

Student Name

Mathematics Stage 5 Paths

# ALGEBRAIC TECHNIQUES B

**Book 2**      Factorise algebraic expressions by taking out a  
common algebraic factor

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## OVERVIEW

### SYLLABUS CONTENT

**MA5-ALG-P-01** simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions

#### Factorise algebraic expressions by taking out a common algebraic factor

- Factorise algebraic expressions, including those involving indices, by determining the highest common factors (HCF) including negative coefficients/pronumerals

### PRIOR KNOWLEDGE CHECK

1 Factorise the following expressions:

**a**  $4x + 12$

**b**  $9y - 6$

**c**  $6 - 12x$

**d**  $24a - 16$

**e**  $24x + 72$

**f**  $36x + 48$

**g**  $x^2 - 4x$

**h**  $5pq + 20p$

**i**  $4x^2 + 12x$

**j**  $6a^2 + 9a$

**k**  $3p^2 - 4pq$

**l**  $10x^2 + 4xy^2$

# FACTORISING A NEGATIVE HCF

- To **factorise** with a **negative HCF**:
  - Identify the negative **highest common factor** (HCF) of the terms. Write it outside brackets.
  - Divide each term by the HCF. Put the results inside the brackets.

**Example** Factorise by finding a negative HCF.

|                                                      |                                                                                                                    |
|------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| $-8x - 16$<br>1. $-8( \quad )$<br>2. $-8(x + 2)$     | Why is the inside of the bracket $(x + 2)$ ?<br><i>We divided <math>-8x</math> and <math>-16</math> by .....</i>   |
| $12x - 4xy$<br>1. $-4x( \quad )$<br>2. $-4x(-3 + y)$ | Why is the inside of the bracket $(-3 + y)$ ?<br><i>We divided <math>12x</math> and <math>-4xy</math> by .....</i> |

Factorise these expressions by finding a negative HCF.

|                                       |                                        |                                    |
|---------------------------------------|----------------------------------------|------------------------------------|
| <b>a</b> $-8x - 4$<br>$= -4( \quad )$ | <b>b</b> $-2m + 4n$<br>$= -2( \quad )$ | <b>c</b> $-3xy + 18x$              |
| <b>d</b> $8x^2 - 12x$                 | <b>e</b> $6x - 20x^2$                  | <b>f</b> $5a^3x + 10a^2$           |
| <i><math>-4(2x + 1)</math></i>        | <i><math>-2(m - 2n)</math></i>         | <i><math>-3x(y - 6)</math></i>     |
| <i><math>-4x(-2x + 3)</math></i>      | <i><math>-2x(-3 + 10x)</math></i>      | <i><math>-5a^2(-ax - 2)</math></i> |

**Note:** You should factor out a negative HCF if both terms are negative, or if the question asks for it.  
 This skill is used in a later topic (factorising by grouping in pairs).

## Key ideas

- Factorising is the reverse of ..... brackets.
- To factorise, divide both terms by the ..... and have the ..... outside the brackets.
- It is possible to factorise out a ..... HCF if both terms are negative, or if the question asks for it.

1 Complete these factorisations.

|                                                                                                       |                                                                                                       |                                                                                                       |
|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| <b>a</b><br>$7x + 7 = 7(\underline{\quad} + \underline{\quad})$<br><small><math>x, 1</math></small>   | <b>b</b><br>$14x + 7 = 7(\underline{\quad} + \underline{\quad})$<br><small><math>2x, 1</math></small> | <b>c</b><br>$14x - 7 = 7(\underline{\quad} - \underline{\quad})$<br><small><math>2x, 1</math></small> |
| <b>d</b><br>$-9x + 9 = -9(\underline{\quad} - \underline{\quad})$<br><small><math>x, 1</math></small> | <b>e</b><br>$-9x + 9 = 9(\underline{\quad} + \underline{\quad})$<br><small><math>-x, 1</math></small> | <b>f</b><br>$6 - 12x = 6(\underline{\quad} - \underline{\quad})$<br><small><math>1, 2x</math></small> |

2 Factorise by taking out a negative HCF.

|                                                                     |                                                                      |                                                                     |
|---------------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------|
| <b>a</b><br>$-8x - 4$<br><small><math>-4(2x + 1)</math></small>     | <b>b</b><br>$-4x - 2$<br><small><math>-2(2x + 1)</math></small>      | <b>c</b><br>$-10x - 5y$<br><small><math>-5(2x + y)</math></small>   |
| <b>d</b><br>$-7a - 14b$<br><small><math>-7(a + 2b)</math></small>   | <b>e</b><br>$-9x - 1$<br><small><math>-(9x + 1)</math></small>       | <b>f</b><br>$-6y - 8$<br><small><math>-2(3y + 4)</math></small>     |
| <b>g</b><br>$-10x - 15y$<br><small><math>-5(2x + 3y)</math></small> | <b>h</b><br>$-4m - 20n$<br><small><math>-4(m + 5n)</math></small>    | <b>i</b><br>$-3x^2 - 18x$<br><small><math>-3x(x + 6)</math></small> |
| <b>j</b><br>$-6x + 8$<br><small><math>-2(3x - 4)</math></small>     | <b>k</b><br>$-12x + 16$<br><small><math>-4(3x - 4)</math></small>    | <b>l</b><br>$-60x + 45$<br><small><math>-15(4x + 3)</math></small>  |
| <b>m</b><br>$-9x - 12$<br><small><math>-3(3x + 4)</math></small>    | <b>n</b><br>$-20xy + 15x$<br><small><math>-5x(4y - 3)</math></small> | <b>o</b><br>$21x - 15xy$<br><small><math>3x(7 - 5y)</math></small>  |

3 Read the examples carefully and answer the questions.

Fei's work

$$-4x - 2$$

$$-4x - 2$$

$$-2(2x + 1)$$


Where did the +1 come from in Fei's factored expression?

-2 divided by ..... is 1

Factorise:

$$-9x^2 - 3x$$

$$-3x(3x + 1)$$

4 Factorise the following. For these questions, always take out a negative HCF.

|                                                                             |                                                                          |                                                                             |
|-----------------------------------------------------------------------------|--------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| <b>a</b> $3x - 18$<br><br><small><math>-3(-x + 6)</math></small>            | <b>b</b> $-12a - 3$<br><br><small><math>-3(4a + 1)</math></small>        | <b>c</b> $-5x - 30$<br><br><small><math>-5(x + 6)</math></small>            |
| <b>d</b> $10a - 5a^2$<br><br><small><math>-5a(-2 + a)</math></small>        | <b>e</b> $4x^2 + x$<br><br><small><math>-x(-4x - 1)</math></small>       | <b>f</b> $6b^2 - 18b$<br><br><small><math>-6b(-b + 3)</math></small>        |
| <b>g</b> $12x - 30x^2$<br><br><small><math>-6x(-2 + 5x)</math></small>      | <b>h</b> $-2x - x^2$<br><br><small><math>-x(2 + x)</math></small>        | <b>i</b> $-4y - 8y^2$<br><br><small><math>-4y(1 + 2y)</math></small>        |
| <b>j</b> $6xyz^2 + 3z^2$<br><br><small><math>-3z^2(-2xy - 1)</math></small> | <b>k</b> $2x^2yz - 4xy$<br><br><small><math>-2xy(-xz + 2)</math></small> | <b>l</b> $-12m^2n - 12mn^2$<br><br><small><math>-12mn(m + n)</math></small> |
| <b>m</b> $-16y^2 - 6y$<br><br><small><math>-2y(8y + 3)</math></small>       | <b>n</b> $-5a^2 - 10a$<br><br><small><math>-5a(a + 2)</math></small>     | <b>o</b> $-6x - 20x^2$<br><br><small><math>-2x(3 + 10x)</math></small>      |
| <b>p</b> $-6p - 15p^2$<br><br><small><math>-3p(2 + 5p)</math></small>       | <b>q</b> $-16b - 8b^2$<br><br><small><math>-8b(2 + b)</math></small>     | <b>r</b> $-9x - 27x^2$<br><br><small><math>-9x(1 + 3x)</math></small>       |

5 Many of the following expressions are not fully factorised. Finish the factorisation.

|                                                                                                  |                                                                              |                                                                                      |
|--------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| <b>a</b> $2(3x + 9)$<br>$= 2 \times 3(x + 3)$<br>$=$<br><br><small><math>6(x + 3)</math></small> | <b>f</b> $2(6x - 12)$<br><br><small><math>12(x - 2)</math></small>           | <b>k</b> $4(12 - 6x + 4y)$<br><br><small><math>8(6 - 3x + 2y)</math></small>         |
| <b>b</b> $2(-3x - 9)$<br><br><small><math>-6(x + 3)</math></small>                               | <b>g</b> $2(12 - 6x)$<br><br><small><math>12(2 - x)</math></small>           | <b>l</b> $4(12 - 6x + 5y)$<br><br><small>can't factorise further</small>             |
| <b>c</b> $2(6x + 10)$<br><br><small><math>4(3x + 5)</math></small>                               | <b>h</b> $4(-12 - 6x)$<br><br><small><math>-24(2 + x)</math></small>         | <b>m</b> $4(12x - 6x^2 + 5xy)$<br><br><small><math>4x(12 - 6x + 5y)</math></small>   |
| <b>d</b> $2(6x + 11)$<br><br><small>can't factorise further</small>                              | <b>i</b> $4(12 - 6x + 2y)$<br><br><small><math>8(6 - 3x + y)</math></small>  | <b>n</b> $4(12x - 6x^2 + 4xy)$<br><br><small><math>4x^2(6 - x^2 + 2y)</math></small> |
| <b>e</b> $2(-6x - 12)$<br><br><small><math>-12(x + 2)</math></small>                             | <b>j</b> $4(12 - 6x + 3y)$<br><br><small><math>12(4 - 2x + y)</math></small> | <b>o</b> $4(12x^2 - 6x^2 + 5x^2y)$<br><br><small><math>4x^2(6 + 5y)</math></small>   |

- 6** Common factors can also be removed from expressions with more than two terms.  
For example:  $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$ . Factorise these expressions.

|                            |                                |                                |
|----------------------------|--------------------------------|--------------------------------|
| <b>a</b> $3a + 9b + 12$    | <b>b</b> $5z^2 - 10z + zy$     | <b>c</b> $x^2 - 2xy + x^2y$    |
| $3(a + 3b + 4)$            | $z(5z - 10 + y)$               | $x(x - 2y + xy)$               |
| <b>d</b> $4by - 2b + 6b^2$ | <b>e</b> $-12xy - 8yz - 20xyz$ | <b>f</b> $3ab + 4ab^2 + 6a^2b$ |
| $2b(2y - 1 + 3b)$          | $-4y(3x + 2z + 5xz)$           | $ab(3 + 4b + 6a)$              |

- 7** Sometimes we can choose to factor out a negative or a positive HCF.  
Both factorisations are correct. For example:

$$\begin{aligned} -13x + 26 &= -13(x - 2) \quad (\text{HCF is } -13) \\ \text{or } -13x + 26 &= 13(-x + 2) \quad (\text{HCF is } 13) \\ &= 13(2 - x) \end{aligned}$$

Factorise these expressions in two ways.

|                                       |                                             |                                                       |
|---------------------------------------|---------------------------------------------|-------------------------------------------------------|
| <b>a</b> $-4x + 12$                   | <b>b</b> $-3x + 9$                          | <b>c</b> $-8n + 8$                                    |
| $4(-x + 3),$<br>$-4(x - 3)$           | $3(-x + 3),$<br>$-3(x - 3)$                 | $8(-n + 1),$<br>$-8(n - 1)$                           |
| <b>d</b> $-5m + 5m^2$                 | <b>e</b> $-7x + 7x^2$                       | <b>f</b> $-4y + 22y^2$                                |
| $5m(-1 + m),$<br>$-5m(1 - m)$         | $7x(-1 + x),$<br>$-7x(1 - x)$               | $2y(-2 + 11y),$<br>$-2y(2 - 11y)$                     |
| <b>g</b> $-8n + 12n^2$                | <b>h</b> $-8y + 20$                         | <b>i</b> $-15mn + 10$                                 |
| $4n(-2 + 3n),$<br>$-4n(2 - 3n)$       | $4(-2y + 5),$<br>$-4(2y - 5)$               | $5xy(-2y + 3x),$<br>$-5xy(2y - 3x)$                   |
| <b>j</b> $9x - 27x^2$                 | <b>k</b> $16ab - 8ab^2$                     | <b>l</b> $-10xy^2 + 15x^2y$                           |
| $9x(1 - 3x),$<br>$-9x(-1 + 3x)$       | $8ab(2 - b),$<br>$-8ab(-2 + b)$             | $5xy(-2y + 3x),$<br>$-5xy(2y - 3x)$                   |
| <b>m</b> $x^2y - 4xy + xy^2$          | <b>n</b> $6ab - 10a^2b + 8ab^2$             | <b>o</b> $-3a^3b + 12ab^2 + 9a^2b^3$                  |
| $xy(x - 4 + y),$<br>$-xy(-x + 4 - y)$ | $2ab(3 - 5a + 4b),$<br>$-2ab(-3 + 5a - 4b)$ | $3ab(-a^2 + 4b + 3ab^2),$<br>$-3ab(a^2 - 4b - 3ab^2)$ |