

Student Name

Mathematics Stage 5 Core

LINEAR RELATIONSHIPS B

Book 1 Examine gradient-intercept form

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OVERVIEW

SYLLABUS CONTENT

MA5-LIN-C-02 graphs and interprets linear relationships using the gradient/slope-intercept form

Examine the gradient-intercept form

- Interpret the coefficient of $x(m)$ as the gradient/slope, and the constant (c) as the y -intercept for equations of the form $y = mx + c$
- Find the equation of a straight line in the form $y = mx + c$, given the gradient/slope and the y -intercept of the line
- Graph equations of the form $y = mx + c$ by using the gradient and the y -intercept
- Determine the gradient and y -intercept of a straight line from its graph and apply these values to determine the equation of the line
- Explain the effect of increasing or decreasing the gradient with or without digital tools
- Recognise and describe linear relationships in real-life contexts

REVIEW OF PRIOR KNOWLEDGE

1 Identify the coefficient of x and the constant term in these algebraic expressions.

a $2x + 5$

Coefficient of x :

Constant term:

b $4x - 3$

Coefficient of x :

Constant term:

c $-5x + 13$

Coefficient of x :

Constant term:

d $\frac{2}{3}x - 7$

Coefficient of x :

Constant term:

e $\frac{2x}{3} - 7$

Coefficient of x :

Constant term:

f $-3x$

Coefficient of x :

Constant term:

2 Plot these points on the Cartesian plane.

a A $(-3, 2)$

b B $(1, 4)$

c C $(2, -1)$

d D $(-2, -4)$

e F $(-1, 4)$

f G $(-3, -1)$

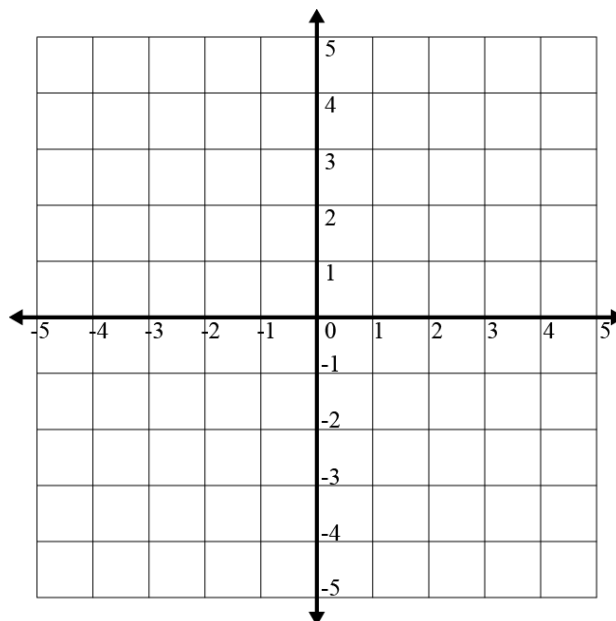
g H $(1.5, -2.5)$

h I $(3.5, -2\frac{1}{2})$

i J $(-2.5, 1.5)$

j K $(-1\frac{1}{2}, -1.5)$

k L $(\frac{1}{3}, 5)$



GRADIENT INTERCEPT FORM

- Every linear relationship has this equation:

Gradient-Intercept Equation of a Straight Line	
$y = mx + c$	
$\uparrow$ Coefficient of x Gradient	$\uparrow$ Constant term y-intercept

Example Determine the gradient and y-intercept from an equation.

$y = 2x - 7$ $y = \textcircled{2}x - \boxed{7}$ Gradient: 2 y-intercept: (0, -7)	How do we determine the gradient? <i>The gradient is the of x; the number that x is by.</i> Why is the y-intercept -7 rather than 7 ? <i>$y = 2x - 7$ is the same as $y = 2x + (-7)$, the constant term is</i>
$y = \frac{2x}{3} + 5$ $y = \textcircled{\frac{2x}{3}} + \boxed{5}$ Gradient: $\frac{2}{3}$ y-intercept: (0, 5)	Why is the gradient in this example $\frac{2}{3}$ rather than 2? <i>$\frac{2x}{3}$ is the same as $\frac{2}{3}x$, x has been multiplied by</i>

Determine the gradient and y – intercept of these lines.

a $y = 3x + 2$ Gradient: y- intercept: 3 2	b $y = -3x + 2$ Gradient: y- intercept: -3 2
c $y = 3x - 2$ Gradient: y- intercept: 3 -2	d $y = \frac{5}{4}x - 1$ Gradient: y- intercept: $\frac{5}{4}$ -1
e $y = 2 + 3x$ Gradient: y- intercept: 3 2	f $y = 2 - 3x$ Gradient: y- intercept: -3 2

- To find a line's equation given the gradient (m) and y -intercept (c), substitute m and c into $y = mx + c$

Example Determine equation of a line given gradient and y -intercept.

Find the equation of a line with gradient 3 and y-intercept (0,4). $m = 3$ $c = 4$ $y = 3x + 4$	<i>The gradient is the of x</i> <i>The y-intercept is the term.</i> Would $y = 4 + 3x$ also be correct?
Find the equation of a line with gradient $-\frac{1}{2}$ and y-intercept (0,0). $m = -\frac{1}{2}$ $c = 0$ $y = -\frac{x}{2}$	Does it matter if we write $y = -\frac{1}{2}x$ or $y = -\frac{x}{2}$? Why didn't we write $y = -\frac{x}{2} + 0$?

Determine the equation of the line if:

a Gradient is 6 and y -intercept is 17

$$y = 6x + 17$$

b Gradient is -2 and y -intercept is 3

$$y = -2x + 3$$

c y -intercept is -1 and gradient is 5

$$y = 5x - 1$$

d y -intercept is 2 and gradient is $\frac{2}{3}$

$$y = \frac{2}{3}x + 2 \text{ or } y = \frac{2x}{3} + 2$$

e Gradient is 1 and y -intercept is -4

$$y = x - 4$$

f y -intercept is -9 and gradient is $-\frac{6}{5}$

$$y = -\frac{6}{5}x - 9 \text{ or } y = -\frac{6x}{5} - 9$$

Key ideas

- Every straight-line graph has the equation
- m is the of the line.
- c is the of the line.

1 Determine the gradient and y-intercept of these lines.

a $y = 4x + 6$ $m =$ $c =$ 4, 6	b $y = -x - 1$ $m =$ $c =$ -1, -1
c $y = 6 + 4x$ $m =$ $c =$ 4, 6	d $y = \frac{3}{4}x - 8$ $m =$ $c =$ $\frac{3}{4}, -8$
e $y = -4x + 6$ $m =$ $c =$ -4, 6	f $y = \frac{3x}{4} - 8$ $m =$ $c =$ $\frac{3}{4}, -8$
g $y = -4x - 6$ $m =$ $c =$ -4, -6	h $y = -\frac{3x+9}{8}$ $m =$ $c =$ $-\frac{3}{8}, -\frac{9}{8}$
i $y = -6 - 4x$ $m =$ $c =$ -4, -6	j $y = \frac{x+5}{6}$ $m =$ $c =$ $\frac{1}{6}, \frac{5}{6}$
k $y = -4x$ $m =$ $c =$ -4, 0	l $y = 5.3x + 6.5$ $m =$ $c =$ 5.3, 6.5
m $y = x$ $m =$ $c =$ 1, 0	n $y = \frac{3}{4}x - \frac{8}{3}$ $m =$ $c =$ $\frac{3}{4}, -\frac{8}{3}$

2 For each line, state the gradient and the coordinates of the y-intercept.

a $y = 3x + 5$ $m = 3, (0, 5)$	b $y = 7 - 2x$ $m = -2, (0, 7)$	c $y = 0.5 - 4x$ $m = -4, (0, 0.5)$
d $y = 19 + 3x$ $m = 3, (0, 19)$	e $y = 11 + \frac{3}{5}x$ $m = \frac{3}{5}, (0, 11)$	f $y = 54 - \frac{2x}{3}$ $m = -\frac{2}{3}, (0, 54)$
g $7 - 11x = y$ $m = -11, (0, 7)$	h $-100 - 134x = y$ $m = -134, (0, -100)$	

3 Write the equations of the lines with these gradients and y-intercepts.

a Gradient 7 and y-intercept (0,2) $y = 7x + 2$	b y-intercept (0, -9) and gradient 4 $y = 4x - 9$
c Gradient -5 and y-intercept (0,0) $y = -5x$	d y-intercept (0,11) and gradient -1 $y = -x + 11$
e Gradient -0.65 and y-intercept (0, -1.2) $y = -0.65x - 1.2$	f y-intercept $(0, \frac{7}{12})$ and gradient $\frac{5}{6}$ $y = \frac{5x}{6} + \frac{7}{12}$

4 Consider these four graphs:

$y = -4x$

$y = -5x$

$y = 4 - x$

$y = 5 - x$

a Which two graphs have the same gradient?

$y = 4 - x, y = 5 - x$

b Which two graphs have same y -intercept?

$y = -4x, y = -5x$

c What do all four graphs have in common?

Negative gradient

5 Consider these four graphs:

$y = 3x + 1$

$y = 4 + 3x$

$y = 3x - 1$

$y = 3x - 5$

a Which two graphs have a y -intercept above the x -axis?

$y = 3x + 1, y = 4 + 3x$

b Which two graphs have a y -intercept below the x -axis?

$y = 3x - 1, y = 3x - 5$

c What do all four graphs have in common?

Same gradient of 3

6 Consider these four graphs:

$y = 5x + 4$

$y = -2x + 4$

$y = 3x + 4$

$y = 4 - 5x$

a Which two graphs slope downwards from left to right? (i.e, decreasing)

$y = -2x + 4, y = 4 - 5x$

b Which two graphs slope upwards from left to right? (i.e., increasing)

$y = 5x + 4, y = 3x + 4$

c What do all four graphs have in common?

Same y -intercept of 4

7 Consider the equation $y = 56 - 14x$

a Find the value of y when $x = -3$

$y = 98$

b Factorise the right-hand side of the equation.

$y = 14(4 - x)$

c Solve the equation for $y = 50.5$

$x = \frac{11}{28}$

d Write down the gradient and y -intercept of the line.

$m = -14, (0.56)$

8 Parallel lines have the same gradient. Decide whether each pair of lines will be parallel.

a $y = 4x + 5$ and $y = 3x + 5$

Not parallel

b $y = 7 - x$ and $y = -x + 4$

Parallel

c $y = \frac{2}{3}x - 9$ and $y = 4 + \frac{2x}{3}$

Parallel

d $y = 18x - 11$ and $y = 11 - 18x$

Not parallel

e $y = 4(3x + 1)$ and $y = 3x - 8$

Not parallel

f $y = 2x + 7$ and $y = 2(x - 9)$

Parallel

9 Decide whether each pair of lines will have the same y-intercept.

a $y = 4x + 5$ and $y = 3x + 5$

Yes

b $y = 7 - x$ and $y = -x + 4$

No

c $y = \frac{2}{3}x - 9$ and $y = 4 + \frac{2x}{3}$

No

d $y = 18x - 11$ and $y = 11 - 18x$

No

e $y = 4(3x + 1)$ and $y = 3x - 8$

No

f $y = 2x + 7$ and $y = 2(x - 9)$

No

10 Here are the equations for eight lines,

i $y = 20x - 17$

ii $y = 32x + 11$

iii $y = 19 - 32x$

iv $y = 19 - 20x$

v $y = 11 - 20x$

vi $y = -17 + 32x$

vii $y = 0.75 + 20x$

viii $\frac{3}{4} - 32x = y$

a Identify four pairs of parallel lines.

i and vii

ii and vi

iii and viii

iv and v

b Identify four pairs of lines that have the same y-intercept.

i and vi

ii and v

iii and iv

vii and viii

11 Write down *any* equation of a line that:

a Is parallel to $y = 23x + 9$

b Is parallel to $y = 1 - 11x$

c intersect the y-axis at the same point as the line $y = 23x + 9$

d intersect the y-axis at the same point as the line $y = 1 - 7x$

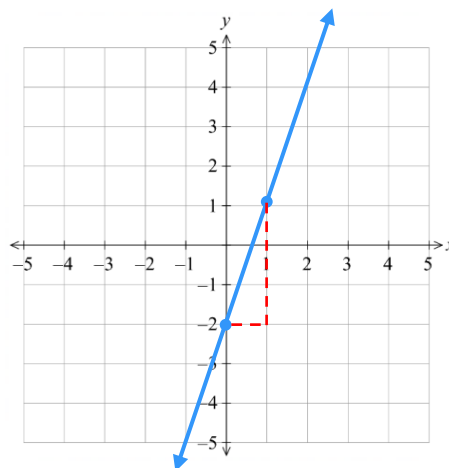
SKETCH GRAPHS FROM THE EQUATION

1. Plot the y -intercept.
2. Starting from the y -intercept, use the gradient to find another point.
 - a. The denominator is how much to move right (1 if the gradient is a whole number).
 - b. The numerator is how much to move up or down.
3. Draw a straight line through both points and extend it.

Example Sketch graphs from the equation using gradient and intercept.

Sketch $y = 3x - 2$

1. Plot a point at $(0, -2)$
2. The gradient is 3. Starting at $(0, -2)$:
Move right 1 unit.
Move up 3 units.
3. Join the two points and extend the line.



How did we know to plot a point at $(0, -2)$?

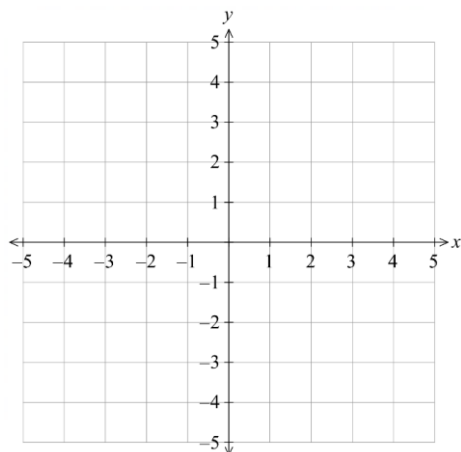
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How does a gradient of 3 tell us to move right 1 unit and up 3 units?

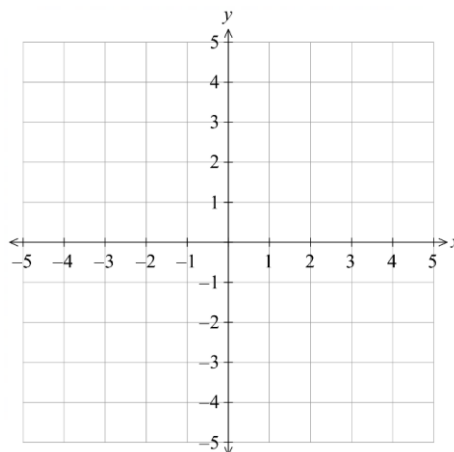
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a Sketch $y = -2x + 1$



b Sketch $y = \frac{3x}{4} - 1$

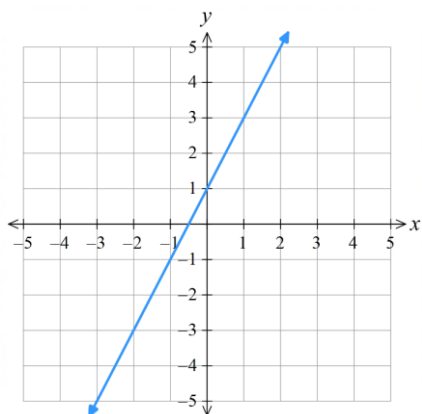


Key ideas

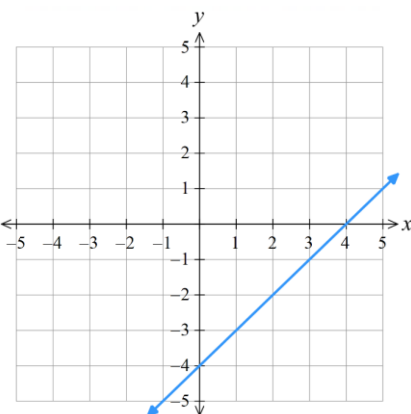
1. To sketch a line from its equation, first plot the-.....
2. Then use the to find another point.
3. Then the two points with a line.

1 Sketch these graphs without a table of values.

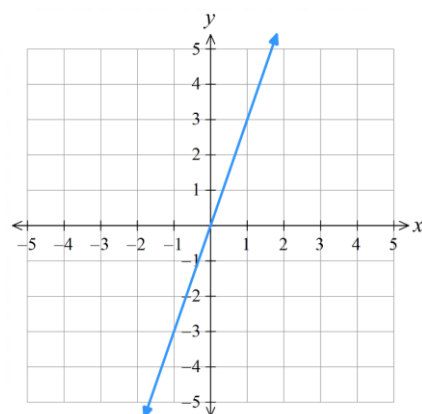
a $y = 2x + 1$



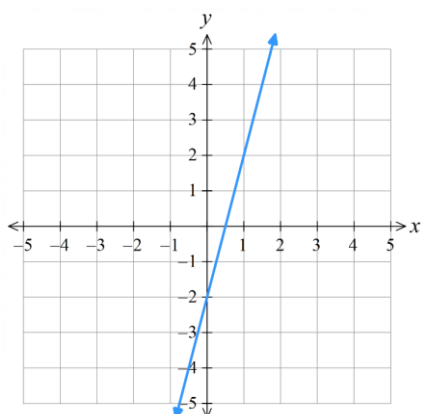
b $y = x - 4$



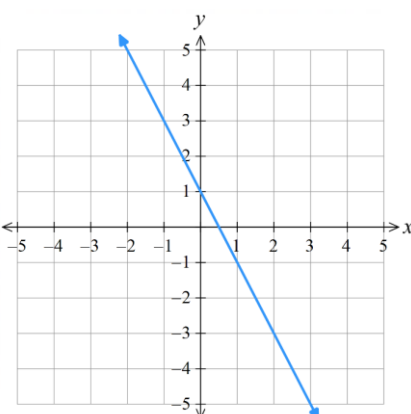
c $y = 3x$



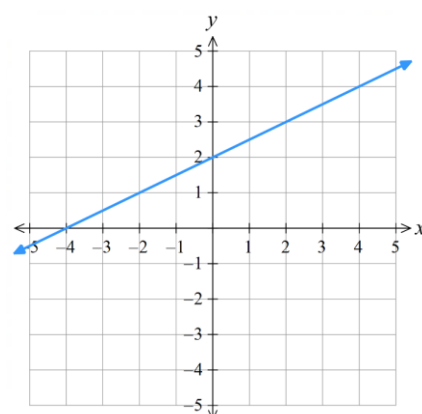
d $y = 4x - 2$



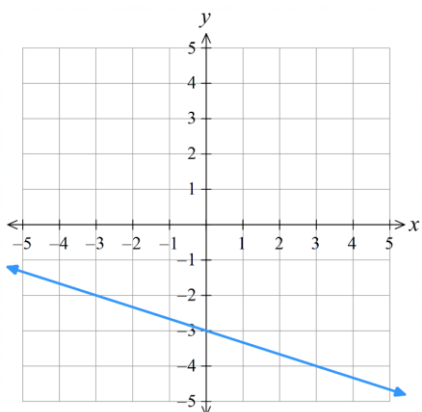
e $y = -2x + 1$



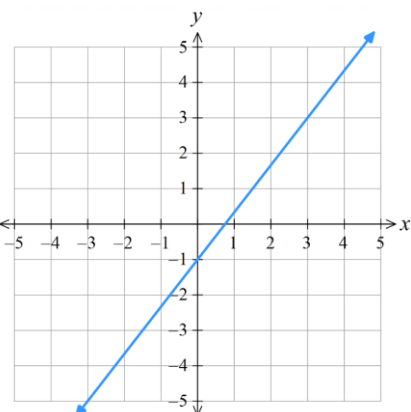
f $y = \frac{1}{2}x + 2$



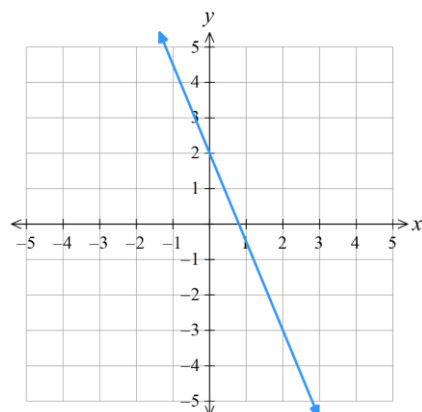
g $y = -\frac{1}{3}x - 3$



h $y = \frac{4}{3}x - 1$

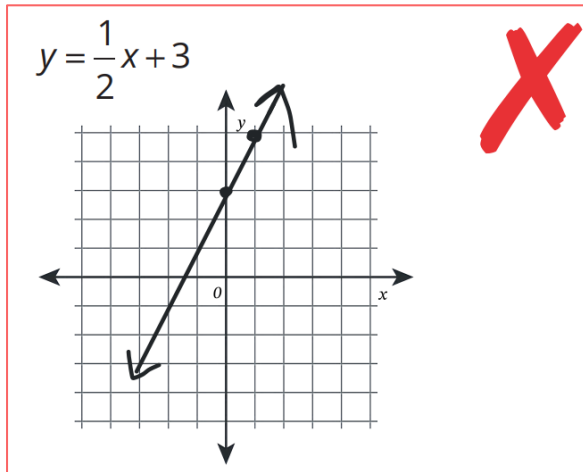


i $y = -\frac{5}{2}x + 2$



2

Mark graphed this equation:



Mark incorrectly graphed the equation.
What did he get wrong?

Gradient is 2, not $\frac{1}{2}$

How many times up and to the right from point (0, 3) should Mark have gone when graphing this slope? up right

up 1 right 2

Draw the correct graph of $y = \frac{1}{2}x + 3$ on the axis on the left.

3

a At each point on the line $y = 2x$, the-coordinate is two times the-coordinate.

The y-coordinate is two times the x-coordinate

b Sketch the graph of $y = 2x$ on the Cartesian plane below and label the line.

c On the same axis, sketch and label:

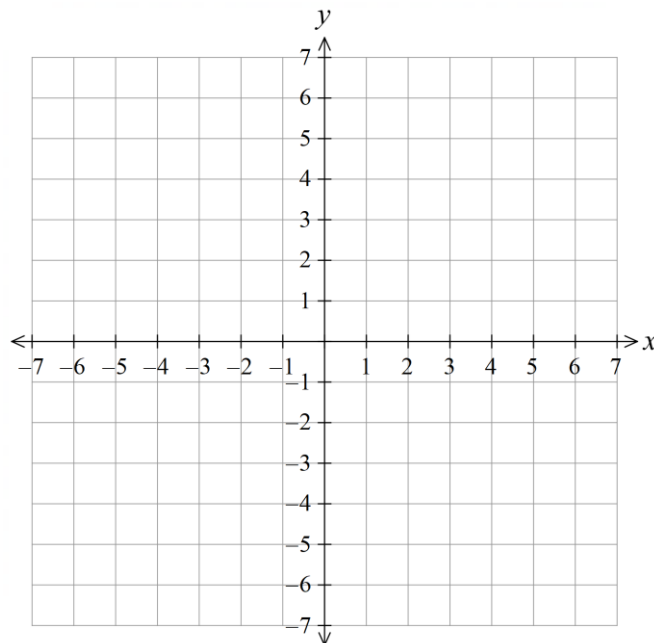
i $y = 3x$

ii $y = 5x$

iii $y = 6x$

iv $y = \frac{1}{2}x$

v $y = \frac{1}{4}x$



d On a graph of the form $y = mx$, the number m is the of the line.

gradient

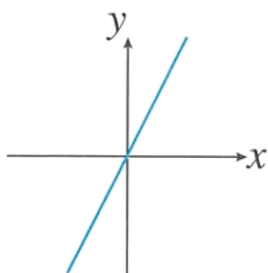
The greater the value of m , the the line.

steeper

The lower the value of m , the the line.

Less steep

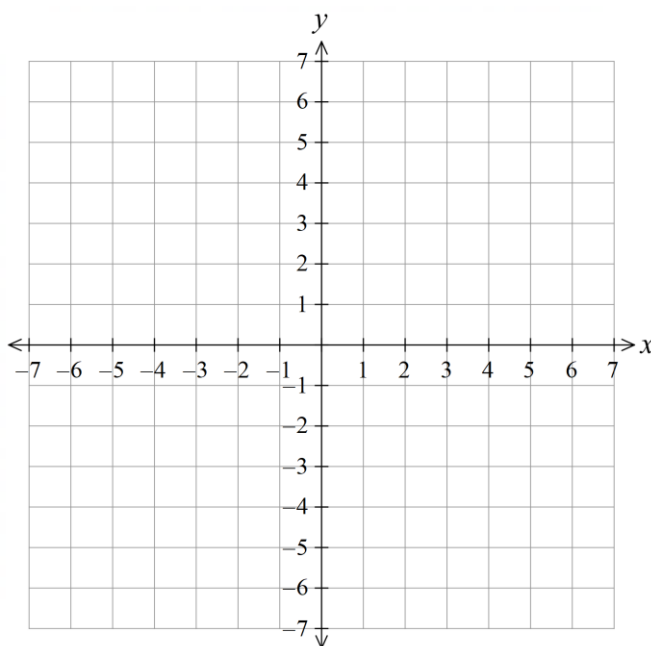
- 4 Here is a sketch of $y = 12x$.



On the same axis, sketch $y = 4x$ and $y = 15x$.
Explain your reasoning.

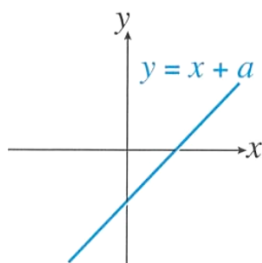
- 5 On the same axis, sketch and label:

- a $y = x + 1$
- b $y = x + 2$
- c $y = x + 3$
- d $y = x + 4$
- e $y = x - 1$
- f $y = x - 2$
- g $y = x - 3$
- h $y = x - 4$



- i All of these graphs are parallel because
They have the same gradient, 1
- j On a graph of the form $y = x + c$, the number c is the of the line.
y-intercept
If c is, the y-intercept is above the x -axis.
positive
If c is negative, the y-intercept is the x -axis.
below

- 6 Here is the graph of $y = x + a$.

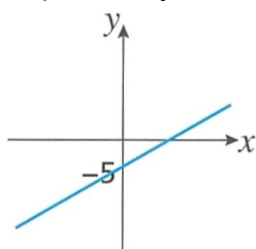


- a Explain why a is negative.

y-intercept is below the x -axis.

- b If the value of a is -5 , On the same axis, sketch $y = x + 3$ and $y = x - 12$

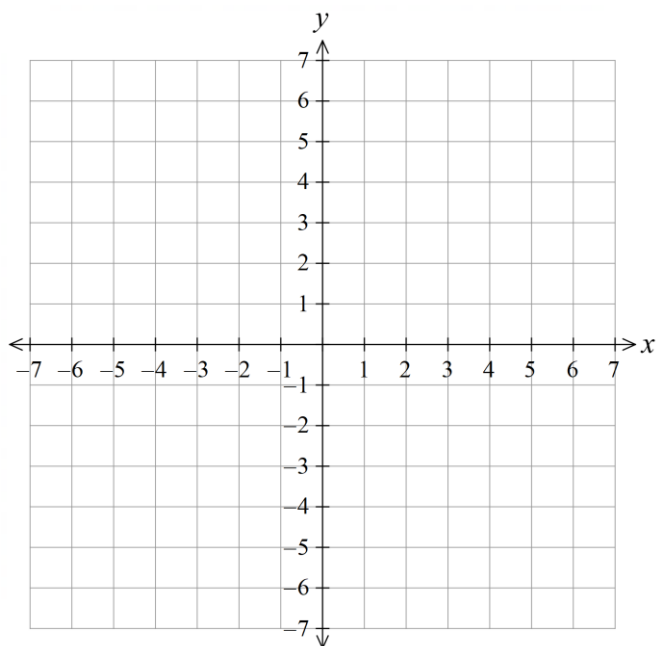
- 7 Explain why this cannot be the graph of $y = 7x + 3$



The equation has a y-intercept of 3. On the graph, the y-intercept is -5 .

8 On the same axis, sketch and label:

- a $y = x$
- b $y = -x$
- c $y = 2x$
- d $y = -2x$

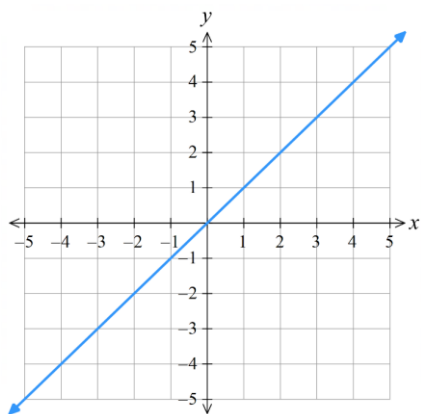


What does a negative gradient mean?

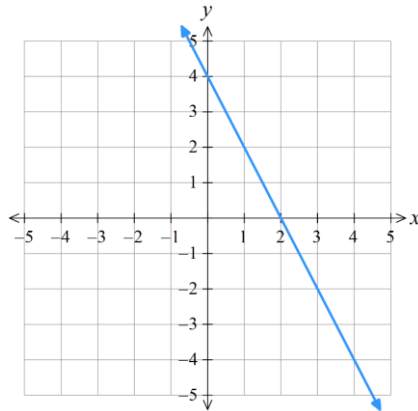
The line slopes down from left to right.

9 Sketch these graphs without a table of values. (Rearrange to $y = mx + c$)

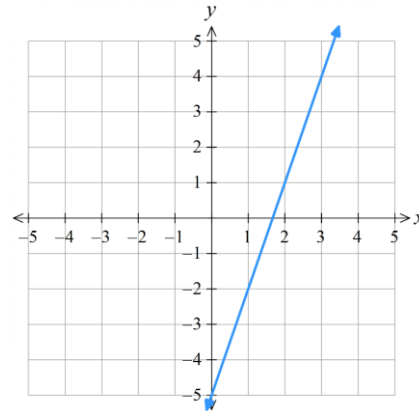
a $y - x = 0$



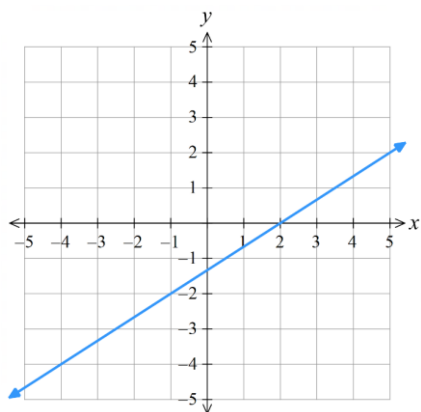
b $4x + 2y = 8$



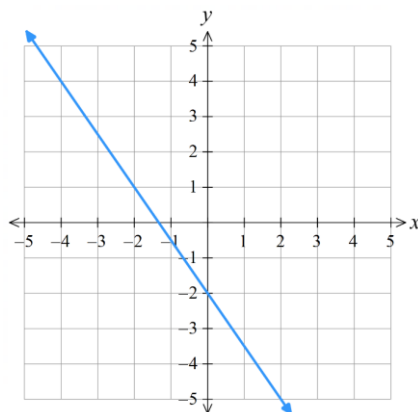
c $6x - 2y = 10$



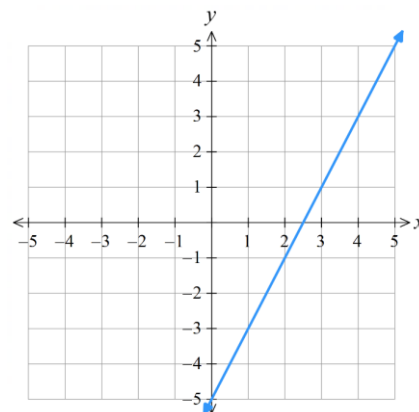
d $6y - 4x = -8$



e $3x + 2y = -4$

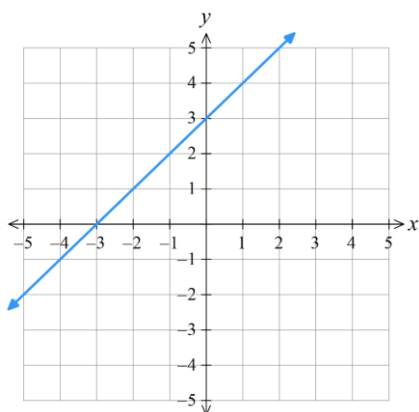


f $2x - y = 5$

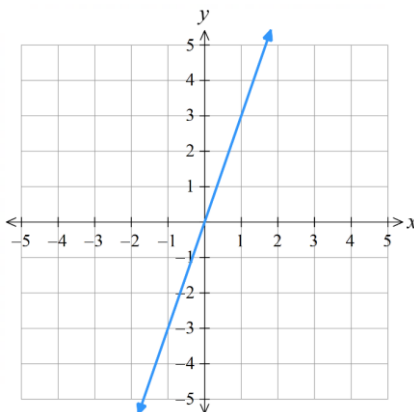


10 Sketch these graphs.

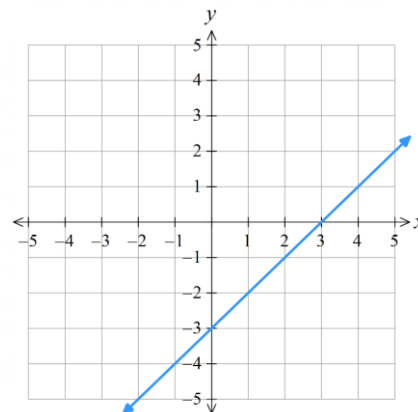
a $y = x + 3$



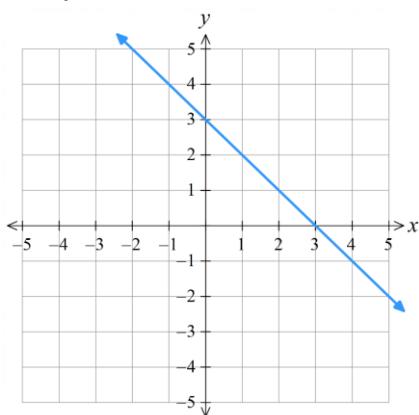
b $y = 3x$



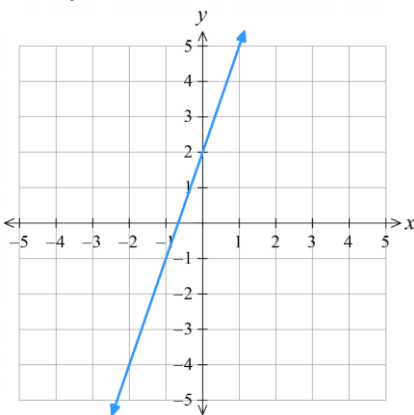
c $y = x - 3$



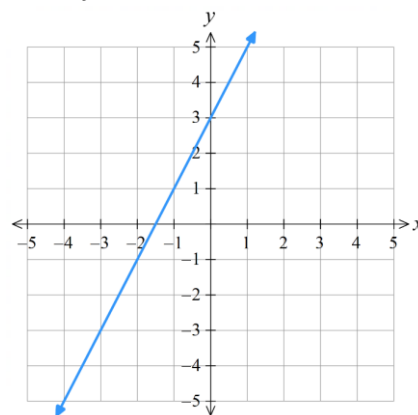
d $y = 3 - x$



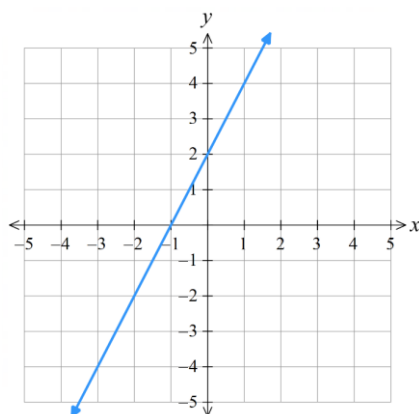
e $y = 3x + 2$



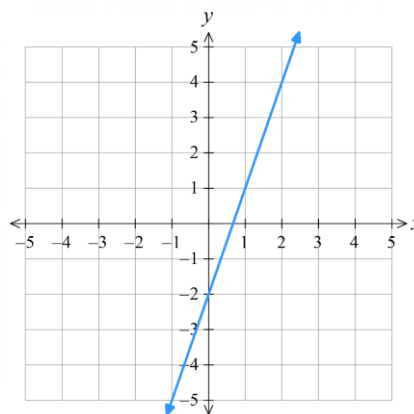
f $y = 3 + 2x$



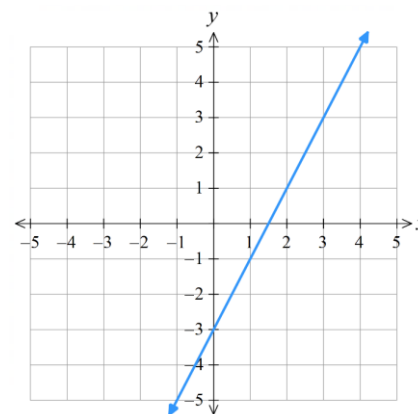
g $y = 2(x + 1)$



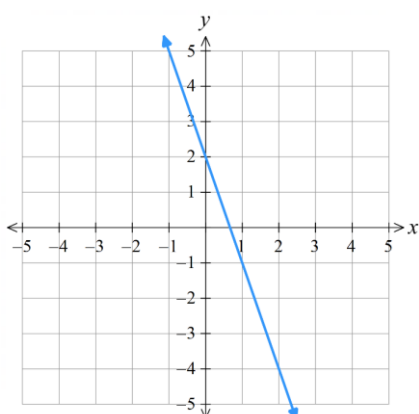
h $y = 3x - 2$



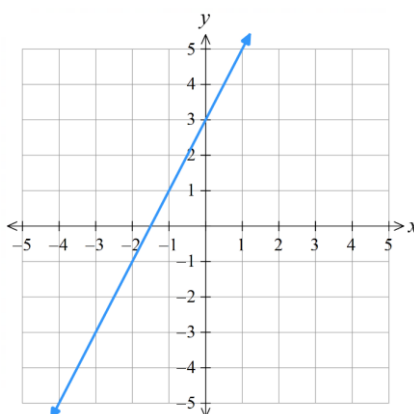
i $y = 2x - 3$



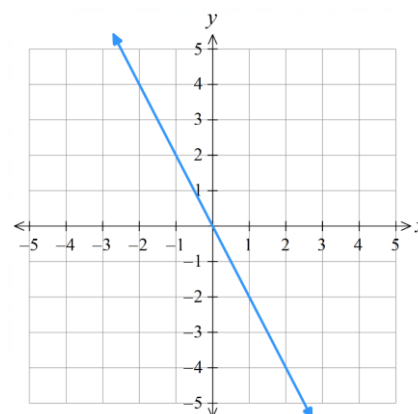
j $y = 2 - 3x$



k $y = 2x + 3$



l $y = -2x$



FINDING THE EQUATION

1. Identify the y -intercept, this is c .
2. Calculate the gradient, this is m .
 - a. Choose two points on the line and create a right-angled triangle.
 - b. Going from left to right, determine the rise and the run.
 - c. Calculate gradient using the formula $m = \frac{\text{Rise}}{\text{Run}}$
3. Write the equation $y = mx + c$.

Example Determine the equation of a line from its graph.

1. The y -intercept is at $(0, -1)$

$$c = -1$$

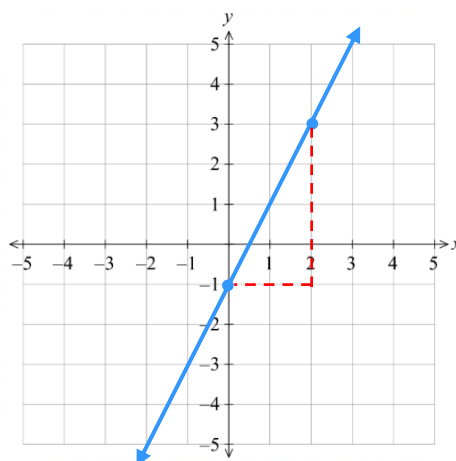
2. Calculate the gradient using another point on the line: $(2, 3)$

Rise: 4

Run: 2

$$m = \frac{4}{2} = 2$$

3. $y = 2x - 1$

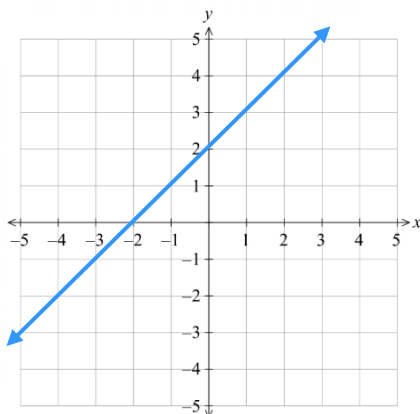


Why did we choose $(2, 3)$ for calculating the gradient?

$(2, 3)$ is a point that can be easily, as it lies on the gridlines.

Determine the equation of these lines.

a



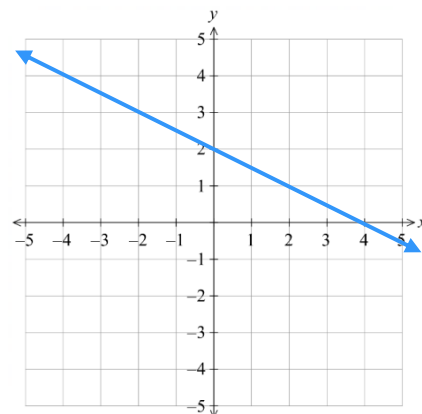
y -int: $c = \dots\dots\dots$

Gradient: $m = \dots\dots\dots$

Equation: $\dots\dots\dots$

$$y = x + 2$$

b



y -int: $c = \dots\dots\dots$

Gradient: $m = \dots\dots\dots$

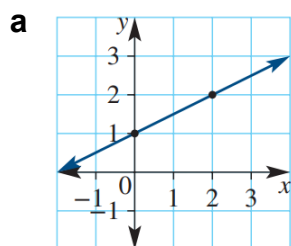
Equation: $\dots\dots\dots$

$$y = -0.5x + 2$$

Key ideas

1. c is the of the line. It can be read off the graph.
2. To find m , choose two points on the graph and calculate the of the interval.

1 Determine the equation of these lines.

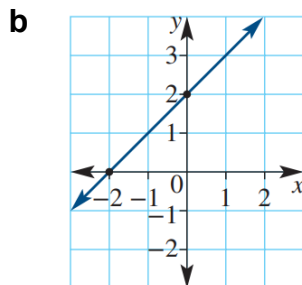


$c = \dots\dots\dots$

$m = \dots\dots\dots$

Equation: $\dots\dots\dots$

$y = \frac{x}{2} + 1$

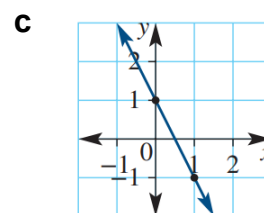


$c = \dots\dots\dots$

$m = \dots\dots\dots$

Equation: $\dots\dots\dots$

$y = x + 2$

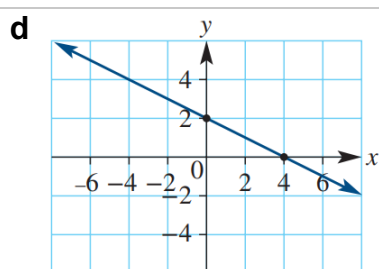


$c = \dots\dots\dots$

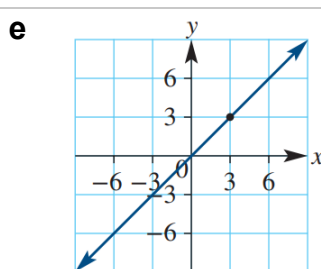
$m = \dots\dots\dots$

Equation: $\dots\dots\dots$

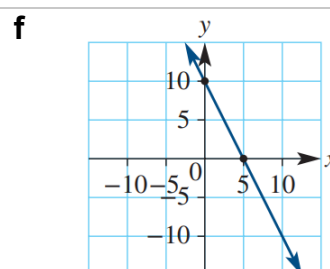
$y = -2x + 1$



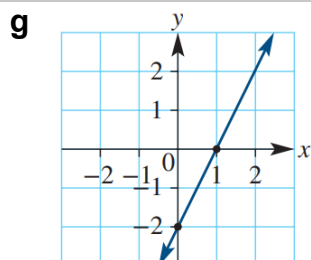
$y = -\frac{x}{2} + 2$



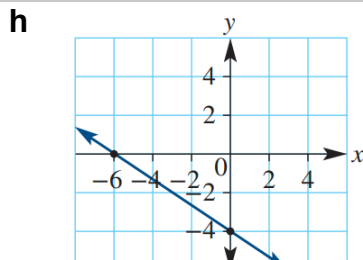
$y = x$



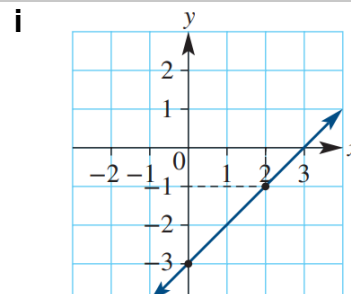
$y = -2x + 10$



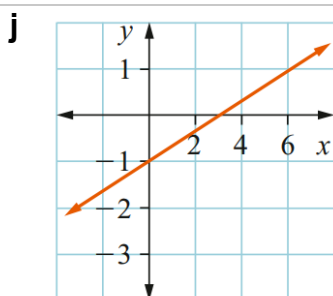
$y = 2x - 2$



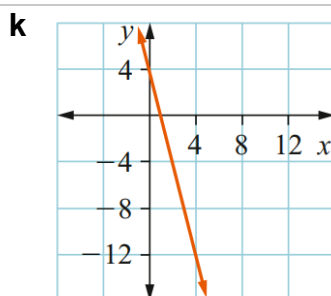
$y = -\frac{2x}{3} - 4$



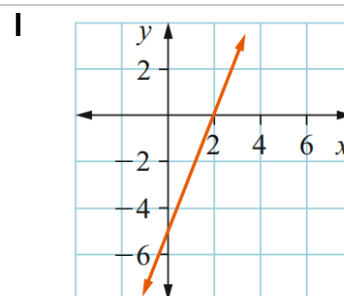
$y = x - 3$



$y = \frac{x}{3} - 1$



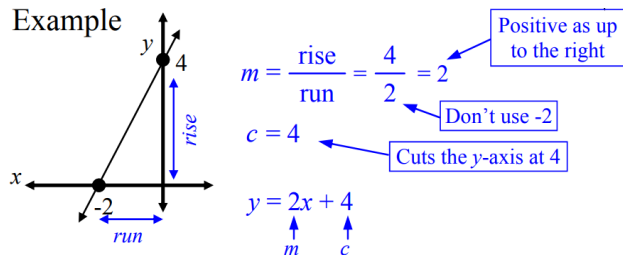
$y = -4x + 4$



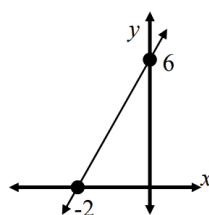
$y = \frac{5x}{2} - 5$

- 2 Find the gradient and y-intercept, then write the equation.
Remember that the run is always positive.

a Example



b

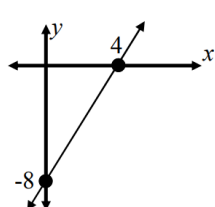


$$m = \frac{\square}{\square} = \square \quad c = \square$$

$$y = \square x + \square$$

$$y = 3x + 6$$

c

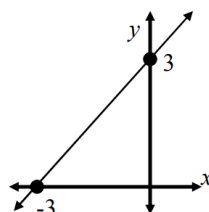


$$m = \frac{\square}{\square} = \square \quad c = \square$$

$$y = \square x - \square$$

$$y = 2x - 8$$

d

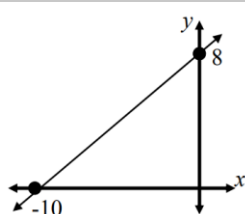


$$m = \frac{\square}{\square} = \square \quad c = \square$$

$$y = \square$$

$$y = x + 3$$

e

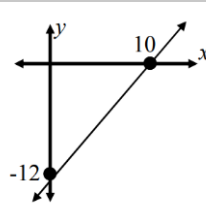


$$m = \frac{\square}{\square} = \frac{\square}{\square} \quad c = \square$$

$$y = \frac{\square}{\square} x + \square$$

$$y = \frac{4}{5}x + 8$$

f

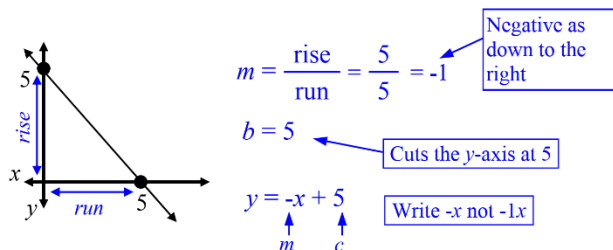


$$m = \frac{\square}{\square} = \frac{\square}{\square} \quad c = \square$$

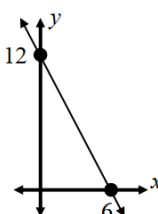
$$y = \square$$

$$y = \frac{6}{5}x - 12$$

g



h

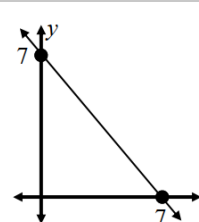


$$m = \frac{\square}{\square} = \square \quad c = \square$$

$$y = \square$$

$$y = -2x + 12$$

i

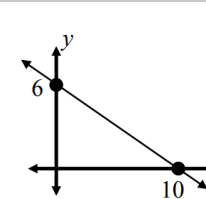


$$m = \frac{\square}{\square} = \square \quad c = \square$$

$$y = \square$$

$$y = -x + 7$$

j

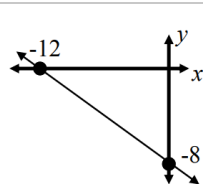


$$m = \frac{\square}{\square} = \frac{\square}{\square} \quad c = \square$$

$$y = \frac{\square}{\square} x + \square$$

$$y = -\frac{3}{5}x + 6$$

k

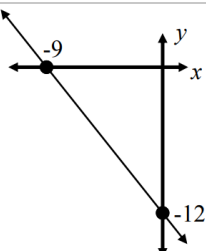


$$m = \frac{\square}{\square} = \frac{\square}{\square} \quad c = \square$$

$$y = \square$$

$$y = -\frac{2}{3}x - 8$$

l

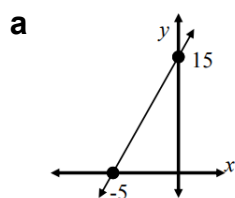


$$m = \frac{\square}{\square} = \frac{\square}{\square} \quad c = \square$$

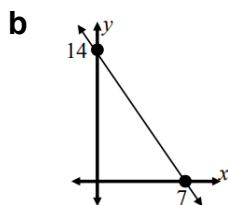
$$y = \square$$

$$y = -\frac{4}{3}x - 12$$

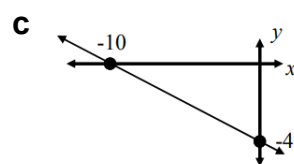
3 Find the gradient and y-intercept, then write the equation.



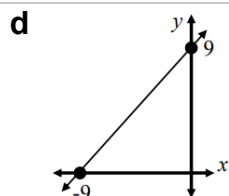
$$y = 3x + 15$$



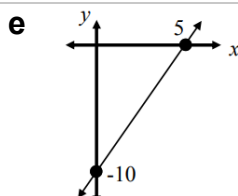
$$y = -2x + 14$$



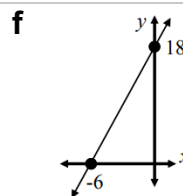
$$y = -\frac{2x}{5} - 4$$



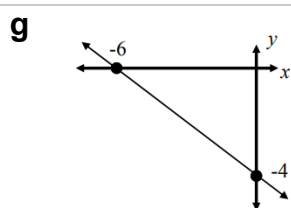
$$y = x + 9$$



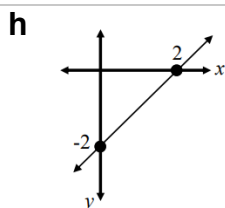
$$y = 2x - 10$$



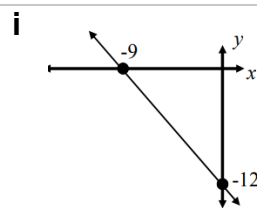
$$y = 3x + 18$$



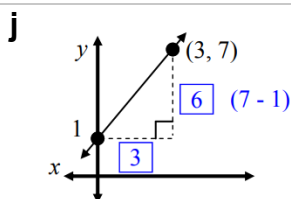
$$y = -\frac{2x}{3} - 4$$



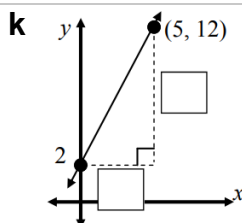
$$y = x - 2$$



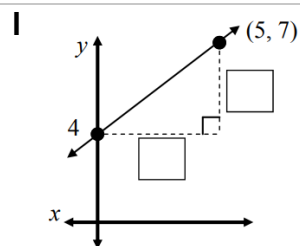
$$y = -\frac{4x}{3} - 12$$



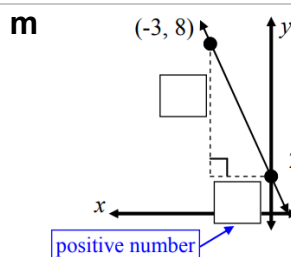
$$y = 2x + 1$$



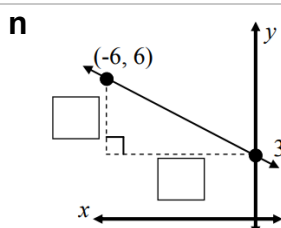
$$y = 2x + 2$$



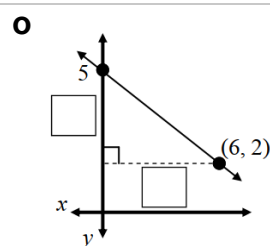
$$y = \frac{3x}{5} + 4$$



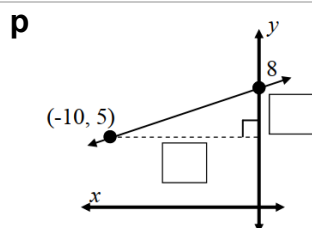
$$y = -2x + 2$$



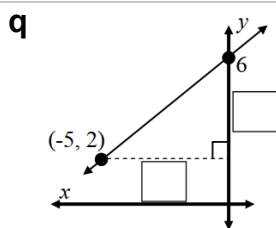
$$y = -\frac{x}{2} + 3$$



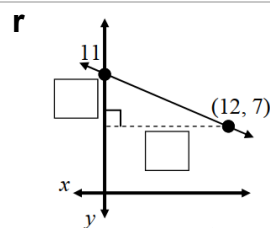
$$y = -\frac{x}{2} + 5$$



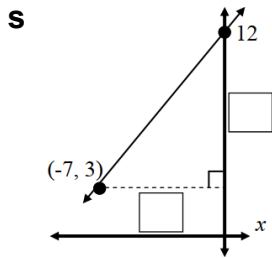
$$y = \frac{3x}{10} + 8$$



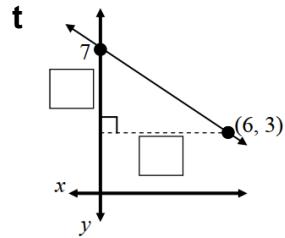
$$y = \frac{4x}{5} + 6$$



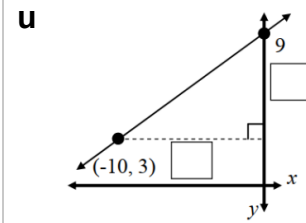
$$y = -\frac{x}{3} + 11$$



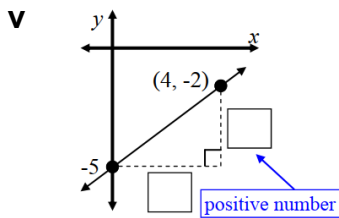
$$y = \frac{9x}{7} + 12$$



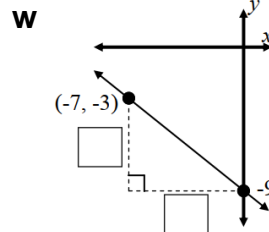
$$y = -\frac{2x}{3} + 7$$



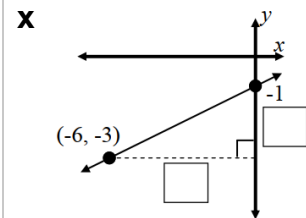
$$y = \frac{3x}{5} + 9$$



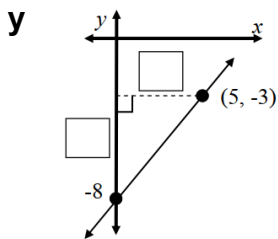
$$y = \frac{3x}{4} - 5$$



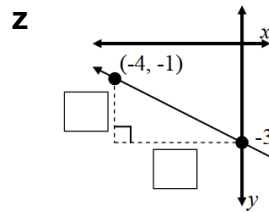
$$y = -\frac{6x}{7} - 9$$



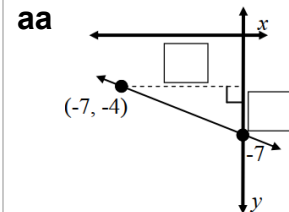
$$y = \frac{x}{3} - 1$$



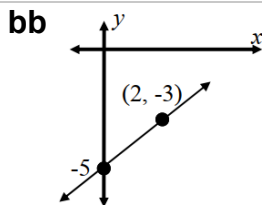
$$y = x - 8$$



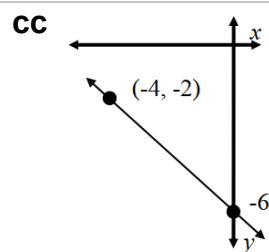
$$y = -\frac{x}{2} - 3$$



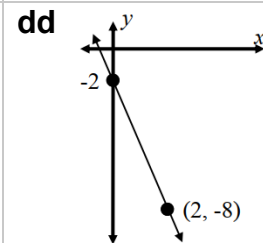
$$y = -\frac{3x}{7} - 7$$



$$y = x - 5$$

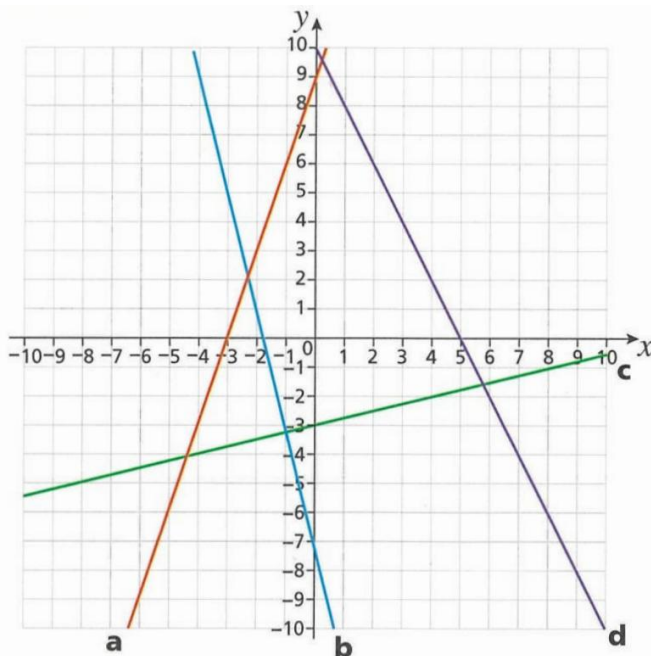


$$y = -x - 6$$



$$y = -3x - 2$$

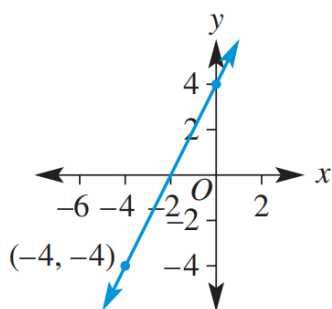
4 Find the equation for each line.



a: $y = 3x + 9$
b: $y = -4x - 7$
c: $y = \frac{x}{4} - 3$
d: $y = -2x + 10$

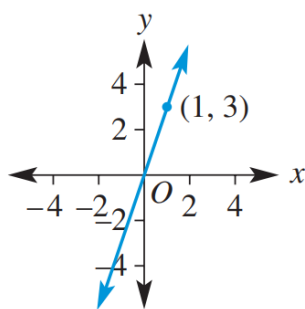
5 Determine the gradient and y -intercept of these lines and hence find the equation.

a



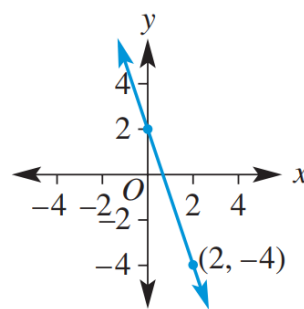
$$y = 2x + 4$$

b



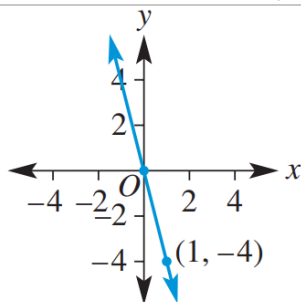
$$y = 3x$$

c



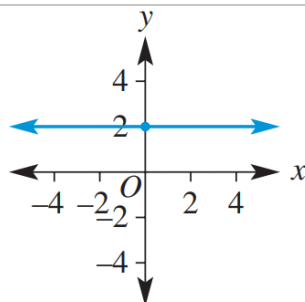
$$y = -3x + 2$$

d



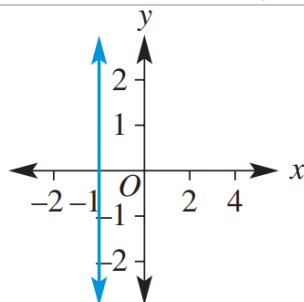
$$y = -4x$$

e



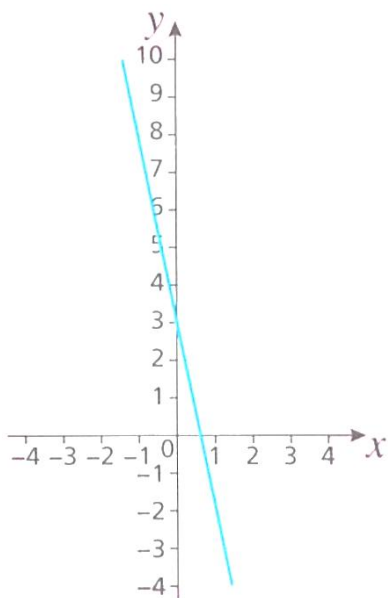
$$y = 2$$

f



$$x = -1$$

6



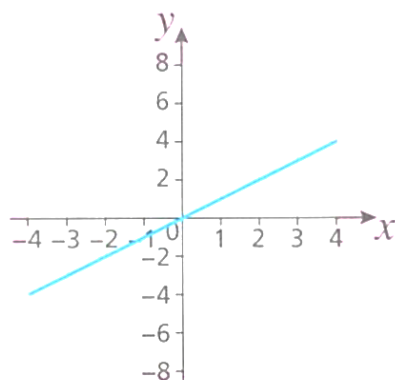
- a Nick has found the gradient of this line to be 5.
Without calculation, explain why Nick must be incorrect.

The gradient is negative

- b Nicole has found the gradient of the line to be $-\frac{1}{5}$
This is incorrect. What mistake do you think Nicole has made?

Nicole did $\frac{\text{run}}{\text{rise}}$ instead of $\frac{\text{rise}}{\text{run}}$

12 Beca says that the equation of this line is $y = \frac{1}{2}x$.



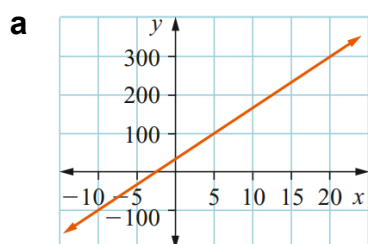
a Explain why she is incorrect.

The gradient is 1

b What makes this question tricky?

The y-axis and x-axis scales are different.

7 Determine the equation of these lines.

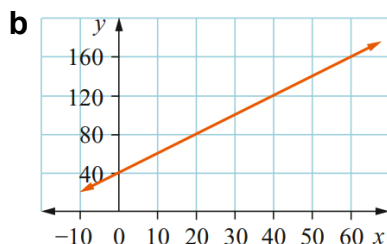


$$c = 40$$

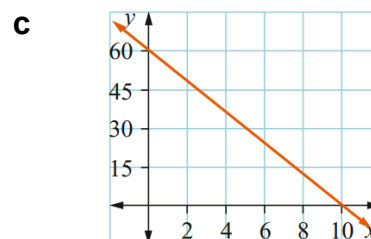
$$m = \dots\dots\dots$$

$$y = \dots\dots\dots$$

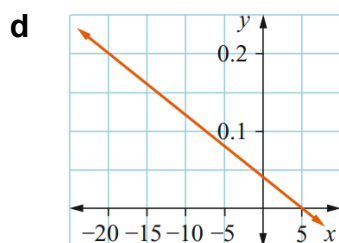
$$y = 13x + 40$$



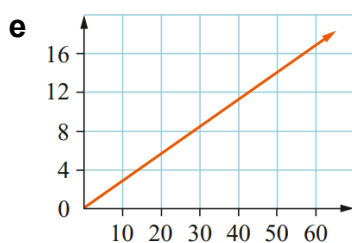
$$y = 2x + 40$$



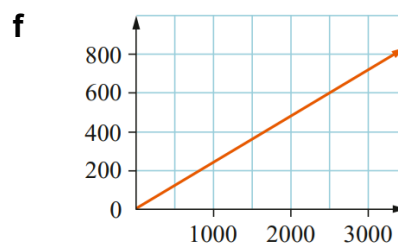
$$y = -6x + 60$$



$$y = -0.008x + 0.04$$



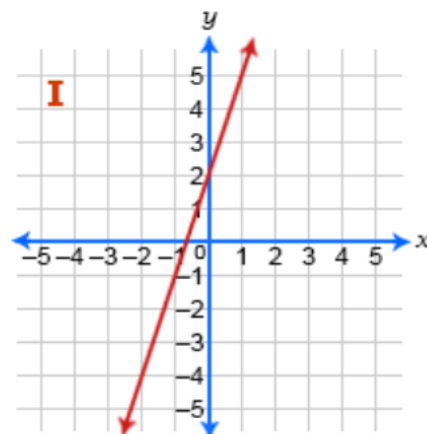
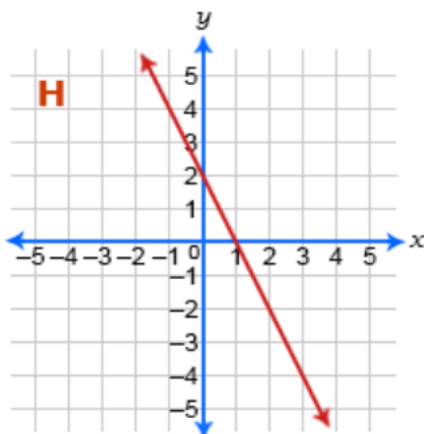
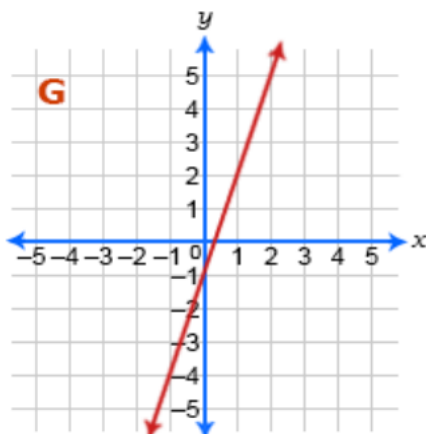
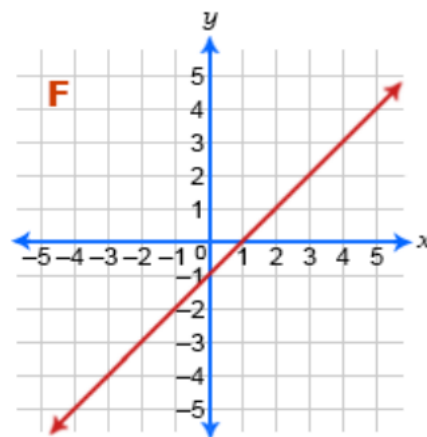
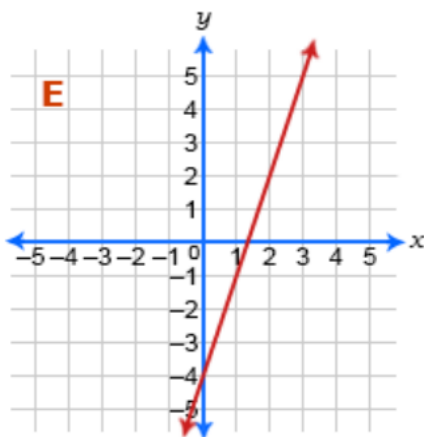
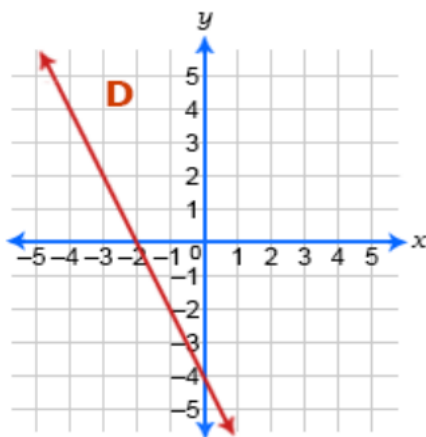
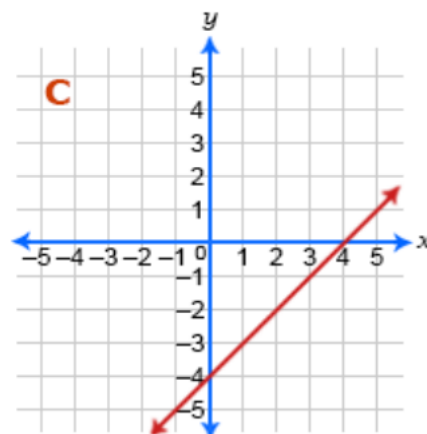
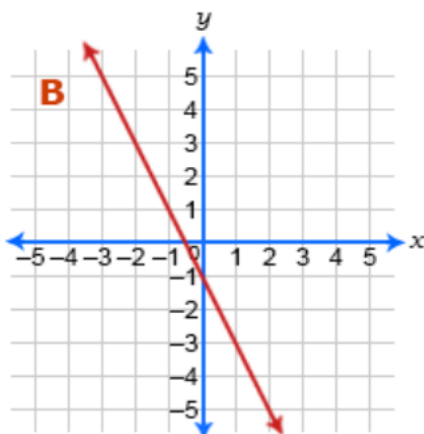
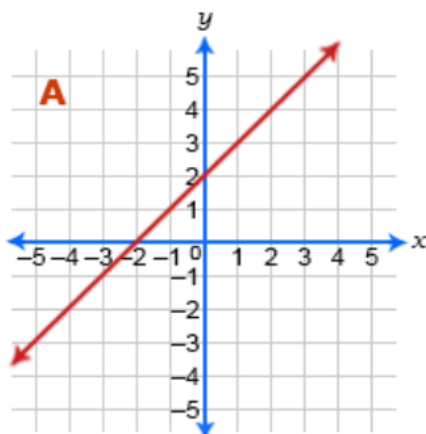
$$y = \frac{7x}{25}$$



$$y = \frac{6x}{25}$$

8 Match the equations with the graph. Write the letter of the matching graph alongside each equation.

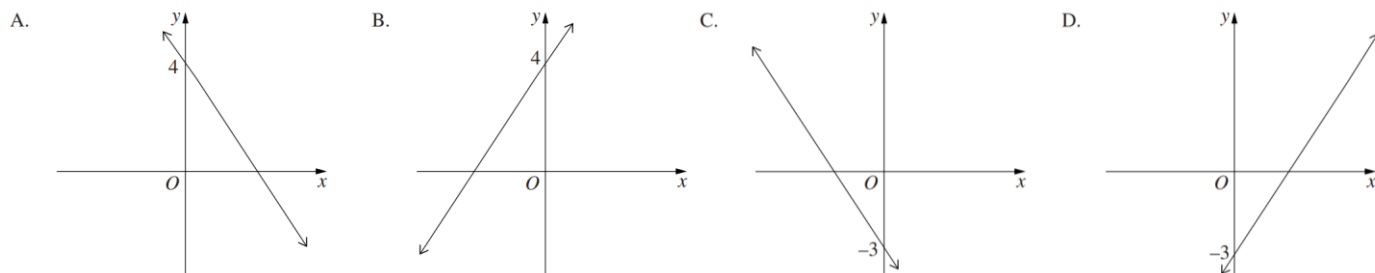
$y = x - 1$	_____	$y = 3x + 2$	_____	$y = -2x - 1$	_____
$y = -2x - 4$	_____	$y = 3x - 4$	_____	$y = x - 4$	_____
$y = 3x - 1$	_____	$y = x + 2$	_____	$y = 2 - 2x$	_____



F I B
D E C
G A H

9 2021 HSC Standard 1

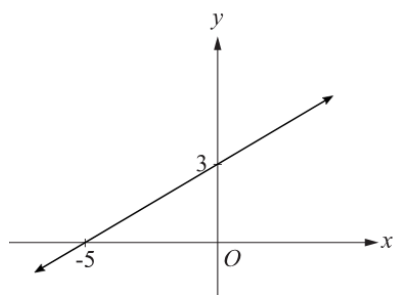
Which of the following best represents the graph of $y = -3x + 4$



A

10 HSC Standard 2 Sample Question Band 4

What is the equation for the line shown?

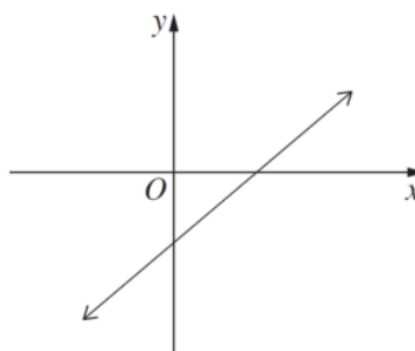


$$y = \frac{3x}{5} + 3$$

11 2016 HSC Standard 2 Band 4

The graph shows a line of the form $y = mx + b$. Which statement is true?

- (A) m is positive and b is negative
- (B) m is negative and b is positive
- (C) m and b are both positive
- (D) m and b are both negative



A

12 2006 HSC Standard 2 Band 4

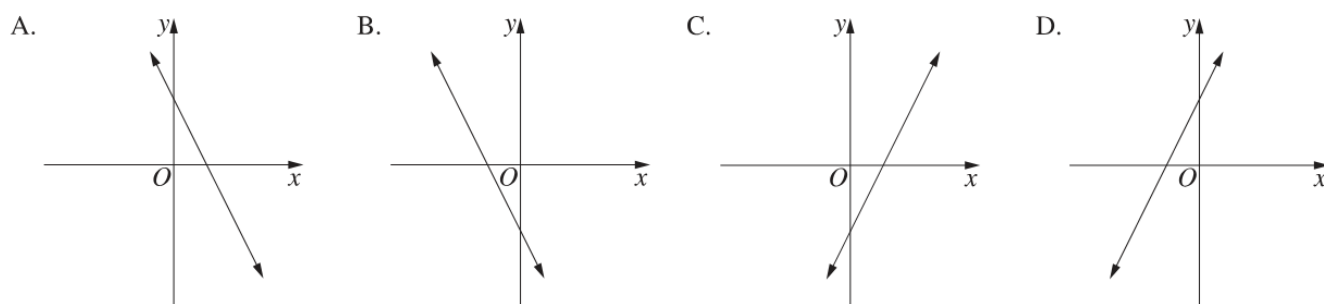
What equation is shown by the table?

x	0	2	4	6	8
y	1	2	3	4	5

$$y = \frac{x}{2} + 1$$

13 2022 HSC Standard 2 Band 5

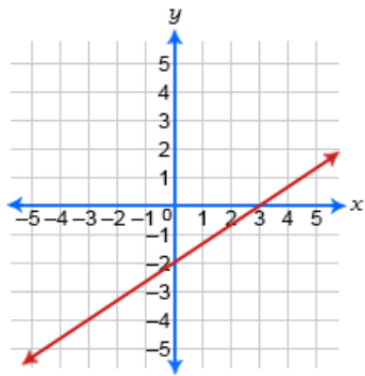
Which of the following could be the graph of $y = -2x + 2$?



A

14 Determine the equation of the line, then use it to find the point on the line.

a



$m = \dots\dots\dots$ $c = \dots\dots\dots$

$y = \dots\dots\dots$

(30 ,) would lie on this line.

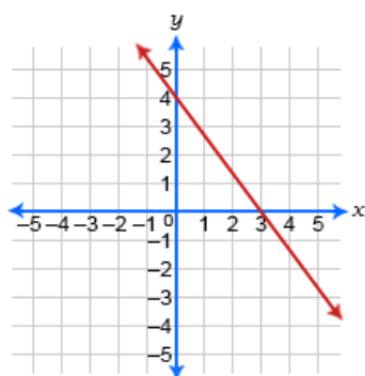
(, 4) would lie on this line.

$y = \frac{2x}{3} - 2$

18

9

b



(-6 ,) would lie on this line.

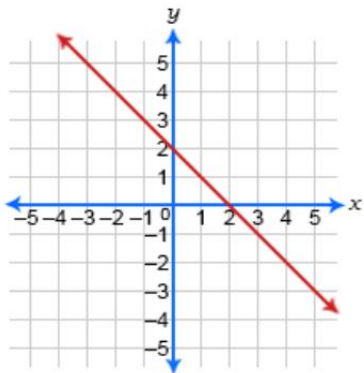
(, -8) would lie on this line.

$y = -\frac{4x}{3} + 4$

12

9

c



(-3.7,) would lie on this line.

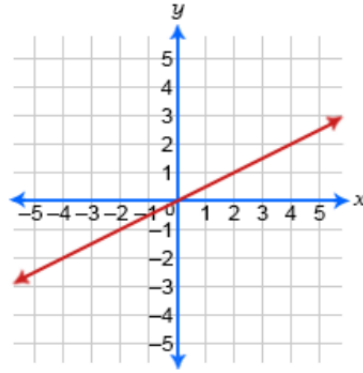
(, 18) would lie on this line.

$y = -x + 2$

5.7

-16

d



(, 1.25) would lie on this line.

(25 ,) would lie on this line.

$y = \frac{x}{2}$

2.5

12.5

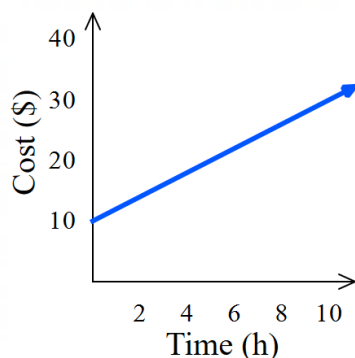
LINEAR MODELLING

- Linear relationships model many real-life scenarios.
 - The y -axis is the dependent variable. It changes based on another variable.
 - The x -axis is the independent variable. It causes the change.
 - The **gradient is the rate**; it represents the **change in y per unit increase in x** .
 - The **y -intercept** is the **initial or fixed value at $x = 0$** .

Example Interpret the gradient and y -intercept of a linear model.

Identify the meaning of the gradient and y -intercept of these linear models.

a Taxi hire cost



Gradient: 2

Meaning: 2 per

y -intercept: 10

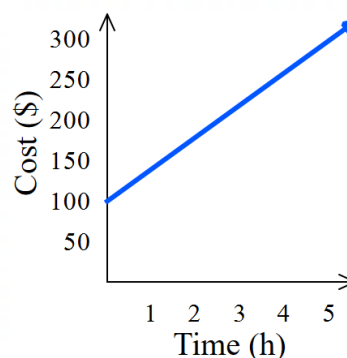
Meaning: for

Equation relating C and t :

$$y = mx + c$$

$$\text{.....} = \text{.....} + \text{...}$$

b Plumber hire cost



Gradient: 40

Meaning:

y -intercept: 100

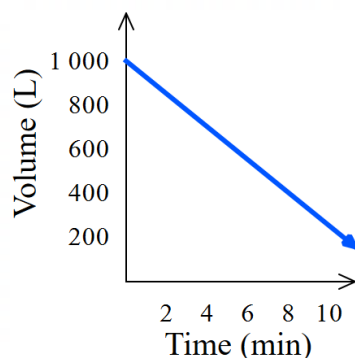
Meaning:

Equation relating C and t :

$$y = mx + c$$

$$\text{.....} = \text{.....} + \text{...}$$

c Volume of water in a tank



Gradient: -75

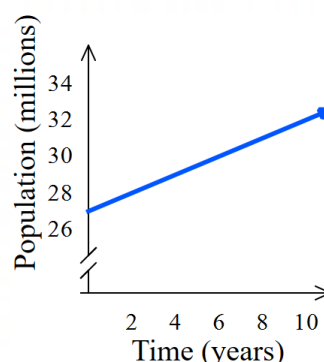
Meaning:

y -intercept: 1000

Meaning:

Equation relating V and t :

d Population of Australia



Gradient: 0.5

Meaning:

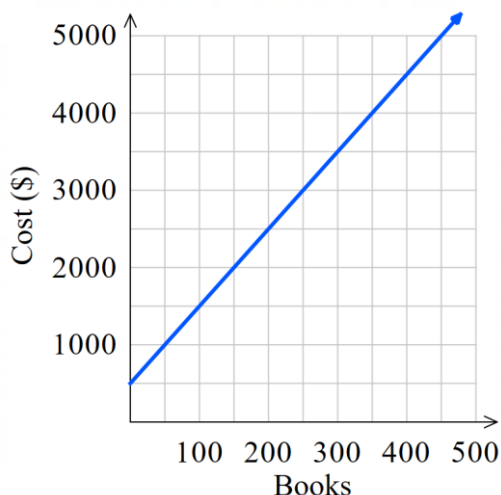
y -intercept: 27

Meaning:

Equation relating P and t :

Example Interpret a linear model.

The linear model shows the cost of setting up a book printing business.



- a** Find the cost of printing 400 books.
\$4500
- b** Find the number of books that can be printed for \$3000.
250 books
- c** Find the y-intercept. What is its meaning?
\$500.
This is the fixed cost for setting up the printing.

- d** Find the gradient of the line, what is its meaning?

By choosing (0,500) and (400, 4500):

$$m = \frac{\text{cost}}{\text{books}} = \frac{4000}{400} = 10$$

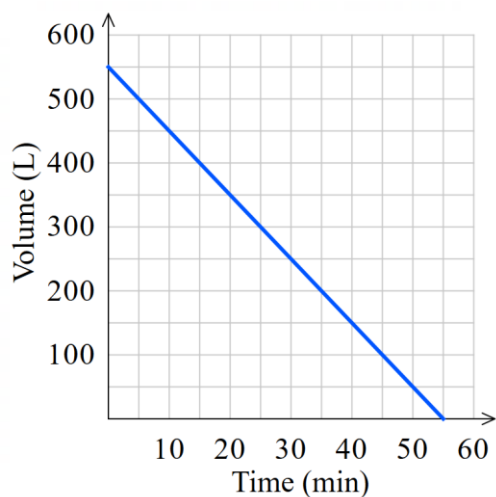
cost per book is \$10.

- e** Write an equation relating cost C and number of books n .

$$m = 10, c = 500$$

$$C = 10n + 500$$

The linear model shows the volume of a chemical tank over time.



- a** Find the volume after 30 minutes.
250 L
- b** When is the volume 200 L?
35 minutes
- c** Find the y-intercept. What is its meaning?
550 L; Initial volume.

- d** Find the gradient of the line, what is its meaning?

-10 litres per minute

- e** Write an equation relating volume V and time t .

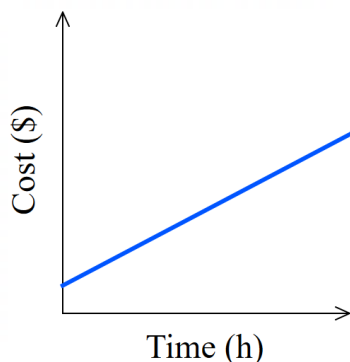
$$V = -10t + 550$$

Key ideas

1. The ...-..... represents the initial or fixed amount.
2. The represents the change in y per unit increase in x .

1 Identify what the gradient and y -intercepts of these graphs represent, then write the equation.

a



Gradient: 0.5

Meaning:

0.5 per

0.5 dollars per hour

y -intercept: 2

Meaning:

.....

\$2 for 0 hours (initial cost)

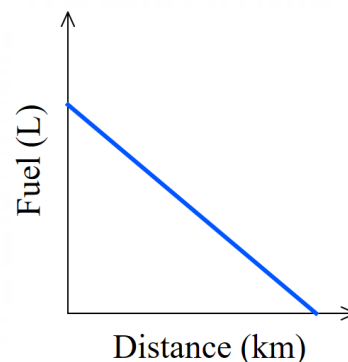
Equation relating cost C and time t :

$$y = mx + c$$

$$\dots = \dots + \dots$$

$$C = 0.5t + 2$$

b



Gradient: -0.8

Meaning:

-0.8 L per km

y -intercept: 15

Meaning:

15 L at 0 km (initial volume)

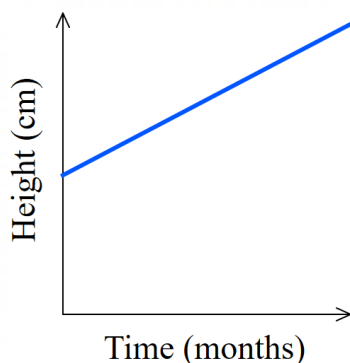
Equation relating fuel f and distance d :

$$y = mx + c$$

$$\dots = \dots + \dots$$

$$f = -0.8t + 15$$

c



Gradient: 2.5

Meaning:

2.5 cm per month

y -intercept: 50

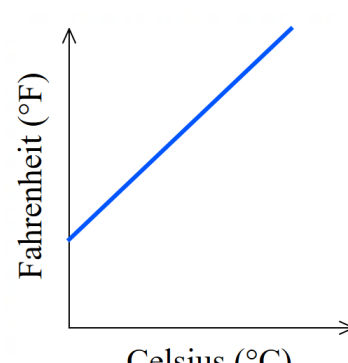
Meaning:

50 cm at 0 months (initial height)

Equation relating height h and time t :

$$h = 2.5t + 50$$

d



Gradient: 1.8

Meaning:

1.8 °F per 1°C

y -intercept: 32

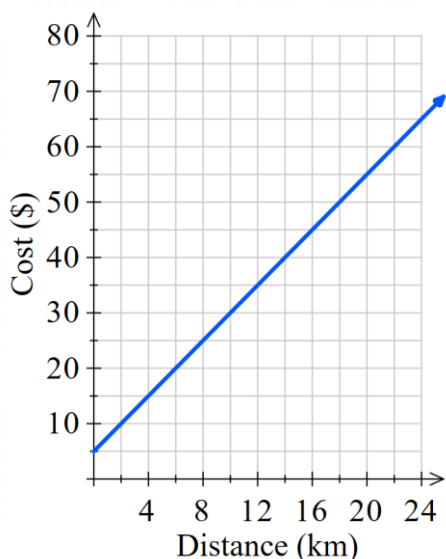
Meaning:

32°F at 0°C

Equation relating F and C :

$$F = 1.8C + 32$$

- 2 The linear model shows the cost of hiring a taxi.



- a From the graph, find the cost of travelling 22 km.

\$60

- b How far can a passenger travel for \$30?

10 km

- c Find the y -intercept.
What is its meaning in this situation?

\$5. Initial cost of hiring the taxi.

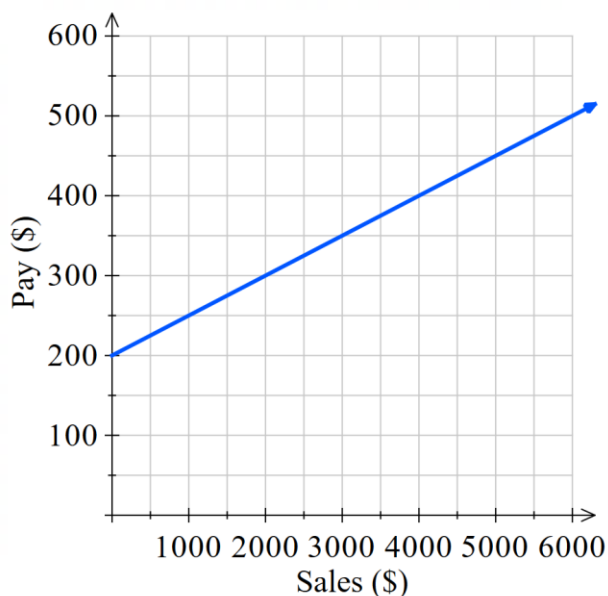
- d Identify the gradient. What is its meaning in this situation?

2.5. \$2.50 per km.

- e Write an equation relating cost C and distance d .

$$C = 2.5d + 5$$

- 3 The linear model shows how an employee is paid on commission.



- a What was the value of sales if the salesperson was paid \$300 for the week?

\$2000

- b What is the salesperson's pay if she sold \$5000 worth of goods?

\$450

- c What retainer is this employee paid?

\$200

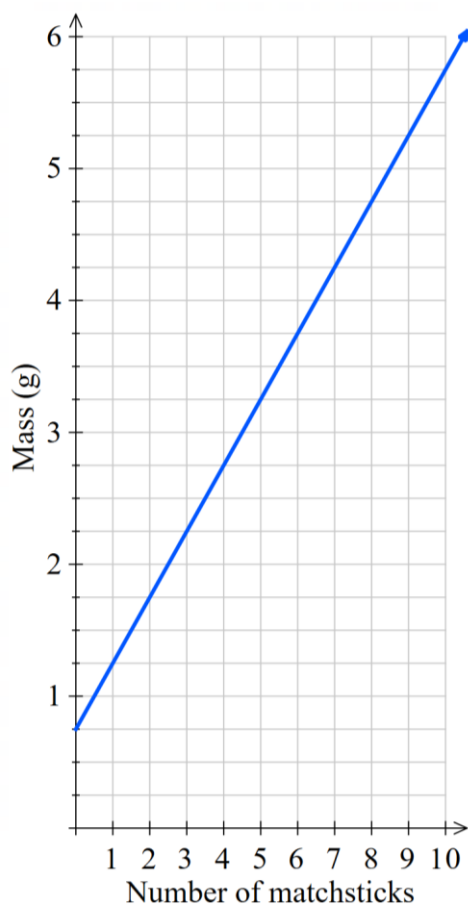
- d What is this employee's rate of commission?

5%

- e Write an equation relating pay P and sales S .

$$P = 0.05S + 200$$

- 4 The graph shows the mass of a box when some identical matchsticks are put into it.



- a Use the graph to determine the mass of
- the empty box

0.75 g

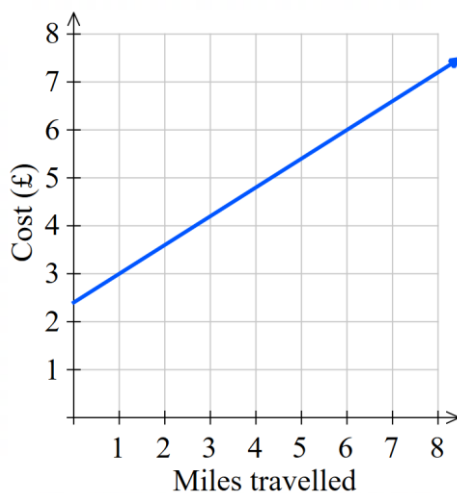
- one matchstick

0.5 g

- b Find the equation relating mass M and number of matchsticks n .

$$M = 0.5n + 0.75$$

- 5 The graph shows the cost of a taxi journey for a given number of miles,



- a Use the graph to find the cost of
- a 1-mile journey

£3

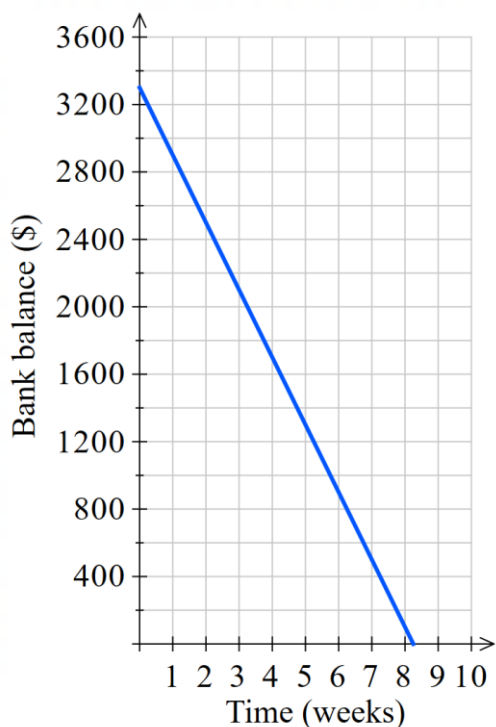
- a 6-mile journey

£6

- b Using the two data points in part a, find the gradient and interpret what it means in the context of the question.

$$m = 0.6. \text{ £}0.60 \text{ per mile}$$

- 6 Nikki's balance in her bank account is shown in the graph.



- a How much money does she have after 3 weeks?

\$2100

- b When did she have \$700 in his account?

After 6.5 weeks

- c What is the y -intercept?
What does it represent in this scenario?

\$3300. Initial amount in the account.

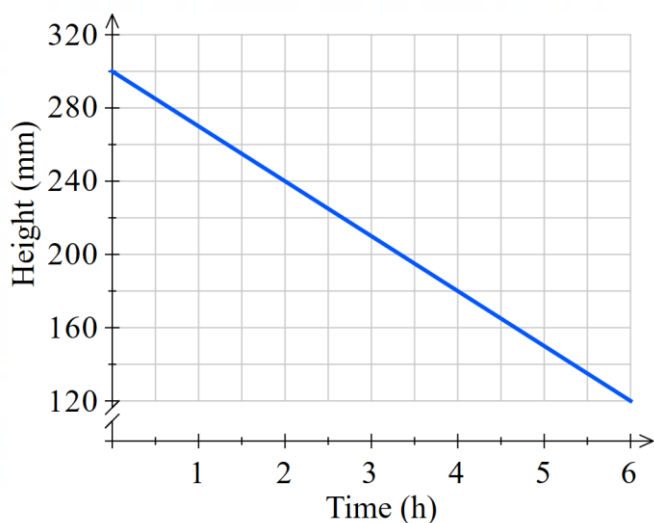
- d What is the gradient?
What does it represent in this scenario?

-400. Her bank balance is reducing by \$400 per week.

- e Write an equation relating bank balance b and time in weeks t .

$$b = -400t + 3300$$

- 7 This model shows the height of a burning candle.



- a What is the height of the candle initially?

300 mm

- b How long does it take for the height to decrease to 180 mm?

4 hours

- c How long does it take for 180 mm of the candle to be used?

6 hours

- d At what rate is the candle's height decreasing per hour?

30 mm / hour

- e Write an equation relating height h and time t .

$$h = -30t + 300$$

- f Predict when the candle will be completely used up.

10 hours

- 8 The Supertight Tie Company has a fixed cost of \$1200 per day and a variable production cost of \$3 per tie. The daily cost can be modelled using the equation $C = 3n + 1200$.

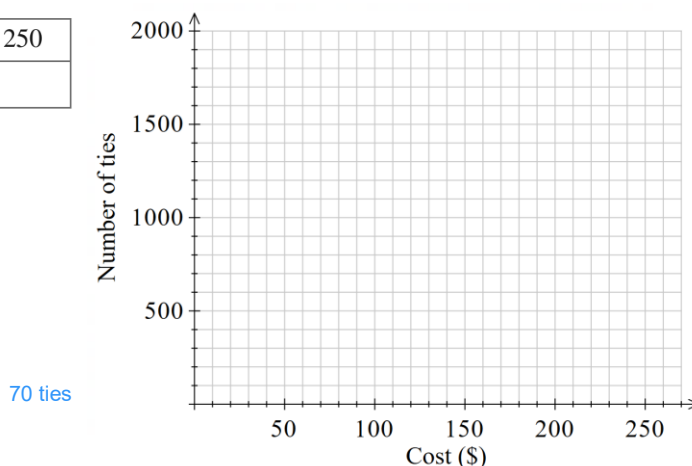
a Explain what each term in the equation $C = 3n + 1200$ represents.

C : n :
total cost number of ties
 3 : 1200 :
cost per tie fixed cost

b Complete the table of values for $C = 3n + 1200$ and draw the graph.

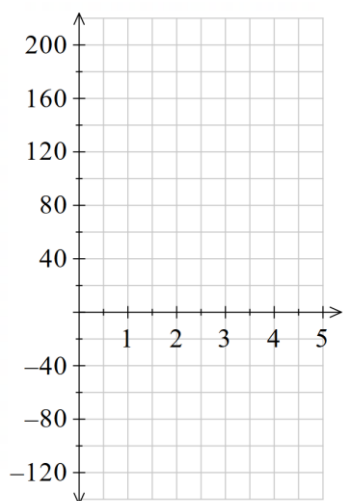
n	0	50	100	150	200	250
C						

c Use the graph to find the number of ties produced when daily cost is \$1400.



- 9 Tina travels 200 km to her friend's home at an average speed of 80 km/h. Tina's distance from her friend's home after travelling for t hours can be modelled using the equation $D = 200 - 80t$.

a Draw the graph of $D = 200 - 80t$. Remember to label the axes.



b Use the graph to find how far Tina is from her friend's home after 1.5 hours.

c Use the graph to find how long it takes for Tina to reach her friend's home.

d What would be the value of D after 4 hours? Hence explain the limits of this model.

80 km

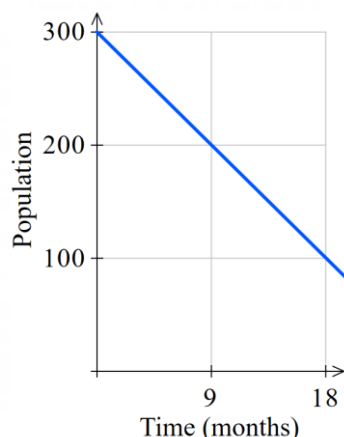
2.5 hours

-120 km.

Distance won't become negative, since Tina would stop at her friend's house.

The model is only valid between at $0 \leq t \leq 2.5$.

- 10** The number of deer in a forest has decreased months according to the graph shown.



- a** How many deer were there to begin with?

300

- b** How many deer were there after 18 months?

100

- c** How many deer disappeared during the 18-month period?

200

- d** How many deer disappear each month?

$11\frac{1}{11}$ deer per month

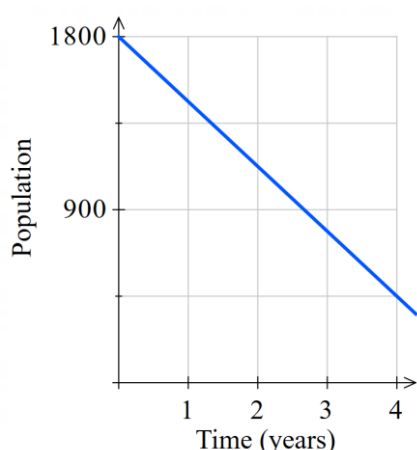
- e** Write an equation relating number of deer d and time t .

$$d = -11\frac{1}{11}t + 300$$

- f** Use your equation to estimate when the deer will be come extinct in this forest.

after a total of 27 months

- 11** The number of people in a village has decreased over recent years according to the graph.



- a** Write an equation relating number of people p and time t .

$$p = -337.5t + 1800$$

- b** Use your equation to determine the number of people in the village after 2 years.

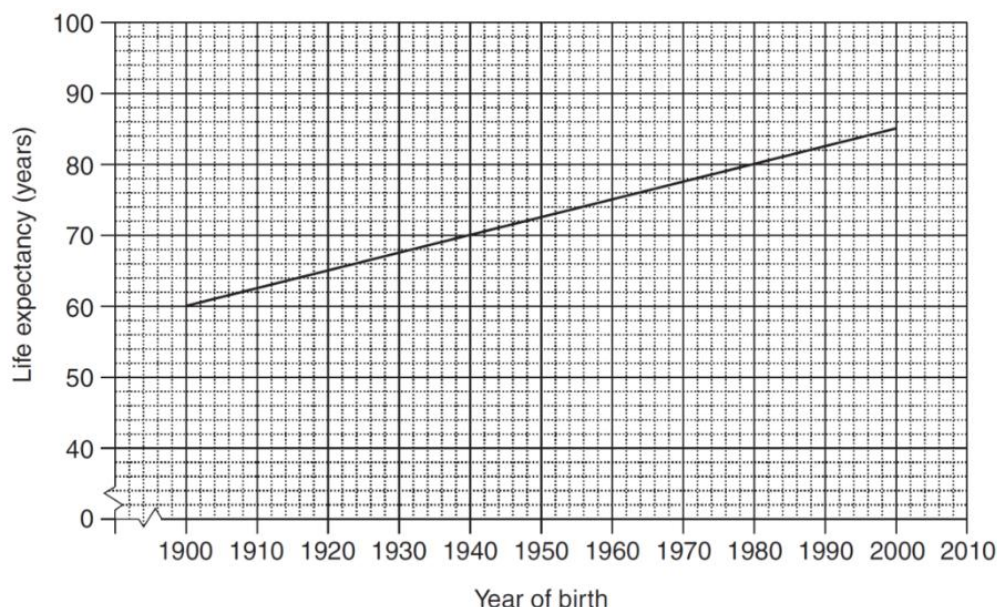
1125 people

- c** Use your equation to determine when the population will first reach 180 people.

4.8 years

12 2016 HSC Standard 2 Band 5

The graph shows the life expectancy of people born between 1900 and 2000.



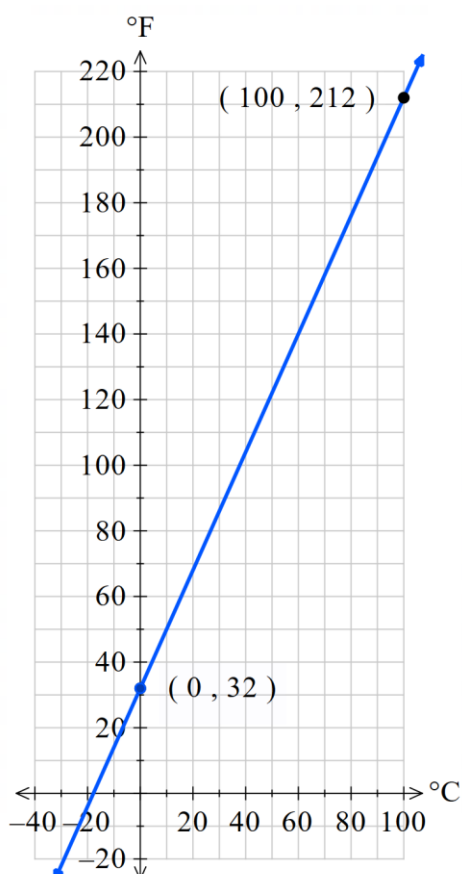
According to the graph, what is the life expectancy of a person born in 1932?

68 years

With reference to the value of the gradient, explain the meaning of the gradient in this context.

0.25 years increase in life expectancy per year

- 13** This graph shows the relationship between degrees Celsius and degrees Fahrenheit. The freezing point of water is 0°C or 32°F , while the boiling point is at 100°C or 212°F .



- a** What is the y-intercept of the line?

32

- b** What is the gradient of the line?

1.8

- c** Write an equation $y = mx + c$ to relate Celsius and Fahrenheit.

$F = 1.8C + 32$

- d** Use your equation to convert 48°C to Fahrenheit. Does this match the graph?

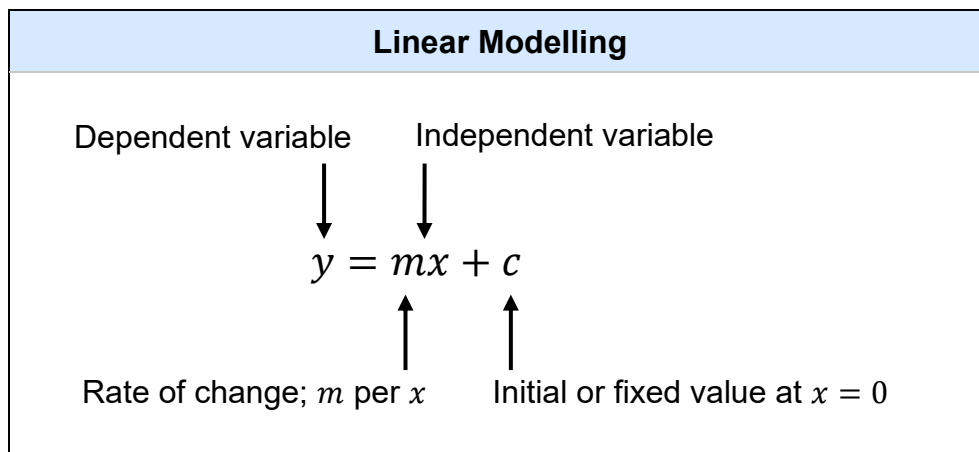
107.8. Yes

- e** Use your equation to convert 200°F to Celsius. Does this match the graph?

93.33. Yes

CONSTRUCTING LINEAR MODELS

1. Read the scenario in its entirety. Highlight the variables and the numbers.
2. Identify the dependent variable y . It changes based on another variable.
3. Identify the independent variable x . It causes the change.
4. Determine the gradient m . This is the rate given by the question. " m per x "
5. Determine the y -intercept c . This is the initial or fixed value given by the question.



Example Construct a linear model from a given scenario.

A muffin shop has fixed costs of \$100 a day and a production cost of \$1.50 per muffin.

Write an equation relating cost C and number of muffins n .

Dependent var: Cost C

Independent var: Number of muffins n

Variable cost $m = 1.5$

Fixed cost $c = 100$

$$y = mx + c$$

$$C = 1.5n + 100$$

Use your equation to determine the number of

Muffins that can be produced for \$300.

$$300 = 1.5n + 100$$

$$200 = 1.5n$$

$$\frac{200}{1.5} = n$$

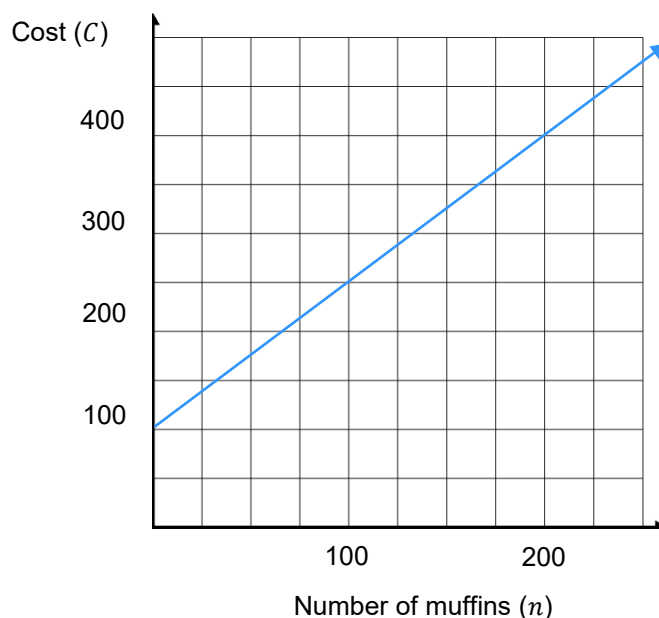
$$133 \approx n$$

How do you know what number to let be m ?

m is the cost, it changes the total cost based on the independent variable x .

How do you know what number to let be c ?

c is the cost, it does not change based on the independent variable x .



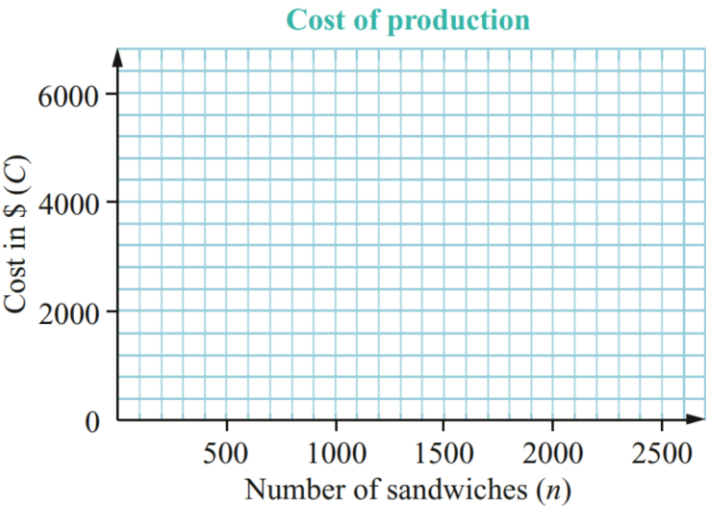
- 1 A sandwich shop has a monthly fixed expense of \$2000 and the cost of making each sandwich is \$1.20.
- a Write an equation modelling this information. Use C for monthly cost in dollars and n for the number of sandwiches made.

$C = 1.2n + 2000$

- b Draw the straight-line graph for this linear model.

n	0	500	1000	1500
c				

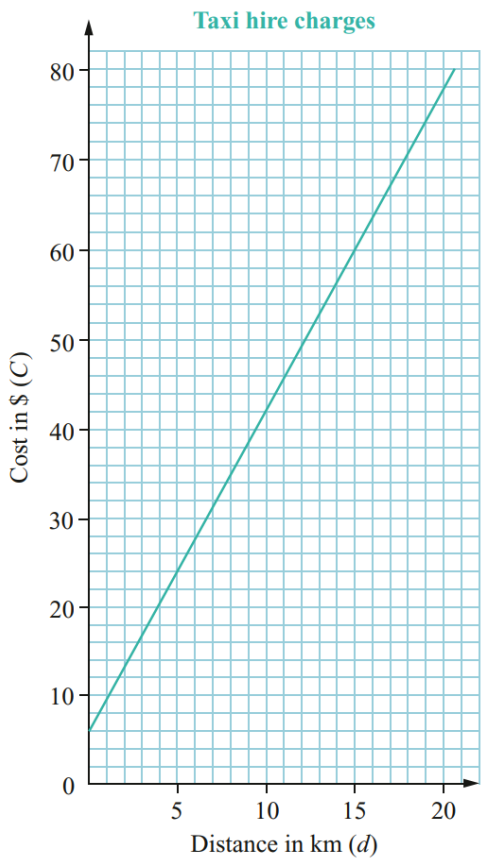
- c Use the graph to find the number of sandwiches made if the cost is \$3500.
- d Use the equation to find the sandwiches made if the cost is \$3500. Which answer is more accurate?



1250. The equation.

- 2 The cost of hiring a taxi is \$6 flagfall and \$3.60 per kilometre travelled.
- a Write a linear equation to relate cost C and distance d travelled.

- b Use the equation to determine how far you could travel for \$75. How does this compare with the graph?



$C = 3.6d + 6$

19.17 km
Matches the graph.

- 3 The weekly wage, $\$w$, of a vacuum cleaner salesperson consists of a fixed sum of $\$350$ plus $\$20$ for each cleaner sold. If n cleaners are sold per week,

a Write an equation that describes the weekly wage of the salesperson.

$$w = 20n + 350$$

b How much would the salesperson make if he sold 23 vacuums that week?

$$\$810$$

c How many cleaners would he have to sell to make at least $\$1215$ in a week?

$$44 \text{ vacuums}$$

- 4 A telecommunications company's rates for local calls from private telephones consist of a quarterly rental fee of $\$40$ plus $25c$ for every call.

a Write an equation that describes the quarterly rental fee.

$$C = 40 + 0.25n$$

b How many calls could you make for $\$90.50$?

$$202$$

- 5 Blake converted 100 Australian dollars (AUD) to 60 euros (EUR).

a Draw a conversion graph with AUD on the x axis and EUR on the y axis.

b How many euros is 25 Australian dollars?

c How many Australian dollars is 45 euros?

d Determine the gradient. What is its meaning?

$$0.6, \text{ how much EUR for 1 AUD}$$

e Determine the y -intercept. What is its meaning?

$$0. \text{ How much EUR for 0 AUD}$$

f Write an equation relating AUD to EUR.

$$\text{EUR} = 0.6 \text{ AUD}$$

g How much EUR can be purchased with $\$1855$ AUD?

$$\$1113$$

- 6** A petrol tank holds 66 litres of fuel. If it contains 12 litres of petrol initially and the petrol pump fills it at 3 litres every 10 seconds, find:
- a** linear equation for the amount of fuel (F litres) in the tank after t minutes
- $F = 12 + 18x$
- b** how long it will take to fill the tank.
- 3 minutes
- c** how long it will take to put 45 litres into the petrol tank.
- 2 min 30 sec
- 7** A tank is initially full with 4000 L of water and water is being used at a rate of 20 L per minute.
- a** Write a rule for the volume, V litres, of water after t minutes.
- $V = 4000 - 20x$
- b** Calculate the volume after 1.5 hours.
- 2200 L
- c** How long will it take for the tank to be emptied?
- 200 min
- d** How long will it take for the tank to have only 500 L?
- 175 min
- 8** The rule for distance travelled, d km, over a given time, t hours, for a moving vehicle is given by $d = 50 + 80t$.
- a** What is the speed of the vehicle?
- 80 km/h
- b** If the speed was 70 km per hour, how would this change the rule? Give the new rule.
- $d = 50 + 70t$
- 9** The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.
- a** What is the helicopter's initial height?
- 350 m
- b** How is the helicopter's height changing?
- Decreasing by 20 m/s
- c** If instead the rule was $h = 350 + 20t$, describe what the helicopter would be doing.
- Ascending by 20 m/s

10 2020 HSC Standard 2 Band 5

A plumber charges a call-out fee of \$90 as well as \$2 per minute while working.

Suppose the plumber works for t hours.

What equation expresses the amount the plumber charges (\$ c) as a function of time (t hours)?

$$C = 90 + 120t$$

11 HSC Standard 2 Sample Question Band 5

The average height, h , in centimetres, of a girl between the ages of 6 years and 11 years can be represented by a line with equation $C = 6A + 79$, where A is age in years.

For this line, the gradient is 6.

What does this indicate about the heights of girls aged 6 to 11?

Increases by 6 cm per year.

Give ONE reason why this equation is not suitable for predicting heights of girls older than 12.

In teenage years, puberty changes the growth rate.

12 2007 HSC Standard 2 Band 5

A clubhouse uses four long-life light globes for five hours every night of the year.

The purchase price of each light globe is \$6.00 and they each cost \$ d per hour to run.

Write an equation for the total cost (\$ c) of purchasing and running these four light globes for one year in terms of d .

$$C = 4(6 + 1825d) \text{ or } C = 24 + 7300d$$

Find the value of d (correct to three decimal places) if the total cost of running these four light globes for one year is \$250.

$$d = \$0.031$$

If the use of the light globes increases to ten hours per night every night of the year, does the total cost double? Justify your answer with appropriate calculations.

No, as $C = 4(6 + 3650d)$ is not double $C = 4(6 + 1825d)$

CHECK YOUR UNDERSTANDING

Gradient intercept form

1 What is the gradient of $y = 2x - 3$

a 2

b $2x$

c 3

d -3

2 What is the y -intercept of $y = 2x - 3$

a $(3,0)$

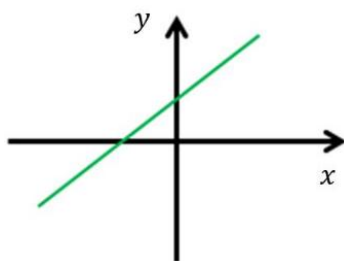
b $(0,3)$

c $(-3,0)$

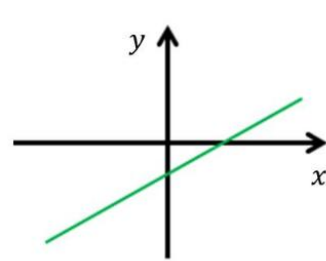
d $(0,-3)$

3 Which of these could represent $y = 2x - 3$?

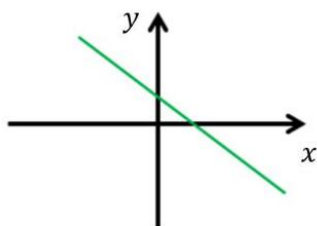
a



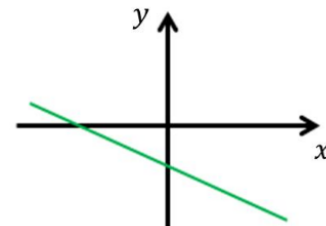
b



c

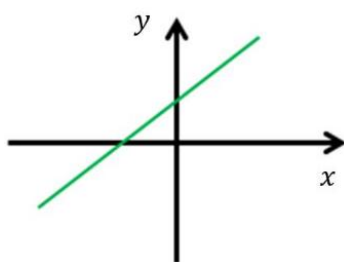


d

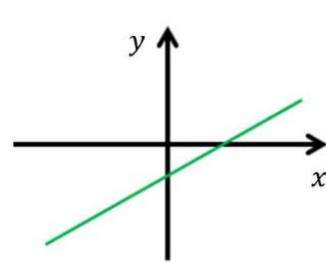


4 Which of these could represent $y = 2 - 3x$?

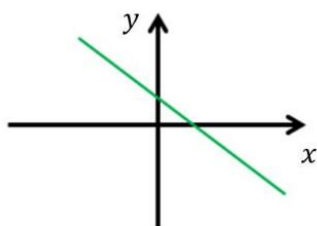
a



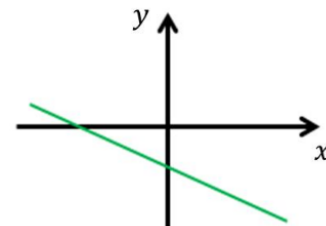
b



c



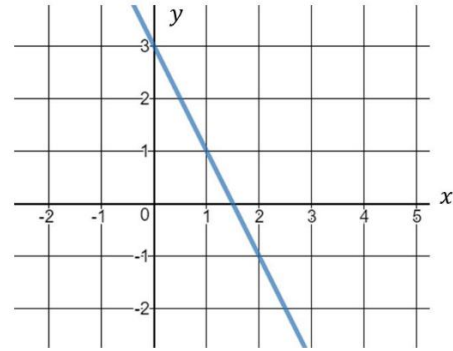
d



Finding the equation

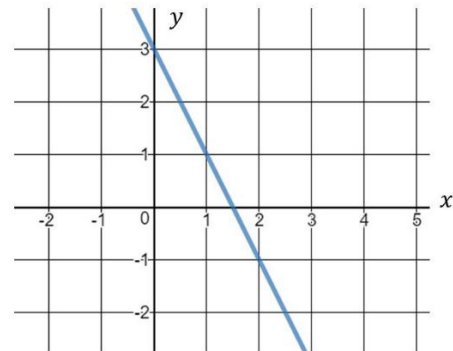
- 1 Jon is finding the equation of this line $y = mx + c$.
What is the value of m ?

- a 2
- b 3
- c -2
- d -1.5



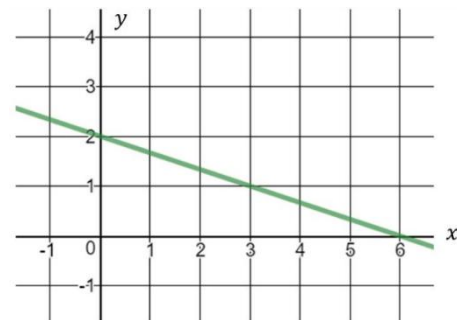
- 2 Jon is finding the equation of this line $y = mx + c$.
What is the value of c ?

- a 2
- b 3
- c -2
- d -1.5



- 3 Shelly is finding the equation of this line $y = mx + c$.
What is the value of m ?

- a $-\frac{1}{3}$
- b 2
- c -3
- d 6



- 4 Shelly is finding the equation of this line $y = mx + c$.
What is the value of c ?

- a $-\frac{1}{3}$
- b 2
- c -3
- d 6

