

Student Name

Mathematics Stage 4

ALGEBRAIC TECHNIQUES

Book 4 Extend and apply the distributive law
to the expansion of algebraic expressions

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OVERVIEW

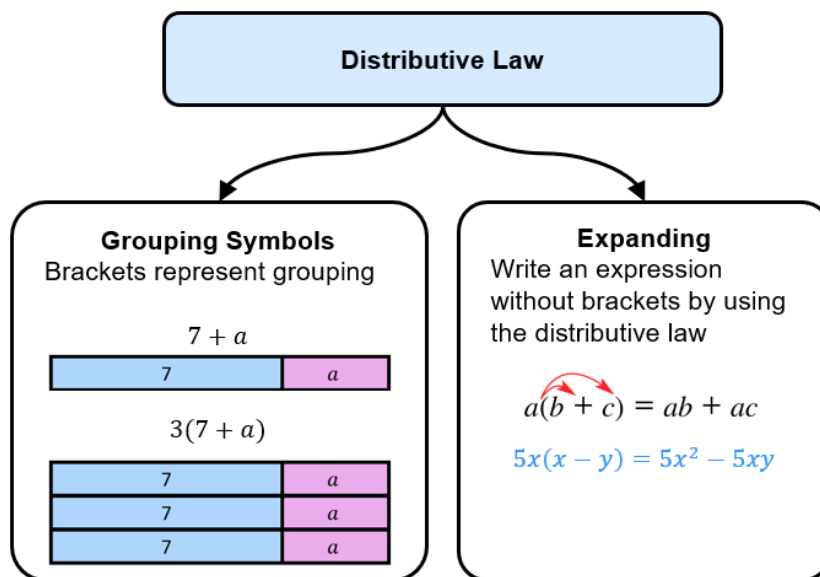
SYLLABUS CONTENT

MA4-ALG-C-01 generalises number properties to operate with algebraic expressions including expansion and factorisation

Extend and apply the distributive law to the expansion of algebraic expressions

- Explain the role and meaning of grouping symbols in algebraic expressions
- Apply the distributive law to expand and simplify algebraic expressions by removing grouping symbols

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Complete each multiplication using the distributive law.

a 15×11
 $= 15 \times (10 + 1)$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

b 34×9
 $= 34 \times (10 - 1)$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

c 45×8
 $\dots\dots\dots$
 $\dots\dots\dots$
 $\dots\dots\dots$

2 Write these expressions using algebra.

a 5 is added to x ,
then the result is multiplied by 3.

b The sum of x and 2 is
then multiplied by 5.

c 4 is subtracted from x ,
then the result is multiplied by 7.

d x is subtracted from 12,
then the result is multiplied by 0.5.

3 Simplify these expressions.

a $3 \times 2a$

b $7y \times 3$

c $-3 \times 4y$

d $4 \times -3x$

e $8 \times -2a$

f $-9b \times 3a$

g $5x \times 4x$

h $3xy \times -6x$

4 The expression $3(a + 2)$ can be written as $(a + 2) + (a + 2) + (a + 2)$
Simplify this expression by collecting like terms.

5 Write out $3(p - 4)$ in full and simplify.

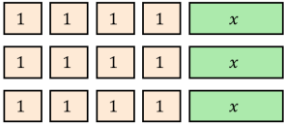
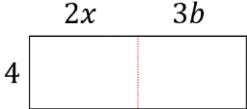
6 Write out $4(2p - 1)$ in full and simplify.

EXPANDING BRACKETS: PART 1

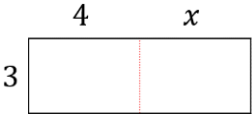
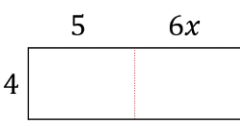
- We **group** terms using **brackets**.
- **Expanding brackets** means simplifying an expression by **writing without brackets**.
- To expand brackets, we multiply each term inside the brackets by the term outside. This is called the **distributive law**.

Distributive Law
To expand brackets, we use the distributive law: $a(b + c) = ab + ac$

Example Expand algebraic expressions using the distributive law.

$3(4 + x)$ $= 12 + 3x$ 	How is $3(4 + x)$ different to $3(4) + x$? $3(4) + x$: <i>you are only multiplying by 3.</i> <i>In $3(4 + x)$: you are multiplying by 3, $(4 + x)$ is treated as a g.....</i>
$4(2x + 3b)$ $= 8x + 12b$ 	Complete the area model to represent this calculation.

Expand these expressions.

a $2(1 + 3x)$  $2 + 6x$	b $3(4 + x)$  $12 + 3x$	c $4(5 + 2x)$  $20 + 8x$
d $12(a + 3)$ $12a + 36$	e $7(a + 2b)$ $7a + 14b$	f $5(3p + 2)$ $15p + 10$

Distributive Law

$$a(b - c) = ab - ac$$

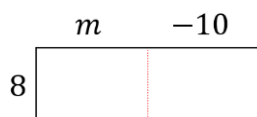
Example Expand algebraic expressions using the distributive law.

$$\begin{aligned} &3(4a - 5) \\ &= 12a - 15 \end{aligned}$$

$$\begin{aligned} &4(2x - y) \\ &= 8x - 4y \end{aligned}$$

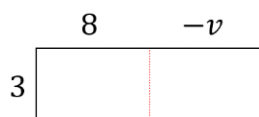
Expand these expressions.

a $8(m - 10)$



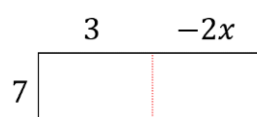
$$8m - 80$$

b $3(8 - v)$



$$24 - 3v$$

c $7(3 - 2x)$



$$21 - 14x$$

d $2(3c - v)$

$$6c - 2v$$

e $5(2x - y)$

$$10x - 5y$$

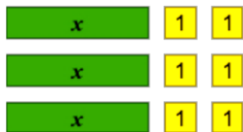
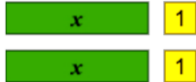
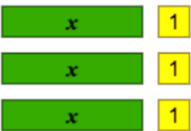
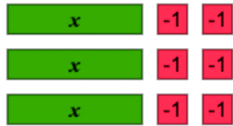
f $6(2h - 9m)$

$$12h - 54m$$

Key ideas

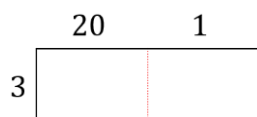
1. Brackets in algebraic expression represent
2. To “expand” an algebraic expression means to write it without
3. To expand, we use the law:
Multiply term inside the brackets by the term the brackets.

1 Complete the table.

Expression	Diagram	Expanded Expression
$3(x + 2)$		$3x + 6$
$2(x + 1)$		$2x + 2$
$2(x - 2)$		$2x - 4$
$2(x + 4)$		$2x + 8$
$3(x + 1)$		$3x + 3$
$3(x - 2)$		$3x - 6$

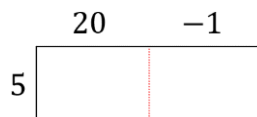
2 Expand these expressions using the distributive law.

a $3 \times (20 + 1)$



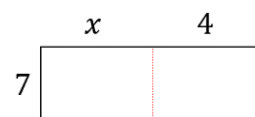
63

b $5 \times (20 - 1)$



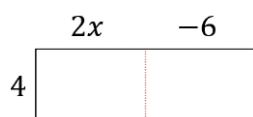
95

c $7(x + 4)$



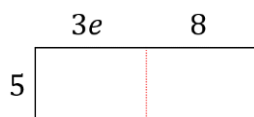
$7x + 28$

d $4(2x - 6)$



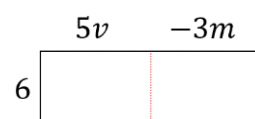
$8x - 24$

e $5(3e + 8)$



$15e + 40$

f $6(5v - 3m)$



$30v - 18m$

3 Expand these expressions using the distributive law.

a $6(y + 8)$

b $8(s + 7)$

c $-7(x + 5)$

d $9(x - 9)$

$6y + 48$

e $8(y - 4)$

$8s + 56$

f $-6(3 - e)$

$-7x - 35$

$9x - 81$

$8y - 32$

$-18 + 6e$

4 Expand these expressions using the distributive law.

a $10(6g - 7)$

b $5(7w + 10)$

c $-7(8x - 2)$

d $7(q - 7)$

$60g - 70$

e $2(2u - 6)$

$35w + 50$

f $-6(8l + 5)$

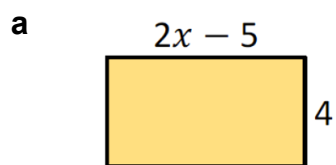
$-56x + 14$

$7q - 49$

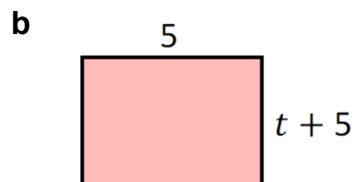
$4u - 12$

$-48l - 30$

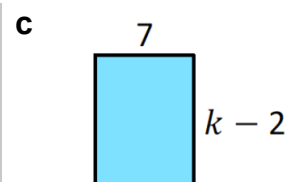
5 The area of a rectangle is $A = \text{length} \times \text{width}$.
Write an expression for the area of these rectangles.



$8x - 20$



$5t + 25$



$7k - 14$

6 Expand the brackets.

a $3(x + 4)$

b $3(x - 4)$

c $3(2x + 4)$

$3x + 12$

$3x - 12$

$6x + 12$

d $3(2x - 4)$

$6x - 12$

e $3(2x - 5y + 4)$

$6x - 15y + 12$

7 Write an expression for each of the following and then expand it.

a A number, x , has 3 added to it and the result is multiplied by 5.

$$5x + 15$$

b A number, b , has 6 added to it and the result is doubled.

$$2b + 12$$

c A number, z , has 4 subtracted from it and the result is multiplied by 3.

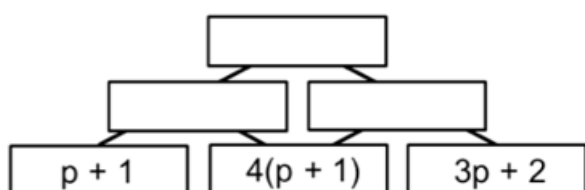
$$3z - 12$$

d A number, y , is subtracted from 10 and the result is multiplied by 7.

$$7y - 70$$

8 Complete each pyramid. Each expression is the sum of the two expressions below it.

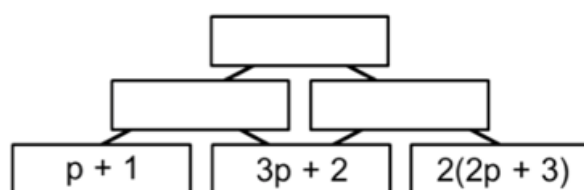
a



$$12p + 11$$

$$5p + 5, 7p + 6$$

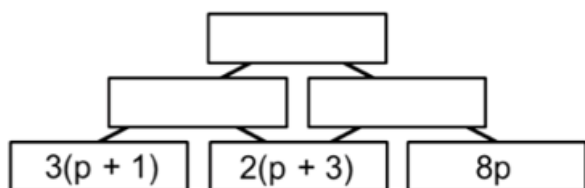
b



$$11p + 11$$

$$4p + 3, 7p + 8$$

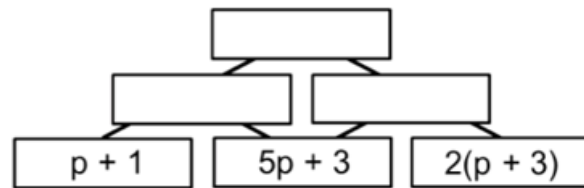
c



$$15p + 15$$

$$5p + 9, 10p + 6$$

d



$$13p + 13$$

$$6p + 4, 7p + 9$$

9 The number of boys in a classroom is b and the number of girls is g . Each boy has 5 pencils and each girl has 3 pencils.

a Write an expression for the total number of pencils in the class.

$$5b + 3g$$

b If the pencils cost \$2 each, write and expand an expression for the total cost of all the pencils in the room.

$$10b + 6g$$

c Each boy and girl also has one pencil case, costing \$4 each. Write a simplified and expanded expression for the total cost of all pencils and cases.

$$14b + 10g$$

d If there are 10 boys and eight girls in the room, what is the total cost for all the pencils and cases in the room?

10 Complete these sequences of operations.

a

$k + 2$

$\rightarrow$

$+4$

$\rightarrow$

$\rightarrow$

$\times 3$

$\rightarrow$

$k + 6, 3k + 18$

b

$k + 2$

$\rightarrow$

$\times 3$

$\rightarrow$

$\rightarrow$

$+4$

$\rightarrow$

$3k + 6, 3k + 10$

c

$2m + 1$

$\rightarrow$

$\times 2$

$\rightarrow$

$\rightarrow$

$+3$

$\rightarrow$

$4m + 2, 4m + 5$

d

$2m + 1$

$\rightarrow$

$+3$

$\rightarrow$

$\rightarrow$

$\times 2$

$\rightarrow$

$2m + 4m, 4m + 8$

e

$6y + 9$

$\rightarrow$

$\div 3$

$\rightarrow$

$\rightarrow$

$+12$

$\rightarrow$

$2m + 3, 2m + 15$

f

$6y + 9$

$\rightarrow$

$+12$

$\rightarrow$

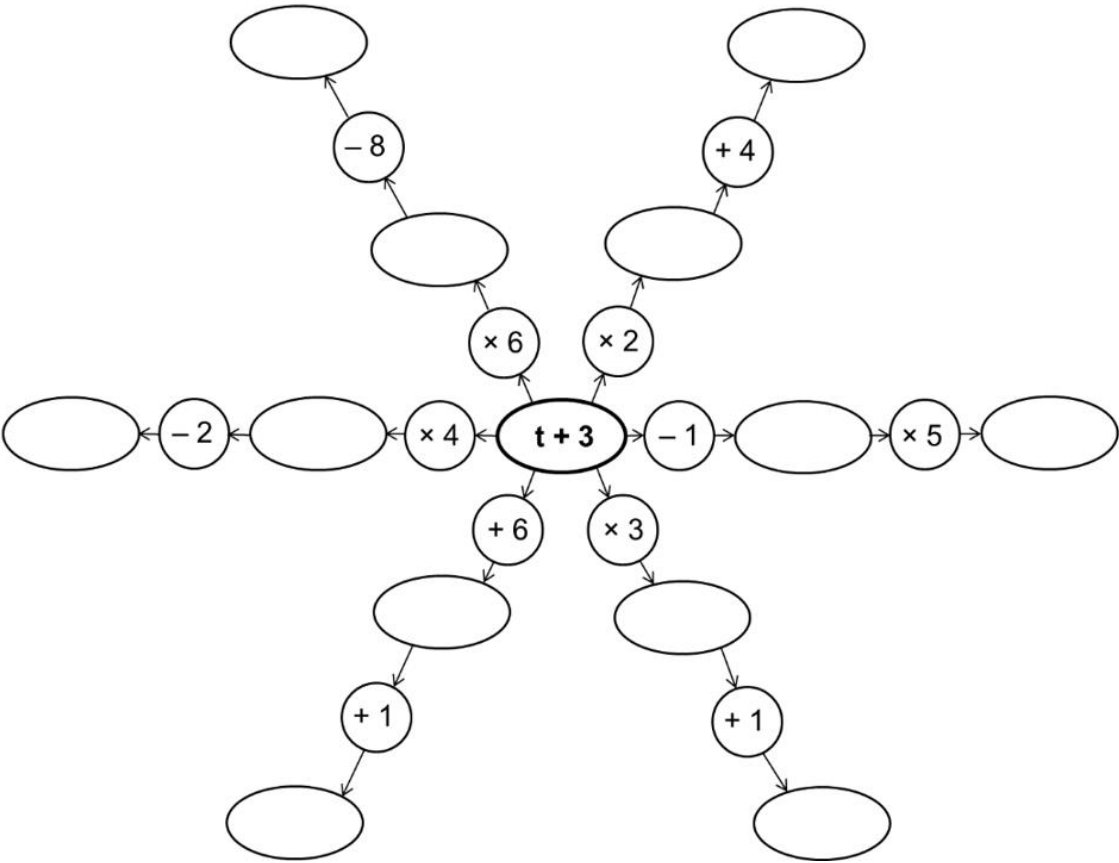
$\rightarrow$

$\div 3$

$\rightarrow$

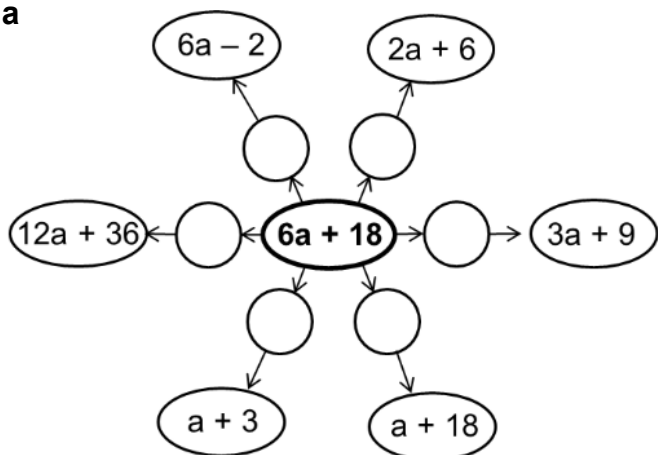
$6y + 21, 2y + 7$

12 Start in the middle and work outwards. Fill in the missing expressions.



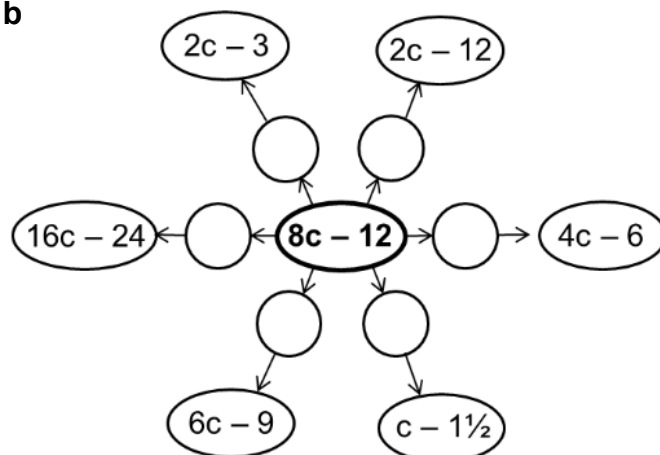
11 Start in the middle and work outwards. What are the missing operations?

a



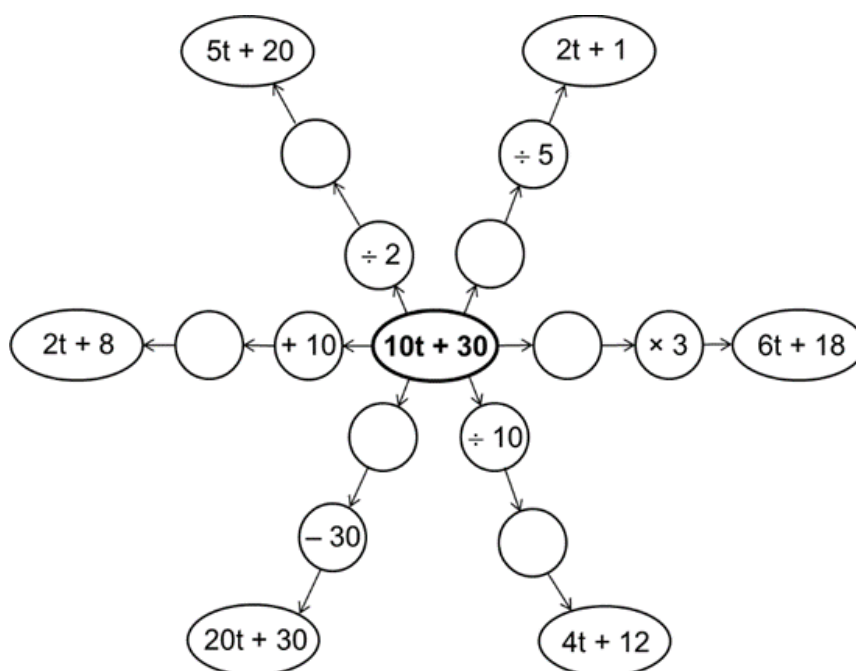
(clockwise from top left): $\div 2$, $-3b$, $\times 2$, -10 , $\div 4$, $\times 5$

b



(clockwise from top left): $\div 4$, $-6c$, $\div 2$, $\div 8$, $\times \frac{3}{4}$, $\times 2$

12 Start in the middle and work outwards. What are the missing operations?



(clockwise from top left): $+5$, -25 , $\div 5$, $\times 4$, $\times 2$, $\div 5$

13

a The side length of a square is $x + 2$. Write an expression for the perimeter.

$4(x + 2)$ or $4x + 8$

b The side length of a regular hexagon is $2x + 5$. Write an expression for the perimeter.

$6(2x + 5)$ or $12x + 30$

14 Expand these expressions involving fractions.

a

$$\frac{1}{3}(6x + 15)$$

$2x + 5$

b

$$\frac{1}{4}(12x + 8y)$$

$3x + 2y$

c

$$\frac{3}{4}(16 - 12y)$$

$12 - 8y$

d

$$\frac{4x + 10y}{2}$$

$2x + 5y$

EXPANDING BRACKETS: PART 2

- The term outside brackets can be algebraic.
- We expand using the same method as before.

Example Expand algebraic expressions using the distributive law.

$x(y + 5)$ $= xy + 5x$	$4x(2x - 3y)$ $= 8x^2 - 12xy$
------------------------	-------------------------------

FOUNDATION

1 Expand these expressions.

a $x(y - 3)$	b $d(x + 9s)$	c $5a(2x + 3)$
d $5c(2w - 4)$	e $-7y(4x + 7)$	f $5j(r + 7p)$
g $-8d(t - 3d)$	h $-7k(3x + 2)$	i $14k(y + 1)$

2 Read the example carefully and answer the questions.

Pablo's work:

$$4w(5+12)$$

$$4w(5+12)$$

$$20w+48$$

What did Pablo forget when distributing the $4w$?

He forgot to write the with 48.

What should Pablo's final expression be?

Expand the following: $4x(3 + 11)$

3 Read the examples carefully and answer the questions.

Michael's work:

$$x(x-4)$$

$$x^2 - 4x$$

Where did x^2 come from in Michael's answer?

$$\dots \times \dots = x^2$$

Expand the following: $3x(x - 4)$

$$3x^2 - 12x$$

4 Expand the brackets.

a $a(a + 4)$

b $a(b + 4)$

c $a(b - 4)$

d $a(4 - b)$

e $a(4 - a)$

$a^2 + 4a$

$ab + 4a$

$ab - 4a$

$4a - ab$

$4a - a^2$

f $a(3a - 4)$

g $a(4 - 3a)$

h $3a(a - 4)$

i $3a(2a - 4)$

j $3a(4 - 2a + 5b)$

$3a^2 - 4a$

$4a - 3a^2$

$3a^2 - 12a$

$6a^2 - 12a$

$12a - 6a^2 + 15ab$

5 Expand these expressions, then simplify the result.

a $2(x + 1) + 3(x + 4)$

b $3(x + 4) + 2(x + 1)$

$5x + 14$

$5x + 14$

c $x(x - 1) + 1(x - 1)$

d $a(b + 3) + 2(b + 3)$

$x^2 - 1$

$ab + 3a + 2b + 6$

6 Fill in the empty boxes.

a $\square(x + 4) = 2x + 8$

2

b $\square(2x - 3) = 8x - 12$

4

c $\square(2x + 3) = 6x^2 + 9x$

3x

d $4(\square + 5) = 12x + 20$

3x

e $4y(\square - \square) = 4y^2 - 4y$

y, 1

f $-2x(\square + \square) = -4x^2 - 6xy$

2x, 3y

7 Find three ways to fill in the empty boxes.
(You can use decimals and fractions!)

a

$$\square(\square x + \square y) = 10x + 20y$$
$$\square(\square x + \square y) = 10x + 20y$$
$$\square(\square x + \square y) = 10x + 20y$$

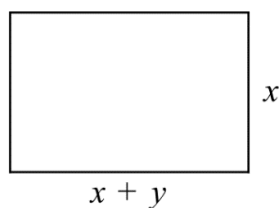
b

$$\square(\square + \square) = 12x + 18xy$$
$$\square(\square + \square) = 12x + 18xy$$
$$\square(\square + \square) = 12x + 18xy$$

8 Fill in the table.

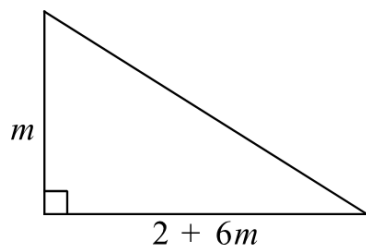
Start	+ 3x	+6y	× 4	× 2y	× 5x	× (2x + 7y)
10x	13x	10x + 6y	40x	20xy	50x ²	20x ² + 70xy
3y	3y + 3x	9y	12y	6y ²	15xy	6xy + 21y ²
-x	2x	-x + 6y	-4x	-2xy	-5x ²	-2x ² - 7xy
-4y	3x - 4y	2y	-16y	-8y ²	-20xy	-8xy
5x	8x	5x + 6y	20x	10xy	25x ²	10x ² + 35xy
7y	7y + 3x	13y	28y	14y ²	35xy	14xy + 49y ²
9xy	9xy + 3x	9xy + 6y	36xy	18xy ²	45x ² y	18x ² y + 63xy ²

- 9 Write an algebraic expression and expand it to represent the area of the rectangle.



$$x^2 + xy$$

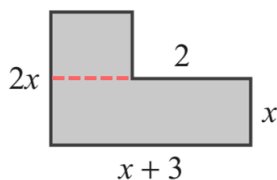
- 10 Write a fully simplified expression to represent the area of the triangle.



$$m + 3m^2$$

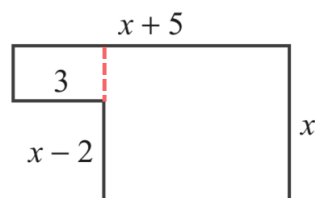
- 11 Find the area of these shapes in simplest expanded form.

a



$$2x^2 + 4x$$

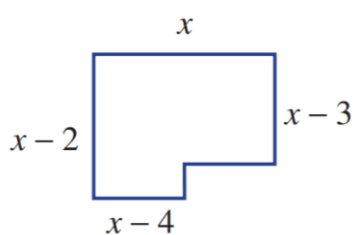
b



$$6 + x^2 + 2x$$

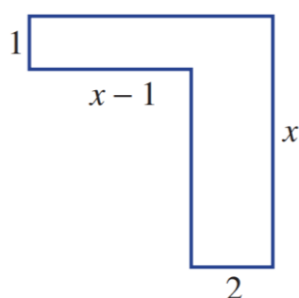
- 12 Find expressions, in simplest form, for the perimeter (P) and area (A) of these shapes.

a



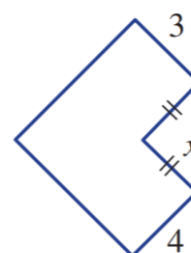
$$\begin{aligned} \text{P: } &4x - 4 \\ \text{A: } &x^2 - 2x - 4 \end{aligned}$$

b



$$\begin{aligned} \text{P: } &4x + 2 \\ \text{A: } &3x - 1 \end{aligned}$$

c



$$\begin{aligned} \text{P: } &4x + 14 \\ \text{A: } &7x + 4 \end{aligned}$$

CHECK YOUR UNDERSTANDING

Expanding brackets

1	Expand $3(p + 1)$	a	$3p + 3$	b	$3p + 1$	c	$p + 3$	d	$3p + 4$
2	Expand $3(p - 1)$	a	$p - 3$	b	$3p - 1$	c	$3p - 3$	d	$3p - 2$
3	Expand $3(2p - 1)$	a	$2p - 3$	b	$6p - 1$	c	$5p - 3$	d	$6p - 3$
4	Expand $3(1 - 2p)$	a	$4 - 6p$	b	$3 - 6p$	c	$1 - 6p$	d	$6p - 3$
5	Expand $4(3c + 2)$	a	$7c + 6$	b	$12c + 8$	c	$12c + 6$	d	$12c + 2$
6	Expand $5(2a - 4)$	a	$10a - 20$	b	$7a - 4$	c	$10a - 4$	d	$-10a$
7	Expand $4c(5d - 3)$	a	$8c^2d$	b	$20cd - 12c$	c	$9cd + c$	d	$20cd^2 - 12c$
8	Expand $3x(5x - 1)$	a	$12x^2$	b	$15x^2 - 3x$	c	$8x^2 - 3x$	d	$12x$