Student Name

Mathematics Stage 5 Core

NON-LINEAR RELATIONSHIPS A

Book 1 Examine the connection between algebraic and graphical representations of quadratics and exponentials

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OVERVIEW

SYLLABUS CONTENT

MA5-NLI-C-01 identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts

Examine the connection between algebraic and graphical representations of quadratics and exponentials

- Represent quadratic and exponential relationships using graphing applications
- Construct a table of values to graph non-linear relationships involving quadratics and exponentials
- Explain that quadratic relationships are represented by parabolas
- Identify graphs and equations of parabolas and exponential curves
- Recognise quadratics and exponentials in real-life contexts

REVIEW OF PRIOR KNOWLEDGE

1 Evaluate these expressions for the given value of x.

a	$x = 4$ $x^2 + 5$	**b**	$x = -1$ $2x^2 - 3$	**c**	$x = 2$ $3 - x^2$
d	$x = 0$ 2^x	**e**	$x = -2$ 3^x	**f**	$x = 1$ -23^x
g	$x = 3$ $3 - 2^x$	**h**	$x = -1$ 4^x	**i**	$x = 2$ $x^2 - 3x - 4$

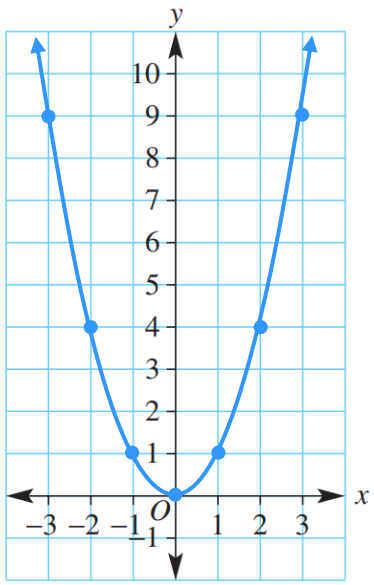
QUADRATIC RELATIONSHIPS

- **Linear relationships** are in the form $y = mx + c$.
  The graph is a straight line; so, the relationship is **linear.**
- **Quadratic relationships** are in the form $y = ax^2 + bx + c$.
  Their graph will not be a straight line; **non-linear.**
- We graph quadratic relationships the same way as linear relationships:
  1. Construct a table of values using the equation.
  2. Graph the table by using the columns as coordinates.
  3. Join the points to form a smooth curve.

Example Graph quadratic relationships using a table of values.

Draw the graph of $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



What does power of 2 (squaring) mean?

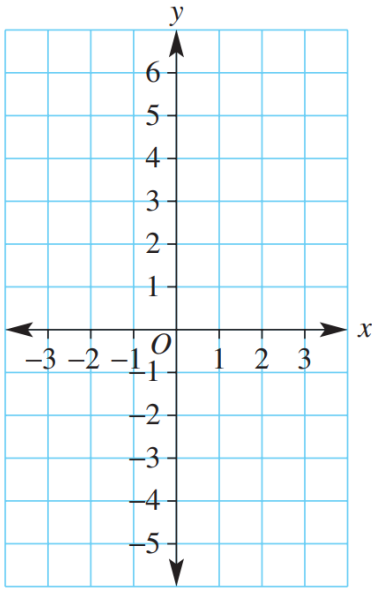
Power of 2 means to multiply a number by .....

What does non-linear mean?

Not a ..... line.

Draw the graph of $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
y							



Key ideas

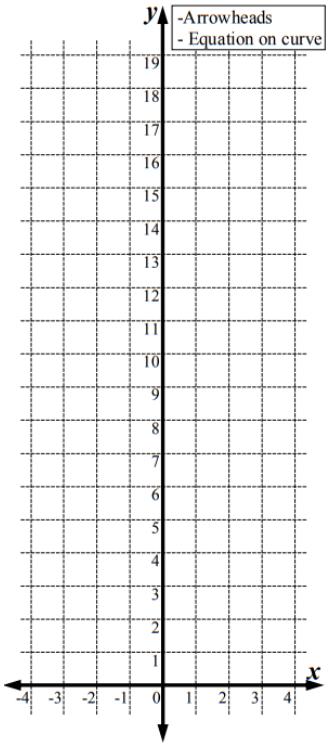
1. ..... equations have an equation with the highest power being x^2, and have a graph in a shape called a .....
2. Parabolas are non-..... curves and are symmetrical.

1 Sketch these parabolas.

a $y = x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

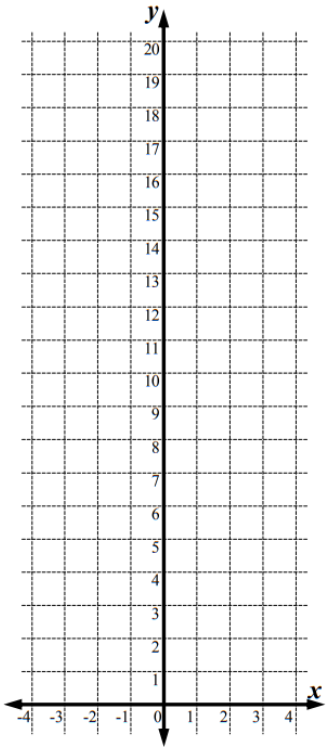
Calc (- 3) x^2 + 5 =



b $y = 2x^2 + 2$

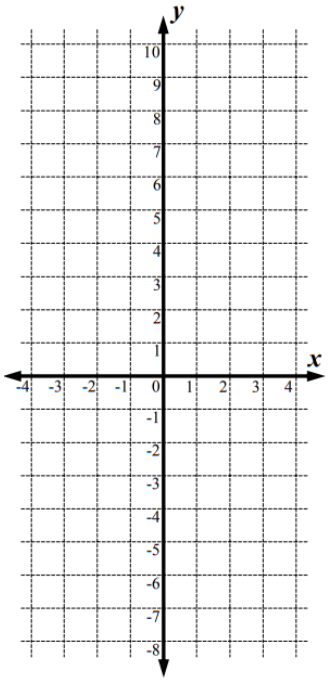
x	-3	-2	-1	0	1	2	3
y							

Calc: 2 × (- 3) x^2 + 2 =



c $y = x^2 - 7$

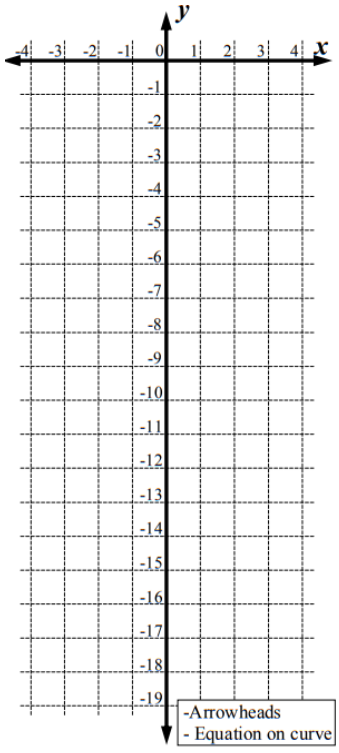
x	-4	-3	-2	-1	0	1	2	3	4
y									



d $y = -x^2 - 6$

x	-3	-2	-1	0	1	2	3
y							

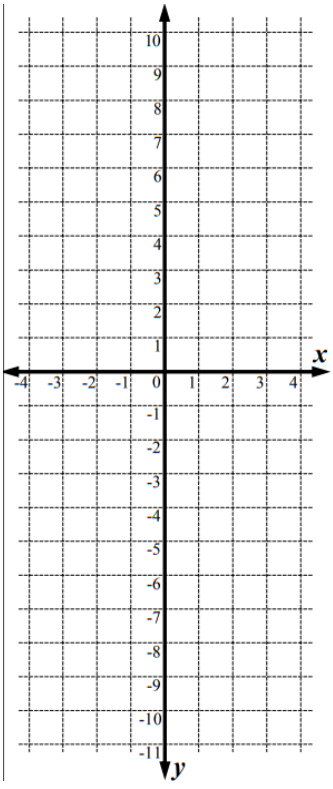
Calc: - (- 3) x^2 - 6 =



e $y = -2x^2 + 7$

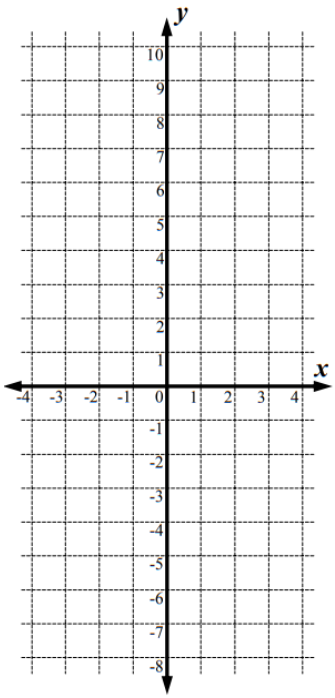
x	-3	-2	-1	0	1	2	3
y							

Calc: - 2 × (- 3) x^2 + 7 =



f $y = -x^2 + 8$

x	-4	-3	-2	-1	0	1	2	3	4
y									

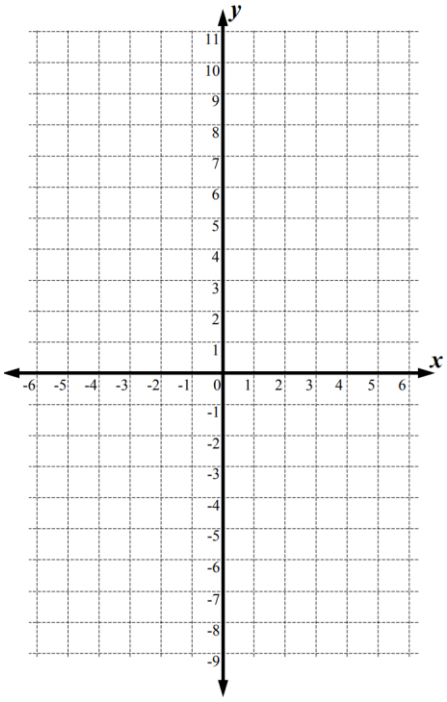


2 Sketch these graphs.

a $y = \frac{1}{2}x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

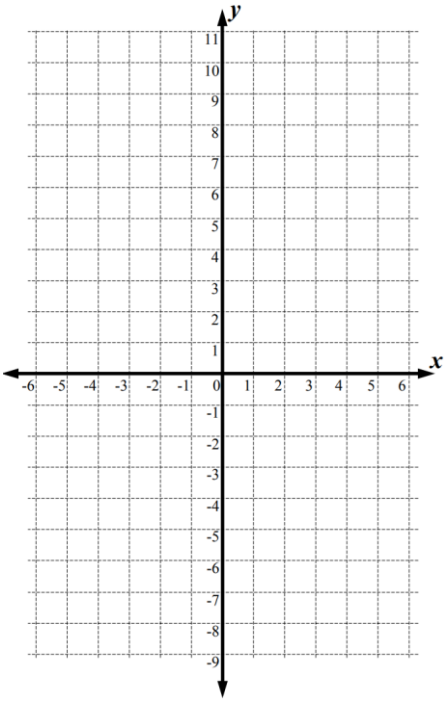
Calc: 1 2 × (- 4) x^2 =



b $y = -\frac{1}{2}x^2 + 10$

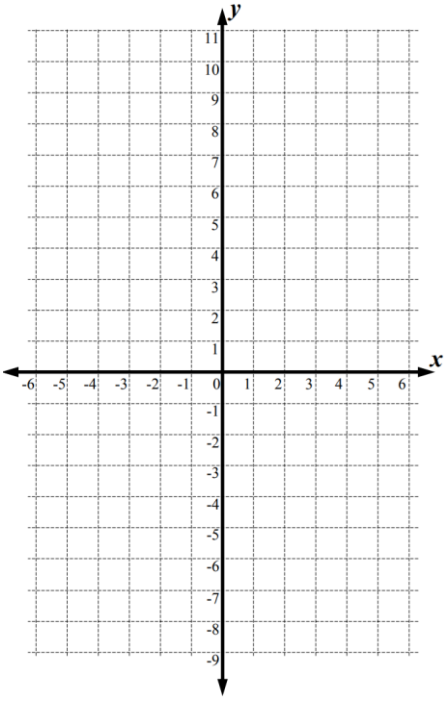
x	-6	-4	-2	-1	0	1	2	4	6
y									

Calc: - 1 2 × (- 6) x^2 + 1 0 =



c $y = \frac{1}{4}x^2 - 2$

x	-6	-4	-2	0	2	4	6
y							



3 Plot these on the same axis and label each parabola.

$y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

$y = x^2$

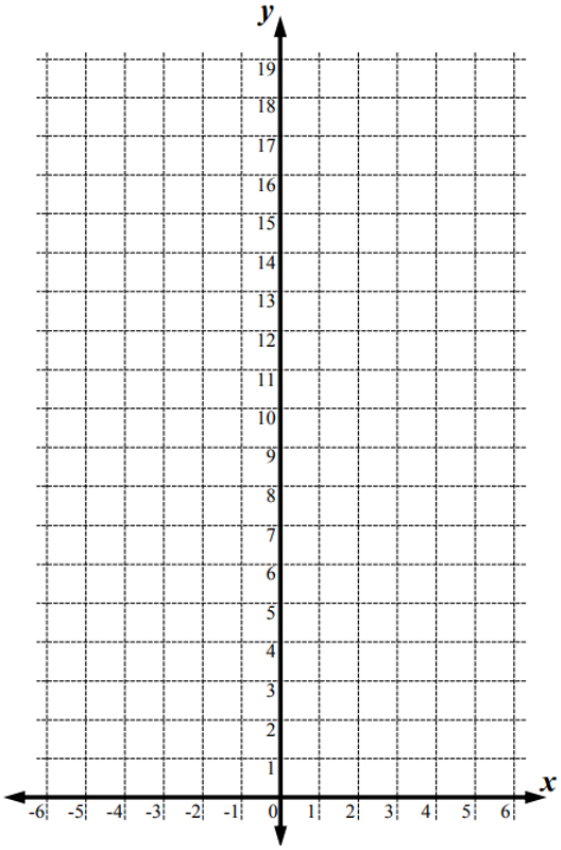
x	-4	-3	-2	1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

$y = \frac{1}{2}x^2$

x	-6	-4	-2	-1	0	1	2	4	6
y	18	8	2	½	0	½	2	8	18

$y = \frac{1}{4}x^2$

x	-6	-4	-2	0	2	4	6
y	9	4	1	0	1	4	9



Circle the correct option:

As the coefficient of x^2 increases, the parabola gets:

A. Wider

B. Narrower

Narrower

4 Plot these on the same axis and label each parabola.

$y = -x^2$

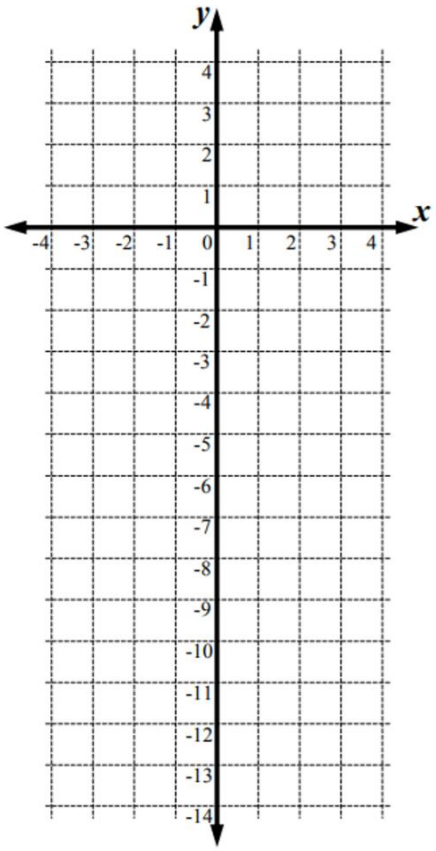
x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

$y = -x^2 + 4$

x	-4	-3	-2	1	0	1	2	3	4
y	-12	-5	0	3	4	3	0	-5	-12

$y = -x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	-12	-7	-4	-3	-4	-7	-12



Circle the correct option:

The constant term affects the parabola by:

A. Shifting the parabola up or down.
B. Determining where the parabola cuts the y-axis.
C. Both of the above.

C

5 Which of the cards show the equation of a straight line and which show the equation of a parabola?

A $y = 3x + 5$ B $y = 3x + 5x^2$ C $y = 3 + x$ D $y = 3 + x^2$

E $y = 3 + x + x^2$ F $y = 6 - 2x$ G $y = 6 - 2x^2$ H $y = (6 - 2x)^2$

Straight line: A, C, F

Parabola: B, D, E, G, H

6 "When you square a negative number you get a positive number, so the graph of $y = -x^2$ is the same as $y = x^2$"

Explain why this statement is incorrect.

……

……

$y = x^2$ is the same as $y = (-x)^2$, but not $y = -x^2$

The negative sign is not in brackets, so it doesn't get squared.

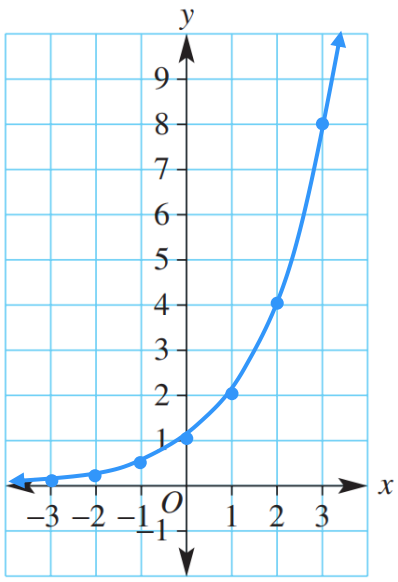
EXPONENTIAL RELATIONSHIPS

- **Exponential relationships** are in the form $y = a^x$.
  Their graph will not be a straight-line; **non-linear.**
- We graph exponential relationships the same way as before:
  1. Construct a table of values using the equation.
  2. Graph the table by using the columns as coordinates.
  3. Join the points to form a smooth curve.
- Consecutive y values change by a **common ratio**, which is the base number a.

Example Graph exponential relationships using a table of values.

Draw the graph of $y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



As x increases by 1, how is are the y values changing?

The y values are being ..... by a common ratio.

How does the common ratio above relate to the equation?

The common ratio is the ..... number.

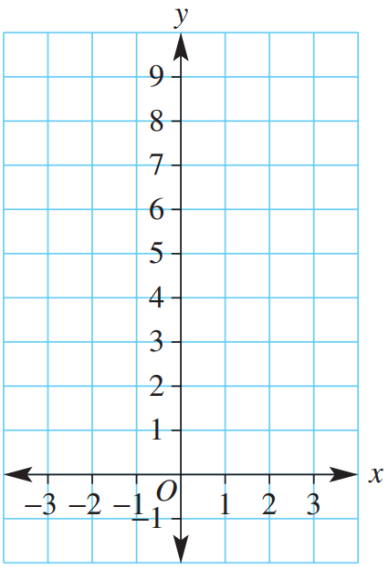
What is the $y-$ intercept? .....

Will the graph ever reach $y = 0$? Explain.

....., if you keep ..... by a number, you ..... reach zero.

Draw the graph of $y = 3^x$

x	-3	-2	-1	0	1	2	3
y							



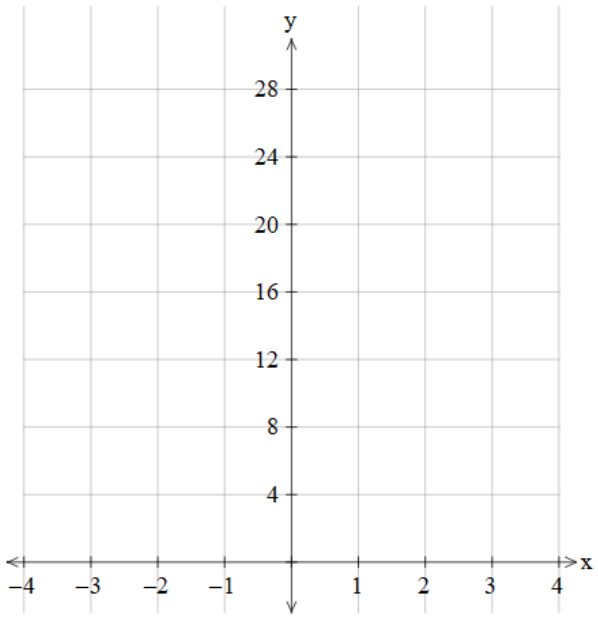
Key ideas

1. Exponential equations $y = a^x$ have x in the .....
2. Exponential curves are non-..... curves.
3. Consecutive y values change by a common r.....
4. $y = a^x$ can never reach $y =$ .....

1 Sketch these exponential curves.

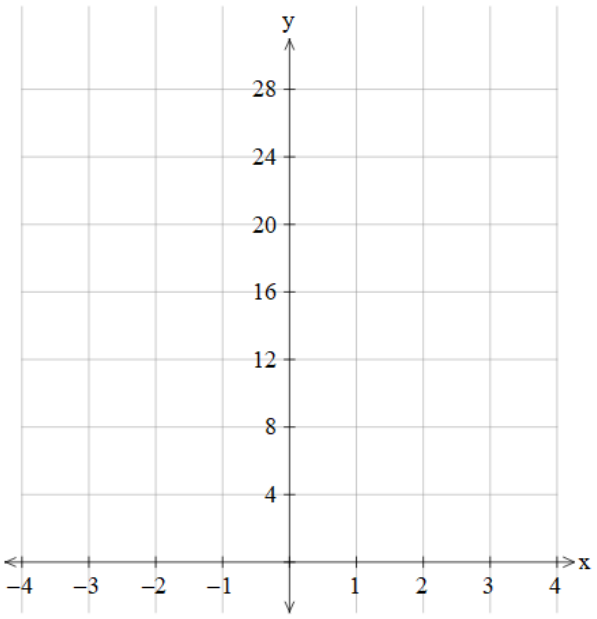
a $y = 2^x$

x	-3	-2	-1	0	1	2	3
y							



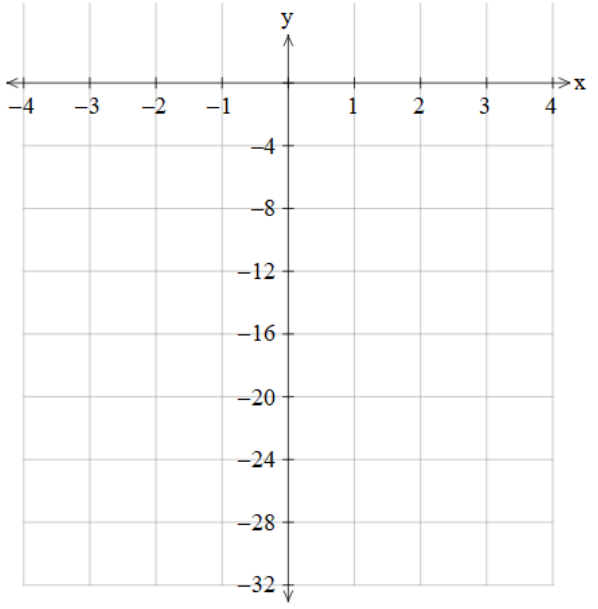
a $y = 3^x$

x	-3	-2	-1	0	1	2	3
y							



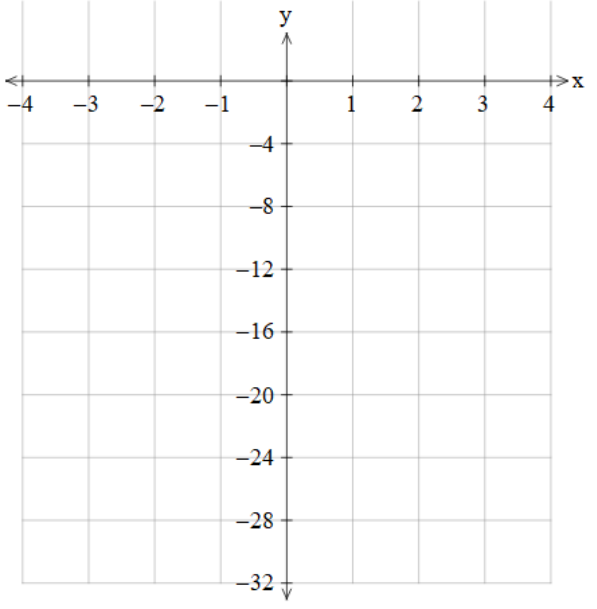
b $y = -2^x$

x	-3	-2	-1	0	1	2	3
y							



c $y = -3^x$

x	-3	-2	-1	0	1	2	3
y							



2 Sketch these exponential curves on the same axis.

a $y = 4^x$

x	-3	-2	-1	0	1	2	3
y							

b $y = -4^x$

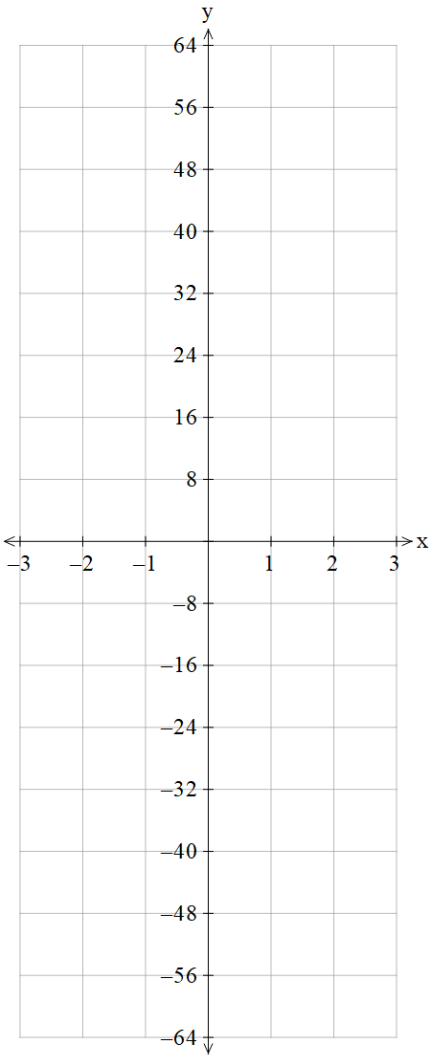
x	-3	-2	-1	0	1	2	3
y							

c $y = 4^{-x}$

x	-3	-2	-1	0	1	2	3
y							

d $y = -4^{-x}$

x	-3	-2	-1	0	1	2	3
y							



What do you notice?

……

……