Student Name

Mathematics Stage 5 Core

NUMBERS OF ANY MAGNITUDE

Book 2 Find absolute and percentage error

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-MAG-C-01 solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures

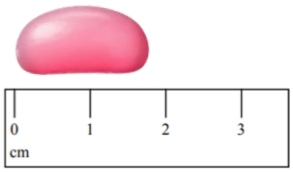
Find absolute and percentage error

- Determine the precision of a measuring instrument by finding the smallest division on the instrument
- Find the absolute error of measuring instruments ($absolute\ error = \frac{1}{2} \times precision$)
- Calculate the percentage error of a given measurement by applying the formula: $percentage\ error = \frac{absolute\ error}{measurement} \times 100\%$

ABSOLUTE AND PERCENTAGE ERROR

INACCURACY

- The ruler to the right is measuring the length of a jellybean.
- Since the ruler measures to the nearest centimetre, it is impossible to say precisely what the length is.

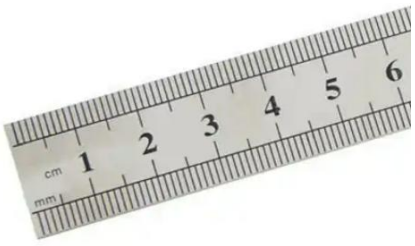


- All measuring instruments have some inaccuracy, depending on how *precise* it is.

PRECISION

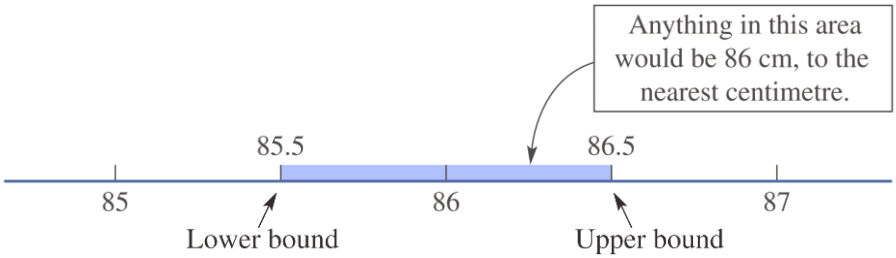
- The **precision** of a measuring instrument is the **smallest unit** it can measure.
- Unless otherwise stated, it is the **smallest place value** of the stated measurement.

Example Identify the precision of a measuring instrument.

a  1 mm or 0.1 cm	**b**  0.01 g	**c**  1 second
d 18 cm 1 cm	**e** 2.4 kg 0.1 kg	**f** 13.15 L 0.01 L
g 263 ms 1 ms	**h** 134.7 g 0.1 g	**i** 130 m (nearest 10) 10 m
j 1460 g (nearest 10) 10 g	**k** 25.50 mm 0.01 mm	**l** 3.0 mg 0.1 mg

LIMITS OF ACCURACY

- The **limits of accuracy** describe the possible values that a measurement could be, considering the inaccuracies of the measuring instrument.



Limits of Accuracy

$$\text{Absolute Error} = \frac{1}{2} \times \text{Precision}$$

$$\text{Lower Bound} = \text{Measurement} - \text{Absolute Error}$$

$$\text{Upper Bound} = \text{Measurement} + \text{Absolute Error}$$

- To find the limits of accuracy:
  1. Identify the precision.
  2. Identify the absolute error.
  3. Find the upper bound.
  4. Find the lower bound.

Example Calculate the upper and lower bound of a measurement.

State the limits of accuracy for a measurement of 16 s 1. Precision 1 s 2. Absolute error 0.5 s 3. Upper limit 16 + 0.5 = 16.5 s 4. Lower limit 16 – 0.5 = 15.5 s	Describe what the precision represents. *The precision is the …… unit an instrument can measure.* How do we find the absolute error? *The absolute error is …… the precision.* How do we find the limits of accuracy? *We add and subtract the …… …… from the ……*
State the limits of accuracy for a measurement of 9.38 m 1. Precision 0.01 m 2. Absolute error 0.005 m 3. Upper limit 9.38 + 0.005 = 9.385 m 4. Lower limit 9.38 – 0.005 = 9.375 m	Why is the precision 0.01 m? *The smallest …… …… is 0.01.*

Determine the limits of accuracy for these measurements.

a 72 cm

Precision: 1 cm
Absolute Error: .....
Lower limit: .....
Upper limit: .....

71.5 – 72.5 cm

b 86.6 mm

Precision: .....
Absolute Error: .....
Lower limit: .....
Upper limit: .....

86.55 – 86.65 mm

c 0.57 cm

Precision: .....
Absolute Error: .....
Lower limit: .....
Upper limit: .....

0.565 – 0.575 cm

d 90 m (nearest 10 metres)

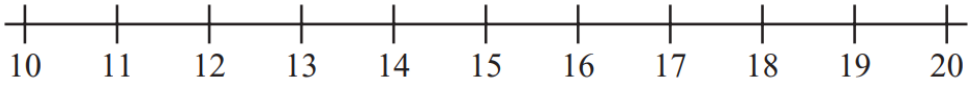
Precision: .....
Absolute Error: .....
Lower limit: .....
Upper limit: .....

85 – 95 m

Key ideas

1. The precision of a measuring instrument is the ..... unit it can measure,
2. The absolute error is ..... of the precision.
3. The lower bound of a measurement is found by ..... the absolute error from the measurement.
4. The upper bound of a measurement is found by adding the .... to the measurement.

1 Complete the following, referring to the diagram below.



a When using the measurement scale, we are measuring to the nearest ……

1

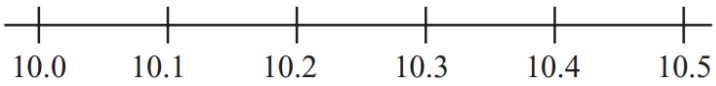
b The absolute error is 0.5 × …… = ……

0.5 × 1 = 0.5

c If a measurement is quoted as 16, it could be anything from …… up to ……

15.5 to 16.5

2 Complete the following, referring to the diagram below.



a When using the measurement scale, we are measuring to the nearest ……

0.1

b The absolute error is 0.5 × …… = ……

0.5 × 0.1 = 0.05

c If a measurement is quoted as 10.4, it could be anything from …….. up to ……

10.35 to 10.45

3 Complete the following for a measurement of 9.45 m

a This measurement of length has been made to the nearest ……,
so the smallest unit on the measuring instrument is ……
Precision = ……

0.01 m, 0.01 m, 0.01 m

b The absolute error = 0.5 × …… = ……

0.005

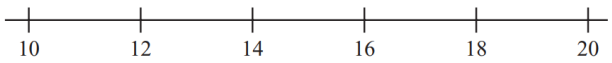
c Limits of accuracy = 9.45 ± …… = …… and ……

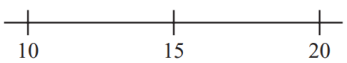
9.45 ± 0.0-5 = 9.445 and 9.455

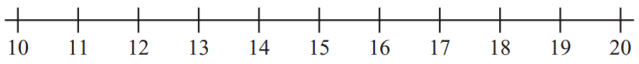
d …… ≤ true length ≤……

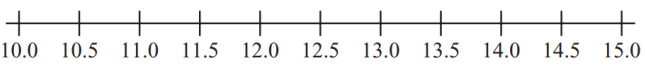
9.445 and 9.455

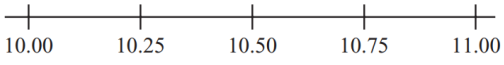
4 Match the scale with the absolute error.

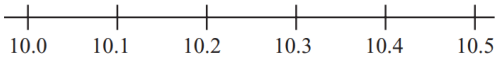












● ±0.125

● ±0.25

● ±0.05

● ±0.5

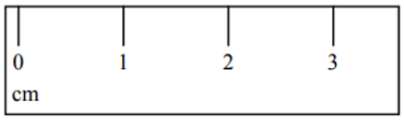
● ±1

● ±2.5

a – E, b – F, c – D, d – A, e – C

5 Determine the precision and absolute error for these two rulers.

a



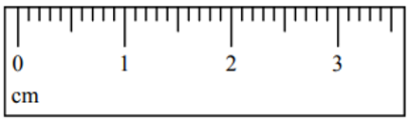
Precision: .....

1

Absolute error: .....

0.5

b



Precision: .....

0.1

Absolute error: .....

0.05

6 The length, x, of a field is given as 90 m.
Write the error interval for x if this measurement is correct to

a the nearest metre

P: 1 m
AE: 0.5 m
$89.5 \leq x \leq 90.5$

b the nearest 10 metres

P: 10 m
AE: 5 m
$85 \leq x \leq 95$

7 Complete the error interval for each statement.

a = 5000 to the nearest integer	$\leqslant a <$		4999.5 to 5000.5
b = 5000 to the nearest ten	$\leqslant b <$		4995 to 5005
c = 5000 to the nearest hundred	$\leqslant c <$		4950 to 5050
d = 5000 to the nearest thousand	$\leqslant d <$		4500 to 5500

8 Write the limits of accuracy for each of these numbers.

a $p = 60$ to the nearest 10

$55 \leq p \leq 65$

b $q = 24.7$ to 1 decimal place

$24.65 \leq q \leq 24.75$

c $x = 85$, to the nearest integer

$84.5 \leq x \leq 85.5$

d $x = 85.0$, correct to 1 decimal place.

$84.95 \leq x \leq 85.05$

9 The time taken for a plane flight is $6\frac{1}{2}$ hours, to the nearest $\frac{1}{2}$ hour.

Find the lower and upper bounds of the true measurement.

P: 0.5 hours
AE: 0.25 hours
6.75 to 6.25 hours

10 **2019 HSC Standard 2 Band 5**

A person's weight is measured as 79.3 kg. What is the absolute error of this measurement?

P: 0.1 kg

AE: 0.05 kg

11 The size of an angle a is given by the error interval $87.5 \leq a < 88.5°$

a Write three possible values of a correct to 1 decimal place.

87.8, 87.9, 88.3°

b What is the size of angle a correct to the nearest integer?

88°

12 Determine the upper and lower limits of these measurements.

a A section of road to be repaired is measured as 870 m in length, to the nearest 10 m.

865 to 875 m

b A horse is measured to weigh 485 kg, to the nearest 5 kg.

482.5 to 487.5 kg

c A solution is measured to have 4.72 g of copper sulphate, to the nearest 0.01 g.

4.715 to 4.725 g

d A chocolate bar is measured as 91.0 g, to the nearest 0.5 g.

90.75 to 91.25 g

13 Cody measures the mass of an object to be 6 kg.
Jacinta says the same object is 5.8 kg and Lachlan gives his answer as 5.85 kg.

a Explain how all three people could have different answers for the same measurement, assuming they all measured correctly.

They used measurement tools with different precision.

b Write down the possible precision of the measuring instrument being used by each person.

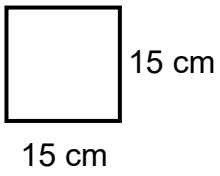
Cody: 1 kg, Jacinta: 0.1 kg, Lachlan: 0.01 kg

14 A square's side length was measured to be 15 cm, to the nearest cm.

a What is the lower limit of the length?

14.5 cm

b Therefore, what is the lower limit of the area?



$14.5 \times 14.5 = 210.25$ cm²

c What is the upper limit of the length?

15.5 cm

d Therefore, what is the upper limit of the area?

$15.5 \times 15.5 = 240.25$ cm²

e Therefore, what are the limits of accuracy for the area?

210.25 to 240.25 cm²

15 Determine the upper limit for the area of this rectangle.

$14.5 \times 6.5 = 94.25$ cm²

16 A rectangular room is 5.4 m long and 3.2 m wide.
What is the lower limit for the area of the room?

$5.35 \times 3.15 = 16.8525$ m²

17 A pizza tray was measured to be 28.6 m in diameter.
What is the lower and upper limit for the area of the pizza tray?

$$\text{Lower} = \pi\left(\frac{28.55}{2}\right)^2 = 640.18 \text{ m}^2$$

$$\text{upper} = \pi\left(\frac{28.65}{2}\right)^2 = 644.72 \text{ m}^2$$

PERCENTAGE ERROR

- The **percentage error** compares the absolute error to the measurement.

Percentage Error
$\text{Percentage Error} = \frac{\text{Absolute Error}}{\text{Measurement}} \times 100$

1. Identify the **precision**. This is the smallest place value of the measurement.
2. Identify the **absolute error**; the absolute error is half the precision.
3. Calculate the percentage error using the formula.

Example Calculate the percentage error of a measurement.

18 kg

1. Precision 1 kg
2. Absolute error 0.5 kg
3. Percentage error $\frac{0.5}{18} \times 100 = 2.778\%$

18.4 kg

1. Precision 0.1 kg
2. Absolute error 0.05 kg
3. Percentage error $\frac{0.05}{18.4} \times 100 = 0.272\%$

Why is the percentage error much smaller in the second example?

Since the ..... is smaller,

the .... is smaller.

This makes the percentage error smaller.

Calculate the percentage error of these measurements. Give your answer to 3 d.p.

a 49.6 kg

Precision: .....

Absolute Error: .....

Percentage Error: .....

0.101%

b 251 g

Precision: .....

Absolute Error: .....

Percentage Error: .....

0.199%

c 50 mL (nearest 10 mL)

Precision: .....

Absolute Error: .....

Percentage Error: .....

10%

d 5.256 L

Precision: .....

Absolute Error: .....

Percentage Error: .....

0.010%

Key ideas

1. The percentage error is the absolute error divided by the ....., then multiplied by 100.

1 Vivaan measured the mass of an iPhone as 251 g.

a Identify the precision.

1 g

b Calculate the absolute error.

0.5 g

c Find the limits of accuracy.

250.5 to 251.5 g

d Find the percentage error. Answer correct to two decimal places.

$\frac{0.5}{251} \times 100 = 0.20\%$

2 Abbey measured the mass of a dishwasher as 49.6 kg.

a Find the limits of accuracy.

P: 0.1 kg

AE: 0.05 kg

49.55 to 49.65 kg

b Find the percentage error correct to two decimal places.

$\frac{0.05}{49.6} \times 100 = 0.10\%$

3 Saliha measured the mass of an LCD screen as 2.71 kg.

a Find the limits of accuracy.

P: 0.01 kg

AE: 0.005 kg

2.705 to 2.715 kg

b Find the percentage error correct to three decimal places.

$\frac{0.005}{2.71} \times 100 = 0.185\%$

4 Find the percentage error for an object measured as 13.0 cm, to the nearest 0.5 cm.
Answer to 2 d.p.

P: 0.5

AE: 0.25

$\frac{0.25}{13} \times 100 = 1.92\%$

5 2015 HSC Standard 2 Band 5

The length of a fish was measured to be 49 cm, correct to the nearest cm.

What is the percentage error in this measurement, correct to two decimal places?

Precision: 1 cm

AE: 0.5 cm

$$\% \text{ error} = \frac{0.5}{49} \times 100 = 1.02\%$$

6 2014 HSC Standard 2 Band 5

The top of the Sydney Harbour Bridge is measured to be 138.4 m above sea level.

What is the percentage error in this measurement?

P = 0.1 m

AE = 0.05 m

$$\% \text{ error} = \frac{0.05}{138.4} \times 100 = 0.036\%$$

7 2008 HSC Standard 2 Band 4

The capacity of a bottle is measured as 1.25 litres correct to the nearest 10 millilitres.

What is the percentage error for this measurement?

Precision: 10 mL

AE: 5 mL

$$\% \text{ error} = \frac{5 \text{ mL}}{1250 \text{ mL}} \times 100 = 0.4\%$$

8 2017 HSC Standard 2 Band 5

The length of a netball court is measured to be 30.50 metres, correct to the nearest centimetre.

What is the lower limit for the length of the netball court?

Precision: 1 cm

AE: 0.5 cm

Lower limit = 30.5 m – 0.5 cm = 30.495 m