Student Name

Mathematics Stage 5 Paths

ALGEBRAIC TECHNIQUES C

Book 2 Expand, factorise and simplify algebraic expressions including special products

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-ALG-P-02 selects and applies appropriate algebraic techniques to operate with algebraic fractions, and expands, factorises and simplifies algebraic expressions

Expand, factorise and simplify algebraic expressions including special products

- Prove and apply these special products $(a-b)(a+b)=a^2-b^2$, $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$
- Expand and simplify a variety of algebraic expressions including binomial products and the special products
- Factorise algebraic expressions involving the special products and strategies, including common factors, grouping in pairs for 4-term expressions and quadratic trinomials (monic and non-monic)
- Simplify algebraic expressions involving algebraic fractions using factorisation

REVIEW OF PRIOR KNOWLEDGE

1 Factorise the following expressions.

a $7x + 7 = 7(\underline{\qquad\qquad})$

b $7x + 14 = 7(\underline{\qquad\qquad})$

c $14x + 7 = 7(\underline{\qquad\qquad})$

d $14x - 7 = 7(\underline{\qquad\qquad})$

e $-9x + 9 = -9(\underline{\qquad\qquad})$

f $-9x + 9 = 9(\underline{\qquad\qquad})$

g $6 - 12x = 6(\underline{\qquad\qquad})$

h $12 - 6x = 6(\underline{\qquad\qquad})$

2 Factorise by taking out the negative highest common factor.

a $-8x - 4$

b $-4m - 2n$

c $-3x^2 + 18x$

d $-8x^2 + 12x$

e $-6x + 20x^2$

f $-5a^3x - 10a^2$

3 Factorise:

a $5x + yx$

b $5(a + b) + y(a + b)$

c $5(a - 2b) - y(a - 2b)$

4 Expand and simplify these expressions.

a $(p + 4)(q + 6)$

$= p(q + 6) + 4(q + 6)$

$=$ ....

$=$ ....

	p	4
q		
6		

b $(x + 2)(x + 6)$

$= x(\qquad) + 2(\qquad)$

$=$ ....

$=$ ....

	x	6
x		
2		

c $(2a - b)(3a + 2)$

$=$ ....

$=$ ....

$=$ ....

	$3a$	2
$2a$		
$-b$		

d $(4x + 3)(2x + 5)$

$=$ ....

$=$ ....

$=$ ....

	$2x$	5
$4x$		
3		

5 Factorise these expressions by noticing the binomial factor.

a $5(x+1)-x(x+1)$	**b** $m(5m-2)+4(5m-2)$
c $(7-3x)+x(7-3x)$	**d** $b(b-2)+(b-2)$
e $x(2x+1)-(2x+1)$	**f** $(x+4)^2+5(x+4)$

6 Expand these perfect squares using the rule $(a+b)^2=a^2+2ab+b^2$

a $(x+2)^2$	**b** $(3x+4)^2$	**c** $(4+3x)^2$
d $(y-9)^2$	**e** $(5a-4)^2$	**f** $(x+y)^2$
g $(3x+5y)^2$	**h** $(7x-3z)^2$	**i** $(z-n)^2$

7 Expand these differences of two squares using the rule $(a+b)(a-b)=a^2-b^2$

a $(k+7)(k-7)$	**b** $(x+8)(x-8)$	**c** $(12+w)(12-w)$
d $(7b-5)(7b+5)$	**e** $(3h+7)(3h-7)$	**f** $(11+4y)(11-4y)$
g $\left(\frac{2}{3}+y\right)\left(\frac{2}{3}-y\right)$	**h** $\left(\frac{4}{m}-x\right)\left(\frac{4}{m}+x\right)$	**i** $\left(3g+\frac{1}{g}\right)\left(3g-\frac{1}{g}\right)$

8 Simplify the following.

a $\frac{3x}{4y} \times \frac{8y}{5x}$	**b** $\frac{5a}{2b} \times \frac{6b}{15a^2}$	**c** $\frac{12p}{7q} \div \frac{24p}{14q}$
d $\frac{15m}{8n} \times \frac{4n}{9m^3}$	**e** $\frac{21r^2}{10s} \div \frac{3r}{5s}$	**f** $\frac{9x^2}{16y} \div \frac{27x}{12y^2}$

9 Simplify the following.

a $\frac{4}{h} + \frac{7}{2h}$	**b** $\frac{6}{7k} + 8$	**c** $\frac{8w+7}{2w} - \frac{3}{5w}$
d $\frac{x+3}{4} + \frac{x+2}{5}$	**e** $\frac{x+2}{3} + \frac{x+1}{4}$	**f** $\frac{x-3}{4} - \frac{x+2}{2}$
g $\frac{2x+1}{2} - \frac{x-2}{3}$	**h** $\frac{3x+1}{5} + \frac{2x+1}{10}$	**i** $\frac{5x+3}{10} - \frac{2x-2}{4}$

EXPANDING PERFECT SQUARES

Investigation Expand the following.

a $(x+1)^2 = (x+1)(x+1) =$	**a** $(x-1)^2 = (x-1)(x-1) =$
b $(x+2)^2 =$	**b** $(x-2)^2 =$
c $(x+3)^2 =$	**c** $(x-3)^2 =$
d $(x+4)^2 =$	**d** $(x-4)^2 =$
e $(x+5)^2 =$	**e** $(x-5)^2 =$
f $(x+6)^2 =$	**f** $(x-6)^2 =$
g $(x+7)^2 =$	**g** $(x-7)^2 =$
h $(x+8)^2 =$	**h** $(x-8)^2 =$
i $(x+9)^2 =$	**i** $(x-9)^2 =$
j $(x+10)^2 =$	**j** $(x-10)^2 =$

Complete the sentence:

When you square an expression in the form of $(x+a)^2$ or $(x-a)^2$, the coefficient of x is .... and the constant term is ....

Complete the following:

k $(x+12)^2 =$	**k** $(x-13)^2 =$
l $(x+20)^2 =$	**l** $(x-15)^2 =$
m $(\qquad)^2 = x^2 + 28x + 196$	**m** $(\qquad)^2 = x^2 - 22x + 121$
n $(\qquad)^2 = x^2 + 60x + 900$	**n** $(\qquad)^2 = x^2 - 80x + 1600$

EXPANDING PERFECT SQUARES

Perfect Square Expansion
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Example Expand perfect squares.

$(x+4)^2$ $= x^2 + 8x + 16$	$(2x-3y)^2$ $= 4x^2 - 12xy + 9y^2$

Expand these expressions using the perfect square rule.

a $(x+y)^2$ $x^2 + 2xy + y^2$	**b** $(x+3)^2$ $x^2 + 6x + 9$	**c** $(x-7)^2$ $x^2 - 14x + 49$
d $(3x+1)^2$ $9x^2 + 6x + 1$	**e** $(2b-4)^2$ $4b^2 - 16b + 16$	**f** $(8+3f)^2$ $64 + 48f + 9f^2$

Key ideas

1. Expressions of the form $(a+b)^2$ and $(a-b)^2$ are called ……………………. ………………
2. To quickly expand perfect squares, follow the rule:

   $(a+b)^2 =$ ………………………… and $(a-b)^2 =$ …………………………

1 Jimmy expanded $(x+5)^2 = x^2 + 25$. Explain why he is incorrect.

He is missing the middle term $x^2 + 10x + 25$

2 Expand these perfect squares.

a $(x+1)^2$ x^2+2x+1	**b** $(x+3)^2$ x^2+6x+9	**c** $(x+4)^2$ $x^2+8x+16$
d $(x+7)^2$ $x^2+14x+49$	**e** $(x+10)^2$ $x^2+20x+100$	**f** $(x+9)^2$ $x^2+18x+81$
g $(x-2)^2$ x^2-4x+4	**h** $(x-6)^2$ $x^2-12x+36$	**i** $(x-7)^2$ $x^2-14x+49$
j $(x-4)^2$ $x^2-8x+16$	**k** $(x-12)^2$ $x^2-24x+144$	**l** $(x-9)^2$ $x^2-18x+81$

3 Expand these perfect squares.

a $(3-x)^2$ $9-6x+x^2$	**b** $(5-x)^2$ $25-10x+x^2$	**c** $(1-x)^2$ $1-2x+x^2$
d $(11-x)^2$ $121-22x+x^2$	**e** $(7-x)^2$ $49-14x+x^2$	**f** $(8-x)^2$ $64-16x+x^2$

4 Fill in the missing numbers.

a $(x+3)^2 = x^2 + \square x + \square$ $6, 9$	**b** $(x-7)^2 = x^2 - 14x + \square$ 49
c $(x-2)^2 = x^2 \,\_\_\, \square + \square$ $-4x, 4$	**d** $(x-\square)^2 = x^2 - 6x + \square$ $3, 9$
e $(x-\square)^2 = x^2 - 10x + \square$ $5, 25$	**f** $(x+\square)^2 = x^2 + 12x + \square$ $6, 36$
g $(x+\square)^2 = x^2 \,\_\_\, \square + 49$ $7, +14x$	**h** $(x+\square)^2 = x^2 + 18x + \square$ $9, 81$

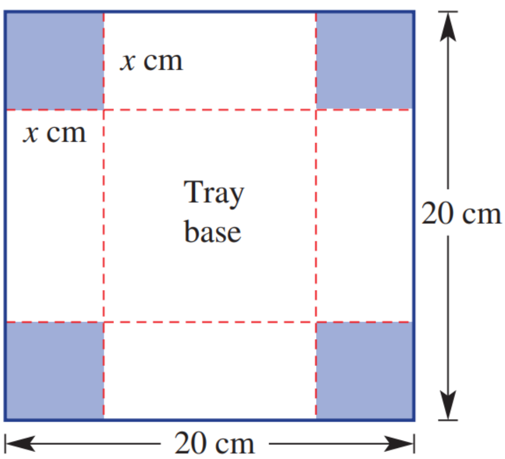
5 Expand these perfect squares.

a $(2x+3)^2$ $4x^2+12x+9$	**b** $(2x+5)^2$ $4x^2+20x+25$	**c** $(3x+2)^2$ $9x^2+12x+4$
d $(5x+2)^2$ $25x^2+20x+4$	**e** $(7+2x)^2$ $49+28x^2+4x^2$	**f** $(5+3x)^2$ $25+30x+9x^2$
g $(5x-2)^2$ $25x^2-20x+4$	**h** $(7-2x)^2$ $49-28x^2+4x^2$	**i** $(5-3x)^2$ $25-30x+9x^2$
g $(2-3x)^2$ $4-12x+9x^2$	**h** $(9-2x)^2$ $81-36x^2+4x^2$	**i** $(10-4x)^2$ $100-80x+16x^2$
j $(3x+5y)^2$ $9x^2+30xy+25y^2$	**k** $(7x+3y)^2$ $49x^2+42xy+9y^2$	**l** $(4x-9y)^2$ $100-72xy+81y^2$
m $(3x-10y)^2$ $9x^2-60xy+100y^2$	**n** $(4x-6y)^2$ $16x^2-48xy+36y^2$	**o** $(6x+5y)^2$ $36x^2+60xy+25y^2$

6 Expand the following.

a $2(x+3)^2$ $2x^2+12x+18$	**b** $4(m+5)^2$ $4m^2+40m+100$	**c** $2(a-7)^2$ $2a^2-28a+98$
d $-3(y+5)(y+5)$ $-3y^2-30y-75$	**e** $3(2b-1)^2$ $12b^2-12b+3$	**f** $-3(2y-6)^2$ $-12y^2+72y-108$

7 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.



a Write an expression for the side length of the base of the tray.

$20-2x$

b Write an expression for the area of the base of the tray. Expand your answer.

$400 = 80x + 4x^2$

c Find the area of the tray base if $x = 3$.

196 cm^2

8 Expand the following perfect squares.

a $\left(x+\frac{1}{3}\right)^2$ $x^2+\frac{2x}{3}+\frac{1}{9}$	**b** $\left(x-\frac{2}{5}\right)^2$ $x^2-\frac{4x}{5}+\frac{4}{25}$	**c** $\left(3x+\frac{5}{7}\right)^2$ $9x^2+\frac{30x}{7}+\frac{25}{49}$
d $\left(3x+\frac{2}{5}\right)^2$ $9x^2+\frac{12x}{5}+\frac{4}{25}$	**e** $\left(\frac{x}{5}-\frac{3}{8}\right)^2$ $\frac{x^2}{25}-\frac{3x}{20}+\frac{9}{64}$	**f** $\left(\frac{2x}{3}+\frac{9y}{8}\right)^2$ $\frac{4x^2}{9}+\frac{3xy}{2}+\frac{81y^2}{64}$

9 Consider this pattern when squaring numbers ending in .5

$1.5^2 = 2.25$	$2.5^2 = 6.25$	$7.5^2 = 56.25$
$1 \times 2 + 0.25 = 2.25$	$2 \times 3 + 0.25 = 6.25$	$7 \times 8 + 0.25 = 56.25$

a How do the whole numbers in the multiplication relate to the original decimal being squared?

The two numbers being used in the multiplication are the number before the decimal and the next consecutive number.

b Use the pattern to find the following. Check your answers using a calculator.

$3.5^2 = 3 \times \square + 0.25$ $= 12.25$ 4	$4.5^2 = \square \times \square + 0.25$ $= 20.25$ 4, 5
$5.5^2 = \square \times \square + 0.25$ $=$ 5, 6, 30.25	$6.5^2 = \square \times \square + 0.25$ $=$ 6, 7, 42.25
$9.5^2 = \square \times \square + \square$ $=$ 8, 19, 0.25, 90.25	$10.5^2 =$ 110.25

c Prove algebraically that this pattern exists for all decimals ending in .5.

$(x+0.5)^2 =$

$(x+0.5)^2 = x^2 + x + 0.25 = x(x+1) + 0.25$

d Prove algebraically that the pattern $25^2 = 20 \times 30 + 25$, $205^2 = 200 \times 210 + 25$ ... etc. exists for all numbers ending in 5.

$(x+5)^2 = x^2 + 10x + 25 = x(x+10) + 25$

EXPANDING DIFFERENCE OF TWO SQUARES

Investigation Expand the following.

a $(x+1)(x-1) =$	**a** $(x+2)(x-2) =$
b $(x+3)(x-3) =$	**b** $(x+4)(x-4) =$
c $(x+5)(x-5) =$	**c** $(x+6)(x-6) =$
d $(x+7)(x-7) =$	**d** $(x+8)(x-8) =$
e $(x+9)(x-9) =$	**e** $(x+10)(x-10) =$

Complete the sentence:

When you have a difference of two squares in the form of $(x+b)(x-b)$, the coefficient of x is ……………………………………… and the constant term is …………………………………

Complete the following:

a $(x+12)(x-12) =$	**a** $(x-20)(x+20) =$
b $(\quad)(\quad) = x^2 - 121$	**b** $(\quad)(\quad) = x^2 - 139$

EXPAND DIFFERENCE OF TWO SQUARES

Difference of Two Squares
$(a-b)(a+b)=a^2-b^2$

Example Expand difference of two squares.

$(y-3)(y+3)$ $=y^2-9$	$(6-2t)(6+2t)$ $=36-4t^2$

Expand these expressions using the difference of two squares rule.

a $(x-4)(x+4)$

x^2-16

b $(x+8)(x-8)$

x^2-64

c $(2x+3)(2x-3)$

$4x^2-9$

d $(7-3x)(7+3x)$

$49-9x^2$

e $(x^2+5)(x^2-5)$

x^4-25

f $(3x^2-2)(3x^2+2)$

$9x^4-4$

Key ideas

1. An expression in the form $(a+b)(a-b)$ is called a ……………… …… ……… ……………
2. To quickly expand, we use the rule:

   $(a+b)(a-b)=$ ……………………………

1 Expand the following using the difference of two squares.

a $(x+4)(x-4) = \dots\dots\dots - \dots\dots\dots$ x^2-16	**b** $(x-10)(x+10) = \dots\dots\dots - \dots\dots\dots$ x^2-100
c $(2x+1)(2x-1) = \dots\dots\dots - \dots\dots\dots$ $4x^2-1$	**d** $(3x+4)(3x-4) = \dots\dots\dots - \dots\dots\dots$ $9x^2-16$

2 Expand these differences of two squares.

a $(x+1)(x-1)$ x^2-1	**b** $(x+3)(x-3)$ x^2-9	**c** $(x+8)(x-8)$ x^2-64
d $(x-6)(x+6)$ x^2-36	**e** $(6-x)(6+x)$ $36-x^2$	**f** $(5+x)(5-x)$ $25-x^2$
g $(x+11)(x-11)$ x^2-121	**h** $(12+x)(12-x)$ $(144-x^2)$	**i** $(7+x)(7-x)$ $49-x^2$

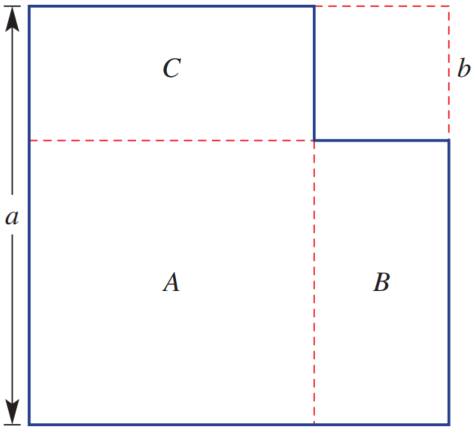
3 Expand these differences of two squares.

a $(3x-2)(3x+2)$ $9x^2-4$	**b** $(5x-4)(5x+4)$ $25x^2-16$	**c** $(4x-3)(4x+3)$ $16x^2-9$
d $(9x-5y)(9x+5y)$ $81x^2-25y^2$	**e** $(11x-y)(11x+y)$ $121x^2-y^2$	**f** $(8x+2y)(8x-2y)$ $64x^2-4y^2$
g $(7x-5y)(7x+5y)$ $49x^2-25y^2$	**h** $(8x-3y)(8x+3y)$ $64x^2-9y^2$	**i** $(9x-4y)(9x+4y)$ $81x^2-16y^2$
j $(x^2+7)(x^2-7)$ x^4-49	**k** $(x^2-11)(x^2+11)$ x^4-121	**l** $(x^3+2y)(x^3-2y)$ x^6-4y^2
m $(2x^3+3y^2)(2x^3-3y^2)$ $4x^6-9y^4$	**n** $(5x^2-14x)(5x^2+14x)$ $25x^4-196x^2$	**o** $(3x^8+5y^3)(3x^8-5y^3)$ $9x^{16}-25y^6$

4 A square of side length x has one side reduced by 1 unit and the other increased by 1 unit. By forming an expression for the area, explain why the area of the resulting rectangle is not the same as the original square.

$(x+1)(x-1) = x^2 - 1$. The new area is 1 less than the original.

5 A square of side length b is removed from a square of side length a.



a Using subtraction, write down an expression for the remaining area.

$a^2 - b^2$

b Write expressions for the area of the regions:

i A

$(a-b)^2$

ii B

$b(a-b)$

iii C

$b(a-b)$

c Add all the expressions from part **b** and confirm you get your answer from part **a**.

6 Expand these differences of two squares.

a $(x+\sqrt{7})(x-\sqrt{7})$

$x^2 - 7$

b $(x-\sqrt{5})(x+\sqrt{5})$

$x^2 - 5$

c $(3x+\sqrt{17})(3x-\sqrt{17})$

$9x^2 - 17$

d $(x^2+\sqrt{21})(x^2-\sqrt{21})$

$x^4 - 21$

e $(2x^3-\sqrt{10})(2x^3+\sqrt{10})$

$4x^6 - 10$

f $(\sqrt{6}+3x)(\sqrt{6}-3x)$

$6 - 9x^2$

7 Expand these differences of two squares involving fractional terms.

a $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$

$x^2 - \frac{1}{9}$

b $\left(x+\frac{3}{4}\right)\left(x-\frac{3}{4}\right)$

$x^2 - \frac{9}{16}$

c $\left(\frac{x}{3}+\frac{4}{5}\right)\left(\frac{x}{3}-\frac{4}{5}\right)$

$\frac{x^2}{9} - \frac{16}{25}$

d $\left(\frac{x}{2}+\frac{y}{7}\right)\left(\frac{x}{2}-\frac{y}{7}\right)$

$\frac{x^2}{4} - \frac{y^2}{49}$

e $\left(2x-\frac{3}{7}\right)\left(2x+\frac{3}{7}\right)$

$4x^2 - \frac{9}{49}$

f $\left(\frac{7}{3}-5x\right)\left(\frac{7}{3}+5x\right)$

$\frac{49}{9} - 25x^2$

8 Compute the following using the difference of two squares.

a $27 \times 33 = (30 - 3) \times (30 + 3)$

$= 30^2 - \boxed{}^2$

$= 900 - \_\_\_\_\_$

$=$

891

b $78 \times 82 = (80 - \_\_\_\_\_) \times (80 + \_\_\_\_\_)$

6396

c 96×104

9984

d 189×211

39879

9 Compute the following using perfect squares.

a $53^2 = (50 + 3)^2$

$= 50^2 + 2 \times \boxed{} \times \boxed{} + 3^2$

$=$

2809

b $98^2 = (100 - 2)^2$

$= \boxed{} - 2 \times 100 \times \boxed{} + 2^2$

$=$

9604

c 82^2

6724

d 39^2

1521

e 102^2

10404

f 197^2

38809

10 Which is larger? 77×77 or 78×76?
Compute each using perfect squares and the difference of two squares.

a 77×77

5929

b 78×76

5928

10 Consider that that for any two consecutive integers, the difference of their squares is their sum.

$4^2 - 3^2 = 4 + 3, \quad 5^2 - 4^2 = 5 + 4, \quad 10^2 - 9^2 = 10 + 9,$

Prove that this is always true using algebra.

$$(n+1)^2 - n^2$$
$$= n^2 + 2n + 1 - n^2$$
$$= 2n + 1$$
$$= (n+1) + n$$

11 Consider that for any three consecutive integers, the product of the smallest integer and the largest integer, minus the square of the middle integer, is always −1.

$3 \times 5 - 4^2 = -1, \quad 11 \times 13 - 12^2 = -1, \quad 18 \times 20 - 19^2 = -1$

Prove that this is always true using algebra.

$$(n-1)(n+1) - n^2$$
$$= n^2 - 1 - n^2$$
$$= -1$$

12 “The square of a number is the product of one more and one less than the number, add one.“

Show this is true for 10^2

$$10^2 = 11 \times 9 + 1$$

Prove this is always true using algebra.

WTP $n^2 = (n+1)(n-1) + 1$

RHS $= n^2 - 1 + 1$

$= n^2$

$=$ LHS

MIXED EXPANSION PRACTICE

FOUNDATION

1 Match the equivalent expressions.

a	$x(x+4)$	**A**	$x^2 - 8x + 16$
b	$(x+1)(x+4)$	**B**	$x^2 + 5x + 4$
c	$(x+4)^2$	**C**	$x^2 + 4x$
d	$(x-4)^2$	**D**	$x^2 - 16$
e	$(x+4)(x-4)$	**E**	$x^2 + 8x + 16$

a - C, b - B, c - E, d - A, e - D

2 Expand the following using the appropriate rule.

a $(x-10)^2$

$x^2 - 20x + 100$

b $(x+7)^2$

$x^2 + 14x + 49$

c $(x+8)(x-8)$

$x^2 - 64$

d $(x+11)(x-11)$

$x^2 - 121$

e $(x+11)(x+11)$

$x^2 + 22x + 121$

f $(x-7)(x-7)$

$x^2 - 14x + 49$

g $(x+12)(x-12)$

$x^2 - 144$

h $(x-3)(x-3)$

$x^2 - 6x + 9$

i $(x+6)(x-6)$

$x^2 - 36$

3 Expand the following.

a $2(x+3)^2$

$2x^2 + 12x + 18$

b $4(m+5)^2$

$4m^2 + 40m + 100$

c $2(a-7)^2$

$2a^2 - 28a + 98$

d $-3(y+5)(y+5)$

$-3y^2 - 30y - 75$

e $3(2b-1)^2$

$12b^2 - 12b + 3$

f $-3(2y-6)^2$

$-12y^2 + 72y - 108$

g $2(x-3)^2$

$2x^2 - 12x + 18$

h $-2(x+8)(x-8)$

$-2x^2 + 128$

i $-4(x+9)(x+9)$

$-4x^2 - 72x - 324$

j $3(x+5)(x-3)$

$3x^2 + 6x - 45$

k $-3(a+2)(a-7)$

$-3a^2 + 15a + 42$

l $-2(x-7)(x+7)$

$-2x^2 + 98$

m $3(2x+3)(2x+3)$

$12x^2 + 36x + 27$

n $6(3x-4)(3x+4)$

$54x^2 - 96$

o $-2(x+4)(3x-7)$

$-6x^2 - 10x + 56$

4 Expand and simplify the following.

a $3(x+2)$ $3x+6$	**b** $x(x-1)$ x^2-x	**c** $-2(3-x)$ $-6+2x$
d $-2x(5-3x)$ $-10x+6x^2$	**e** $x^2(x-3)$ x^3-3x^2	**f** $x^2(x^2-3x)$ x^4-3x^2
g $3xy(3x-y)$ $9x^2y-3xy^2$	**h** $-3xy(4y^2-2xy)$ $-12xy^3+6x^2y62$	**i** $(x+1)(x+2)$ x^2+3x+2
j $(x-3)(x+2)$ x^2-x-6	**k** $(x-5)(x-4)$ $x^2-9x+20$	**l** $2(x+1)(x-2)$ $2x^2-2x-4$
m $x(x-3)(x+2)$ x^3-x^2-6x	**n** $2x(3-x)(x^2+2)$ $6x^3+12x-2x^4-4x^2$	**o** $3x^2(3x^2-5)(4-x^2)$ $51x^4-9x^6-60x^2$
p $-(x+3)(2-x)$ x^2+x-6	**q** $(x+3)(x^2+3)$ $x^3+3x+3x^2+9$	**r** $(2x-3)(x^2-3)$ $2x^3-6x-3x^2+9$
s $(x+1)^2$ x^2+2x+1	**t** $(2-x)^2$ $4-4x+x^2$	**u** $-(2x^2-3)^2$ $-4x^4+12x^2-9$
v $-2x(x^2+2)^2$ $-2x^5-8x^3-8x$	**w** $\left(x+\frac{1}{x}\right)^2$ $x^2+2+\frac{1}{x^2}$	**x** $\left(x+\frac{1}{\sqrt{x}}\right)^2$ $x^2+2\sqrt{x}+\frac{1}{x}$

5 All of these are common mistakes students make when expanding.
Identify the mistake in each set of working, then give the correct answer.

a $-x(x-7) = -x^2 - 7x$

Mistake:

………………………………………………………

………………………………………………………

Correct expansion:

$-x^2 + 7x$

b $3a - 7(4-a) = 3a - 28 - 7a$

$= -4a - 28$

Mistake:

………………………………………………………

………………………………………………………

Correct expansion:

$10a - 28$

c $(2x+3)^2 = 4x^2 + 9$

Mistake:

………………………………………………………

………………………………………………………

Correct expansion:

$4x^2 + 12x + 9$

d $(x+2)^2 - (x^2+4) = 0$

Mistake:

………………………………………………………

………………………………………………………

Correct expansion:

$4x$

6 Expand and simplify the following.

a $\left(t + \frac{1}{t}\right)^2$

$t^2 + 2 + \frac{1}{t^2}$

b $\left(t - \frac{1}{t}\right)^2$

$t^2 - 2 + \frac{1}{t^2}$

c $\left(t + \frac{1}{t}\right)\left(t - \frac{1}{t}\right)$

$t^2 - \frac{1}{t^2}$

d $\left(\frac{4}{m} - m\right)\left(\frac{4}{m} + m\right)$

$\frac{16}{m^2} - m^2$

e $\left(xy - \frac{1}{y}\right)\left(xy + \frac{1}{y}\right)$

$x^2y^2 - \frac{1}{y^2}$

f $\left(\frac{4a}{3} + 2\right)\left(\frac{4a}{3} - 2\right)$

$\frac{16a^2}{9} - 4$

7 Expand and simplify the following.

a $3a^2(4a^3 - b^4) + 2a^2(5a^3 + 3b^4)$

$22a^5 + 3a^2b^4$

b $2y^3(5y^2 + 3y^4) - 4y^2(2y^2 - y^4)$

$6y^7 + 4y^6 + 10y^5 - 8y^4$

c $(x + 2)^2 - 4$

$x^2 + 4x$

d $(x + 3)(x - 3) + 6x$

$x^2 - 9 + 6x$

e $(x + 4)(x + 3) + x - 4$

$x^2 + 8x + 8$

f $3x^2 + (2x - 1)(x - 2)$

$5x^2 - 5x + 2$

g $(2a + 3)(a - 5) - (a + 6)$

$2a^2 - 8a - 21$

h $x^2 + 2x - 3 + (2 + x)(2 + x)$

$2x^2 + 6x + 1$

i $x^2 - (x + 1)(x - 1)$

1

j $(3x - 2)(3x + 2) - (3x + 2)^2$

$-12x - 8$

k $(2x - 3)^2 + (2x + 3)^2$

$8x^2 + 18$

l $2(3x - 4)^2 - (3x - 4)(3x + 4)$

$9x^2 - 48x + 48$

m $(5x - 1)^2 - (5x + 1)(5x - 1)$

$2 - 10x$

n $2(x + y)^2 - 2(x - y)^2$

$8xy$

o $(x + y)^2 - (x - y)^2 + (x + y)(x - y)$

$x^2 + 4xy - y^2$

p $(2 - x)^2 - (2 + x)^2$

$-8x$

q $(x^2 + 2)(x^2 - 2)$

$x^4 - 4$

r $(x^2 + 5)^2$

$x^4 + 10x^2 + 25$

s $(3ab - 4c)(3ab + 4c)$

$9a^2b^2 - 16c^2$

t $\left(x + \frac{2}{x}\right)^2$

$x^2 + 4 + \frac{4}{x^2}$

u $\left(a - \frac{2}{a}\right)\left(a + \frac{2}{a}\right)$

$a^2 - \frac{4}{a^2}$

v $[x + (y - 2)]\,[x - (y - 2)]$

$x^2 - y^2 + 4y - 4$

w $(a + 3)^2 - (a - 3)^2$

$12a$

x $16 - (z - 4)(z + 4)$

$32 - z^2$

y $(x^2 + y^2) - 4x^2y^2$

$x^2 + y^2 - 4x^2y^2$

z $(2a^2 + 5b^3)^2$

$4a^4 + 20a^2b^3 + 25b^6$

aa $(a + b)(a^2 - ab + b^2)$

$a^3 + b^3$

bb $(a - b)(a^2 + ab + b^2)$

$a^3 - b^3$

cc $[(x + 3)(x - 3)]^2$

$x^4 - 18x^2 + 81$

dd $\left[\left(\frac{x}{3} - 5\right)\left(\frac{x}{3} + 5\right)\right]^2$

$\frac{x^4}{81} - \frac{50x^2}{9} + 625$

ee $(m + 2)^2 - (m + 2)(m - 2) - (m - 2)^2$

$-m^2 + 8m + 4$

ff $2(6 - d)^2 - (d + 5)^2 - (6 - d)(d + 5)$

$2d^2 - 35d + 17$

FACTORISING PERFECT SQUARES

Factorising Perfect Squares	
$a^2 + 2ab + b^2$ $= (a + b)^2$	$a^2 - 2ab + b^2$ $= (a - b)^2$

- To factorise a perfect square $a^2 + 2ab + b^2$
1. Find the square roots of the first and last term.
   Check that the middle term matches twice the product of the square roots.
2. Write $(a + b)^2$ or $(a - b)^2$ based on the sign of the middle term.

Example Factorise by recognising difference of two squares.

Worked Example	Your Turn	
$x^2 + 10x + 25$ $\sqrt{x^2} = x$ $\sqrt{25} = 5$ $= (x + 5)^2$	$x^2 + 18x + 81$ $\sqrt{x^2} =$ $\sqrt{81} =$ $= (\quad)^2$ $(x + 9)^2$	$p^2 + 2pq + q^2$ $(p + q)^2$
$y^2 - 6y + 9$ $\sqrt{y^2} = y$ $\sqrt{9} = 3$ $= (y - 3)^2$	$x^2 - 16x + 64$ $\sqrt{x^2} =$ $\sqrt{64} =$ $(x - 8)^2$	$x^2 - 8x + 16$ $(x - 4)^2$
$4x^2 + 20x + 25$ $\sqrt{4x^2} = 2x$ $\sqrt{25} = 5$ $= (2x + 5)^2$	$25x^2 + 60x + 36$ $\sqrt{25x^2} =$ $\sqrt{36} =$ $(5x + 6)^2$	$16x^2 - 72xy + 81y^2$ $(4x - 9y)^2$

Key ideas

1. A perfect square is a quadratic t………………… of the form $a^2 + 2ab + b^2$:
   the first and third term are p………………… s………………… and the middle term is d…………… the p……………… of the square roots of the first and third term.

1 Factorise these perfect squares.

a $x^2 + 14x + 49$ $(x+7)^2$	**b** $x^2 + 2x + 1$ $(x+1)^2$	**c** $y^2 - 6y + 9$ $(y-3)^2$
d $y^2 - 2y + 1$ $(y-1)^2$	**e** $x^2 + 4x + 4$ $(x+2)^2$	**f** $p^2 - 8p + 16$ $(p-4)^2$
g $m^2 + 6m + 9$ $(m+3)^2$	**h** $a^2 - 10a + 25$ $(a-5)^2$	**i** $x^2 - 4x + 4$ $(x-2)^2$

2 Factorise these perfect squares.

a $9x^2 + 48x + 64$ $(3x+8)^2$	**b** $9x^2 - 30x + 25$ $(3x-5)^2$	**c** $4x^2 + 44x + 121$ $(2x+11)^2$
d $49x^2 + 140x + 100$ $(7x+10)^2$	**e** $25x^2 - 20x + 4$ $(5x-2)^2$	**f** $9x^2 - 30x + 25$ $(3x-5)^2$
g $m^2 - 2mn + n^2$ $(m-n)^2$	**h** $r^2 + 2rt + t^2$ $(r+t)^2$	**i** $9x^2 + 12xy + 16y^2$ $(3x+4y)^2$

3 Factorise these perfect squares.

a $25a^4 - 10a^2 + 1$ $(5a^2-1)^2$	**b** $16x^4 + 24x^2 + 9$ $(4x^2+3)^2$	**c** $36 - 12x + x^2$ $(6-x)^2$
d $49x^6 - 14x^3 + 1$ $(7x^3-1)^2$	**e** $81x^2 - 36xz + 4z^2$ $(9x-2z)^2$	**f** $144m^6 - 72m^3 + 9$ $(12m^3-3)^2$

FACTORISING DIFFERENCE OF TWO SQUARES

Factorising Difference of Two Squares
$a^2 - b^2$ $= (a-b)(a+b)$

- To factorise $a^2 - b^2$:

1. Find the square roots of each of the two terms.
2. Write $(a+b)(a-b)$.

Example Factorise by recognising difference of two squares.

Worked Example	Your Turn	
$a^2 - 36$ $\sqrt{a^2} = a$ $\sqrt{36} = 6$ $= (a-6)(a+6)$	$a^2 - 1$ $(a-1)(a+1)$	$25 - y^2$ $(5-y)(5+y)$
$16x^2 - y^2$ $\sqrt{16x^2} = 4x$ $\sqrt{y^2} = y$ $= (4x-y)(4x+y)$	$4c^2 - 9$ $(2c-3)(2c+3)$	$49x^2 - 4y^2z^2$ $(7x-2yz)(7x+2yz)$
$a^2 - \frac{1}{9}$ $\sqrt{a^2} = a$ $\sqrt{\frac{1}{9}} = \frac{1}{3}$ $= \left(a - \frac{1}{3}\right)\left(a + \frac{1}{3}\right)$	$x^2 - \frac{1}{4}$ $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$	$\frac{y^2}{16} - \frac{4}{9}$ $\left(\frac{y}{4} - \frac{2}{3}\right)\left(\frac{y}{4} + \frac{2}{3}\right)$

Key ideas

1. If an expression involves a ……………… of two terms, both of which are ………………… ……………, you should recognise them as a difference two squares and factorise by using the rule: $a^2 - b^2 = (a-b)(a+b)$

1 Complete these factorisations.

a $x^2 - 16 = x^2 - 4^2$ $= (x+4)(\_\_ - \_\_)$ $(x+4)(x-4)$	**b** $x^2 - 144 = x^2 - (\_\_)^2$ $= (\_\_ + 12)(x - \_\_)$ $(x+12)(x-12)$
c $16x^2 - 1 = (\_\_)^2 - (\_\_)^2$ $= (4x + \_\_)(\_\_ - 1)$ $(4x+1)(4x-1)$	**d** $9a^2 - 4b^2 = (\_\_)^2 - (\_\_)^2$ $= (3a + \_\_)(\_\_ - 2b)$ $(9a+2b)(9a-2b)$

2 Factorise the following using difference of two squares.

a $x^2 - 9$ $(x-3)(x+3)$	**b** $y^2 - 25$ $(y-5)(y+5)$	**c** $y^2 - 1$ $(y-1)(y+1)$
d $x^2 - 16$ $(x-4)(x+4)$	**e** $a^2 - 81$ $(a-9)(a+9)$	**f** $x^2 - y^2$ $(x-y)(x+y)$
g $16 - a^2$ $(4-a)(4+a)$	**h** $25 - x^2$ $(5-x)(5+x)$	**i** $1 - b^2$ $(1-b)(1+b)$
j $36 - y^2$ $(6-y)(6+y)$	**k** $121 - b^2$ $(11-b)(11+b)$	**l** $x^2 - 400$ $(x-20)(x+20)$

3 Factorise the following using difference of two squares.

a $4x^2 - 25$ $(2x-5)(2x+5)$	**b** $25b^2 - 4$ $(5b-2)(5b+2)$	**c** $100y^2 - 9$ $(10y-3)(10y+3)$
d $1 - 4x^2$ $(1-2x)(1+2x)$	**e** $16 - 9y^2$ $(4-3y)(4+3y)$	**f** $4x^2 - 25y^2$ $(2x-5y)(2x+5y)$
g $4p^2 - 25q^2$ $(2p-5q)(2p+5q)$	**h** $25a^2 - 49b^2$ $(5a-7b)(5a+7b)$	**i** $81m^2 - 4n^2$ $(9m-2n)(9m+2n)$

4 Factorise each of the following by first taking out the common factor.

a $3x^2 - 108$ $= 3(x^2 - 36)$ $=$ $3(x-6)(x+6)$	**b** $6x^2 - 24$ $6(x-2)(x+2)$	**c** $10a^2 - 10$ $10(a-1)(a+1)$
d $4y^2 - 64$ $4(y-4)(y+4)$	**e** $98 - 2x^2$ $2(7-x)(7+x)$	**f** $32 - 8m^2$ $8(2-m)(2+m)$
g $5x^2y^2 - 5$ $5(xy-1)(xy+1)$	**h** $3 - 3x^2y^2$ $3(1-xy)(1+xy)$	**i** $63 - 7a^2b^2$ $7(3+ab)(3-ab)$

5 Factorise these expressions, if possible.

a $x^2 - 9x$

$x(x-9)$

b $x^2 + 9x$

$x(x+9)$

c $x^2 + 9$

can't simplify

d $x^2 - 9$

$(x-3)(x+3)$

e $x^2 - 8$

can't simplify

f $2x^2 - 50$

$2(x-5)(x+5)$

g $50 - 2x^2$

$2(5-x)(5+x)$

h $200 - 2x^2$

$2(10-x)(10+x)$

i $300 - 3x^2$

$3(10-x)(10+x)$

j $300 - 27x^2$

$3(10-3x)(10+3x)$

k $12x^2 - 3y^2$

$3(2x-y)(2x+y)$

l $3x^2 - 3y^2$

$3(x-y)(x+y)$

m $3x^2 - 27y^2$

$3(x-3y)(x+3y)$

n $x^2 - 27y^2$

can't simplify

6 $-4 + 9x^2$ may not seem like a difference of two squares, but

$$-4 + 9x^2 = 9x^2 - 4$$
$$= (3x + 2)(3x - 2)$$

Use this idea to factorise these expressions.

a $-9 + x^2$ $(x - 3)(x + 3)$	**b** $-121 + 16x^2$ $(4x - 11)(4x + 11)$	**c** $-25a^2 + 4$ $(2 - 5a)(2 + 5a)$
d $-25a^2 + 4b^2$ $(2b - 5a)(2b + 5a)$	**e** $-36a^2b^2 + c^2$ $(c - 6ab)(c + 6ab)$	**f** $-16x^2 + y^2z^2$ $(yz - 4x)(yz + 4x)$

7 Olivia factorises $16x^2 - 4$ to get $(4x + 2)(4x - 2)$ but the answers say $4(2x + 1)(2x - 1)$.

a What should Olivia do to get from her answer to the actual answer?

Factorise 2 out of both sets of brackets.

$2(2x + 1)2(2x - 1) = 4(2x + 1)(2x - 1)$

b What should Olivia have done initially to avoid this issue?

Factorise out the HCF: 4

8 Amy factorises $25 - x^2$ to get $(5 + x)(5 - x)$ but the answers say $-(x + 5)(x - 5)$.

Are these the same? Justify your answer.

Yes. You can check by expanding both expressions, or notice that we factorised -1 out of the second set of brackets.

9 Some expressions with fractions can be simplified as a difference of two squares.
Factorise these.

a $x^2 - \frac{1}{4}$ $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$	**b** $x^2 - \frac{4}{25}$ $\left(x - \frac{2}{5}\right)\left(x + \frac{2}{5}\right)$	**c** $25x^2 - \frac{9}{16}$ $\left(5x - \frac{3}{4}\right)\left(5x + \frac{3}{4}\right)$
d $\frac{x^2}{9} - 1$ $\left(\frac{x}{3} - 1\right)\left(\frac{x}{3} + 1\right)$	**e** $\frac{a^2}{4} - \frac{b^2}{9}$ $\left(\frac{a}{2} - \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)$	**f** $\frac{5x^2}{9} - \frac{5}{4}$ $5\left(\frac{x}{3} - \frac{1}{2}\right)\left(\frac{x}{3} + \frac{1}{2}\right)$

10 Some expressions can be factorised as a difference of two squares using surds.

a $x^2 - 7$ $= (x + \sqrt{7})(x - \sqrt{7})$	**b** $x^2 - 5$ $(x - \sqrt{5})(x + \sqrt{5})$	**c** $x^2 - 19$ $(x - \sqrt{19})(x + \sqrt{19})$
d $3x^2 - 4$ $(\sqrt{3}x - 2)(\sqrt{3}x + 2)$	**e** $5x^2 - 9$ $(\sqrt{5}x - 3)(\sqrt{5}x + 3)$	**f** $-9 + 2x^2$ $(\sqrt{2}x - 3)(\sqrt{2}x + 3)$

11 Read the example for factorising $25 - (x + 2)^2$

Worked Solution	Explanation
$25 - (x + 2)^2$ $= (5 + (x + 2))(5 - (x + 2))$ $= (7 + x)(3 - x)$	How can we recognise this as a difference of two squares? Explain why $5 - (x + 2)$ becomes $3 - x$

12 Factorise the following.

a $(x + 5)^2 - 9$ $(x + 2)(x + 8)$	**b** $(x + 10)^2 - 16$ $(x + 6)(x + 14)$	**c** $(x - 7)^2 - 1$ $(x - 8)(x - 6)$
d $49 - (x - 1)^2$ $(8 - x)(x + 6)$	**e** $9 - (x + 3)^2$ $-x(x + 6)$	**f** $16 - (x + 4)^2$ $-x(x + 8)$
g $25 - (x + 2)^2$ $(3 - x)(x + 7)$	**h** $100 - (x + 4)^2$ $(6 - x)(x + 14)$	**i** $25 - (x - 5)^2$ $-x(x - 1)$
j $49 - (x + 3)^2$ $(4 - x)(x + 10)$	**k** $4 - (x + 2)^2$ $-x(x + 4)$	**l** $81 - (x + 8)^2$ $(1 - x)(x + 17)$
m $(3x + 5)^2 - x^2$ $(2x + 5)(4x + 5)$	**n** $(2y + 7)^2 - y^2$ $(y + 7)(3y + 7)$	**o** $(3x - 5y)^2 - 25y^2$ $3x(3x - 10y)$
p $(x - 6)^2 - 15$ $(x - 6 - \sqrt{15})(x - 6 + \sqrt{15})$	**q** $21 - (x + 4)^2$ $(\sqrt{21} - x - 4)(\sqrt{21} + x + 4)$	**r** $19 - (x - 1)^2$ $(\sqrt{19} - x + 1)(\sqrt{19} + x - 1)$

13 Richard factorised this expression but made a mistake.

$$9 - (x - 1)^2 = (3 + x - 1)(3 - x - 1)$$
$$= (2 + x)(2 - x)$$

a Identify the mistake.

He should have kept the brackets: $(3 + x - 1)(3 - (x - 1))$

b Write the correct factorisation.

$(2 + x)(4 - x)$

14 Some expressions with powers of 4 can be factorised as a difference of two squares.
Look at the example and factorise the following.

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$$
$$= (x^2 + y^2)(x + y)(x - y)$$

a $1 - w^4$

$(1 - w)(1 + w)(w^2 + 1)$

b $\dfrac{x^4}{3} - \dfrac{y^4}{3}$

$\dfrac{(x - y)(x + y)(x^2 + y^2)}{3}$

c $2a^4 - 32b^4$

$2(a - 2b)(a + 2b)(a^2 + 4b^2$

d $\dfrac{3a^8}{16} - \dfrac{3b^4}{625}$

$3\left(\dfrac{a^2}{2} - \dfrac{b}{5}\right)\left(\dfrac{a^2}{2} + \dfrac{b}{5}\right)\left(\dfrac{a^4}{4} + \dfrac{b^2}{25}\right)$

e $5w^5 - 80w$

$5w(w - 2)(w + 2)(w^2 + 4)$

f $w^6 - w^4 - w^2 + 1$

$(w - 1)^2(w + 1)^2(w^2 + 1)$

15 Read the example for factorising $x^2 + 5y - y^2 + 5x$

Worked Solution	**Explanation**
$x^2 + 5y - y^2 + 5x$	How did rearranging help us recognise a difference of two squares?
$= x^2 - y^2 + 5y + 5x$	
$= (x + y)(x - y) + 5(x + y)$	What happened to get to this line?
$= (x + y)(x - y + 5)$	

Use a similar method to factorise these.

a $x^2 + 7x + 7y - y^2$

$(x - y + 7)(x + y)$

b $x^2 - 2x - 2y - y^2$

$(x - y - 2)(x + y)$

c $4x^2 + 4x + 6y - 9y^2$

$(2x - 3y + 2)(2x + 3y)$

d $25y^2 + 15y - 4x^2 + 6x$

$(5y + 2x)(5y - 2x + 3)$

16 Factorise using surds, then simplify the surds.

a $x^2 - 8$

$(x - 2\sqrt{2})(x + 2\sqrt{2})$

b $x^2 - 45$

$(x - 3\sqrt{5})(x + 3\sqrt{5})$

c $x^2 - 20$

$(x - 2\sqrt{5})(x + 2\sqrt{5})$

d $x^2 - 48$

$(x - 4\sqrt{3})(x + 4\sqrt{3})$

e $x^2 - 50$

$(x - 5\sqrt{2})(x + 5\sqrt{2})$

f $x^2 - 200$

$(x - 10\sqrt{2})(x + 10\sqrt{2})$

17 Factorise fully by first factoring a common factor, then simplifying any surds.

a $5x^2 - 120$

$5(x - 2\sqrt{6})(x + 2\sqrt{6})$

b $3x^2 - 162$

$3(x - 3\sqrt{6})(x + 3\sqrt{6})$

c $7x^2 - 126$

$7(x - 3\sqrt{2})(x + 3\sqrt{2})$

d $2(x + 3)^2 - 10$

$2(x + 3 - \sqrt{5})(x + 3 + \sqrt{5})$

e $3(x - 1)^2 - 21$

$3(x + \sqrt{7} - 1)(x - 1 + \sqrt{7})$

f $4(x - 4)^2 - 48$

$4(x \pm 4 + 2\sqrt{3})(x - 4 - 2\sqrt{3})$

18 Factorise using the difference of two squares.

a $x^2 - \frac{1}{9}$	**b** $x^2 - \frac{3}{4}$	**c** $x^2 - \frac{7}{16}$
d $x^2 - \frac{5}{16}$	**e** $(x-2)^2 - 20$	**f** $(x+4)^2 - 27$
g $(x-7)^2 - 40$	**h** $3x^2 - 4$	**i** $5x^2 - 9$
j $5x^2 - 120$	**k** $2(x+3)^2 - 10$	**l** $2x^2 - 96$
m $-9 + 2x^2$	**n** $-10 + 3x^2$	**o** $-7 + 13x^2$
p $3(x-1)^2 - 21$	**q** $4(x-4)^2 - 48$	**r** $5(x+6)^2 - 90$
s $9 - (x+3)^2$	**t** $16 - (x+4)^2$	**u** $25 - (x-5)^2$
v $49 - (x-1)^2$	**w** $(x+2)^2 - (x+3)^2$	**x** $(y-7)^2 - (y+4)^2$
y $(b+5)^2 - (b-5)^2$	**z** $(a+3)^2 - (a-5)^2$	**aa** $(2w+3x)^2 - (3w+4x)^2$

FACTORISING BY NOTICING BINOMIAL COMMON FACTORS

DIVIDING BY A BINOMIAL FACTOR

Example Divide by a binomial factor.

Simplify these expressions.

a $3a \div a$	**b** $3xy \div xy$	**c** $3xy \div 3y$
d $3(x+y) \div 3$	**e** $3(x+y) \div (x+y)$	**f** $3x(a+b) \div (a+b)$
g $5x(2a+3b) \div (2a+3b)$	**h** $-2x(p-q) \div (p-q)$	**i** $4(x+y)^2 \div (x+y)$

Binomial Common Factors
$a(c+d) + b(c+d)$ $= (a+b)(c+d)$

1. Identify the **binomial** that divides into all terms. Write it outside a second set of brackets.
2. Divide each term by the binomial HCF. Write the results inside a second set of brackets.

Example Factorise by noticing binomial HCF.

$3(x+y) + x(x+y)$	$5(2x-y) - 3x(2x-y)$
1. $(x+y)(\quad)$	1. $(2x-y)(\quad)$
2. $(x+y)(3+x)$	2. $(2x-y)(5-3x)$

Factorise these expressions.

a $4(x+3) + x(x+3)$ $= (x+3)(\quad)$ $(x+3)(4+x)$	**b** $3(x+1) - x(x+1)$ $(x+1)(3-x)$	**c** $x(x-7) + 2(x-7)$ $(x-7)(x+2)$
d $8(a-4) - a(a-4)$ $(a-4)(8-a)$	**e** $x(7+3x) + 1(7+3x)$ $(7+3x)(x+1)$	**f** $y(2x-5) - 1(2x-5)$ $(2x-5)(y-1)$

- Sometimes the binomial common factor is hidden.

1. Factorise a pair of terms such that the expression is made up of two terms.
2. Factorise by noticing the binomial HCF.

Example Factorise by noticing binomial HCF after factorising one pair of terms.

$7-2x+x(7-2x)$ 1. $1(7-2x)+x(7-2x)$ 2. $(7-2x)(1+x)$	Why did we rewrite $(7-2x)$ as $1(7-2x)$? *Rewriting $7-2x$ as $1(7-2x)$ makes it easier to see the common factor: ……………*
$7x(x-4)-x+4$ 1. $7x(x-4)-(x-4)$ 2. $(x-4)(7x-1)$	How should we factorise if the terms are the same, but the signs are different? E.g. $(x-4)$ and $(-x+4)$ *If the terms are the same but with opposite signs, we can factor out ……… to make the terms identical.*

Factorise these expressions.

a $7+3x+x(7+3x)$ $=1(7+3x)+x(7+3x)$ $=$ $(7+3x)(1+x)$	**b** $y(4y-1)-(4y-1)$ $(4y-1)(y-1)$	**c** $y(2x-5)-2x+5$ $(2x-5)(y-1)$
d $2x(x+1)-x-1$ $(x+1)(2x-1)$	**e** $5y(x-7)-x+7$ $(x-7)(5y-1)$	**f** $12x^2(y-3)+3-y$ $(y-3)(12x^2-1)$

Key ideas

1. We can factorise an expression in the form $a(c+d)+b(c+d)$ by taking out the ……………… highest common factor, $(c+d)$, then writing ……………………………
2. Sometimes, you may need to factor out 1 or …… for a pair of terms to see the binomial common factor.

1 Fill in the missing information to finish factorising.

a $2(x+1)+a(x+1)=(x+1)(\_\_\_\_\_\_)$

$2+a$

b $3(x+3)-a(x+3)=(x+3)(\_\_\_\_\_\_)$

$3-a$

c $5(x+5)-a(x+5)=(x+5)(\_\_\_\_\_\_)$

$5-a$

d $a(x+7)+4(x+7)=(x+7)(\_\_\_\_\_\_)$

$a+4$

e $a(x-3)+(x-3)=(x-3)(\_\_\_\_\_\_)$

$a+1$

f $a(x+4)-(x+4)=(x+4)(\_\_\_\_\_\_)$

$a-1$

g $(x-3)-a(x-3)=(x-3)(\_\_\_\_\_\_)$

$1-a$

h $(4-x)+2a(4-x)=(4-x)(\_\_\_\_\_\_)$

$1+2a$

2 Factorise the following expressions by noticing the binomial factor.

a $4(x+3)+x(x+3)$

$(x+3)(4+x)$

b $3(x+1)+x(x+1)$

$(x+1)(3+x)$

c $7(m-3)+m(m-3)$

$(m-3)(7+m)$

d $x(x-7)+2(x-7)$

$(x-7)(x+2)$

e $8(a+4)-a(a+4)$

$(a+4)(8-a)$

f $y(y+3)-2(y+3)$

$(y+3)(y-2)$

g $a(x+2)+(x+2)$

$(x+2)(a+1)$

h $t(2t+5)+3(2t+5)$

$(2t+5)(t+3)$

i $y(4y-1)-(4y-1)$

$(4y-1)(y-1)$

3 Factorise these expressions.

a $x(x+1)+3(x+1)$

$(x+1)(x+3)$

b $x(x+1)+2(x+1)$

$(x+1)(x+2)$

c $x(x+1)+1(x+1)$

$(x+1)^2$

d $3x(x+1)+(x+1)$

$(x+1)(3x+1)$

e $3x(x+1)-(x+1)$

$(x+1)(3x-1)$

f $3x(2x+1)-(2x+1)$

$(2x+1)(3x-1)$

g $3x(1+2x)-(2x+1)$

$(1+2x)(3x-1)$

h $3x(1-2x)-(1-2x)$

$(1-2x)(3x-1)$

i $2x(1-2x)-(1-2x)$

$(1-2x)(2x-1)$ or $-(2x-1)^2$

j $2x(1-2x)-(2x-1)$

$(1-2x)(2x+1)$

4 Using the fact that $a - b = -(b - a)$, you can factorise $x(x-2) - 5(2-x)$ by following these steps:

$$x(x-2) - 5(2-x) = x(x-2) + 5(x-2)$$
$$= (x-2)(x+5)$$

Use this idea to factorise these expressions.

a $x(x-4) + 3(4-x)$ $= x(x-4) - 3(x-4)$ $=$ $(x-4)(x-3)$	**b** $x(x-5) - 2(5-x)$ $(x-5)(x+2)$	**c** $x(x-3) - 3(3-x)$ $(x-3)(x+3)$
d $3x(x-4) + 5(4-x)$ $(x-4)(3x-5)$	**e** $3(2x-5) + x(5-2x)$ $(2x-5)(3-x)$	**f** $2x(x-2) + (2-x)$ $(x-2)(2x-1)$
g $-4(3-x) - x(x-3)$ $(x-3)(4-x)$	**h** $x(x-5) + (10-2x)$ $(x-2)(x-5)$	**i** $x(x-3) + (6-2x)$ $(x-2)(x-3)$

5 Factorise these expressions using an appropriate method.

a $6a + 30$ $6(a+5)$	**b** $5x - 15$ $5(x-3)$	**c** $8b + 18$ $2(4b+9)$
d $x^2 - 4x$ $x(x-4)$	**e** $y^2 + 9y$ $y(y+9)$	**f** $a^2 - 3a$ $a(a-3)$
g $x^2y - 4xy + xy^2$ $xy(x-4+y)$	**h** $6ab - 10a^2b + 8ab^2$ $2ab(3-5a+4b)$	**i** $m(m+5) + 2(m+5)$ $(m+5)(m+2)$
j $x(x+3) - 2(x+3)$ $(x+3)(x-2)$	**k** $x(2x-1) + (-2x+1)$ $(2x-1)(x-1)$	**l** $y(3-2y) - 5(2y-3)$ $(3-2y)(y+5)$

FACTORISING BY GROUPING IN PAIRS

Factorising by Grouping in Pairs
$ac + ad + bc + bd$
$= (ac + ad) + (bc + bd)$
$= a(c + d) + b(c + d)$
$= (a + b)(c + d)$

- Factorise expressions with **four terms** using **grouping in pairs**:
  1. Consider the expression as two pairs of binomial terms.
     Be careful of negatives.
  2. Factorise each pair separately.
  3. Further factorise by noticing the binomial common factor.
     You may need to factorise out a negative to get a binomial common factor.

Example Factorise by grouping in pairs.

Worked Example	Your Turn	
$6xy + 9x + 4y + 6$ 1. $6xy + 9x + 4y + 6$ 2. $3x(2y + 3) + 2(2y + 3)$ 3. $(2y + 3)(3x + 2)$	$2x + 8 + bx + 4b$ $2x + 8 + bx + 4b$ $= 2(\quad) + b(\quad)$ $(x + 4)(2 + b)$	$m^2 - 2m + 3m - 6$ $(m - 2)(m + 3)$
$x^2 + 5x - 2x - 10$ 1. $x^2 + 5x - 2x - 10$ 2. $x(x + 5) - 2(x + 5)$ 3. $(x + 5)(x - 2)$	$3x^2 + 6x - 2x - 4$ $3x^2 + 6x - 2x - 4$ $= \quad(\quad) - \quad(\quad)$ $(x + 2)(3x - 2)$	$2ax - bx - 2ay + by$ $(2a - b)(x - y)$
$2x - 4 + 6y - 3xy$ 1. $2x - 4 + 6y - 3xy$ 2. $2(x - 2) + 3y(2 - x)$ 3. $2(x - 2) - 3y(x - 2)$ $(x - 2)(2 - 3y)$	$3x - 15 + 5y - xy$ $(x - 5)(3 - y)$	$6 - 2a + a^3 - 3a^2$ $(3 - a)(2 - a^2)$

Key ideas

1. If there are four terms that share no common factor, factorise by ……………… in ………..
2. Factorising by grouping in pairs is the ……………… of expanding two binomial terms.

1 Fill in the missing information to finish factorising.

a

$x^2 + 3x + 4x + 12 = (x^2 + 3x) + (\_\_\_\_\_\_\_\_\_)$

$= \_\_\_(x + 3) + \_\_\_(x + 3)$

$= (x + \_\_\_)\ (x + \_\_\_)$

$(x+3)(x+4)$

b

$x^2 + 3x - 4x - 12 = (x^2 + 3x) - (\_\_\_\_\_\_\_\_\_)$

$= x(\_\_\_ + \_\_\_) - 4(\_\_\_ + \_\_\_)$

$= (\_\_\_\_\_\_)(\_\_\_\_\_\_)$

$(x+3)(x-4)$

2 Factorise the following expressions by grouping in pairs.

a $x^2 + 3x + 2x + 6$

$= x(x+3) + 2(x+3)$

$(x+3)(x+2)$

b $x^2 + 4x + 3x + 12$

$(x+4)(x+3)$

c $x^2 + 7x + 2x + 14$

$(x+7)(x+2)$

d $x^2 - 6x + 4x - 24$

$(x-6)(x+4)$

e $x^2 - 4x + 6x - 24$

$(x-4)(x+6)$

f $x^2 - 3x + 10x - 30$

$(x-3)(x+10)$

g $x^2 + 2x - 18x - 36$

$(x+2)(x-18)$

h $x^2 + 3x - 14x - 42$

$(x+3)(x-14)$

i $x^2 - 3x - 3xc + 9c$

$(x-3)(x-3c)$

j $x^2 - 2x - xa + 2a$

$(x-2)(x-a)$

k $x^2 - 5x - 3xa + 15a$

$(x-5)(x-3a)$

l $x^2 + 4x - 18x - 72$

$(x+4)(x-18)$

3 Factorise these expressions.

a $3ab + 5bc + 3ad + 5cd$

$(3a+5c)(b+d)$

b $4ab - 7ac + 4bd - 7cd$

$(4b-7)(a+d)$

c $2xy - 8xz + 3wy - 12wz$

$(3w+2x)(y-4z)$

d $5rs - 10r + st - 2t$

$(s-2)(5r+t)$

e $4x^2 + 12xy - 3x - 9y$

$(4x-3)(x+3y)$

f $2ab - a^2 - 2bc + ac$

$(a-2b)(c-a)$

4 Factorise these expressions. Remember to use a factor of 1 where necessary, for example,

$$x^2 - 3x + x - 3 = x(x-3) + 1(x-3)$$

a $x^2 - bx + x - b$

$(x+1)(x-b)$

b $x^2 - cx + x - c$

$(x+1)(x-c)$

c $x^2 + bx + x + b$

$(x+1)(b+x)$

d $x^2 + cx - x - c$

$(x-1)(c+x)$

e $x^2 + ax - x - a$

$(x-1)(x+a)$

f $x^2 - bx - x + b$

$(x-1)(x-b)$

5 Factorise these expressions.

a $x^2 + 2x + 3x + 6$

$(x+2)(x+3)$

b $x^2 + 3x + 2x + 6$

$(x+3)(x+2)$

c $3x^2 + 3x + 2x + 2$

$(x+1)(3x+2)$

d $3x^2 + 3x + 4x + 4$

$(x+1)(3x+4)$

e $3x^2 - 3x + 4x - 4$

$(x-1)(3x+4)$

f $3x^2 - 3x - 4x + 4$

$(x-1)(3x-4)$

g $3x^2 + 3x - 4x - 4$

$(x+1)(3x+4)$

h $3x^2 - 4x + 3x - 4$

$(3x-4)(x+1)$

i $5x^2 - 4x + 5x - 4$

$(5x-4)(x+1)$

j $-5x^2 + 4x - 5x + 4$

$-(5x-4)(x+1)$

6 The expression $xa - 21 + 7a - 3x$ currently do not have common factors between pairs. You can rearrange the expression in two ways and factorise. You should get the same answer.

Method 1	Method 2
$xa + 7a - 3x - 21 = a(x+7) - 3(\_\_\_\_\_\_)$	$xa - 3x + 7a - 21 = x(a-3) + 7(\_\_\_\_\_\_)$
$= \_\_\_\_\_\_\_\_\_\_\_\_$	$= \_\_\_\_\_\_\_\_\_\_\_\_$

Complete the factorisation of the following by rearranging in two different ways.

a $xb - 6 - 3b + 2x$

$xb - 3b - 6 + 2x$

$xb + 2x - 6 - 3b$

$(b+2)(x-3)$

b $4m^2 - 15n + 6m - 10mn$

$(2m+3)(2m-5n)$

c $4a - 6b^2 + 3b - 8ab$

$(2b-1)(-4a-3b)$

7 Factorise these expressions by first rearranging the terms.

a $2x^2 - 7 - 14x + x$

$= 2x^2 - 14x - 7 + x$

$=$

$(2x+1)(x-7)$

b $5x + 2x + x^2 + 10$

$(x+5)(x+2)$

c $2x^2 - 3 - x + 6x$

$(2x-1)(x+3)$

d $3x - 8x - 6x^2 + 4$

$(1-2x)(3x+4)$

e $11x - 5a - 55 + ax$

$(a+11)(x-5)$

f $12y + 2x - 8xy - 3$

$(2x-3)(1-4y)$

g $6m - n + 3mn - 2$

$(3m-1)(n+2)$

h $15p - 8r - 5pr + 24$

$(5p+8)(3-r)$

i $16x - 3y - 8xy + 6$

$(8x+3)(2-y)$

8 These expressions are not fully factorised. Finish factorising by taking out a common factor.

a $(2x+4)(x+5)$

$= 2(x+2)(x+5)$

b $(x+2)(2x+10)$

$2(x+2)(x+5)$

c $(3x+6)(2x+11)$

$3(x+2)(2x+11)$

d $(9x+6)(2x+11)$

$3(3x+2)(2x+11)$

e $(9x+6)(2x+10)$

$6(3x+2)(x+5)$

f $(9x+6)(5x+10)$

$15(3x+2)(x+2)$

g $(9x+6)(15x+10)$

$15(3x+2)^2$

h $2(9x+6)(15x+10)$

$30(3x+2)^2$

i $2(6x-9x^2)(15x+10)$

$30x(2-3x)(3x+2)$

j $2x^3(6x-9x^2)(15x+10)$

$30x^4(2-3x)(x+2)$

9 Factorise by grouping.

a $2(a-3)-x(a-3)-c(a-3)$

$(a-3)(2-x-c)$

b $b(2a+1)+5(2a+1)-a(2a+1)$

$(2a+1)(b+5-a)$

c $x(a+1)-4(a+1)-ba-b$

$(a+1)(x-4-b)$

d $c(1-a)-x+ax+2-2a$

$(1-a)(c-x+2)$

e $a^2-3ac-2ab+6bc+3abc-9bc^2$

$(a-3c)(a-2b+3bc)$

f $8z-4y+3x^2+xy-12x-2xz$

$(x-4)(3x+y-2z)$

10 Factorise by noticing binomial factors, then simplify.
This skill will be helpful when applying chain rule and product rule in Year 11 calculus.

a $3x(5x+2)^2 - 2(5x+2)$

$= (5x+2)(3x(5x+2)-2)$

$= (5x+2)(15x^2+6x-2)$

b $3x(7x-1)^2 - 2(7x-1)$

$= (7x-1)(\text{ .... })$

$=$

$(7x-1)(21x^2-3x-2)$

c $(x+3)^2 + 5(x+3)$

$(x+3)(x+8)$

d $(y+1)^2 - 4(y+1)$

$(y+1)(y-3)$

e $25x(2x+1) + 15(2x+1)$

$5(2x+1)(5x+3)$

f $25x(2x+1) - 15(2x+1)$

$5(2x+10(5x-3)$

g $25x(2x+1)^2 + 15(2x+1)$

$5(2x+1)(10x^2+5x+3)$

h $25x(2x+1) + 15(2x+1)^2$

$5(2x+1)(11x+3)$

i $50x(2x+1) - 15(2x+1)^2$

$5(2x+1)(4x-3)$

j $25x(2x+1)^3 - 15(2x+1)^2$

$5(2x+1)^2(10x^2+5x-3)$

k $5ax(2x+1)^3 - 3a(2x+1)^2$

$a(2x+1)^2(10x^2+5x-3)$

l $5x\sqrt{2x+1} - 3\sqrt{2x+1}$

$\sqrt{2x+1}(5x-3)$

m $5x(2x+1)(5x-2) - 3(2x+1)^2(5x-2)^2$

$-2(5x-2)(2x+1)(15x^2-x-3)$

n $5x(2x-1)^2 - 3x(1-2x)$

$2x(5x-1)(2x-1)$

FACTORISING NON-MONIC QUADRATIC TRINOMIALS

- **Non-monic** means the coefficient of x^2 is not 1.
- **Non-monic quadratic trinomials** are of the form $ax^2 + bx + c$.

Factorising Non-Monic Quadratic Trinomials (PSF)

To factorise a non-monic quadratic trinomial: $ax^2 + bx + c = 0$

1. Identify the product: ac
2. Identify the sum: b
3. Find a factor pair, α and β, that has a product of ac and sum to b.
4. Split bx to $\alpha x + \beta x$
5. Factorise by grouping in pairs.

Example Factorise non-monic quadratic trinomials.

$5x^2 - 13x + 6$		$4y^2 + 4y - 3$	
4. $5x^2 - 10x - 3x + 6$	1. P: 30	4. $4y^2 + 6y - 2y - 3$	1. P: -12
5. $5x(x-2) - 3(x-2)$	2. S: -13	5. $2y(2y+3) - (2y+3)$	2. S: 4
$(x-2)(5x-3)$	3. F: $10, -3$	$(2y+3)(2y-1)$	3. F: $6, -2$

Factorise these expressions.

a $3x^2 + 4x + 1$

P: 3

S: 4

F: …………

$(3x+1)(x+1)$

b $2x^2 + 11x + 5$

P: …………

S: …………

F: …………

$(2x+1)(x+5)$

c $6x^2 + 5x - 6$

P: …………

S: …………

F: …………

$(3x-2)(2x+3)$

d $2x^2 - 7x - 15$

P: …………

S: …………

F: …………

$(2x+3)(x-5)$

Key ideas

1. Non-monic quadratic means the coefficient of …… is not ………
2. To factorise non-monic quadratic t………………, split the …… term, then factorise by ……………………

1 Factorise these expressions.

a $6x^2 + 13x + 6$

P: ....
S: ....
F: ....

$(3x + 2)(2x + 3)$

b $3x^2 - 11x - 4$

P: ....
S: ....
F: ....

$(3x + 1)(x - 4)$

c $12x^2 + x - 6$

P: ....
S: ....
F: ....

$(3x - 2)(4x + 3)$

d $10x^2 - 11x - 6$

P: ....
S: ....
F: ....

$(5x + 2)(2x - 3)$

e $21x^2 - 20x + 4$

P: ....
S: ....
F: ....

$(7x - 2)(3x - 2)$

f $15x^2 - 13x + 2$

P: ....
S: ....
F: ....

$(5x - 1)(3x - 2)$

g $13x^2 - 7x - 6$

$(13x + 6)(x - 1)$

h $2x^2 + 5x - 12$

$(2x - 3)(x + 4)$

i $6x^2 - 13x + 6$

$(3x - 2)(2x - 3)$

j $9x^2 + 9x - 10$

$(3x - 2)(3x + 5)$

2 Factorise $4x^2 - 4x - 15$ in two different ways to show that the order you split the x term doesn't change the answer.

a
$$\begin{aligned} 4x^2 - 4x - 15 &= 4x^2 - 10x + 6x - 15 \\ &= 2x(\_\_\_\_) + 3(\_\_\_\_) \\ &= (2x - 5)(\_\_\_\_) \end{aligned}$$

$(2x - 5)(2x + 3)$

b
$$\begin{aligned} 4x^2 - 4x - 15 &= 4x^2 + 6x - 10x - 15 \\ &= 2x(\_\_\_\_) - 5(\_\_\_\_) \\ &= (\_\_\_\_)(\_\_\_\_) \end{aligned}$$

$(2x - 5)(2x + 3)$

3 Factorise these expressions.

a $2x^2 + 3x + 1$

$(2x+1)(x+1)$

b $2x^2 - 3x + 1$

$(2x-1)(x-1)$

c $2x^2 - 3x - 2$

$(2x+1)(x-2)$

d $2x^2 + 3x - 2$

$(2x-1)(x+2)$

e $2x^2 + 5x + 2$

$(2x+1)(x+2)$

f $2x^2 - 5x + 3$

$(2x-3)(x-1)$

g $2x^2 + 5x + 3$

$(2x+1)(x+1)$

h $2x^2 - x - 3$

$(2x-3)(x+1)$

i $2x^2 + x - 3$

$(2x+3)(x-1)$

j $2x^2 + x - 1$

$(2x-1)(x+1)$

k $2x^2 - x - 1$

$(2x+1)(x-1)$

l $4x^2 - 1$

$(2x-1)(2x+1)$

m $4x^2 + 4x + 1$

$(2x+1)^2$

n $4x^2 - 4x + 1$

$(2x-1)^2$

o $4x^2 + 5x + 1$

$(4x+1)(x+1)$

p $4x^2 - 5x + 1$

$(4x-1)(x-1)$

q $4x^2 + 3x - 1$

$(4x-1)(x+1)$

r $4x^2 - 3x - 1$

$(4x+1)(x-1)$

4 Factorise by first taking out the common factor.

a $6x^2 + 38x + 40$

$2(3x + 4)(x + 5)$

b $6x^2 - 15x - 36$

$3(2x + 3)(x - 4)$

c $48x^2 - 18x - 3$

$3(8x + 1)(2x - 1)$

d $32x^2 - 88x + 60$

$4(4x - 5)(2x - 3)$

e $16x^2 - 24x + 8$

$8(2x - 1)(x - 1)$

f $90x^2 + 90x - 100$

$10(3x - 2)(3x + 5)$

g $-50x^2 - 115x - 60$

$-5(5x + 4)(2x + 3)$

h $12x^2 - 36x + 27$

$3(2x - 3)^2$

5 Factorise the following.

a $18x^2 + 27x + 10$

$(6x + 5)(3x + 2)$

b $21x^2 + 22x - 8$

$(7x - 2)(3x + 4)$

c $40x^2 - x - 6$

$(8x + 3)(5x - 2)$

d $45x^2 - 46x + 8$

$(9x - 2)(5x - 4)$

6 Show that $-12x^2 - 5x + 3$ factorises to $(1 - 3x)(4x + 3)$.

....

....

....

Hence factorise these expressions.

a $-8x^2 + 2x + 15$

$(3 - 2x)(4x + 5)$

b $-6x^2 + 11x + 10$

$(5 - 2x)(3x + 2)$

c $-12x^2 + 13x + 4$

$(4 - 3x)(4x + 1)$

d $-8x^2 + 18x - 9$

$(3 - 4x)(2x - 3)$

e $-14x^2 + 39x - 10$

$(2 - 7x)(2x - 5)$

f $-15x^2 - x + 6$

$(3 - 5x)(3x + 2)$

7 A cable is suspended across a farm channel. The height (h), in metres, of the cable above the water surface is modelled by the equation $h = 3x^2 - 21x + 30$, where x metres is the distance from one side of the channel.

a Factorise the right-hand side of the equation.

$3(x - 2)(x - 5)$

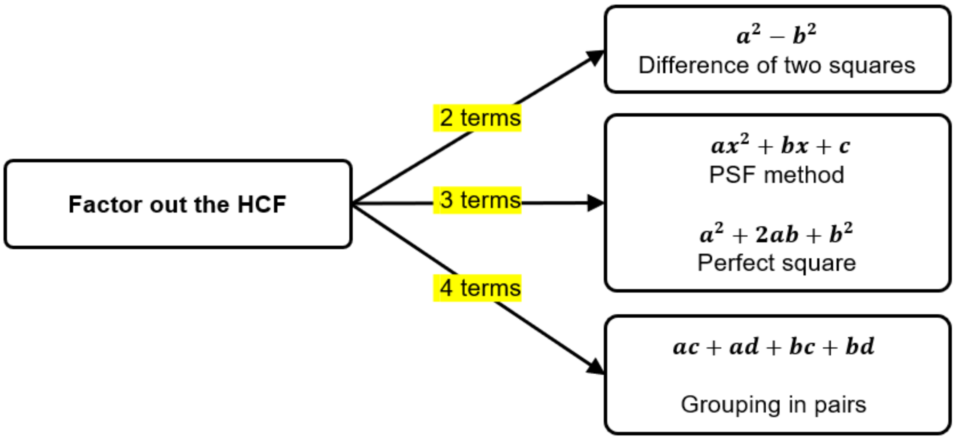
b Determine the height of the cable when $x = 3$. Interpret this result.

– 6, the cable is 6 m below the water.

c Determine where the cable is at the level of the water surface.

$x = 2$ and $x = 5$

MIXED FACTORISATION PRACTICE



FOUNDATION

1 Factorise the following expressions.

a $a^2 - 25$ $(a+5)(a-5)$	**b** $b^2 - 25b$ $b(b-25)$	**c** $c^2 - 25c + 100$ $(c-5)(c-20)$
d $2d^2 + 25d + 50$ $(d+10)(2d+5)$	**e** $e^3 + 5e^2 + 5e + 25$ $(e+5)(e^2+5)$	**f** $16 - f^2$ $(4+f)(4-f)$
g $16g^2 - g^3$ $g^2(16-g)$	**h** $h^2 + 16h + 64$ $(h+8)^2$	**i** $i^2 - 16i - 36$ $(i-18)(i+2)$
j $5j^2 + 16j - 16$ $(5j-4)(j+4)$	**k** $4k^2 - 16k - 9$ $(2k+1)(2k-9)$	**l** $2k^3 - 16k^2 - 3k + 24$ $(k-8)(2k^2-3)$
m $2a^2 + ab - 4a - 2b$ $(a-2)(2a+b)$	**n** $6m^3n^4 + 9m^2n^5$ $3m^2n^4(2m+3n)$	**o** $49p^2 - 121q^2$ $(7p+11q)(7p-11q)$
p $r^2 - 12r + 32$ $(r-4)(r-8)$	**q** $3t^2 + 2t - 40$ $(3t-10)(t+4)$	**r** $5t^2 + 54t + 40$ $(5t+4)(t+10)$
s $5t^2 + 33t + 40$ $(5t+8)(t+5)$	**t** $5t^3 + 10t^2 + 15t$ $5t(t^2+2t+3)$	**u** $u^2 + 15u - 54$ $(u+18)(u-3)$
v $3x^3 - 2x^2y - 15x + 10y$ $(x^2-5)(3x-2y)$	**w** $(p+q)^2 - r^2$ $(p+q+r)(p+q-r)$	**x** $4a^2 - 12a + 9$ $(2a-3)^2$

2 Factorise the following expressions. Take out a common factor first.

a $3a^2 - 12$ $3(a+2)(a-2)$	**b** $x^3 - x$ $x(x+1)(x-1)$	**c** $25y - y^3$ $-y(y+5)(y-5)$
d $9d^2 - 2\frac{1}{4}$ $\left(3d+\frac{3}{2}\right)\left(3d-\frac{3}{2}\right)$	**e** $3x^4 - 27p^2$ $3(x^2+3p)(x^2-3p)$	**f** $4x^2 + 14x - 30$ $2(2x-3)(x+5)$
g $a^4 + a^3 + a^2 + a$ $a(a+1)(a^2+1)$	**h** $x^3 - 8x^2 + 7x$ $x(x-1)(x-7)$	**i** $ax^2 - a - 2x^2 + 2$ $(a-2)(x+1)(x-1)$

3 Factorise the following.

a $4a^3 - 36a$

$4a(a+3)(a-3)$

b $2x^2 - 18$

$2(x+3)(x-3)$

c $3p^2 - 3p - 36$

$3(p+3)(p-4)$

d $5y^2 - 5$

$5(y+1)(y-1)$

e $5a^2 - 10a + 5$

$5(a-1)^2$

f $3z^3 + 27z^2 + 60z$

$3z(z+4)(z+5)$

g $9ab - 4a^3b^3$

$ab(3+2ab)(3-2ab)$

h $x^3 - x$

$x(x+1)(x-1)$

i $6x^2 + 8x - 8$

$2(x+2)(3x-2)$

j $y^2(y+5) - 16(y+5)$

$(y+5)(y+4)(y-4)$

k $x^4 + 8x^3 - x^2 - 8x$

$x(x+8)(x+1)(x-1)$

l $y^6 - 4$

$(y^3-2)(y^3+2)$

m $x^3 - 3x^2 - 10x$

$x(x+2)(x-5)$

n $x^3 - 3x^2 - 9x + 27$

$(x-3)(x+3)(x-3)$

o $4x^2y^3 - y$

$y(2xy+1)(2xy-1)$

p $24 - 6b^2$

$6(2+b)(2-b)$

q $18x^2 + 33x - 30$

$3(3x+5)(2x-2)$

r $3x^2 - 6x + 3$

$3(x-1)^2$

s $49y^2 + 14y + 1$

$(7y+1)^2$

t $9y^2 - 30y + 25$

$(3y-5)^2$

u $16k^2 - 24k + 9$

$(4k-3)^2$

v $9 - 16x^2$

$(3+4x)(3-4x)$

w $36x^2 - y^2$

$(6x+y)(6x-y)$

x $4a^2 - 81b^2$

$(2a+9b)(2a-9b)$

y $t^2 + t + \frac{1}{4}$

$\left(t+\frac{1}{2}\right)^2$

z $x^2 - \frac{4x}{3} + \frac{4}{9}$

$\left(x-\frac{2}{3}\right)^2$

aa $9y^2 + \frac{6y}{5} + \frac{1}{25}$

$\left(3y+\frac{1}{5}\right)^2$

bb $x^2 + 2 + \frac{1}{x^2}$

$\left(x+\frac{1}{x}\right)^2$

cc $25k^2 - 20 + \frac{4}{k^2}$

$\left(5k-\frac{2}{k}\right)^2$

dd $(x+2)^2 - y^2$

$(x+2+y)(x+2-y)$

ee $(a-1)^2-(b-2)^2$ $(a+b-3)(a-b+1)$	**ff** $z^2-(1+w)^2$ $(z+w+1)(z-w-1)$	**gg** $(x+2)^2-(2y+1)^2$ $(x+2y+3)(x-2y+1)$
hh x^4-1 $(x^2+1)(x+1)(x-1)$	**ii** $9x^6-4y^2$ $(3x^3+2y)(3x^3-2y)$	**jj** x^4-16y^4 $(x^2+4y^2)(x+2y)(x-2y)$
kk $125a^2b^2-20v^2$ $5(5ab+2v)(5ab-2v)$	**ll** $16-a^4$ $(4+a^2)(2+a)(2-a)$	**mm** $54g^3p-6gp^3$ $6gp(3g+p)(3g-p)$
nn $28c-63cw^2$ $7c(2+3w)(2-3w)$	**oo** $144e^5-36e^3$ $36e^3(2e+1)(2e-1)$	**pp** x^4-y^4 $(x^2+y^2)(x+y)(x-y)$
qq $100k^2-6\frac{1}{4}$ $(10k+5/2)(10k-5/2)$	**rr** $108m^3n^2-27m$ $27m(2mn+1)(2mn-1)$	**ss** $c^2-1\frac{7}{9}$ $(c+4/3)(c-4/3)$
tt $\frac{1}{25}-x^2$ $\left(\frac{1}{5}+x\right)\left(\frac{1}{5}-x\right)$	**uu** $\frac{64b^2}{25}-\frac{4y^2}{81}$ $\left(\frac{8b}{5}+\frac{2y}{9}\right)\left(\frac{8b}{5}-\frac{2y}{9}\right)$	**vv** $3p^2-\frac{48}{p^2}$ $3\left(p+\frac{4}{p}\right)\left(p-\frac{4}{p}\right)$
ww $(m+3)^2-25$ $(m+8)(m-2)$	**xx** $(x-y)^2-y^2$ $x(x-2y)$	**yy** $(p+x)^2-(p-x)^2$ $4px$
zz $\frac{4c^2}{9}-\frac{e^2}{36}$ $\left(\frac{2c}{3}+\frac{e}{6}\right)\left(\frac{2c}{3}-\frac{e}{6}\right)$	**aaa** $4g^2-(g-2k)^2$ $(3g-2k)(g+2k)$	**bbb** $5w^5-80w$ $5w(w-2)(w+2)(w^2+4)$
ccc $a^2(a+2)-4(a+2)$ $(a+2)(a+2)(a-2)$	**ddd** $6x^2-5x-6$ $(2x+3)(3x-2)$	**eee** $6x^2-5xy-6y^2$ $(3x+2y)(2x-3y)$
fff $x(y-1)+3(y-1)$ $(y-1)(x+3)$	**ggg** $3x(y-2)+5(2-y)$ $(y-2)(3x-5)$	**hhh** $(m-n)^2-(m-n)$ $(m-n)(m-n-1)$

8 Factorise

a $4x+4$

$4(x+1)$

b $4x+x$

$x(4+1)=5x$

c $4x+8$

$4(x+2)$

d $4x+8x$

$4x(1+2)=12x$

e x^2+x

$x(x+1)$

f $4x^2+2x$

$2x(2x+1)$

g $-4x-4$

$-4(x+1)$

h $-4x+x$

$x(-4+1)=-3x$

i $-4x^2-8x$

$-4x(x+2)$

j x^3+x^2

$x^2(x+1)$

k $-4x^3-8x^4$

$-4x^3(1+2x)$

l $4xy+8x$

$4x(y+2)$

m $4xy^2+8xy$

$4xy(y+2)$

n $-4x^2y-8xy^2$

$-4xy(x+2y)$

o $(x+4)(x+8)+x+4$

$(x+4)(x+9)$

p $(x+4)(x+8)+(x+4)(x-2)$

$2(x+4)(x+3)$

q $(x+4)(x+8)+(x+4)^2(x+8)$

$(x+8)(x+5)(x+4)$

r $(x+4)(x+8)+(x+4)(x+8)$

$2(x+4)(x+8)$

s $(x+4)(x+8)-(-x-4)(x+8)$

$2(x+4)(x+8)$

t $(x+4)(x+8)-(-x-4)(x+2)$

$2(x+4)(x+5)$

u $(2x+8)(x+8)+(3x+12)(x+8)$

$5(x+4)(x+8)$

v $-8x^2y^3(x+4)(2x+16)-4xy^4(x+4)(x+8)$

$-4xy^3(x+4)(x+8)(4x+y)$

w 4^n+16^n

$4^n(1+4^n)$

x $4^{n+1}+4^n$

5×4^n

9 Finish factorising the following:

a $(x+3)(x^2-16)$ $(x+3)(x+4)(x-4)$	**b** $(x+3)(x^2+10x+16)$ $(x+3)(x+2)(x+8)$	**c** $(x+3)(x^2+8x+16)$ $(x+3)(x+4)^2$
d $(x+3)(4x^2+8x+16)$ $4(x+3)(x^2+2x+4)$	**e** $(x+3)(4x^2+8x+4)$ $4(x+3)(x+1)^2$	**f** $(x+3)(4x^2-8x+4)$ $4(x+3)(x-1)^2$
g $5(x+3)(4x^2+8x-5)$ $5(x+3)(2x-1)(2x+5)$	**h** $5(x+3)(4x^2+8x-12)$ $20(x+3)^2(x-1)$	**i** $5(x+3)(4x^2+8x)$ $20x(x+3)(x+2)$

10 Show that:

a n^3-n is divisible by 6 for all integers n.
(hint: a number is divisible by 6 if it is divisible by 2 and 3)

$n^3-n=n(n+1)(n-1)$

n^3-n is the product of 3 consecutive integers; one of the integers must be even, and one must be divisible by 3. Therefore, divisible by 6.

b $n^4+2n^3-n^2-2n$ is divisible by 24 for all integers n.

$n^4+2n^3-n^2-2n=n(n+1)(n-1)(n+2)$; a product of 4 consecutive integers.

Two of the numbers are even, and one of them must be divisible by 4.

One of the numbers must be divisible by 3.

Therefore, divisible by 24 as it is divisible by 8 (from the two even numbers, one divisible by 4) and 3.

Note: it is not enough to prove divisibility by 4 and 6, as they are not coprime (e.g. 12 is divisible by 4 and 6 but not 24)

SIMPLIFYING ALGEBRAIC FRACTIONS BY FACTORISING 1

- To simplify algebraic fractions by factorising:
  1. **Factorise** the numerator and denominator.
  2. **Cancel** any common factors.

Example Simplifying algebraic fractions by factorising.

$\frac{6x+8}{6}$ 1. $\frac{2(3x+4)}{6}$ 2. $\frac{3x+4}{3}$	Why did we factorise the numerator and denominator? *We factorise to see if the numerator and denominator have common ………………* Why can't $\frac{3x+4}{3}$ be simplified further? *The numerator and denominator do not share any common ……………………*

Simplify these expressions.

a $\frac{5a+10}{5}$

$= \frac{5(\quad\quad)}{5}$

$=$

$a+2$

b $\frac{8}{4d-2}$

$= \frac{8}{2(\quad\quad)}$

$=$

$\frac{4}{2d-1}$

c $\frac{2p+2q}{p+q}$

2

d $\frac{10}{2x^2+4x}$

$\frac{5}{x(x+2)}$

e $\frac{x^2-4}{x+2}$

$x-2$

f $\frac{9-x^2}{3-x}$

$3+x$

Key ideas

1. To simplify algebraic fractions, first ………………… the numerator and denominator, then cancel any ……………… …………………

1 Simplify these expressions by cancelling common factors.

a $\frac{2x}{4}$ $\frac{x}{2}$	**b** $\frac{6a}{5a}$ $\frac{6}{5}$	**c** $\frac{4y}{20y}$ $\frac{1}{5}$
d $\frac{3(x+1)}{9(x+1)}$ $\frac{1}{3}$	**e** $\frac{2(x-2)}{8(x-2)}$ $\frac{1}{4}$	**f** $\frac{8(x+4)}{12(x+4)}$ $\frac{2}{3}$
g $\frac{(x+5)(x-5)}{(x+5)}$ $x-5$	**h** $\frac{6(x-1)(x+3)}{9(x+3)}$ $\frac{2(x-1)}{3}$	**i** $\frac{8(x-2)}{4(x-2)(x+4)}$ $\frac{2}{x+4}$

2 Complete these factorisations.

a $\frac{2x-4}{8} = \frac{2(\quad)}{8}$ $\frac{2(x-2)}{8}$	**b** $\frac{12-18x}{2x-3x^2} = \frac{6(\quad)}{x(\quad)}$ $\frac{6(2-3x)}{x(2-3x)}$

3 Simplify by factorising before cancelling.

a $\frac{5x-5}{5}$ $x-1$	**b** $\frac{4x-12}{10}$ $\frac{2(x-3)}{5}$	**c** $\frac{2x-4}{3x-6}$ $\frac{2}{3}$
d $\frac{15x+9y}{3}$ $5x+3y$	**e** $\frac{6}{12r-18}$ $\frac{1}{2r-3}$	**f** $\frac{7-y}{y-7}$ -1
g $\frac{x^2-3x}{x}$ $x-3$	**h** $\frac{4x^2+10x}{5x}$ $\frac{2(2x+5)}{5}$	**i** $\frac{3x+3y}{2x+2y}$ $\frac{3}{2}$
j $\frac{x^2+x}{x}$ $x+1$	**k** $\frac{x^2-2x}{x}$ $x-2$	**l** $\frac{2x-4xy}{2x}$ $1-2y$
m $\frac{2x+4}{5x+10}$ $\frac{2}{5}$	**n** $\frac{7x-7}{2x-2}$ $\frac{7}{2}$	**o** $\frac{3ac+5a}{a+2ab}$ $\frac{3c+5}{1+2b}$
p $\frac{7a-21}{2a-6}$ $\frac{7}{2}$	**q** $\frac{12p}{80+2pq}$ $\frac{6p}{40+pq}$	**r** $\frac{100-10x}{20-2x}$ 5

4 This is a common mistake when cancelling. Explain why it is incorrect.

$$\frac{2x + \cancel{4}^{2}}{\cancel{2}^{1}} = 2x + 2 \quad ✗$$

You must divide every term in the numerator by 2.

5 Simplify the following by first factorising using the difference of two squares.

a $\dfrac{x^2-100}{x+10}$ $x-10$	**b** $\dfrac{x^2-49}{x+7}$ $x-7$	**c** $\dfrac{x^2-25}{x+5}$ $x-5$
d $\dfrac{2(x-20)}{x^2-400}$ $\dfrac{2}{x+20}$	**e** $\dfrac{5(x-6)}{x^2-36}$ $\dfrac{5}{x+6}$	**f** $\dfrac{3x+27}{x^2-81}$ $\dfrac{3}{x-9}$

6 The expression below can be simplified by factoring out -1:

$$\frac{5-2x}{2x-5} = \frac{-1(-5+2x)}{2x-5} = \frac{-1(2x-5)}{2x-5} = -1$$

Factorise the following using the same method.

a $\dfrac{7-3x}{3x-7}$ -1	**b** $\dfrac{4x-1}{1-4x}$ -1	**c** $\dfrac{8x+16}{-2-x}$ -8
d $\dfrac{x+3}{-9-3x}$ $-\dfrac{1}{3}$	**e** $\dfrac{5-3x}{18x-30}$ $-\dfrac{1}{6}$	**f** $\dfrac{x^2-9}{3-x}$ $-(x+3)$
g $\dfrac{-x^2-4x}{-4x}$ $\dfrac{x+4}{4}$	**h** $\dfrac{5x-2x^2}{2x^2-5x}$ -1	**i** $\dfrac{5y-3xy}{18xy-30y}$ $-\dfrac{1}{6}$
j $\dfrac{x^2-100}{10-x}$ $-(10+x)$	**k** $\dfrac{-5a-20}{4+a}$ -5	**l** $\dfrac{x^2-49}{35-5x}$ $-\dfrac{x+7}{5}$

7 Expand the following.

a $\frac{1}{2}(6y+10)$	**b** $\frac{1}{3}(9x-12)$	**c** $\frac{1}{5}(5s-20t+5)$	**d** $-\frac{1}{10}(20-90k)$
$3y+5$	$3x-4$	$s-4t+1$	$-2+9k$

8 Rewrite each division as multiplication by a fraction, then expand. Simplify the answer.

a $\frac{2y+8}{2}=\frac{1}{2}(2y+8)$ $y+4$	**b** $\frac{12-16y}{4}$ $3-4y$	**c** $\frac{18k-6}{6}$ $3k-1$
d $\frac{30t+15}{5}$ $6t+3$	**e** $\frac{63-14k}{-7}$ $-9+2k$	**f** $\frac{4a+8}{4}+3$ $a+5$
g $2y+\frac{4y-16}{4}$ $3y-4$	**h** $\frac{60y-24}{12}-2(y+3)$ $3y-8$	

SIMPLIFYING ALGEBRAIC FRACTIONS BY FACTORISING 2

- To simplify algebraic fractions by factorising:
  1. **Factorise** the numerator and denominator.
  2. **Cancel** any common factors.

Example Simplifying algebraic fractions by factorising.

$\dfrac{x^2 - x}{x^2 + 2x - 3}$	How did we factorise the denominator?
1. $\dfrac{x(x-1)}{(x+3)(x-1)}$	*The denominator is a q……………… t………………, so we used the ……… method.*
2. $\dfrac{x}{x+3}$	What was the common factor that we "cancelled"? *The common factor was ……………*

Simplify the following.

a $\dfrac{x^2+5x+6}{x+2}$

$x+3$

b $\dfrac{x^2+6x+8}{x+4}$

$x+2$

c $\dfrac{a^2-9}{a^2+a-12}$

$\dfrac{a+3}{a+4}$

d $\dfrac{x^2+11x+28}{x^2+7x+12}$

$\dfrac{x+7}{x+3}$

(Skip these if you haven't learned factorising non-monics yet)

e $\dfrac{2x^2-3x-2}{x^2-4}$

$\dfrac{2x+1}{x+2}$

f $\dfrac{2p^2+7p-15}{6p-9}$

$\dfrac{p+5}{3}$

1 Simplify by first factorising.

a $\dfrac{x^2-x-6}{x-3}$

$x+2$

b $\dfrac{x^2+8x+16}{x+4}$

$x+4$

c $\dfrac{x^2-7x+12}{x-4}$

$x-3$

d $\dfrac{x-2}{x^2+x-6}$

$\dfrac{1}{x+3}$

e $\dfrac{x+7}{x^2+5x-14}$

$\dfrac{1}{x-2}$

f $\dfrac{x-9}{x^2-19x+90}$

$\dfrac{1}{x-10}$

g $\dfrac{c^2+11c+30}{6c+30}$

$\dfrac{c+6}{6}$

h $\dfrac{he-hd+pe-pd}{e^2-d^2}$

$\dfrac{h+p}{e+d}$

i $\dfrac{k^2+5k-24}{2k^2-18}$

$\dfrac{k+8}{2(k+3)}$

j $\dfrac{r^2-2r-24}{r^2+12r+32}$

$\dfrac{r-6}{r+8}$

k $\dfrac{6m-30-pm+5p}{m^2+3m-40}$

$\dfrac{6-p}{m+8}$

l $\dfrac{n^3-5n^2+4n}{n^2-2n+1}$

$\dfrac{n(n-4)}{n-1}$

2 Simplify these expressions.

a $\dfrac{2x}{x}$

2

b $\dfrac{2x+7}{x}$

can't simplify

c $\dfrac{2x+7}{7}$

can't simplify

d $\dfrac{2(x+3)}{3(x+3)}$

$\frac{2}{3}$

e $\dfrac{2(x+3)}{(x+3)}$

2

f $\dfrac{(x+3)}{2(x+3)}$

$\frac{1}{2}$

g $\dfrac{x+3}{2x+6}$

$\frac{1}{2}$

h $\dfrac{(x+3)(x+4)}{2x+6}$

$\frac{x+4}{2}$

i $\dfrac{(x+3)(x+4)}{2x+8}$

$\frac{x+3}{2}$

j $\dfrac{x^2+7x+12}{2x+8}$

$\frac{x+3}{2}$

k $\dfrac{x^2+7x+12}{(x+4)(x-9)}$

$\frac{x+3}{x-9}$

l $\dfrac{x^2+7x+12}{x^2-5x-36}$

$\frac{x+3}{x-9}$

m $\dfrac{2x^2+14x+24}{x^2-5x-36}$

$\frac{2(x+3)}{x-9}$

n $\dfrac{2x^2+14x+24}{3x^2-15x-108}$

$\frac{2(x+3)}{3(x-9)}$

o $\dfrac{2x^2+14x+24}{3x^2+4x-15}$

$\frac{2(x+4)}{3x-5}$

p $\dfrac{14x-24-2x^2}{3x^2+4x-15}$

$-\frac{2(x-3)(x-4)}{(3x-5)(x+3)}$

q $\dfrac{14x-24-2x^2}{3x^2-4x-15}$

$-\frac{2(x-4)}{3x+5}$

3 Simplify these expressions.

a $\frac{(a+1)^2}{a+1}$ $a+1$	**b** $\frac{5(a-3)^2}{a-3}$ $5(a-3)$	**c** $\frac{7(x+7)^2}{14(x+7)}$ $\frac{x+7}{2}$
d $\frac{3(x-1)(x+2)^2}{18(x-1)(x+2)}$ $\frac{x+2}{6}$	**e** $\frac{x^2+6x+9}{2x+6}$ $\frac{x+3}{2}$	**f** $\frac{11x-22}{x^2-4x+4}$ $\frac{11}{x-2}$
g $\frac{x^2-6x+9}{x-3}$ $x-3$	**h** $\frac{4x+20}{x^2+10x+25}$ $\frac{4}{x+5}$	**i** $\frac{x^2-14x+49}{5x-35}$ $\frac{x-7}{5}$

4 Simplify by factorising using the difference of two squares first.

a $\frac{x^2-7}{x+\sqrt{7}}$ $x-\sqrt{7}$	**b** $\frac{x^2-10}{x-\sqrt{10}}$ $x+\sqrt{10}$	**c** $\frac{\sqrt{5}x+3}{5x^2-9}$ $\frac{1}{\sqrt{5}-3}$
d $\frac{\sqrt{3}x-4}{3x^2-16}$ $\frac{1}{\sqrt{3}x+4}$	**e** $\frac{(x+1)^2-2}{x+1+\sqrt{2}}$ $x+1-\sqrt{2}$	**f** $\frac{(x-3)^2-5}{x-3-\sqrt{5}}$ $x-3+\sqrt{5}$

5 Simplify by first factorising.

a $\dfrac{6x^2-x-35}{3x+7}$

$2x-5$

b $\dfrac{8x^2+10x-3}{2x+3}$

$4x-1$

c $\dfrac{10x-2}{15x^2+7x-2}$

$\dfrac{2}{3x+2}$

d $\dfrac{20x-12}{10x^2-21x+9}$

$\dfrac{4}{2x-3}$

e $\dfrac{2x^2+11x+12}{6x^2+11x+3}$

$\dfrac{x+4}{3x+1}$

f $\dfrac{12x^2-x-1}{8x^2+14x+3}$

$\dfrac{3x-1}{2x+3}$

g $\dfrac{9x^2-4}{15x^2+4x-4}$

$\dfrac{3x-2}{5x-2}$

h $\dfrac{14x^2+19x-3}{49x^2-1}$

$\dfrac{2x+3}{7x+1}$

i $\dfrac{8x^2-2x-15}{16x^2-25}$

$\dfrac{2x-3}{4x-5}$

j $\dfrac{25-4a^2}{6a^2-11a-10}$

$-\dfrac{2a+5}{3a+2}$

k $\dfrac{40+11b-2b^2}{8b^2+34b+35}$

$\dfrac{8-b}{4b+7}$

l $\dfrac{x^2y^2-4}{xy^2-3mxy-2y+6m}$

$\dfrac{xy-2}{y-3m}$

MULTIPLYING AND DIVIDING FRACTIONS BY FACTORISING

- To multiply and divide complicated algebraic fractions:
  1. **Change division to multiplication** by the reciprocal.
  2. **Factorise** all numerators and denominators.
  3. **Cancel** any common factors vertically or diagonally.
  4. **Multiply** the numerators and multiply the denominators.

Example Multiply/divide algebraic fractions by factorising.

$\frac{2a}{a^2-9}\times\frac{a-3}{5a}$	$\frac{12x}{x+1}\div\frac{6x}{x^2+2x+1}$
2. $\frac{2a}{(a-3)(a+3)}\times\frac{a-3}{5a}$	1. $\frac{12x}{x+1}\times\frac{x^2+2x+1}{6x}$
3. $\frac{2a}{a+3}\times\frac{1}{5a}$	2. $\frac{12x}{x+1}\times\frac{(x+1)^2}{6x}$
4. $\frac{2}{a+3}\times\frac{1}{5}$	3. $\frac{12x}{1}\times\frac{x+1}{6x}$
$\frac{2}{5(a+3)}$	4. $\frac{2}{1}\times\frac{x+1}{1}=2(x+1)$

Simplify these expressions.

a $\frac{3x+6}{5}\times\frac{10}{x+2}$

$=\frac{3(x+2)}{5}\times\frac{10}{x+2}$

$=$

6

b $\frac{3x+3}{2x}\times\frac{x^2}{x^2-1}$

$=\frac{3(\quad)}{2x}\times\frac{x^2}{(\quad)(\quad)}$

$=$

$\frac{3x}{2(x-1)}$

c $\frac{a^2-4}{3}\times\frac{5b}{a+2}$

$\frac{5b(a-2)}{3}$

d $\frac{6x-12}{5}\div\frac{x-2}{3}$

$\frac{18}{5}$

e $\frac{3-6y}{8}\div\frac{1-2y}{2}$

$\frac{3}{4}$

f $\frac{2}{a-1}\div\frac{3}{2a-2}$

$\frac{4}{3}$

Key ideas

1. To multiply algebraic fractions, first ……………………… the numerator and denominator, and …………………… before you multiply.
2. To divide by a fraction, multiply by the ………………………

1 Simplify.

a $\frac{2x(x-4)}{4(x+1)} \times \frac{x+1}{x}$

$\frac{x-4}{2}$

b $\frac{(x+2)(x-3)}{x-5} \times \frac{x-5}{x+2}$

$x-3$

c $\frac{x-3}{x+2} \times \frac{3(x+4)(x+2)}{x+4}$

$3(x-3)$

2 Simplify.

a $\frac{x(x+1)}{(x+3)} \div \frac{x+1}{x+3}$

x

b $\frac{x+3}{x+2} \div \frac{x+3}{2(x-2)}$

$\frac{2(x-2)}{x+2}$

c $\frac{5(2x-3)}{x+7} \div \frac{(x+2)(2x-3)}{x+7}$

$\frac{5}{x+2}$

3 Simplify.

a $\frac{x^2-4}{x^2-x-6} \times \frac{5x-15}{x^2+4x-12}$

$\frac{5}{x+6}$

b $\frac{x^2+3x+2}{x^2+4x+3} \times \frac{x^2-9}{3x+6}$

$\frac{x-3}{3}$

c $\frac{x^2+2x-3}{x^2-25} \times \frac{2x-10}{x+3}$

$\frac{2(x-1)}{x+5}$

d $\frac{x^2-4x+3}{x^2+4x-21} \times \frac{5x-15}{x^2+4x-12}$

$\frac{5(x-3)(x-1)}{(x+7)(x+6)(x-2)}$

e $\frac{x^2-x-6}{x^2+x-12} \times \frac{x^2+5x+4}{x^2-1}$

$\frac{x+2}{x-1}$

f $\frac{x^2-4x-12}{x^2-4} \times \frac{x^2-6x+8}{x^2-36}$

$\frac{x-4}{x+6}$

g $\frac{x^2+2x-3}{x^2-25} \div \frac{3x-3}{2x+10}$

$\frac{2(x+3)}{3(x-5)}$

h $\frac{x^2+3x+2}{x^2+4x+3} \div \frac{4x+8}{x^2-9}$

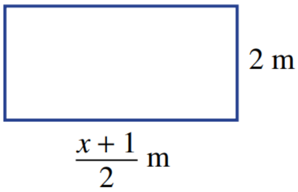
$\frac{x-3}{4}$

i $\frac{x^2+5x-14}{x^2+2x-3} \div \frac{x^2+9x+14}{x^2+x-2}$

$\frac{x-2}{x+3}$

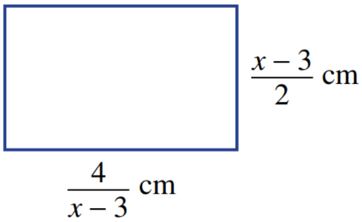
4 Find an expression for the area of these rectangles.

a



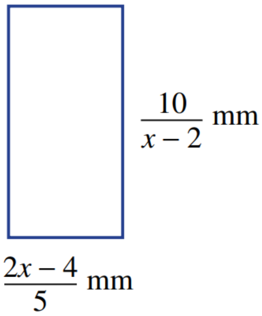
$x+1$ m^2

b



2 cm^2

c



4 mm^2

5 Find the missing algebraic fraction.

a $\frac{x+3}{5} \times \square = 2$

$\frac{10}{x+3}$

b $\frac{1-x}{x} \times \square = 3$

$\frac{3x}{1-x}$

c $\square \div \frac{x}{2} = \frac{3(x+2)}{x}$

$\frac{3(x+2)}{2}$

d $\square \div \frac{2x-2}{3} = \frac{5x}{x-1}$

$\frac{10x}{3}$

e $\frac{1}{x} \div \square \times \frac{x-1}{2} = 1$

$\frac{x-1}{2x}$

f $\frac{2-x}{7} \times \square \div \frac{5x}{x-1} = x$

$\frac{35x^2}{(x-1)(2-x)}$

6 Simplify

a $\frac{a^2+2ab+b^2}{a(a+b)} \div \frac{a^2-b^2}{a^2-2ab+b^2}$

$\frac{a-b}{a}$

b $\frac{a^2-2ab+b^2}{a^2-b^2} \div \frac{a^2-b^2}{a^2+2ab+b^2}$

1

c $\frac{a^2-b^2}{a^2-2ab+b^2} \div \frac{a^2-b^2}{a^2+2ab+b^2}$

$\frac{(a+b)^2}{(a-b)^2}$

d $\frac{a^2+2ab+b^2}{a(a+b)} \div \frac{a(a-b)}{a^2-2ab+b^2}$

$\frac{(a+b)(a-b)}{a^2}$

7 Simplify these expressions.

a $\frac{3x+3}{2x} \times \frac{x^2}{x^2-1}$

$\frac{3x}{2(x-1)}$

b $\frac{a^2+a-2}{a+2} \times \frac{a^2-3a}{a^2-4a+3}$

a

c $\frac{c^2+5c+6}{c^2-16} \div \frac{c+3}{c-4}$

$\frac{c+2}{c+4}$

d $\frac{x^2-x-20}{x^2-25} \times \frac{x^2-x-2}{x^2+2x-8} \div \frac{x+1}{x^2+5x}$

x

e $\frac{a^2-5a}{y^2-4y+4} \div \frac{3a-15}{y^2-4} \times \frac{y^2-y-2}{5ay}$

$\frac{(y+2)(y+1)}{15y}$

f $\frac{x^2-8x+15}{5x^2+10x} \div \frac{x^2-9}{10x^2} \times \frac{x^2+5x+6}{2x-10}$

x

g $\frac{100x^2-25}{2x^2-9x-5} \div \frac{5x^2+10x-75}{2x^2-5x-3}$

$\frac{5(2x-1)(2x+1)}{(x-5)(x+5)}$

h $\frac{3x^2-12}{30x+15} \div \frac{2x^2-3x-2}{4x^2+4x+1}$

$\frac{x+2}{5}$

ADDING FRACTIONS BY FACTORISING DENOMINATORS

IDENTIFYING THE LOWEST COMMON MULTIPLE

- The lowest common multiple is the smallest expression that is divisible by each of the given expressions.

Example Identify the lowest common multiple of algebraic terms.

<table>
<tr><td colspan="12">Identify the lowest common multiple of these terms.</td></tr>
<tr><td colspan="3">3 and 4</td><td colspan="3">5 and 8</td><td colspan="3">7 and 10</td><td colspan="3">x and y</td></tr>
<tr><td colspan="3">4 and 8</td><td colspan="3">6 and 18</td><td colspan="3">8 and 32</td><td colspan="3">x and $2x$</td></tr>
<tr><td colspan="3">x and $3x$</td><td colspan="3">4 and $4x$</td><td colspan="3">x and x^2</td><td colspan="3">$2x$ and x^2</td></tr>
<tr><td colspan="3">4 and $10x$</td><td colspan="3">5 and $(x-9)$</td><td colspan="3">$3x$ and 12</td><td colspan="3">$4x$ and $16x^2$</td></tr>
<tr><td colspan="3">$(x+1)$ and x</td><td colspan="3">$(x-3)$ and $(x+2)$</td><td colspan="3">$x(x+3)$ and x</td><td colspan="3">$(x+1)$ and $(x-2)$</td></tr>
<tr><td colspan="4">$(x-7)^2$ and $(x-7)$</td><td colspan="4">$(x-7)(x-1)$ and $(x-7)$</td><td colspan="4">$(x+2)(x-2)$ and $(x-2)$</td></tr>
<tr><td colspan="4">$(2x+1)$ and $(x+3)$</td><td colspan="4">$(4-x)$ and $(x-1)$</td><td colspan="4">$(3x-4)(3x+4)$ and $(3x-4)$</td></tr>
<tr><td colspan="6">$(x+2)(x-2)$ and $(x-2)(x+3)$</td><td colspan="6">$(x+1)(x+2)$ and $(x+2)(x-1)$</td></tr>
<tr><td colspan="6">$x(x-2)$ and $(x-2)(x+3)$</td><td colspan="6">$4x(x+5)$ and $2x^2(x+5)(x-3)$</td></tr>
</table>

Hint: If you're having trouble noticing the LCM of two terms, use the rule:

1. Multiply the terms.
2. Divide by the HCF.

ADDING SUBTRACTING FRACTIONS BY FACTORISING

- To add and subtract complicated algebraic fractions:
  1. **Factorise** the denominators.
  2. Change denominators to the **Lowest Common Denominator** using equivalent fractions.
  3. Add or subtract the numerators.

  Note: All fractions have invisible brackets in the numerator and denominator $\frac{a+b}{c+d} = \frac{(a+b)}{(c+d)}$

Example Add/subtract algebraic fractions by factorising.

$\frac{1}{x-4} + \frac{1}{x}$ 2. $\frac{}{x(x-4)} + \frac{}{x(x-4)}$ $\frac{x}{x(x-4)} + \frac{x-4}{x(x-4)}$ 3. $\frac{2x-4}{x(x-4)}$	$\frac{2+x}{x^2-x} - \frac{5}{x-1}$ 1. $\frac{2+x}{x(x-1)} - \frac{5}{x-1}$ 2. $\frac{}{x(x-1)} - \frac{}{x(x-1)}$ $\frac{2+x}{x(x-1)} - \frac{5x}{x(x-1)}$ 3. $\frac{2-4x}{x(x-1)}$

Simplify these expressions.

a $\frac{1}{x} + \frac{1}{x+1}$

$= \frac{}{x(x+1)} + \frac{}{x(x+1)}$

$\frac{2x+1}{x(x+1)}$

b $\frac{4}{x+1} + \frac{3}{x-2}$

$\frac{7x-5}{(x+1)(x-2)}$

c $\frac{2}{x-3} - \frac{3}{x-2}$

$\frac{-x+5}{(x-3)(x-2)}$

d $\frac{3}{(x-1)^2} - \frac{2}{x-1}$

$\frac{5-2x}{(x-1)^2}$

e $\frac{2x}{x-5} - \frac{x}{x+1}$

$\frac{x^2+7x}{(x-5)(x+1)}$

f $\frac{2}{x^2-4} - \frac{3}{x+2}$

$\frac{8-3x}{(x-2)(x+2)}$

Key ideas

1. To add or subtract algebraic fractions:
   a. ………………… the denominators.
   b. Change denominators to the …………… ……………… ………………
   c. Add or subtract the ……………………

1 Simplify the following. Change denominators to be the same first.

a $\dfrac{5}{x}+\dfrac{2}{x+4}$

$\dfrac{7x+20}{x(x+4)}$

b $\dfrac{4}{x-7}+\dfrac{3}{x}$

$\dfrac{7(x-3)}{x(x-7)}$

c $\dfrac{1}{x-3}+\dfrac{2}{x+5}$

$\dfrac{3x-1}{(x-3)(x+5)}$

d $\dfrac{3}{x+3}-\dfrac{2}{x-4}$

$\dfrac{x-18}{(x-4)(x+3)}$

e $\dfrac{6}{2x-1}-\dfrac{3}{x-4}$

$\dfrac{-21}{(x-4)(2x-1)}$

f $\dfrac{4}{x-5}+\dfrac{2}{3x-4}$

$\dfrac{2(7x-13)}{(x-5)(3x-4)}$

g $\dfrac{5}{2x-1}-\dfrac{6}{x+7}$

$\dfrac{41-7x}{(x+7)(2x-1)}$

h $\dfrac{8}{3x-2}-\dfrac{3}{1-x}$

$\dfrac{14-17x}{(1-x)(3x-2)}$

2 Simplify the following.

a $\dfrac{1}{(x+3)(x+4)}+\dfrac{2}{(x+4)(x+5)}$

$\dfrac{3x+11}{(x+3)(x+4)(x+5)}$

b $\dfrac{3}{(x+1)(x+2)}-\dfrac{5}{(x+1)(x+4)}$

$\dfrac{2-2x}{(x+1)(x+2)(x+4)}$

c $\dfrac{4}{(x-1)(x-3)}-\dfrac{6}{(x-1)(2-x)}$

$\dfrac{26-10x}{(x_1)(x-3)(2-x)}$

d $\dfrac{5x}{(x+1)(x-5)}-\dfrac{2}{(x-5)}$

$\dfrac{3x-2}{(x+1)(x-5)}$

e $\dfrac{7}{k(k-1)}-\dfrac{4}{k(k+1)}$

$\dfrac{3k+11}{k(k-1)(k+1)}$

f $\dfrac{8}{(x+4)(x-7)}+\dfrac{3}{x(x+4)}$

$\dfrac{11x-21}{x(x-7)(x+4)}$

3 Simplify the following.

a $\dfrac{4}{(x+1)^2}-\dfrac{3}{x+1}$

$\dfrac{1-3x}{(x+1)^2}$

b $\dfrac{2}{(x+3)^2}-\dfrac{4}{x+3}$

$\dfrac{-4x-10}{(x+3)^2}$

c $\dfrac{3}{x-2}+\dfrac{4}{(x-2)^2}$

$\dfrac{3x-2}{(x-2)^2}$

d $\dfrac{3x}{(x-1)^2}+\dfrac{2}{x-1}$

$\dfrac{5x-2}{(x-1)^2}$

e $\dfrac{3x+2}{3x}+\dfrac{7}{12}$

$\dfrac{19x+8}{12x}$

f $\dfrac{2x-1}{4}+\dfrac{2-3x}{10x}$

$\dfrac{10x^2-11x+4}{20x}$

g $\dfrac{3}{4-x}-\dfrac{2x}{x-1}$

$\dfrac{(2x+1)(x-3)}{(4-x)(x-1)}$

h $\dfrac{x}{x+1}-\dfrac{5x+1}{(x+1)^2}$

$\dfrac{x^2-4x-1}{(x+1)^2}$

4 Factorise the denominator before simplifying these fractions.

a $\dfrac{3}{x+2}+\dfrac{5}{2x+4}$

$\dfrac{11}{2(x+2)}$

b $\dfrac{7}{3x-3}-\dfrac{2}{x-1}$

$\dfrac{1}{3(x-1)}$

c $\dfrac{3}{8x-4}-\dfrac{5}{1-2x}$

$\dfrac{23}{4(2x-1)}$

d $\dfrac{4}{x^2-9}-\dfrac{3}{x+3}$

$\dfrac{13-3x}{(x-3)(x+3)}$

e $\dfrac{5}{2x+4}+\dfrac{2}{x^2-4}$

$\dfrac{5x-6}{2(x-2)(x+2)}$

f $\dfrac{10}{3x-4}-\dfrac{7}{9x^2-16}$

$\dfrac{3(10x+11)}{(3x-4)(3x+4)}$

g $\dfrac{1}{x^2+x}+\dfrac{1}{x^2-x}$

$\dfrac{2x}{x(x+1)(x-1)}$

h $\dfrac{1}{x-y}+\dfrac{2x-y}{x^2-y^2}$

$\dfrac{3x}{(x+y)(x-y)}$

i $\dfrac{3}{x^2+2x-8}-\dfrac{2}{x^2+x-6}$

$\dfrac{x+1}{(x-2)(x+3)(x+4)}$

j $\dfrac{1}{x^2-4x+3}+\dfrac{1}{x^2-5x+6}-\dfrac{1}{x^2-3x+2}$

$\dfrac{x}{(x-3)(x-2)(x-1)}$

5 Use the fact that $a-b=-(b-a)$ to help simplify these.

a $\dfrac{3}{1-x}-\dfrac{2}{x-1}$

$=\dfrac{3}{-(x-1)}-\dfrac{2}{x-1}$

$=\dfrac{-3-2}{x-1}$

$=$

$\dfrac{-5}{x-1}$

b $\dfrac{4x}{5-x}-\dfrac{3}{x-5}$

$\dfrac{4x+3}{5-x}$

c $\dfrac{2}{7x-3}-\dfrac{7}{3-7x}$

$\dfrac{9}{7x-3}$

d $\dfrac{1}{4-3x}+\dfrac{2x}{3x-4}$

$\dfrac{1-2x}{4-3x}$

e $\dfrac{-3x}{5-3x}-\dfrac{5}{3x-5}$

1

f $\dfrac{4}{x-6}+\dfrac{4}{6-x}$

0

6 Simplify:

a $\dfrac{2}{x+3}+\dfrac{x}{x^2-x-12}$

$\dfrac{3x-8}{(x-4)(x+3)}$

b $\dfrac{4}{x+2}+\dfrac{3x}{x^2-7x-18}$

$\dfrac{7x-36}{(x-9)(x+2)}$

c $\dfrac{4}{x^2-9}-\dfrac{1}{x^2-8x+15}$

$\dfrac{3x-23}{(x-5)(x-3)(x+3)}$

d $\dfrac{x+4}{x^2-x-6}-\dfrac{x-5}{x^2-9x+18}$

$\dfrac{x-14}{(x-6)(x-3)(x+2)}$

e $\dfrac{7}{x^2+7x+12}+\dfrac{2}{x^2-2x-15}$

$\dfrac{9(x-3)}{(x-5)(x+3)(x+4)}$

f $\dfrac{3}{(x+1)^2-4}-\dfrac{2}{x^2+6x+9}$

$\dfrac{x+11}{(x-1)(x+3)^2}$

g $\dfrac{2}{w^2-9w+20}+\dfrac{3}{w^2+2w-24}$

$\dfrac{5w-3}{(w-5)(w-4)(w+6)}$

h $\dfrac{d}{d^2-3d-4}-\dfrac{7}{2d-8}$

$-\dfrac{5x+7}{2(x-4)(x+1)}$

i $\dfrac{2}{4x^2-1}+\dfrac{1}{6x^2-x-2}$

$\dfrac{8x-5}{(2x-1)(2x+1)(3x-2)}$

j $\dfrac{4}{8x^2-18x-5}-\dfrac{2}{12x^2-5x-2}$

$\dfrac{2}{(2x-5)(3x-2)}$

k $\dfrac{c+1}{10c^2+7c-12}+\dfrac{c}{5c^2-39c+28}$

$\dfrac{3c^2-3c-7}{(c-7)(2c+3)(5c-4)}$

l $\dfrac{n+1}{4n^3-36n}-\dfrac{2}{5n^2+15n}$

$\dfrac{29-3n}{20(n-3)(n+3)}$

7 2008 HSC Advanced Band 4

Simplify $\dfrac{2}{n}-\dfrac{1}{n+1}$

$\dfrac{n+2}{n(n+1)}$

6 Explain the mistake.

Express $\dfrac{3}{x^2-2x}+\dfrac{2}{x^2-7x+10}$ as a single fraction in simplest form. **2**

$\dfrac{3(x^2-7x+10)+2(x^2-2x)}{(x^2-2x)(x^2-7x+10)} = \dfrac{(3x^2-21x+30)+(2x^2-4x)}{(x^2-2x)(x^2-7x+10)}$

$= \dfrac{6x^4-12x^3-42x^3-84x^2+60x^2-\ldots}{(x^2-2x)(x^2-7x+10)}$

$= \dfrac{6x^4-54x^3-24x^2-120x}{(x^2-2x)(x^2-7x+10)}$

They should have factorised before trying to make a common denominator

8 Simplify

$$\frac{1}{n}+\frac{(n+1)(n-1)}{n}-(n+1)$$

-1

9 Consider this pattern:

$$\frac{1}{2}-\frac{1}{3}=\frac{1}{6},\quad \frac{1}{3}-\frac{1}{4}=\frac{1}{12},\quad \frac{1}{4}-\frac{1}{5}=\frac{1}{20}$$

Prove that, for *any* two consecutive integers, the difference of their reciprocals is the reciprocal of their product.

Let the two consecutive integers be n and $n+1$

$$\frac{1}{n}-\frac{1}{n+1}=\frac{n+1-n}{n(n+1)}=\frac{1}{n(n+1)}$$

10 Consider the pattern:

$$\left(\frac{3}{2}+\frac{2}{3}\right)^2-\left(\frac{3}{2}-\frac{2}{3}\right)^2=4,\quad \left(\frac{7}{6}+\frac{6}{7}\right)^2-\left(\frac{7}{6}-\frac{6}{7}\right)^2=4,$$

Write a rule for this pattern and prove it algebraically.

$$\left(\frac{n+1}{n}+\frac{n}{n+1}\right)^2-\left(\frac{n+1}{n}-\frac{n}{n+1}\right)^2$$

$$=\left(\frac{n^2+2n+1+n^2}{n(n+1)}\right)^2-\left(\frac{n^2+2n+1-n^2}{n(n+1)}\right)^2$$

$$=\left(\frac{2n^2+2n+1}{n(n+1)}\right)^2-\left(\frac{2n+1}{n(n+1)}\right)^2$$

Using difference of two squares,

$$=\left(\frac{2n^2+2n+1}{n(n+1)}+\frac{2n+1}{n(n+1)}\right)\left(\frac{2n^2+2n+1}{n(n+1)}-\frac{2n+1}{n(n+1)}\right)$$

$$=\left(\frac{2n^2+4n+2}{n(n+1)}\right)\left(\frac{2n^2}{n(n+1)}\right)$$

$$=\left(\frac{2(n+1)^2}{n(n+1)}\right)\left(\frac{2n^2}{n(n+1)}\right)$$

By cancelling common factors,

$$=4$$

COMPOUND FRACTIONS

- Compound fractions involve fractions inside fractions. Remember that:
  - fractions represent division
  - there are invisible brackets enclosing the numerator and denominator.

1. Create a single fractions in the numerator and denominator.
2. Divide the numerator by the denominator.
3. Simplify.

Example Simplify expressions involving compound fractions.

Worked Example	Your Turn
Simplify $\frac{\frac{2}{5}}{1-\frac{3}{4}}$ 1. $=\frac{\left(\frac{2}{5}\right)}{\left(\frac{1}{4}\right)}$ 2. $=\frac{2}{5}\div\frac{1}{4}$ 3. $=\frac{2}{5}\times\frac{4}{1}$ $=\frac{8}{5}$	Simplify $\frac{4}{3+\frac{5}{8}}$ $\frac{32}{29}$
Simplify $\frac{1+\frac{1}{x}}{1-\frac{1}{x^2}}$ 1. $=\frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x^2-1}{x^2}\right)}$ 2. $=\frac{x+1}{x}\div\frac{x^2-1}{x^2}$ 3. $=\frac{x+1}{x}\times\frac{x^2}{x^2-1}$ $=\frac{x+1}{x}\times\frac{x^2}{(x+1)(x-1)}$ $=\frac{x}{x-1}$	Simplify $\frac{\frac{2}{x}-\frac{2}{3x}}{\frac{1}{x}-\frac{5}{6x}}$ 8

1 Simplify the following to a single fraction.

a $\dfrac{4}{1-\frac{3}{4}}$

16

b $\dfrac{3}{\frac{1}{4}+\frac{1}{3}}$

$\frac{36}{7}$

c $\dfrac{3}{1-\frac{1}{1-\frac{1}{3}}}$

-6

d $\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}$

$\frac{1}{3}$

e $\dfrac{2+\frac{1}{3}}{5-\frac{2}{3}}$

$\frac{7}{13}$

f $\dfrac{\frac{1}{2}-\frac{1}{5}}{1+\frac{1}{10}}$

$\frac{3}{11}$

g $\dfrac{\frac{17}{20}-\frac{3}{4}}{\frac{4}{5}-\frac{3}{10}}$

$\frac{1}{5}$

h $\dfrac{\frac{1}{x}}{1+\frac{2}{x}}$

$\frac{1}{x+2}$

i $\dfrac{1-\frac{1}{t}}{t+\frac{1}{t}}$

$\frac{t-1}{t^2+1}$

j $\dfrac{1}{\frac{1}{b}+\frac{1}{a}}$

$\frac{ab}{a+b}$

k $\dfrac{\frac{x}{y}+\frac{y}{x}}{\frac{x}{y}-\frac{y}{x}}$

$\frac{x^2+y^2}{x^2-y^2}$

2 Simplify

a $\dfrac{1-\frac{1}{x+1}}{\frac{1}{x}+\frac{1}{x+1}}$

$\dfrac{x^2}{2x+1}$

b $\dfrac{\frac{1}{x}+\frac{1}{x+1}}{\frac{1}{x}-\frac{1}{x+1}}$

$2x+1$

c $\dfrac{\frac{2}{x}+\frac{1}{x+3}}{\frac{3}{x}-\frac{1}{x+3}}$

$\dfrac{3(x+2)}{2x+9}$

d $\dfrac{\frac{3}{x+2}-\frac{2}{x+1}}{\frac{5}{x+2}-\frac{4}{x+1}}$

$\dfrac{x-1}{x+3}$

3 Write as a single fraction, without negative indices, and simplify.

a $a^{-1}-b^{-1}$

$\dfrac{b-a}{ab}$

b $\dfrac{1-y^{-1}}{1-y^{-2}}$

$\dfrac{y}{y+1}$

c $(x^{-2}-y^{-2})^{-1}$

$\dfrac{x^2y^2}{x^2-y^2}$

d $\dfrac{a^{-1}+b^{-1}}{a^{-2}-b^{-2}}$

$\dfrac{ab}{b-a}$

e $x^{-2}y^{-2}(x^2y^{-1}-y^2x^{-1})$

$\dfrac{x^3-y^3}{x^3y^3}$

f $\dfrac{(a^2-1)^{-1}}{(a-1)^{-1}}$

$\dfrac{1}{a+1}$

4 If $x = \frac{1}{t}$, $y = \frac{1}{1-x}$, $z = \frac{y}{y-1}$, show that $z = t$.

5 Simplify

$$\left(2 - \frac{3n}{m} + \frac{9n^2 - 2m^2}{m^2 + 2mn}\right) \div \left(\frac{1}{m} - \frac{1}{m - 2n - \frac{4n^2}{m+n}}\right)$$

$$\frac{3n - m}{2}$$

6 Simplify

$$\frac{1}{x + \frac{1}{x+2}} \times \frac{1}{x + \frac{1}{x-2}} \div \frac{x - \frac{4}{x}}{x^2 - 2 + \frac{1}{x^2}}$$

$$\frac{1}{x}$$

CHECK YOUR UNDERSTANDING

Expanding perfect squares

1 Expand and simplify $(x+7)^2$

a x^2+49 **b** $x^2+7x+49$ **c** $x^2+14x+49$ **d** x^2+14

2 Expand and simplify $(x-3)^2$

a x^2-6x-9 **b** x^2-9 **c** x^2+9 **d** x^2-6x+9

3 Expand and simplify $(x-5)^2$

a x^2-25 **b** x^2+25 **c** $x^2+10x-25$ **d** $x^2-10x+25$

4 Expand and simplify $(2y-3)^2$

a $(2y-3)(2y-3)$ **b** $4y^2-9$ **c** $4y^2+9$ **d** $4y^2-12y+9$

5 Expand and simplify $(3y-4)^2$

a $9y^2-16$ **b** $9y^2+16$ **c** $9y^2-24y+16$ **d** $(3y-4)(3y-4)$

Expanding difference of two squares

1 $(m+5)(m-5)$

a m^2-25 **b** m^2+25 **c** $m^2-10m-25$ **d** $m^2+10m-25$

2 $(y+2)(y-2)$

a $y^2+2y-2y-4$ **b** y^2-4 **c** y^2-4y-4 **d** y^2+4

3 $(3x-2)(3x+2)$

a $9x^2+6x-4$ **b** $9x^2-4$ **c** $9x^2+4$ **d** $9x^2-6x+4$

4 $(2x+3y)(2x-3y)$

a $4x^2+9y^2$ **b** $4x^2-9y^2$ **c** $4x^2-12xy+9y^2$ **d** $4x^2+12xy+9y^2$