

Student Name

Mathematics Stage 4

ALGEBRAIC TECHNIQUES

Book 1 Examine the concept of pronumerals
as a way of representing numbers

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OVERVIEW

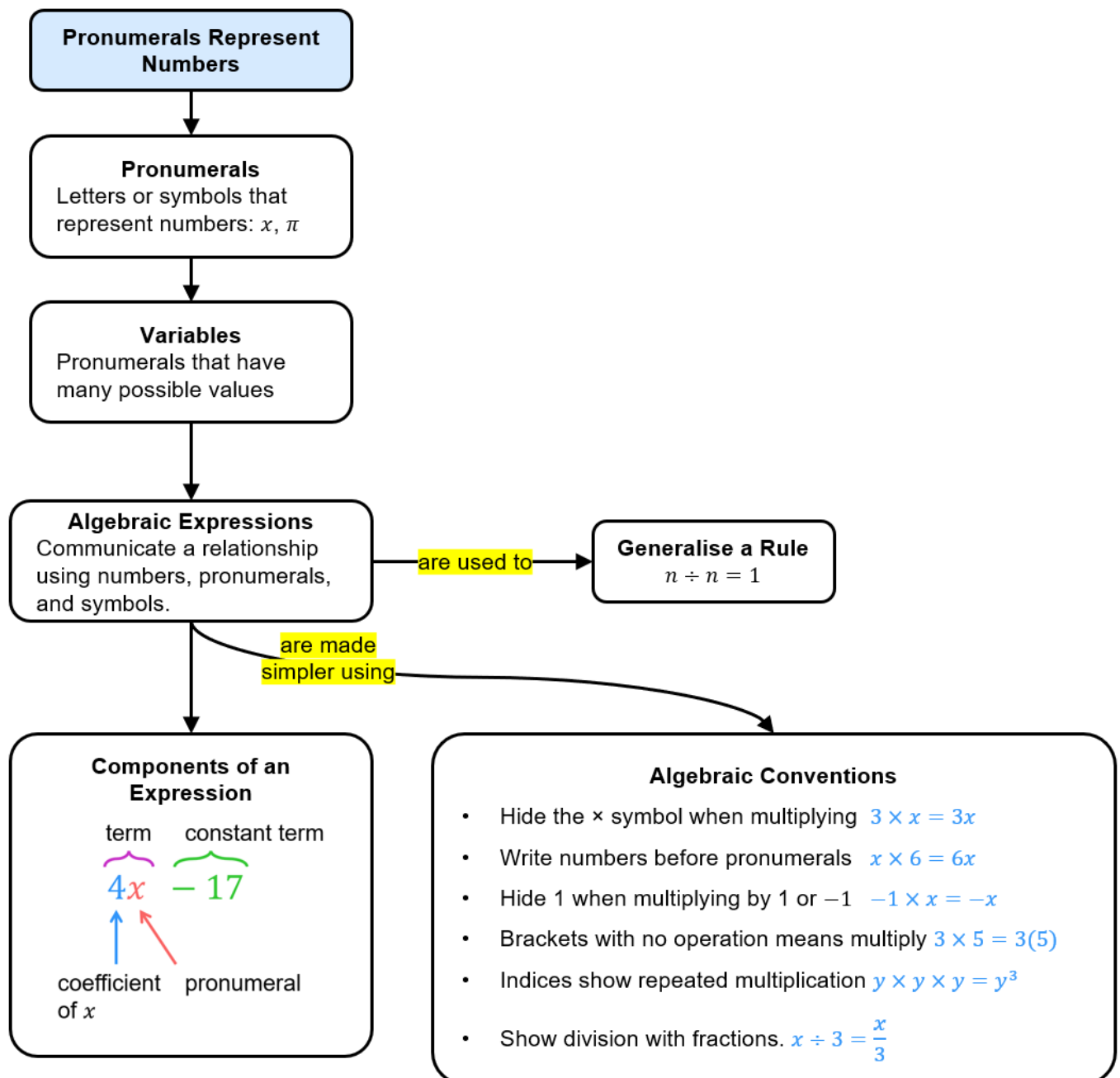
SYLLABUS CONTENT

MA4-ALG-C-01 generalises number properties to operate with algebraic expressions including expansion and factorisation

Examine the concept of pronumerals as a way of representing numbers

- Examine and recognise that pronumerals can be used to represent one or more numerical values and when pronumerals have more than one numerical value, they may then be referred to as variables
- Identify and define an algebraic expression as an expression formed by combining numbers and algebraic symbols using arithmetic operations
- Use concise algebraic notation and conventions for multiplication, division and powers, and explain the meanings for each convention

SUMMARY



GENERALISING RELATIONSHIPS USING ALGEBRA

ALGEBRA

- **Algebra** communicates patterns, rules, and relationships involving numbers.
- **Pronumerals** are letters or symbols that represent numbers, such as x or y .
- **Variables** are pronumerals with many possible numerical values.

GENERALISING

- An **algebraic expression** combines numbers, pronumerals, and symbols.
- Algebra shows relationships that work for any number, not just one specific example.
- Showing a pattern works for any number is called **generalising**.

Example Generalise rules of arithmetic.

Look at the pattern and generalise a rule.

$$5 \times 0 = 0$$

$$6 \times 0 = 0$$

$$51 \times 0 = 0$$

$$\frac{1}{2} \times 0 = 0$$

English expression: Any number times zero is zero.

Algebraic expression: $n \times 0 = 0$

Generalise these relationships.

a

$$4 \times 1 = 4$$

$$7 \times 1 = 7$$

$$(-3) \times 1 = -3$$

English expression:

.....

Any number multiplied by 1 is itself.

Algebraic expression:

$$n \times 1 = n$$

b

$$9 + 9 + 9 = 3 \times 9$$

$$2 + 2 + 2 = 3 \times 2$$

$$(-4) + (-4) + (-4) = 3 \times (-4)$$

English expression:

.....

Any number added to itself twice is three times the number.

Algebraic expression:

$$n + n + n = 3 \times n$$

Key ideas

1. Pronumerals are letters or symbols that represent
2. If a pronumeral represents *any* number, then we call it a
3. Algebra involves using to represent numbers so we can quickly communicate mathematical relationships.

1 Generalise these number facts.

a

$$6 - 6 = 0$$

$$1 - 1 = 0$$

$$17 - 17 = 0$$

English

expression: Any number subtract by itself is zero.

Algebraic

expression:
 $n - n = 0$

b

$$9 + 9 = 2 \times 9$$

$$3 + 3 = 2 \times 3$$

$$(-4) + (-4) = 2 \times (-4)$$

English

expression:

 Any number added to itself is two times the number..

Algebraic

expression:
 $n + n = 2 \times n$

c

$$\frac{1}{100} = 1 \div 100$$

$$\frac{3}{20} = 3 \div 20$$

$$\frac{7}{3} = 7 \div 3$$

$$\frac{11}{7} = 11 \div 7$$

English

expression:

 Any fraction is the numerator divided by the denominator

Algebraic

expression:
 $\frac{a}{b} = a \div b$

d

$$2 + (-5) = 2 - 5$$

$$4 + (-1) = 4 - 1$$

$$9 + (-12) = 9 - 12$$

$$(-3) + (-5) = (-5) - 5$$

English

expression:

 Adding a negative number is the same as subtracting the positive.

Algebraic

expression:
 $a + (-b) = a - b$

e

$$2 \div 2 = 1$$

$$8 \div 8 = 1$$

$$6 \div 6 = 1$$

$$14 \div 14 = 1$$

English

expression:

 Any number divided by itself is 1

Algebraic

expression:
 $n \div n = 1$

f

$$8 \div 1 = 8$$

$$5 \div 1 = 5$$

$$12 \div 1 = 12$$

$$13 \div 1 = 13$$

English

expression:

 Any number divided by 1 is itself.

Algebraic

expression:
 $n \div 1 = n$

2

a

$$\begin{aligned}6 - 0 &= 6 \\1 - 0 &= 1 \\12 - 0 &= 12 \\17 - 0 &= 17\end{aligned}$$

Algebraic
expression:

$$n - 0 = n$$

b

$$\begin{aligned}3 + 4 &= 4 + 3 \\8 + 6 &= 6 + 8 \\16 + 7 &= 7 + 16 \\(-2) + 8 &= 8 + (-2)\end{aligned}$$

Algebraic
expression:

$$a + b = b + a$$

c

$$\begin{aligned}2 \times 3 &= 3 \times 2 \\4 \times 7 &= 7 \times 4 \\111 \times 56 &= 56 \times 111 \\(-2) \times 4 &= 4 \times (-2)\end{aligned}$$

Algebraic
expression:

$$a \times b = b \times a$$

d

$$\begin{aligned}2 + (-2) &= 0 \\3 + (-3) &= 0 \\15 + (-15) &= 0 \\8 + (-8) &= 0\end{aligned}$$

Algebraic
expression:

$$n + (-n) = 0$$

e

$$\begin{aligned}2 \times 2 &= 2^2 \\10 \times 10 &= 10^2 \\(-3) \times (-3) &= (-3)^2 \\24 \times 24 &= 24^2\end{aligned}$$

Algebraic
expression:

$$n \times n = n^2$$

f

$$\begin{aligned}7 \div 2 &= 7 \times \frac{1}{2} \\8 \div 4 &= 8 \times \frac{1}{4} \\11 \div \frac{2}{3} &= 11 \times \frac{3}{2} \\9 \div \frac{5}{4} &= 9 \times \frac{4}{5}\end{aligned}$$

Algebraic
expression:

$$a \div b = a \times \frac{1}{b}$$

3 Change these English expressions to algebraic expressions.

a Any number add zero equals itself.

$$n + 0 = n$$

b Any number added to itself is the same as multiplying that number by 2.

$$n + n = 2n$$

c Doubling any number twice is the same as multiplying it by 4.

$$n \times 2 \times 2 = 4n$$

d Multiplying any number by 5 is the same as multiplying it by 10, then dividing by 2.

$$n \times 5 = n \times 10 \div 2$$

ALGEBRAIC CONVENTIONS

- **Conventions** or guidelines make communicating using algebra simpler.

Example Use algebraic conventions.

Leave out $\times$ symbol when multiplying.

$$3 \times x = 3x$$

Write numbers first, then variables.

$$x \times 6 = 6x$$

Hide the 1 when multiplying by 1 or -1 .

$$1 \times x = x$$

$$-1 \times x = -x$$

a $6 \times a$

b $t \times 3$

c $h \times z$

d $x \times 5 \times y$

$$6a$$

$$3t$$

$$hz$$

$$5xy$$

e $12 \times y \times x$

f $-1 \times y$

g $z \times 1$

h $h \times -1 \times n$

$$12yz$$

$$-y$$

$$z$$

$$-hn$$

Brackets without an operation in front means multiply. $3 \times 5 = 3(5)$

a 6×8

b 12×4

c -2×12

d -3×7

$$6(8)$$

$$12(4)$$

$$-2(12)$$

$$-3(7)$$

e $7 \times 4 \times 5$

f $12 \times (x - 3)$

g $(x - 5) \times -3$

h $-1 \times (x + 4)$

$$7(4)(5)$$

$$12(x - 3)$$

$$-3(x - 5)$$

$$-(x + 4)$$

Indices (powers) show repeated multiplication.

$$x \times x \times x \rightarrow x^3$$

a $w \times w$

b $9 \times w \times w$

c $z \times z \times z \times z$

d $x \times x \times w$

$$w^2$$

$$9w^2$$

$$z^4$$

$$x^2w$$

e $y \times 6 \times y$

f $y \times y \times z$

g $z \times y \times y$

h $x \times 7 \times x \times y$

$$6y^2$$

$$y^2z$$

$$zy^2$$

$$7x^2y$$

Show division using fractions.

$$x \div 3 = \frac{x}{3}$$

a $a \div 7$

b $7 \div a$

c $2x \div 13$

d $n \div y$

$$\frac{a}{7}$$

$$\frac{7}{a}$$

$$\frac{2x}{13}$$

$$\frac{n}{y}$$

Algebraic Convention	Example
Hide the $\times$ symbol when multiplying.	$3 \times x = 3x$
Write numbers before pronumerals.	$x \times 6 = 6x$
Hide 1 when multiplying by 1 or -1 .	$1 \times x = x$ $-1 \times x = -x$
Brackets without an operation means multiply.	$3 \times 5 = 3(5)$
Indices show repeated multiplication.	$x \times x \times x = x^3$
Show division as fractions.	$x \div 3 = \frac{x}{3}$

Key ideas

1. Rules of reading and writing algebra are called algebraic
2. Using algebraic conventions make mathematical expressions,
so communication is more efficient.

1 Simplify these expressions by writing them without multiplication signs.

a $4 \times m$ $4m$	b $6 \times t$ $6t$	c $3 \times k$ $3k$	d $10 \times z$ $10z$
e $p \times 7$ $7p$	f $m \times 5$ $5m$	g $k \times 8$ $8k$	h $n \times 2$ $2n$
i $w \times z$ wz	j $c \times v$ cv	k $p \times q$ pq	l $a \times b$ ab
m $t \times t$ t^2	n $q \times q$ q^2	o $-1 \times s$ $-s$	p $a \times b \times c$ abc
q $2 \times s \times t$ $2st$	r $6 \times q \times p$ $6qp$	s $8 \times p \times q \times r$ $8pqr$	t $a \times a \times c$ a^2c
u $2 \times v \times v$ $2v^2$	v $5 \times w \times w$ $5w^2$	w $p \times p \times 8$ $8p^2$	x $p \times 8 \times p$ $8p^2$

2 Show the meaning of each algebraic expression by inserting multiplication signs.

a $9w$ $9 \times w$	b $5g$ $5 \times g$	c $52m$ $52 \times m$
d tz $t \times z$	e v^2 $v \times v$	f $3pq$ $3 \times p \times q$
g $8ab$ $8 \times a \times b$	h b^2 $b \times b$	i $12mn$ $12 \times m \times n$
j krz $k \times r \times z$	k $3abc$ $3 \times a \times b \times c$	l $35hs$ $35 \times h \times s$
m $-y$ $-1 \times y$	n $-4xy$ $-4 \times x \times y$	o $-x^3$ $-1 \times x \times x \times x$

3 Simplify these expressions by writing them as fractions.

a $y \div 6$	b $8 \div k$	c $6 \div w$
$\frac{y}{6}$	$\frac{8}{k}$	$\frac{6}{w}$
d $2 \div s$	e $9 \div t$	f $ab \div 3$
$\frac{2}{s}$	$\frac{9}{t}$	$\frac{ab}{3}$
g $pq \div 7$	h $3t \div 4$	i $7y \div 3$
$\frac{pq}{7}$	$\frac{3t}{4}$	$\frac{7y}{3}$
j $4k \div 9$	k $7tv \div 2$	l $12xy \div z$
$\frac{4k}{9}$	$\frac{7tv}{2}$	$\frac{12xy}{z}$
m $(z + 3) \div 2$	n $(3 + x) \div -4$	o $-11 \div (x + 1)$
$\frac{z + 3}{2}$	$\frac{3 + x}{-4}$	$\frac{-11}{x + 1}$

4 Show the meaning of each expression by rewriting them with division signs.

a $\frac{3}{10}$	b $\frac{m}{6}$	c $\frac{8}{n}$
$3 \div 10$	$m \div 6$	$8 \div n$
d $\frac{h}{3}$	e $\frac{z}{2}$	f $\frac{2}{p}$
$h \div 3$	$z \div 2$	$2 \div p$
g $\frac{pq}{2}$	h $\frac{km}{5}$	i $\frac{4}{5t}$
$pq \div 2$	$km \div 5$	$4 \div 5t$

5 Simplify these expressions using algebraic conventions.

a $4 \times m$ $4m$	b $6 \times t$ $6t$	c $m \times 5$ $5m$
d $x \times w$ xw	e $5 \times w \times w$ $5w^2$	f $4q \times (n + 5)$ $4q(n + 5)$
g $a \times 3 \times b$ $3ab$	h $h \div 3$ $\frac{h}{3}$	i $3 \div h$ $\frac{3}{h}$
j $a \times b \div 3$ $\frac{ab}{3}$	k $7 \div (3 + y)$ $\frac{7}{3 + y}$	l $(a + 3) \div 4$ $\frac{a + 3}{4}$
m $a \times b \times a \times a$ a^3b	n $c \times c \div 4$ $\frac{c^2}{4}$	o $(a - 4) \div (b + 9)$ $\frac{a - 4}{b + 9}$

6 Write the operations to show the difference between these three expressions.

a $2a$ $2 \times a$	b $\frac{2}{a}$ $2 \div a$	c $\frac{a}{2}$ $a \div 2$
d a^2 $a \times a$	e $2ab$ $2 \times a \times b$	f $(ab)^2$ $ab \times ab$

7 Show the meaning of each expression by writing it out in full.

a $9w$ $9 \times w$	b $48z$ $48 \times z$	c tz $t \times z$
d $3abc$ $3 \times a \times b \times c$	e $2w - 9$ $2 \times w - 9$	f $4(k - 9m)$ $4 \times (k - 9 \times m)$
g d^2 $d \times d$	h $3x^4$ $3 \times x \times x \times x \times x$	i $3ac^2d$ $3 \times a \times c \times c \times d$
j $\frac{1}{9}$ $1 \div 9$	k $\frac{h}{7}$ $h \div 7$	l $\frac{2a}{p}$ $2 \times a \div p$

PARTS OF AN EXPRESSION

TERMS

- A **term** is part of an expression.
- Terms are **separated by addition**.
- Terms can be pronumerals, numbers, or a combination.

Example Identify terms of an expression.

$3a + b + 13c$ The terms are: $3a, b, 13c$	What is meant by “term” of an expression? <i>A term is a of an expression, separated by</i>
$3xy + y - 12$ $= 3xy + y + (-12)$ The terms are: $3xy, y, (-12)$	Why is the negative symbol part of the term? <i>Terms are separated by, so we treat subtraction as the negative</i>

Expression	Terms	Expression	Terms
a $3x + 9$	$3x, 9$	g $9 - 3x - 4y$	$9, -3x, -4y$
b $3x - 9$	$3x, -9$	h $9 - 3a - 4b$	$9, -3a, -4b$
c $3x + 4y - 9$	$3x, 4y, -9$	i $3a^2$	$3a^2$
d $3x - 4y - 9$	$3x, -4y, -9$	j $3a^2 - 4$	$3a^2, -4$
e $3x - 4y + 9$	$3x, -4y, 9$	k $3a^2 - 4b + 9$	$3a^2, -4b, 9$
f $-3x - 4y + 9$	$-3x, -4y, 9$	l $3a^2 - 4a + 9$	$3a^2, -4a, 9$

Key ideas

1. Terms are parts of an algebraic expression that are separated by 2. In algebra, treat subtraction as adding the 3. In the expression $3x - 7$, the terms are: and

COEFFICIENTS

- A **coefficient** is the number multiplying a pronumeral.

Example Identify coefficients.

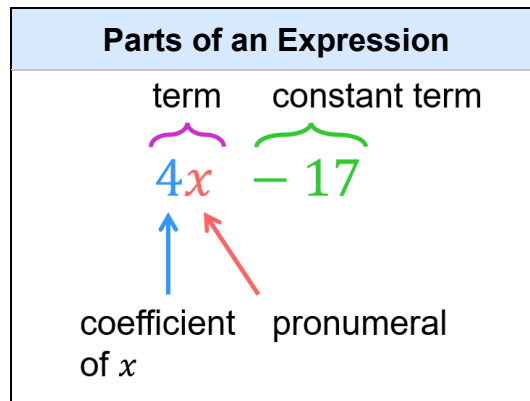
Identify the coefficient of x			
a	$3x$	3	
b	$5x$		
c	$13x$		
d	$2481x$		
e	$0.5x$		
f	$-123x$		
g	$-827.5x$		
h	x		
i	$-x$		
j	$2x + y$		
k	$y - 2x$		
l	$18 - 9x$		
m	y		
n	$2y$		
o	$\frac{1}{2}x$		
p	$\frac{x}{2}$		
q	$\frac{x}{5}$		
r	$\frac{3}{7}x$		
s	$\frac{3x}{7}$		
t	$\frac{9x}{2}$		
u	$1 - \frac{9x}{2}$		

CONSTANT TERM

- Constant terms** are numbers without variables.

Example Identify constant terms.

Identify the constant term.			
a	$3x + 5$	5	
b	$4x + 9$		
c	$12x + 7$		
d	$7 + 12x$		
e	$4x + 0.5$		
f	$4x + \frac{1}{2}$		
g	$4x - 1$		
h	$4x - 48$		
i	$y + 3x + 12$		
j	$2y - 3x + 4$		
k	$2xy - 9$		
l	$9y$		
m	$2x - 4y$		



Example Identify the parts of an algebraic expression.

Example	Your Turn
Identify the following for the expression: $2x - 4y - 9$	Identify the following for the expression: $3x - 6y + 14$
Terms: $2x, -4y, -9$	Terms:
Pronumerals: x, y	Pronumerals:
Coefficient of x : 2	Coefficient of x :
Coefficient of y : -4	Coefficient of y :
Constant term: -9	Constant term:

a $-2a + 4b + 9$

Terms:

$-2a, 4b, 9$

Pronumerals:

a, b

Coefficient of a :

-2

Coefficient of b :

4

Constant term:

9

b $17u + \frac{b}{2} - 12$

Terms:

$17u, \frac{b}{2}, -12$

Pronumerals:

u, b

Coefficient of u :

17

Coefficient of b :

$\frac{1}{2}$

Constant term:

-12

Key ideas

1. A is a symbol or letter that represents a number.
2. A is a pronumeral that represents a value that can
3. A term is a part of an expression; terms are separated by
4. A constant term is a term that has no
5. The coefficient is the number that a pronumeral is by.

1 List all the terms in these expressions.

a $3x + 2y$

$3x, 2y$

b $4a + 2b + c$

$4a, 2b, c$

c $5a + 3b + 2$

$5a, 3b, 2$

d $2x + 5$

$2x, 5$

2 Identify the constant terms in these expressions.

a $7a + 5$

5

b $2x + 3y + 6$

6

c $8 + 3x + 2y$

8

d $5a + 1 + 3b$

1

3 For each of the following expressions, state the coefficient of n .

a $17n + 24$

17

b $31 - 27a + 15n$

15

c $15w + 21n + 1$

21

d $15a + 2n + xy$

2

e $n + 51$

1

f $21 - n$

-1

g $0.5 - 8n$

-8

h $5 - \frac{3n}{4}$

$-\frac{3}{4}$

i

4 The expression $4x + 3y + 24z + 7$ has four terms.

a List all the terms.

$4x, 3y, 24z, 7$

b Identify the constant term.

7

c Identify the coefficient of x .

4

d Which pronumeral has coefficient 24?

z

5 Consider the expression $5a + 3b - 7c + 12$

a List all the terms.

$5a, 3b, -7c, 12$

b Identify the constant term.

12

c Identify the coefficient of c .

-7

6 Consider expression $3a + b + 13c$

a List all the terms.

$3a, b, 13c$

b Identify the constant term.

0

c Identify the coefficient of b .

1

7

Expression	Pronumerals	Coefficient of a	Constant Term
$3a + 9$	a	3	9
$3a - 9$	a	3	-9
$3b - 9$	b	3	-9
$3b$	b	3	0
$3a$	a	3	0
-9	None	0	-9
$2a + 3b - 9$	a, b	2	-9
$2a - 3b - 9$	a, b	2	-9
$2a - 3b + 9$	a, b	2	9
$-2a - 3b + 9$	a, b	-2	9
$9 - 2a - 3b$	a, b	-2	9
$a - 3b + 2$	a, b	1	2
$\frac{a}{3} - 3b + 2$	a, b	$\frac{1}{3}$	2
$\frac{2a}{3} - 3b + 2$	a, b	$\frac{2}{3}$	2
$-\frac{4a}{5} + 3b + 15$	a, b	$-\frac{4}{5}$	15
$3b - \frac{3a}{7} - 14$	a, b	$-\frac{3}{7}$	-14
$3a^2 + 4a + 1$	a	4	1
$3a^2 - 4a$	a	-4	0

CHECK YOUR UNDERSTANDING

Algebraic Conventions

1	What is $\frac{x}{3}$?	a	$x \times 3$	b	$3 \times x$	c	$3 \div x$	d	$x \div 3$
2	What is $\frac{8}{p}$?	a	$8 \div p$	b	$8 - p$	c	$p \div 8$	d	$8 \times p$
3	What is n^2 ?	a	$n + 2$	b	$n \times n$	c	$n \times 2$	d	$2 \times n$
4	What is $6 \times h$?	a	$6h$	b	h^6	c	$h \times 6$	d	$h6$
5	What is $h \times 8$?	a	$h8$	b	$h + 8$	c	h^8	d	$8h$
6	What is $p + p + p + p$	a	p^4	b	$\frac{p}{4}$	c	$4p$	d	$p + 4$
7	What is $y + y + y + y + y + y$	a	$\frac{y}{6}$	b	$6y$	c	$y + 6$	d	y^6
8	Which of these is the same as $2ab$	a	$2 \times a \times b$	b	$2a - b$	c	$2a \times 2b$	d	$2a + b$