

Student Name

Mathematics Stage 5 Paths

ALGEBRAIC TECHNIQUES B

Book 3 Expand binomial products and factorise monic quadratic expressions

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview	2
Expanding Binomial Products	4
Factorising Monic Quadratic Trinomials	10
Check your Understanding.....	18

OVERVIEW

SYLLABUS CONTENT

MA5-ALG-P-01 simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions

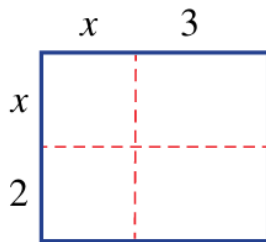
Expand binomial products and factorise monic quadratic expressions

- Expand binomial products
- Factorise monic quadratic trinomial expressions
- Solve problems involving expansion and factorisation of algebraic expressions

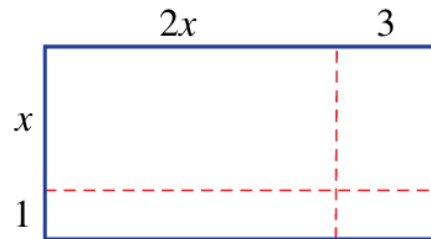
REVIEW OF PRIOR KNOWLEDGE

1 Expand these expressions using the diagrams.

a $(x+2)(x+3)$



b $(x+1)(2x+3)$



2 Find two integers that have:

a a product of 12
sum to 7

b a product of 20
sum to 9

c a product of 6
sum to 7

d a product of 15
sum to 8

e a product of 8
sum to 6

f a product of 24
sum to 11

g a product of 63
sum to 16

h a product of 125
sum to 26

i a product of 36
sum to 13

3 Find two integers that have:

a a product of -12
sum to 1

b a product of -12
sum to -4

c a product of -6
sum to -5

d a product of 20
sum to -9

e a product of -108
sum to 3

f a product of 9
sum to -6

g a product of -16
sum to 0

h a product of -120
sum to 13

i a product of -72
sum to -1

EXPANDING BINOMIAL PRODUCTS

Distributive Law for Binomial Products

$$(a + b)(c + d) = ac + ad + bc + bd$$

Example Expand binomial products.

$$(x + 3)(x + 5)$$

$$(x + 3)(x + 5) = x^2 + 5x + 3x + 15$$

$$= x^2 + 8x + 15$$

$$(2x - 4)(x + 7)$$

$$(2x - 4)(x + 7) = 2x^2 + 14x - 4x - 28$$

$$= 2x^2 + 10x - 28$$

Expand and simplify these expressions.

a $(x + 4)(x + 2)$

=

=

$$x^2 + 6x + 8$$

b $(x - 5)(x - 2)$

=

=

$$x^2 - 7x + 10$$

c $(2x - 1)(x - 6)$

=

=

$$2x^2 - 13x + 6$$

d $(5x - 2)(3x + 7)$

=

=

$$15x^2 + 29x - 14$$

Key ideas

1. Binomial means terms.
2. A binomial product means to multiply binomials.
3. We can multiply binomials using the rule:

$$(a + b)(x + y) = a(\quad) + b(\quad)$$

=

1 Expand these expressions.

a $(x + 3)(x - 4)$

b $(x - 6)(x + 2)$

c $(x - 2)(x + 7)$

$x^2 - x - 12$

$x^2 - 4x - 12$

$x^2 + 5x - 14$

d $(4x + 3)(2x + 5)$

e $(2x - 3)(3x + 5)$

f $(5x + 2)(2x - 7)$

$8x^2 + 26x + 15$

$6x^2 + x - 15$

$10x^2 - 31x - 14$

g $(3x - 2)(6x - 5)$

h $(2x + 3)(3x - 2)$

i $(4x + 1)(4x - 5)$

$18x^2 - 27x + 10$

$6x^2 + 5x - 6$

$16x^2 - 16x - 5$

j $(5x - 2)(3x - 1)$

k $(3x - 2)(2x + 1)$

l $(7x - 3)(3x - 4)$

$15x^2 - 11x + 2$

$6x^2 - x - 2$

$21x^2 - 37x + 12$

m $2(x + 3)(x + 4)$
 $= 2(x^2 + 3x + 4x + 12)$
 $=$

n $-2(x + 8)(x + 2)$

o $-4(x + 9)(x + 2)$

$2x^2 + 14x + 24$

$-2x^2 - 20x - 32$

$-4x^2 - 44x - 72$

p $3(x + 5)(x - 3)$

q $-3(a + 2)(a - 7)$

r $4(a - 3)(a - 6)$

$3x^2 + 6x - 45$

$-3a^2 + 15a + 42$

$4a^2 - 36a + 72$

s $3(2x + 3)(2x + 5)$

t $6(3x - 4)(x + 2)$

u $-2(x + 4)(3x - 7)$

$12x^2 + 48x + 45$

$18x^2 + 12x - 48$

$-6x^2 - 10x + 56$

- 2 Compare the expansions of $(x + 5)(x + 2)$ and $(x - 2)(x - 5)$.
What's the same and what's different?

The sign of the coefficient of x is different

- 3 A room in a house with dimensions 4 m by 5 m is to be extended. Both the length and width are to be increased by x m.
- a Write an expanded expression for the area of the new room.

$$x^2 + 9x + 20$$

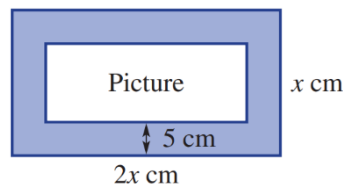
- b If $x = 3$,
- i Find the area of the new room.

$$56 \text{ m}^2$$

- ii By how much the area has increased?

$$36 \text{ m}^2$$

- 4 A picture frame 5 cm wide has a length that is twice the width x cm. :

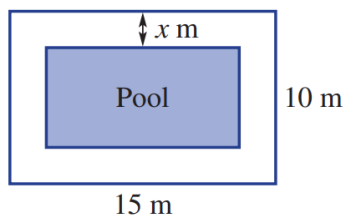


- a Find an expression for the total area of the frame and the picture.
- b Find an expression for the area of the picture, in expanded form.

$$2x^2$$

$$2x^2 - 30x + 100$$

- 5 The outside edge of a path around a rectangular swimming pool is 15 m long and 10 m wide. The path is x metres wide.

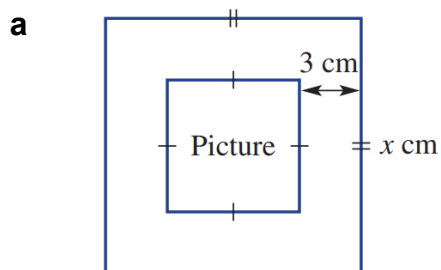


- a Find an expression for the area of the pool in expanded form.
- b Find the area of the pool if $x = 2$.

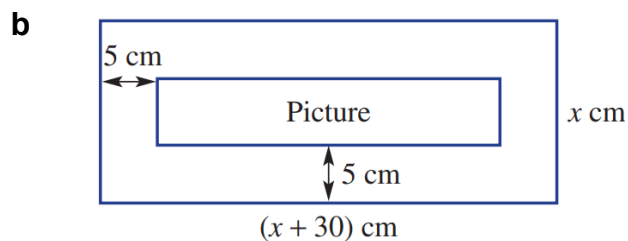
$$(10 - 2x)(15 - 2x) = 4x^2 - 50x + 150$$

$$66 \text{ m}^2$$

6 Find an expanded expression for the area of the pictures centred in these frames.



$$x^2 - 12x + 36$$



$$x^2 + 10x - 200$$

7 Expand and simplify the following.

a $(x + 4)(x + 3) + x - 4$

b $3x^2 + (2x - 1)(x - 2)$

$$x^2 + 8x + 8$$

$$5x^2 - 5x + 2$$

c $(x + 8)(x + 3) + (x + 4)(x + 5)$

d $(y - 7)(y + 4) + (y + 5)(y - 3)$

$$2x^2 + 20x + 44$$

$$2y^2 - y - 43$$

e $(x - 5)(x + 5) + (x + 5)(x + 5)$

f $(2a + 3)(a - 5) - (a + 6)(2a + 5)$

$$2x^2 + 10x$$

$$-24a - 45$$

g $(x - 3)(x - 4) - (x + 2)(x - 1)$

h $3 - (2x - 9)^2$

$$-8x + 14$$

$$-4x^2 + 36x - 78$$

8 Show, by expansion, that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Use this fact to find possible values of a and b for these expansions.

a $(x + a)(x + b) = x^2 + 5x + 6$ $(x + 2)(x + 3)$	b $(x + a)(x + b) = x^2 - 5x + 6$ $(x - 2)(x - 3)$
c $(x + a)(x + b) = x^2 + x - 6$ $(x + 3)(x - 2)$	d $(x + a)(x + b) = x^2 - x - 6$ $(x - 3)(x + 2)$

9 Write the missing terms in these expansions.

a $(x + 2)(x + \underline{\hspace{1cm}}) = x^2 + 5x + 6$ 3	b $(x + \underline{\hspace{1cm}})(x + 5) = x^2 + 7x + 10$ 2
c $(x + 1)(x + \underline{\hspace{1cm}}) = x^2 + 7x + \underline{\hspace{1cm}}$ 6, 6	d $(x + \underline{\hspace{1cm}})(x + 9) = x^2 + 11x + \underline{\hspace{1cm}}$ 2, 18
e $(x + 3)(x - \underline{\hspace{1cm}}) = x^2 + x - \underline{\hspace{1cm}}$ 2, 6	f $(x - 5)(x + \underline{\hspace{1cm}}) = x^2 - 2x - \underline{\hspace{1cm}}$ 3, 15
g $(x + 1)(\underline{\hspace{1cm}} + 3) = 2x^2 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ 2x, 5x, 3	h $(\underline{\hspace{1cm}} - 4)(3x - 1) = 9x^2 - \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ 3x, 15x, 4
i $(x + 2)(\underline{\hspace{1cm}} + \underline{\hspace{1cm}}) = 7x^2 + \underline{\hspace{1cm}} + 6$ 7x, 3, 17x	j $(\underline{\hspace{1cm}} - \underline{\hspace{1cm}})(2x - 1) = 6x^2 - \underline{\hspace{1cm}} + 4$ 3x, 4, 2x

10 Using the area model, or otherwise, expand these expressions.

a $(x + 1)(x^2 + x + 1)$ $x^3 + 2x^2 + 2x + 1$	b $(x - 2)(x^2 - x + 3)$ $x^3 - 3x^2 + 5x - 6$
c $(2x + 1)(2x^2 - x + 4)$ $4x^3 + 7x + 4$	d $(x^2 - x + 1)(x + 3)$ $x^3 + 2x^2 - 2x + 3$
e $(5x^2 - x + 2)(2x - 3)$ $10x^3 - 17x^2 + 7x - 6$	f $(2x^2 - x + 7)(4x - 7)$ $8x^3 - 18x^2 + 35x - 49$
g $(x + 1)(x + 2)(x + 3)$ $x^3 + 6x^2 + 11x + 6$	

- 11 All these expressions are equivalent. Find the values of all the letters, if possible.

$$(x + 5)(x + 4)$$

$$(x + 3)(x + 6) + a$$

$$(x + 2)(x + 10) + bx$$

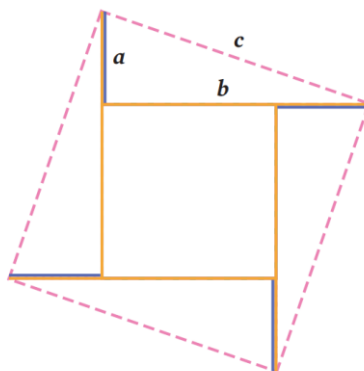
$$(x + 5)^2 + cx + d$$

$$(x + 1)(x + e) + fx + g$$

$$a = 2, b = -3, c = -1, d = -5$$

$$e = 1, f = 1, g = 19 \text{ (many answers are possible)}$$

- 12 Four congruent triangles make a square with side lengths equal to c .



- a What is the base and height of each triangle?

$$a, b$$

- b Find the area of each triangle.

$$\frac{ab}{2}$$

- c What is the side length of the inner square?

$$b - a$$

- d Find the area of the inner square.

$$b^2 - 2ab + a^2$$

- e Write an expression for the combined area of the four triangles and the inner square in terms of a and b .

$$a^2 + b^2$$

- f What theorem have you proven?

$$a^2 + b^2 = c^2$$

Pythagoras' Theorem

FACTORISING MONIC QUADRATIC TRINOMIALS

- A **trinomial** is an expression with **three terms**.
- A **quadratic** expression has highest power x^2 .
- A **monic** quadratic has **1** as the coefficient of x^2 .
- A **monic quadratic trinomial** is an expression of the form $x^2 + bx + c$

Example Identify monic quadratic trinomials.

Determine whether these are monic quadratic trinomials.		
a $x^2 + 3x + 5$	b $2x^2 + 3x + 5$	c $-x^2 + 3x + 5$
d $x^2 - 3x - 9$	e $8 + 2x + x^2$	f $7 + 2x - x^2$
g $5x^2 - 7x + 3$	h $x^3 + x^2 + 12x + 7$	i $(x + 3)(x - 8)$
j $(x - 8)(x - 2)$	k $(2x + 3)(x + 5)$	l $(8 - x)(x + 3)$

Investigation Rule for expanding $(x + \alpha)(x + \beta)$.

a $(x + 5)(x + 3) =$	a $(x + 5)(x + 3) =$
b $(x + 5)(x + 2) =$	b $(x + 5)(x + 2) =$
c $(x + 5)(x + 0) =$	c $(x + 5)(x + 0) =$
d $(x + 5)(x - 1) =$	d $(x + 5)(x - 1) =$
e $(x + 5)(x - 2) =$	e $(x + 5)(x - 2) =$
f $(x + 5)(x - 3) =$	f $(x + 5)(x - 3) =$
Complete the sentence: To find the product of two monic binomials $(x + \alpha)(x + \beta)$, the coefficient of x is and the constant term is Complete the following:	
g $(x + 1)(x - 12) =$	g $(x + 2)(x - 12) =$
h $(x + 3)(x - 20) =$	h $(x + 7)(x - 5) =$
i $(x \quad)(x \quad) = x^2 + 7x + 12$	i $(x \quad)(x \quad) = x^2 + 3x - 10$
j $(x \quad)(x \quad) = x^2 + 17x + 70$	j $(x \quad)(x \quad) = x^2 - 2x - 24$

Factorising Monic Quadratic Trinomials (PSF Method)

To factorise a monic quadratic trinomial: $x^2 + bx + c = 0$:

1. Identify the product: c
2. Identify the sum: b
3. Think of a factor pair, α and β , that has a product of c and sum to b .
4. Factorise $x^2 + bx + c = (x + \alpha)(x + \beta)$

Example Factorise monic quadratic trinomials.

$x^2 - 13x + 36$ 4. $(x - 4)(x - 9)$ 1. P: 36 2. S: -13 3. F: -4, -9	<i>To factorise $x^2 + bx + c$ to $(x + \alpha)(x + \beta)$</i> <i>The two factors, α and β have a product of and sum of</i>
$a^2 + 12a - 28$ 4. $(a + 14)(a - 2)$ 1. P: -28 2. S: 12 3. F: 14, -2	Show why this method works by expanding $(x + \alpha)(x + \beta)$

Factorise these expressions.

a $a^2 + 3a + 2$ $(a \quad)(a \quad)$ P: 2 S: 3 F: $(a + 2)(a + 1)$	b $y^2 + 9y + 20$ P: S: F: $(y + 4)(y + 5)$
c $x^2 - 7x + 12$ P: 12 S: -7 F: $(x - 3)(x - 4)$	d $w^2 - 2w - 8$ P: S: F: $(w - 4)(w + 2)$
e $y^2 - 10y + 21$ P: S: F: $(y - 7)(y - 3)$	f $x^2 - 5x - 24$ P: S: F: $(x - 8)(x + 3)$

Key ideas

1. The expansion of $(x + \alpha)(x + \beta) = \dots\dots\dots$
2. This means that to “un-expand,” or factorise, we need to find two numbers that sum to the of x and whose product is the term.

1 Fill in the table.

Binomial Product	Expansion	Factors	Sum	Product
$(x + 1)(x + 3)$	$x^2 + 3x + x + 3$ $= x^2 + 4x + 3$	1, 3	4	3
$(x + 2)(x + 7)$	$x^2 + 9x + 14$	2, 7	9	14
$(x - 3)(x + 11)$	$x^2 + 8x - 33$	-3, 11	8	-33
$(x + 12)(x - 5)$	$x^2 + 7x - 60$	12, -5	7	-60
$(x - 2)(x - 6)$	$x^2 - 8x + 12$	-2, -6	-8	12

2 Fill in the table.

Factor 1	Factor 2	Sum	Product
5	7	12	35
-5	7	2	-35
5	-7	-2	-35
-5	-7	-12	35
3	5	8	15
-3	5	2	-15
3	-5	-2	-15
-3	-5	-8	15
4	6	10	24
-4	6	2	-24
4	-6	-2	-24
-4	-6	-10	24

3 Factorise each expression using the PSF method.

a $x^2 + 3x + 2$

P:

S:

F:

$(x + 1)(x + 2)$

b $x^2 + 7x + 6$

P:

S:

F:

$(x + 6)(x + 1)$

c $x^2 + 8x + 7$

P:

S:

F:

$(x + 7)(x + 1)$

d $x^2 + 6x + 8$

P:

S:

F:

$(x + 4)(x + 2)$

e $x^2 + 3x + 2$

P:

S:

F:

$(x + 2)(x + 1)$

f $x^2 + 7x + 6$

P:

S:

F:

$(x + 6)(x + 1)$

g $x^2 + 10x + 16$

P:

S:

F:

$(x + 8)(x + 2)$

h $x^2 + 9x + 20$

P:

S:

F:

$(x + 4)(x + 5)$

i $x^2 + 3x - 4$

P:

S:

F:

$(x + 4)(x - 1)$

j $x^2 + 4x - 5$

P:

S:

F:

$(x + 5)(x - 1)$

k $x^2 + 2x - 15$

P:

S:

F:

$(x + 5)(x - 3)$

l $x^2 + 3x - 18$

P:

S:

F:

$(x + 6)(x - 3)$

m $x + x^2 - 12$

P:

S:

F:

$(x - 3)(x + 4)$

n $14 + 15x + x^2$

P:

S:

F:

$(x + 14)(x + 1)$

o $x^2 + 24 + 11x$

P:

S:

F:

$(x + 3)(x + 8)$

p $x^2 - 20 + 8x$

P:

S:

F:

$(x + 10)(x - 2)$

4 Write the missing number.

a $(x + \square)(x + 2) = x^2 + 5x + 6$

3

b $(x + \square)(x + 5) = x^2 + 8x + 15$

3

c $(x + 7)(x - \square) = x^2 + 4x - 21$

3

d $(x + 4)(x - \square) = x^2 - 4x - 32$

8

e $(x - 6)(x - \square) = x^2 - 7x + 6$

1

f $(x - \square)(x - 8) = x^2 - 10x + 16$

2

5 Factorise each expression using the PSF method.

a $x^2 - 6x + 5$

$(x - 5)(x - 1)$

b $x^2 - 5x + 4$

$(x - 4)(x - 1)$

c $x^2 - 4x + 4$

d $x^2 - 11x + 18$

$(x - 2)^2$

$(x - 9)(x - 2)$

e $x^2 - 5x + 6$

f $x^2 - 7x - 8$

$(x - 3)(x - 2)$

$(x - 8)(x + 1)$

g $x^2 - 5x - 6$

h $x^2 - 2x - 24$

$(x + 1)(x - 6)$

$(x + 4)(x - 6)$

i $x^2 - x - 12$

j $x^2 - 4x - 12$

$(x - 4)(x + 3)$

$(x - 6)(x + 2)$

k $12 - 8x + x^2$

l $x^2 + 21 - 10x$

$(x - 2)(x - 6)$

$(x - 7)(x - 3)$

m $x^2 - 15 - 2x$

n $-11x + x^2 - 12$

$(x - 5)(x + 3)$

$(x - 12)(x + 1)$

6 Match each quadratic expression to its factored form.

Expanded form	Factored form
1 $x^2 + 14x + 24$	A $(x - 8)(x + 3)$
2 $x^2 + 10x - 24$	B $(x - 6)(x - 4)$
3 $x^2 + 10x + 24$	C $(x + 12)(x - 2)$
4 $x^2 - 10x + 24$	D $(x + 8)(x - 3)$
5 $x^2 - 5x - 24$	E $(x + 12)(x + 2)$
6 $x^2 + 5x - 24$	F $(x + 6)(x + 4)$

1E, 2C, 3F, 4B, 5A, 6D

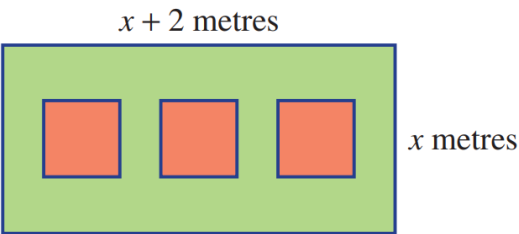
7 Factorise these expressions. Find product, sum, and factors mentally.
Do not write any working out.

a $x^2 + 13x + 36$	j $x^2 + 5x - 24$
(x + 9)(x + 4)	(x + 8)(x - 3)
b $x^2 - 13x + 36$	k $x^2 - 5x - 24$
(x - 9)(x - 4)	(x - 8)(x + 3)
c $x^2 - 20x + 36$	l $x^2 + 20x + 64$
(x - 2)(x - 18)	(x + 4)(x + 16)
d $x^2 + 20x + 36$	m $x^2 + 12x - 64$
(x + 2)(x + 18)	(x + 16)(x - 4)
e $x^2 + 15x + 36$	n $x^2 + 10x + 16$
(x + 12)(x + 3)	(x + 2)(x + 8)
f $x^2 - 15x + 36$	o $x^2 + 10x + 21$
(x - 12)(x - 3)	(x + 3)(x + 7)
g $x^2 + 37x + 36$	p $x^2 + 10x + 25$
(x + 36)(x + 1)	(x + 5) ²
h $x^2 - 12x + 36$	q $x^2 + 6x - 40$
(x - 6) ²	(x + 10)(x - 4)
i $x^2 + 9x - 36$	r $x^2 + 3x - 40$
(x - 12)(x + 3)	(x + 8)(x - 5)

8 Factorise these expressions by first taking out a common factor.

a $2x^2 + 10x + 8$ $= 2(x^2 + 5x + 4)$ $= 2(\quad)(\quad)$	b $3x^2 + 18x + 24$
$2(x + 4)(x + 1)$	$3(x + 4)(x + 2)$
c $2x^2 - 14x - 36$	d $2x^2 + 2x - 12$
$2(x - 9)(x + 2)$	$2(x + 3)(x - 2)$
e $5x^2 - 30x + 40$	f $2x^2 - 6x - 20$
$5(x - 4)(x - 2)$	$2(x - 5)(x + 2)$
g $3x^2 - 33x + 90$	h $-y^2 + 2y + 15$
$3(x - 5)(x - 6)$	$-(y - 5)(y + 3)$
i $-a^2 + 3a + 10$	j $-4x^2 + 8x - 4$
$-(a - 5)(a + 2)$	$-4(x - 1)^2$

9 A backyard, rectangular in area, has a length 2 metres more than its width (x metres). Inside the rectangle are three square paved areas each of area 5 m^2 as shown. The remaining area is lawn. Find an expression for the:



- a** total backyard area
- b** area of lawn in expanded form
- c** area of lawn in factorised form

$x^2 + 2x$

$x^2 + 2x - 15$

$(x + 5)(x - 3)$

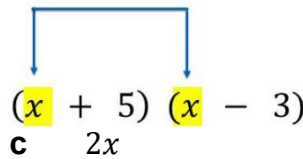
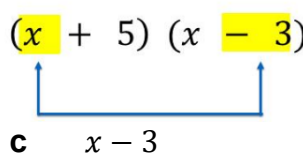
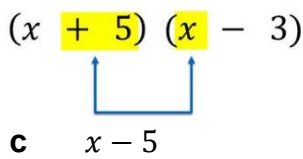
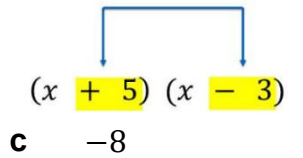
10 Factorise $25x^2 + 70x + 48$

Hint: let $a = 5x$

$(5x + 6)(5x + 8)$

CHECK YOUR UNDERSTANDING

Binomial expansion

1	Identify the product of these two terms.						
a	x	b	x^2	c	$2x$	d	x^2
2	Identify the product of these two terms.						
a	$x + 3$	b	$3x$	c	$x - 3$	d	$-3x$
3	Identify the product of these two terms.						
a	$5x$	b	$x + 5$	c	$x - 5$	d	$-5x$
4	Identify the product of these two terms.						
a	2	b	-15	c	-8	d	15
5	Expand $(x + 4)(x - 1)$						
a	$x^2 - x + 4x + 3$	b	$x^2 + x + 4x - 3$	c	$x^2 - x - 4x + 4$	d	$x^2 - x + 4x - 4$
6	Expand $(x + 5)(x + 9)$						
a	$x^2 + 45$	b	$x^2 + 14x + 45$	c	$60x^2$	d	$x^2 + 59x$
7	Expand $(3x - 2)(x + 1)$						
a	$3x^2 - x - 2$	B	$3x^2 - x + 2$	c	$3x^2 + x - 2$	d	$3x^2 + x + 2$
8	Expand $(x - 2)(1 - x)$						
a	$-x^2 + 3x - 2$	B	$x^2 - 3x + 2$	c	$-x^2 - 3x + 2$	d	$-x^2 - x - 2$

Factorise monic quadratic trinomials

1	When factorising $x^2 + 5x + 6$ we need two numbers whose:						
a	Product is 5 Sum is 6	b	Product is 5 Sum is $6x$	c	Product is 6 Sum is 5	d	Product is 6 Sum is $5x$
2	Niamh and Evan are discussing how to factorise $x^2 + 8x + 12$ Niamh says to list pairs of numbers that add to 12. Evan says to list factor pairs of 12. Whose method is more efficient?						
a	Niamh	b	Evan	c	Both equally efficient	d	Neither method
3	Which factor pair will help factorise $x^2 + 13x + 30$?						
a	1×30	b	2×15	c	3×10	d	5×6
4	$x^2 + 9x + 14 = (x + 2)(\ ? \)$						
a	$x + 12$	b	$x + 8$	c	$x - 7$	d	$x + 7$
5	What signs go in the brackets to correctly factorise this expression? $x^2 + 2x - 8 = (x \star 4)(x \star 2)$						
a	$+ -$	b	$- +$	c	$+ +$	d	$- -$
6	Factorise $x^2 - 18x + 80$						
a	$(x + 8)(x + 10)$	b	$(x - 8)(x - 10)$	c	$(x - 8)(x + 10)$	d	$(x + 8)(x - 10)$