Student Name

Mathematics Stage 4

ALGEBRAIC TECHNIQUES

Book 5 Factorise algebraic expressions by identifying numerical and algebraic factors

Version: 260207

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CONTENTS

OVERVIEW

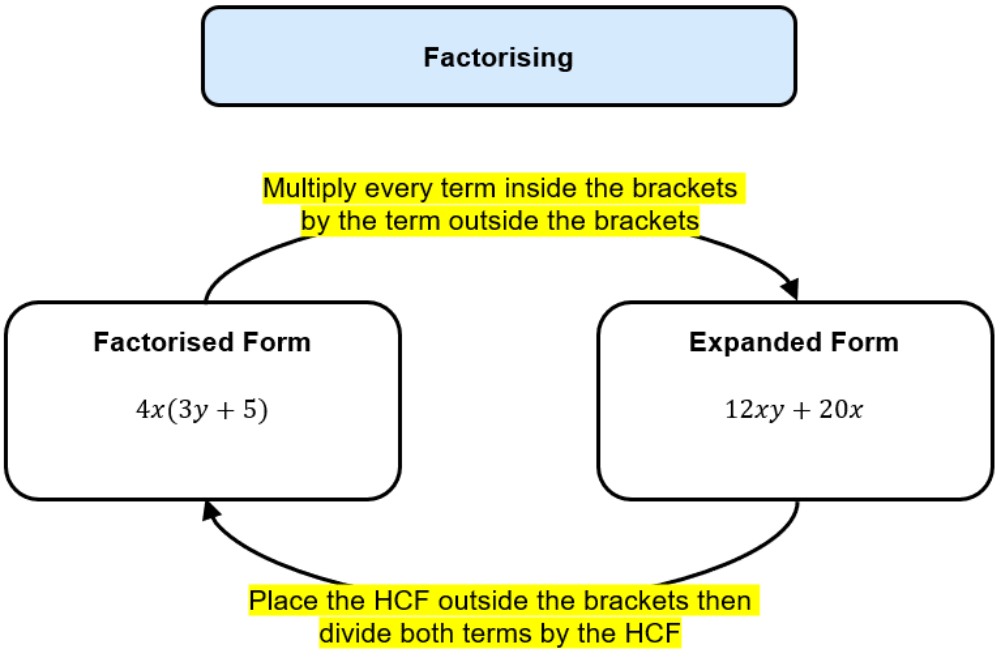
SYLLABUS CONTENT

MA4-ALG-C-01 generalises number properties to operate with algebraic expressions including expansion and factorisation

Factorise algebraic expressions by identifying numerical and algebraic factors

- Identify and list factors of a single term
- Factorise algebraic expressions using knowledge of factors and finding the highest common numerical factor (HCF)
- Factorise algebraic expressions using knowledge of factors by finding a common algebraic factor, including expressions involving more than 2 terms, and verify the result by expansion

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Find the highest common factor of these pairs of terms.

a	$12x$ and 18	**b**	$15b$ and 25	**c**	$12x$ and -20
d	$35x$ and $49b$	**e**	$45a$ and $60w$	**f**	$-14x$ and $28b$
a	$7y$ and $14x$	**b**	$-2y$ and $4x$	**c**	$11y$ and $-33x$
d	$8q$ and $-4r$	**e**	$-3a$ and 6	**f**	$15p$ and $25q$

2 Expand these brackets

a	$3(x+4)$	**b**	$2(3a-5)$	**c**	$5(3x+2)$
d	$x(y+6)$	**e**	$x(3x-12)$	**f**	$5y(3xy+2z)$

3 Find the HCF of these terms.

a	$7x$ and $13x$	**b**	$25a$ and $15a$	**c**	$9a$ and $27ab$
d	$6xy$ and $12x$	**e**	$-2xy$ and $8xy$	**f**	$22xy$ and $-44xy$
g	$12qr$ and $-8r^2$	**h**	$-7a$ and $21a^2$	**i**	$16p^2$ and $24p^2q$

FACTORISATION: NUMERICAL HCF

- **Factorising** reverses **expanding** brackets.

Expanding

$3(2x + 5) = 6x + 15$

Factorising

- To factorise:
  1. Identify the **highest common factor** (HCF) of the terms. Place it outside the brackets.
  2. Divide each term by the HCF. Put the results inside the brackets.

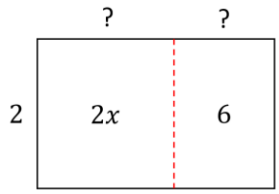
Example Factorise expressions by finding the numerical HCF.

$15a + 10$ 1. $5(\quad)$ 2. $5(3a + 2)$	How did we know to take 5 outside the brackets? *5 is the .... of* $15a$ *and* 10 How did we know the inside is $3a + 2$? *We divided* $15a$ *and* 10 *by .....*
$8a - 12b$ 1. $4(\quad)$ 2. $4(2a - 3b)$	How did we know the inside is $2a - 3b$? *We divided ..... and ..... by 4* Why would $8a - 12b = 2(4a - 6b)$ not be fully factorised? *2 is not the ..... common factor of* $8a$ *and* $-12b$

Factorise these expressions.

a $2x + 6$

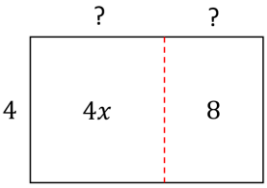
$= 2(\square + \square)$



$2(x + 3)$

b $4x + 8$

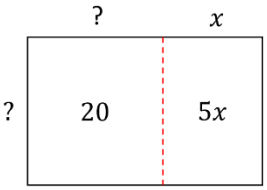
$= 4(\square + \square)$



$4(x + 2)$

c $20 + 5x$

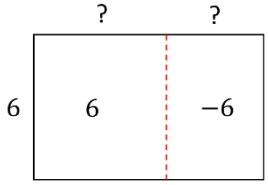
$= \square(\square + x)$



$5(4 + x)$

d $6x - 6$

$= 6(\square - \square)$



$6(x - 1)$

e $8x + 16$

$= \square(\square + \square)$

$8(x + 2)$

f $10x + 15$

$5(2x + 3)$

Key ideas

1. Factorising is the ..... of expanding.
2. To factorise, divide both terms by the ..... and have the ..... outside brackets.

1 Factorise the following by filling in the blanks.

a $3(4x + 1) = \square x + 3$

12

b $5(7 - 2x) = \square - 10x$

35

c $6(2 + 5y) = \square + \square y$

12 , 30

d $7(2a - 3b) = \square - \square$

$14a$, $21b$

e $3(2a + \square) = 6a + 21$

7

f $4(\square - 2y) = 12 - 8y$

3

g $7(\square + \square) = 14 + 7q$

2 , q

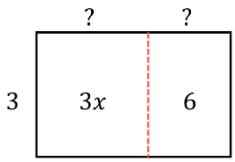
h $\square(2x + 3y) = 8x + 12y$

4

2 Factorise the following.

a $3x + 6$

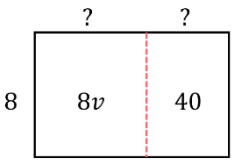
$= 3(\quad)$



$3(x + 2)$

b $8v + 40$

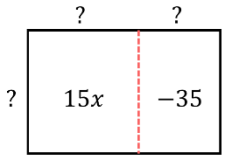
$= 8(\quad)$



$8(x + 5)$

c $15x - 35$

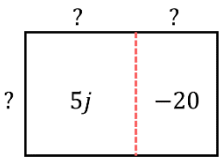
$= \square(\square - \square)$



$5(3x - 7)$

d $5j - 20$

$= \square(\square - \square)$



$5(j - 4)$

e $9b - 15$

$= \square(\square - \square)$

$3(3b - 5)$

f $12 + 16f$

$= \square(\square + \square)$

$4(3 + 4f)$

g $5d - 30$

$5(d - 6)$

h $48 + 12w$

$12(4 + w)$

3 Factorise these expressions, if possible.

a $30x + 18$

$6(5x + 3)$

b $30x - 18$

$6(5x - 3)$

c $30x - 19$

can't factorise

d $30x - 20$

$10(3x - 2)$

e $30x + 20$

$10(3x + 2)$

f $35x + 21$

$7(5x + 3)$

g $30x + 18$

$6(5x + 3)$

h $27x + 18$

$9(3x + 2)$

i $18x + 18$

$18(x + 1)$

j $9x + 18$

$9(x + 2)$

k $9x + 19$

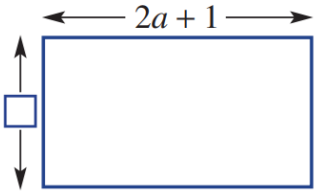
can't factorise

l $19x + 19$

$19(x + 1)$

4 The area of the rectangle is $10a + 5$.
One side's measurement is unknown.

a What is the value of the unknown measurement?



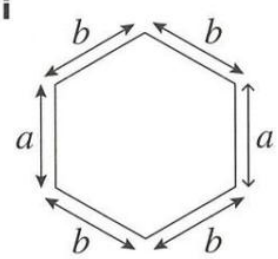
5

b Write an expression for the perimeter of the rectangle.

$4a + 12$

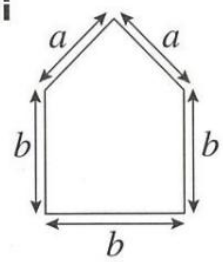
5 Match the shapes with the expressions that show their perimeters.

i



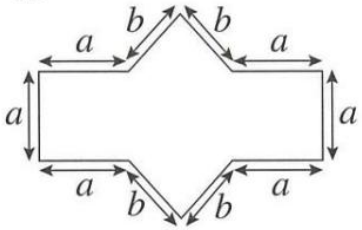
A $2(a + b) + b$

ii



B $2(3a + 2b)$

iii



C $2(a + 2b)$

i-c, ii-A, iii-B

FACTORISATION: ALGEBRAIC HCF

- To factorise:
  1. Identify the **highest common factor** (HCF) of the terms. Write it outside brackets.
  2. Divide each term by the HCF. Put the results inside the brackets.

Example Factorise expressions by finding the algebraic HCF.

$ab - 5b$	$12a^3 + 18a^2b$
1. $b(\quad)$	1. $6a^2(\quad)$
2. $b(a - 5)$	2. $6a^2(2a + 3b)$

Factorise these expressions.

a $xk + xc$

$= x(\square + \square)$

$x(k + c)$

b $x^2 + 3x$

$= x(\square + \square)$

$x(x + 3)$

c $x^2 - 3x$

$= \square(\square - \square)$

$x(x - 3)$

d $6xb + 8b$

$= \square(\square + \square)$

$2b(3x + 4)$

e $3ab - 5b$

$b(3a - 5)$

f $4xy^2 - 6y^2$

$2y^2(2x - 3)$

Key ideas

1. The HCF of two algebraic terms is the largest term that …… into both terms evenly.
2. To find the algebraic HCF, first find the HCF of the ……, then find the HCF of the ……, then multiply them.

1 Complete the factorisation.

a $2x + 2$

$= 2(\quad)$

$2(x + 1)$

b $5x + 15$

$= 5(\quad)$

$5(x + 3)$

c $x^2 + x$

$= x(\quad)$

$x(x + 1)$

d $3x - 6$

$= 3(\quad)$

$3(x - 2)$

e $x^3 - x^2$

$= x^2(\quad)$

$x^2(x - 1)$

f $8x - 4x^2$

$= 4x(\quad)$

$4x(2 - x)$

g $8y^2 - 12y$

$= 4y(\quad)$

$4y(2 - 3)$

h $12d + 9dz$

$= 3d(\quad)$

$3d(4 + 3z)$

i $15x^3 + 9x^2$

$= 3x^2(\quad)$

$3x^2(5x + 3)$

2 Factorise the following.

a $10cn + 12n$

$2n(5c + 6)$

b $24y + 8ry$

$8y(3 + r)$

c $14jn + 10n$

$2n(7j + 5)$

d $24g + 20gj$

$4g(6 + 5j)$

e $10h + 4z$

$2(5h + 2z)$

f $30u - 20n$

$10(3u - 2n)$

g $40y + 56ay$

$8y(5 + 7a)$

h $12d + 9dz$

$3d(4 + 3z)$

i $21hm - 9mx$

$3m(7h - 3x)$

j $49u - 21bu$

$7u(7 - 3b)$

k $28u - 42bu$

$7u(4u - 6b)$

l $21p - 6c$

$3(7p - 2x)$

3 Factorise these expressions.

a $6x + 8$

$2(3x + 4)$

b $6x - 8$

$2(3x - 4)$

c $12x - 16$

$4(3x - 4)$

d $18x - 24$

$6(3x - 4)$

e $9x - 12$

$3(3x - 4)$

f $12 - 9x$

$3(4 - 3x)$

g $12x - 9$

$3(4x - 3)$

h $60x - 45$

$15(4x - 3)$

i $20x - 15$

$5(4x - 4)$

j $20x^2 - 15x$

$5x(4x - 3)$

k $20xy - 15y$

$5y(4x - 3)$

l $20xy - 15x$

$5x(4y - 3)$

m $20 - 15x$

$5(4 - 3x)$

n $21 - 15x$

$3(7 - 5x)$

o $21x - 15xy$

$3x(7 - 5y)$

p $22x - 15xy$

$x(22 - 15y)$

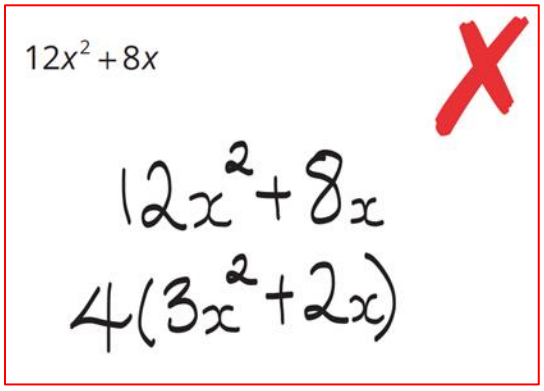
4 Fill in the table.

Expanded	Factorised
$3p + 15$	$3(p + 5)$
$3p + 18$	$3(p + 6)$
$6p + 18$	$6(p + 3)$
$6p + 15$	$3(2p + 5)$
$6p + 10$	$2(3p + 5)$
$6p - 10$	$2(3p - 5)$

Expanded	Factorised
$6p^2 - 10p$	$2p(3p - 5)$
$6p^2 - 10$	$2(3p^2 - 5)$
$6 - 10p^2$	$2(3 - 5p^2)$
$-6 + 10p^2$	$-2(3 - 5p^2)$
$-6 + 12p^2$	$6(-1 + 2p^2)$
$-6 + 11p^2$	can't factorise

6 Read the example carefully and answer the questions.

Caroline's work:



Why is Caroline's expression not factored completely?

$3x^2 + 2x$ still has a common factor of .....

Factorise:

$$12x^2 - 8x$$

$4x(3x - 2)$

5 Factorise the following.

a	$x^2 + 2x$	$x(x + 2)$	**b**	$a^2 - 4a$	$a(a - 4)$
c	$y^2 - 7y$	$y(y - 7)$	**d**	$x - x^2$	$x(1 - x)$
e	$3p^2 + 3p$	$3p(p + 1)$	**f**	$8x - 8x^2$	$8x(1 - x)$
g	$4b^2 + 12b$	$4b(b + 3)$	**h**	$6y - 10y^2$	$2y(3 - 5y)$
i	$12a - 15a^2$	$3a(4 - 5a)$	**j**	$9m + 18m^2$	$9m(1 + 2m)$
k	$16xy - 48x^2$	$16x(y - 3x)$	**l**	$7xy - 28ab^2$	$7(xy - 4ab^2)$

6 Factorise these expressions.

a $6a - 18b + 9c$

$3(2a - 6b + 3c)$

b $6a + 18b - 12c$

$6(a + 3b - 2c)$

c $10a + ab$

$a(10 + b)$

d $10a + ab + a^2$

$a(10 + b + a)$

e $5a^2 + 6ab + 7ac$

$a(5a + 6b + 7c)$

f $5pq - 10qr + 15q^2$

$5q(p - 2r + 3q)$

7 Factorise these expressions.

a $8x - xy$

$x(8 - y)$

b $8x - 2xy$

$2x(4 - y)$

c $8x - 4xy$

$4x(2 - y)$

d $8x - x^2$

$x(8 - x)$

e $x^2 - 8x$

$x(x - 8)$

f $3x^2 - 12x$

$3x(x - 4)$

g $3x^2 - 12x + xy$

$x(3x - 12 + y)$

h $3x^2 - 12x + 6xy$

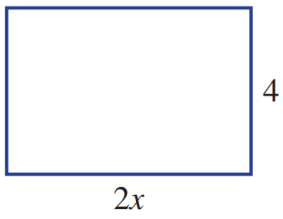
$3x(x - 4 + 2y)$

i $9x^2 + 3x - 12xy$

$3x(3x + 1 - 4y)$

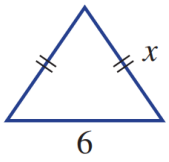
8 Find the perimeters of these shapes and factorise your answer.

a



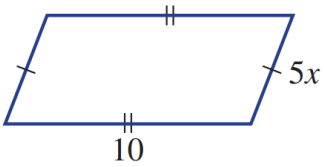
$2(2x + 4)$

b



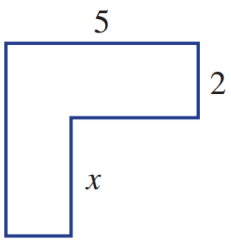
$2(x + 3)$

c



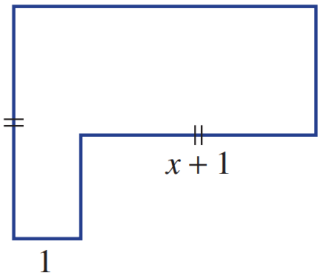
$10(x + 2)$

d



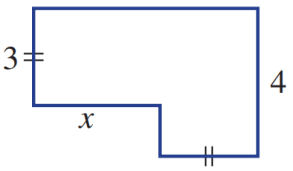
$2(7 + x)$

e



$2(2x + 3)$

f



$2(x + 7)$

9

a If $x = 5$, work out the value of $2x^2 - 4x$

30

b Factorise $2x^2 - 4x$

$2x(x - 2)$

c Determine the value of x in this equation $2x - 4 = 10$

$x = 7$

d Expand $2x(x + 2)$

$2x^2 + 4x$

10 **NAPLAN C**

A rectangle has an area of $9a^2 - 6a$.

By factorising the expression, find the width of the rectangle if the length is $3a$.

$3a - 2$

11 **NAPLAN C**

A rectangle has an area of $2pq^2 + 4p^2q$.

Find the length of the rectangle if the width is $2pq$.

$q + 2p$

12 The expression for the area of a rectangle is $(4x^2 + 8x)$.

Find an expression for its width if the length is $(x + 2)$.

$4x$

13 Factorise the following expressions.

a $3a^2 + 9a + 12$

$3(a^2 + 3a + 4)$

b $5z^2 - 10z + zy$

$5z(z - 2 + y)$

c $x^2 - 2xy + x^2y$

$x(x - 2y + xy)$

d $4by - 2b + 6b^2$

$2b(2y - 1 + 3b)$

e $-12xy - 8yz - 20xyz$

$-4y(3x + 2z + 5xz)$

f $3ab + 4ab^2 + 6a^2b$

$ab(3 + 4b + 6a)$

14 At the end of a lesson using boxes of Smarties, the remaining Smarties are shared equally among the members of each group. In one group there are 6 boxes of Smarties left over and 42 loose Smarties that are not in boxes.

a Write an expression for the total smarties left over.

$6x + 42$

b Factorise the expression in part **a**.

$6(x + 7)$

c If there are 6 people in the group, write an expression for the number of Smarties each person receives.

$x + 7$

d If there are 3 people in the group, write an expression for the number of Smarties each person receives.

$2x + 14$

15 These factorisations are incomplete. Complete the factorisation.

a $2(3x + 9)$ $6(x + 3)$	**b** $7(4x + 12y)$ $28(x + 3y)$	**c** $x(y + 2y)$ $3xy$
d $2(6x + 9)$ $6(2x + 3)$	**e** $3x(16zy + 12y)$ $12xy(4z + 3)$	**f** $12x(x^2 + xy)$ $12x^2(x + y)$

16 **NAPLAN C**

Fully factorise $24p^2qr^3 + 18p^3q^2 + 6p^2qr^2$

$6p^2q(4r^3 + 3pq + r^2)$

17 **NAPLAN C**

Fully factorise $2mn^2 + 6m^2n - 8mn$.

$2mn(n + 3m - 4)$

18 Rosie factorised $16xyz + 24xn = 4(4xyz + 6xn)$.

a Explain why she is not fully correct.

The terms in the brackets share common factors.

b Charlotte says it should have been $8(2xyz + 3xn)$. Explain why she is not fully correct.

The terms in the brackets still share a common factor of x.

19 In senior years, it is often necessary to factorise out a fraction. Complete the following:

a $\frac{x}{2}+\frac{y}{2}$

$=\frac{1}{2}($ $)$

$\frac{1}{2}(x+y)$

b $\frac{3a}{4}-\frac{3b}{4}$

$\frac{3}{4}(a-b)$

c $\frac{2p}{3}-\frac{2q}{3}$

$\frac{2}{3}(p-q)$

d $\frac{x^2}{3}+\frac{2x}{3}$

$\frac{x}{3}(x+2)$

e $\frac{3ab}{5}-\frac{2b}{5}$

$\frac{3b}{5}(a-2)$

f $\frac{5p^2q}{8}-\frac{3pq}{8}$

$\frac{pq}{8}(5p-3)$

20 Sienna factorised $\frac{5x}{3}+\frac{5y}{6}$ as $\frac{5}{6}(2x+y)$

a By expanding, show that she is correct.

b Hence factorise these expressions; there should be no fractions inside brackets.

i $\frac{x}{2}+\frac{y}{4}$

$\frac{1}{4}(2x+y)$

ii $\frac{3a}{4}-\frac{b}{2}$

$\frac{1}{4}(3a-2b)$

iii $\frac{2p}{3}-\frac{q}{6}$

$\frac{1}{6}(4p-q)$

iv $\frac{3x}{8}+\frac{y}{4}$

$\frac{1}{8}(3x+2y)$

v $\frac{xy^2}{5}+\frac{xy}{10}$

$\frac{xy}{5}(y+2)$

vi $\frac{5c^3}{9}-\frac{cd}{3}$

$\frac{c}{9}(5c^2-3d)$

21 Find the value of m if:

$$(x+3)(m-2)=-\frac{5x}{2}-\frac{15}{2}$$

$x=-\frac{1}{2}$

CHECK YOUR UNDERSTANDING

Factorising numerical HCF

1 Fully factorise: $4x + 12$

a $4(x + 3)$ **b** $4(x + 12)$ **c** $2x + 6$ **d** $2(2x + 6)$

2 Fully factorise: $9y - 6$

a $3(3y - 6)$ **b** $6(3y - 1)$ **c** $3(3y - 2)$ **d** $9(y - 6)$

3 Fully factorise: $6 - 12x$

a $6(1 - 12x)$ **b** $6(1 - 2x)$ **c** $12(6 - x)$ **d** $3(2 - 3x)$

4 Fully factorise: $24a - 16$

a $2(12a - 8)$ **b** $4(6a - 4)$ **c** $8(3a - 2)$ **d** $6(4a - 3)$

5 Fully factorise: $24x + 72$

a $2(12x + 36)$ **b** $24(x + 3)$ **c** $6(4x + 12)$ **d** $4(6x + 18)$

6 Fully factorise: $36x + 48$

a $12(3x + 4)$ **b** $6(6x + 8)$ **c** $2(18 + 24)$ **d** $3(12x + 16)$

Factorising algebraic HCF

1 Fully factorise $x^2 - 4x$

a $(x+2)(x-2)$ **b** $x(x-4)$ **c** $x(2-4)$ **d** $x(2x-4)$

2 Fully factorise $5pq + 20p =$

a $5p(q+4)$ **b** $5p(q-4)$ **c** $p(5q+20)$ **d** $5(pq+4p)$

3 Fully factorise $4x^2 + 12x$

a $2(2x^2+6x)$ **b** $x(4x+12)$ **c** $4(x^2+3x)$ **d** $4x(x+3)$

4 Fully factorise $6a^2 + 9a$

a $3a(2a+3)$ **b** $a(6a+9)$ **c** $3(2a^2+3)$ **d** $15a^2$

5 Fully factorise $3p^2 - 4pq$

a $3p^2(1-4q)$ **b** $3p(p-4q)$ **c** $p^2(3-4q)$ **d** $p(3p-4q)$

6 Fully factorise $10x^2 + 4xy^2$

a $2x(5x+2y^2)$ **b** $2(5x+2xy^2)$ **c** $x(10x+4y^2)$ **d** $4x(2.5x+xy^2)$

7 Fully factorise $2p^2q - pq^2$

a $2pq(p-q)$ **b** $pq(2p-q)$ **c** $p(2pq-q^2)$ **d** $q(2p^2-pq)$