to $y = 4$ through (-3, -2)

c parallel to $x = -2$ through (3,0)

d parallel to $x = -2$ through (-3, -3)

e perpendicular to $y = -2$ through (2,0)

f perpendicular to $y = -2$ through (0,0)

g perpendicular to $x = 6$ through (0,7)

h perpendicular to $x = 6$ through $(0, -\frac{1}{2})$

i parallel to y -axis through (-1,4)

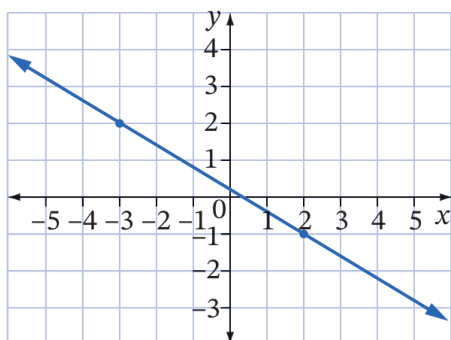
j parallel to x -axis through (0, -2)

k perpendicular to y -axis through (5, -3)

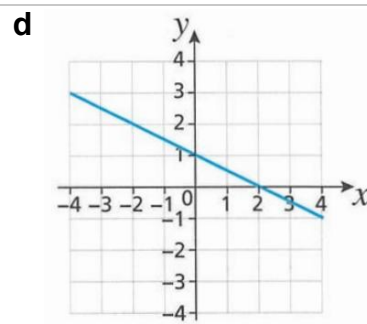
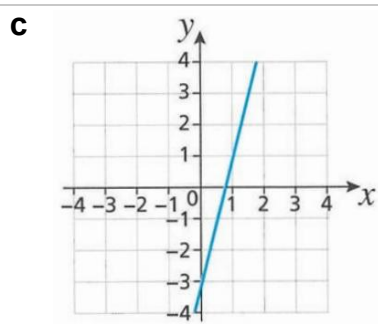
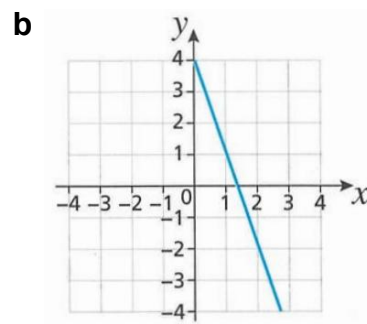
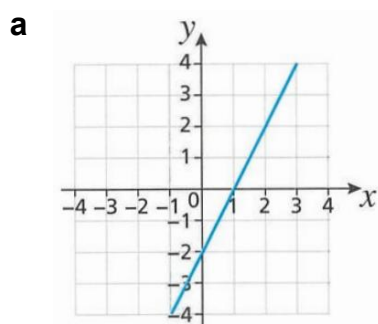
l perpendicular to x -axis through (3, 21)

- 7 Write an equation of a line.
- a perpendicular to $y = 0.75x + 5$
 - b perpendicular to $y = 0.75x + 5$ that passes through $(0,18)$
 - c Explain why there are infinitely many answers to part a, but only one answer to part b.
.....

- 8 What is the gradient of a line perpendicular to the line shown?



- 9 Find an equation for a line that is perpendicular to each of these lines, passing through the same y -intercept.



CHECK YOUR UNDERSTANDING

Parallel and perpendicular lines

1	A line has gradient of 3. A line parallel to this line would have a gradient of:						
a	3	b	-3	c	$-\frac{1}{3}$	d	Not enough information
2	A line has gradient of 3. A line perpendicular to this line would have a gradient of:						
a	3	b	-3	c	$-\frac{1}{3}$	d	Not enough information
3	A line has gradient of $-\frac{3}{4}$. A line perpendicular to this line would have a gradient of:						
a	$\frac{4}{3}$	b	$-\frac{4}{3}$	c	$\frac{3}{4}$	d	$-\frac{3}{4}$
4	A line has gradient of $-\frac{3}{4}$. A line parallel to this line would have a gradient of:						
a	$\frac{4}{3}$	b	$-\frac{4}{3}$	c	$\frac{3}{4}$	d	$-\frac{3}{4}$
5	Tom says $y = 2x + 5$ and $y = 2x - 7$ are parallel. Katie says $y = 2x + 5$ and $y = 3x + 5$ are parallel. Who is correct?						
a	Only Tom	b	Only Katie	c	Both	d	Neither
6	Tom says $y = 3 - x$ and $y = 3 - 2x$ are parallel. Katie says $y = 3 - x$ and $y = 27 - x$ are parallel. Who is correct?						
a	Only Tom	b	Only Katie	c	Both	d	Neither