Student Name

Mathematics Stage 4

FRACTIONS DECIMALS PERCENTAGES

Book 5 Examine the concept of irrational numbers

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CONTENTS

OVERVIEW

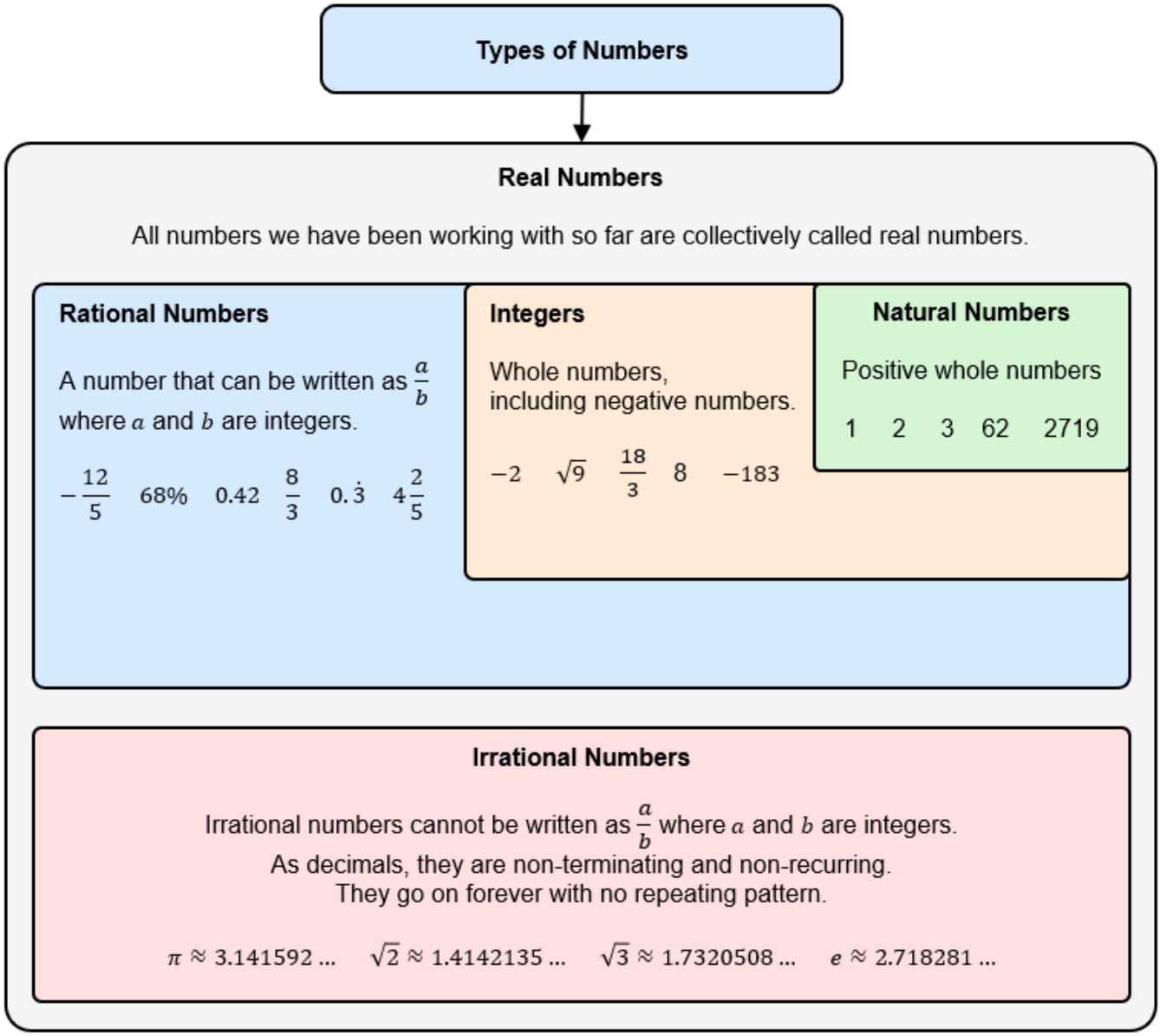
SYLLABUS CONTENT

MA4-FRC-C-01 represents and operates with fractions, decimals and percentages to solve problems

Examine the concept of irrational numbers

- Identify and define irrational numbers as numbers that cannot be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$
- Find approximations of irrational numbers using digital tools
- Locate the approximate position of irrational numbers on a number line

SUMMARY



IRRATIONAL NUMBERS

RATIONAL NUMBERS

- **Rational numbers** are a way to represent values between integers.
- A rational number can be written as a ratio (or fraction):

$$\frac{a}{b}$$

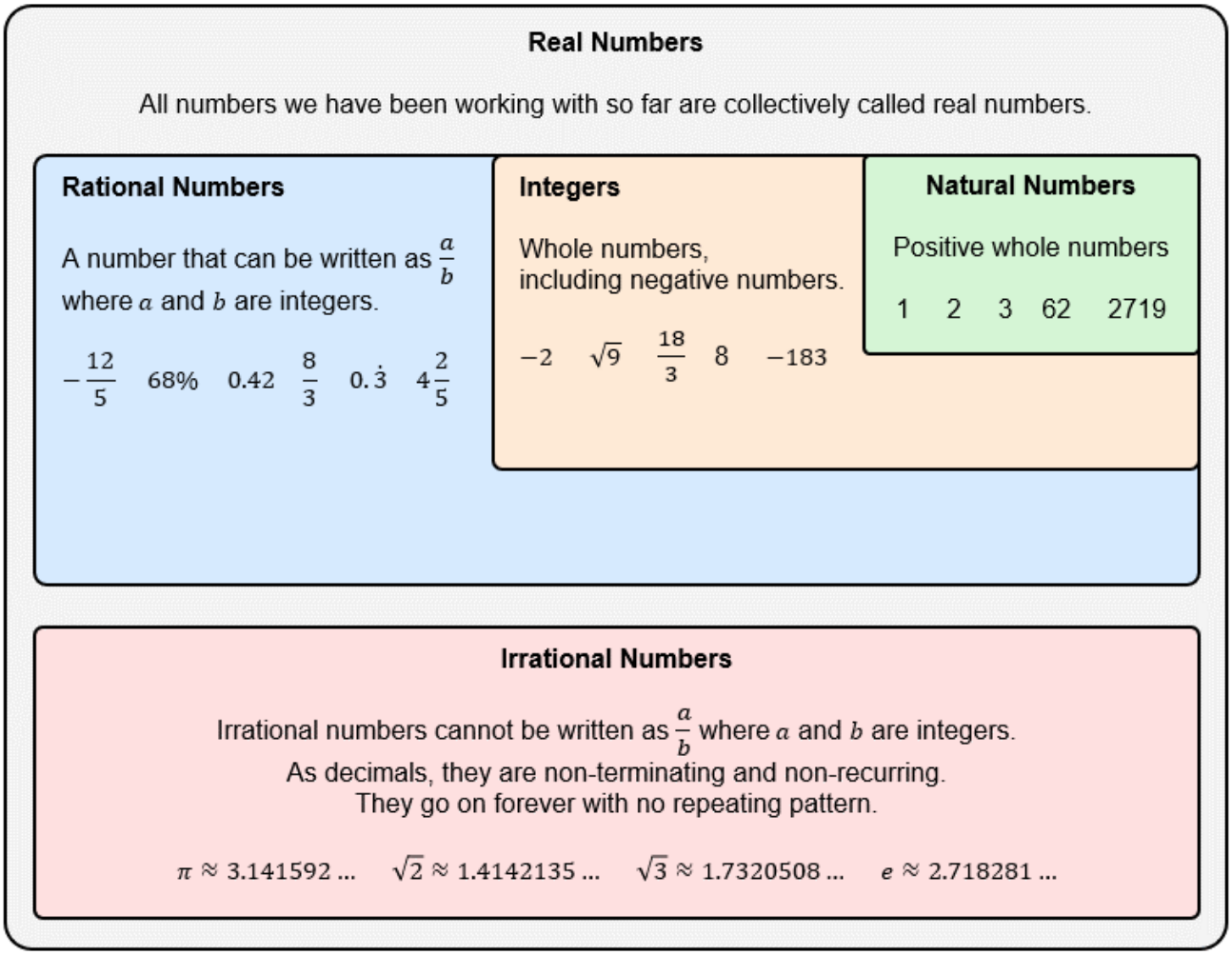
$$\text{where } a \text{ and } b \text{ are integers, and } b \neq 0$$

- Rational numbers include:

Fractions	Integers	Percentages	Terminating Decimals	Recurring Decimals
$\frac{2}{3}$	$2 \rightarrow \frac{2}{1}$	$47\% \rightarrow \frac{47}{100}$	$0.125 \rightarrow \frac{125}{1000}$	$0.333333 \rightarrow \frac{1}{3}$

IRRATIONAL NUMBERS

- **Irrational numbers** are **non-terminating non-recurring** decimals, such as π and $\sqrt{2}$.
- All numbers are collectively known as **real numbers**.



Example Classify numbers as rational or irrational.

Using a calculator, determine whether these numbers are irrational or rational.

a	$\frac{1}{3}$	Rational	Irrational	**i**	$\sqrt{4}$	Rational	Irrational
b	3	Rational	Irrational	**j**	$\sqrt{5}$	Rational	Irrational
c	0.3	Rational	Irrational	**k**	$\sqrt{5}+1$	Rational	Irrational
d	$0.\dot{3}$	Rational	Irrational	**l**	$\frac{\sqrt{2}}{2}$	Rational	Irrational
e	$\sqrt{3}$	Rational	Irrational	**m**	$2\sqrt{2}$	Rational	Irrational
f	$\sqrt{9}$	Rational	Irrational	**n**	π	Rational	Irrational
g	0.25	Rational	Irrational	**o**	$-\frac{22.5}{7}$	Rational	Irrational
h	25%	Rational	Irrational	**p**	$3.\overline{14}$	Rational	Irrational

Key ideas

1. Rational numbers can be written as a fraction $\frac{a}{b}$, where a and b are …… and b …… 0.
2. If a number can be written as a terminating or …… decimal, then it is rational.
3. Irrational numbers cannot be written as a fraction with integers, when written as a decimal, they are non-…… and non-……

Example Approximate irrational numbers.

Approximate $\sqrt{2}$ to 2 decimal places. $\sqrt{2} = 1.414213562\ldots$ ≈ 1.41	When rounding to 2 decimal places, which is the critical digit that we look at? *When rounding to 2 decimal places, the critical digit is the …… decimal place.*
Approximate e to 2 decimal places. $e = 2.718281828\ldots$ ≈ 2.72	How do we know to round up in this example? *The critical digit is ……, which is more than ……, so we round up.*

Approximate these numbers to 2 decimal places.

a $\sqrt{3}$ 1.73

b π 3.14

c $\sqrt{25}$ 5.00

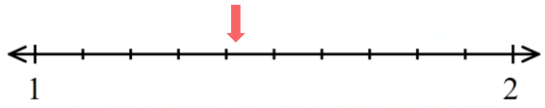
d $\sqrt{18}$ 4.24

e $\frac{1}{3}$ 0.33

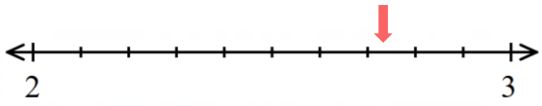
f $2\sqrt{2}$ 2.83

Example Locate the approximate position of irrational numbers on a number line.

$\sqrt{2} \approx 1.41$

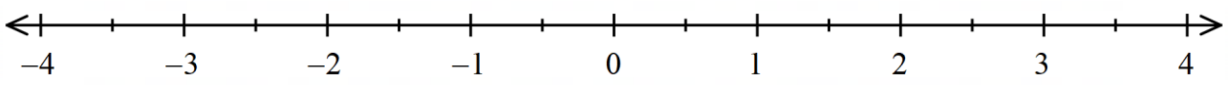


$e \approx 2.72$



Using a calculator, locate these numbers on the number line.

$\sqrt{3}$ $\sqrt[3]{7}$ $-\pi$



1 Circle which of these numbers are integers.

10 9.6 $\frac{3}{4}$ −3 117

10, −3, 117

Explain why all integers are rational numbers.

Integers can be written with a denominator of 1.

2 Circle which of these numbers are rational.

$\frac{7}{2}$ $\sqrt{3}$ 1.5 11

$\frac{7}{2}$, 1.5, 11

Explain how you know whether a number is rational.

The number can be written as a fraction with whole number numerator and denominator.

3 Explain why $\sqrt{4}$ is a rational number.

$\sqrt{4} = 2$, which is an integer, so it's rational.

4 Look at these numbers.

$\sqrt{2}$ $\sqrt{3}$ $\sqrt{4}$ $\sqrt{5}$ $3\sqrt{7}$ $2\sqrt{100}$ $-\sqrt{3}$ $-\sqrt{25}$

a Which numbers are irrational?

$\sqrt{3}, \sqrt{5}, 3\sqrt{7}, -\sqrt{3}$

b Which numbers are rational?

$\sqrt{4}, 2\sqrt{100}, -\sqrt{25}$

5 Emily says:

> 0.7777... recurring is irrational as you cannot write it as a fraction.

Explain why she is incorrect.

0.777… can be written as $\frac{7}{9}$, so it's rational.

6 Darius says:

π is a rational number, since it is roughly equal to $\frac{22}{7}$

Explain why Darius is incorrect.

π is roughly equal to $\frac{22}{7}$ but not exactly. π is a non-terminating, non-recurring decimal, so it is irrational.

7 Becky says:

$\sqrt{25}$ is not rational, because it is a square root.

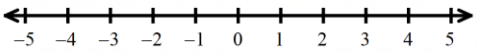
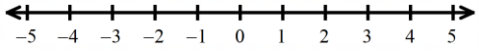
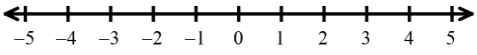
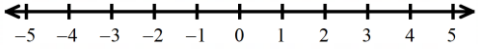
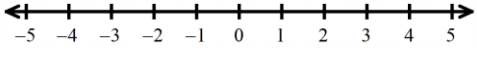
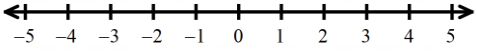
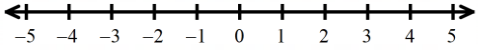
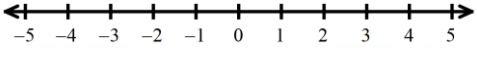
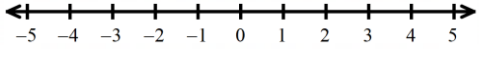
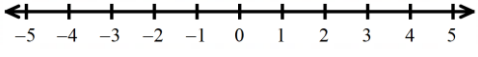
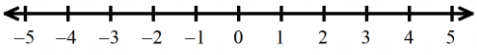
Explain why Becky is incorrect.

$\sqrt{25}$ simplifies to 5, which is rational.

8 Using a calculator, approximate these numbers to 2 decimal places.

a $\sqrt{5}$ 2.24	**b** $2\sqrt{5}$ 4.47	**c** 1.61803399 1.62
d $\frac{22}{7}$ 3.14	**e** $\frac{\pi}{2}$ 1.57	**f** e^2 7.39

9 Complete the table.

Number	Rational/ Irrational?	Approximation (2 d.p.)	Approximation (3 d.p.)	Approximate location on number line
e	Irrational	≈ 2.72	≈ 2.718	
$\sqrt{6}$	Irrational	2.45	2.449	
$2\sqrt{6}$	Irrational	4.90	4.899	
$\sqrt{24}$	Irrational	4.90	4.899	
$\sqrt{13}$	Irrational	3.61	3.606	
$\sqrt{16}$	Rational	4.00	4.000	
π	Irrational	3.14	3.142	
3π	Irrational	9.42	9.425	
$0.\dot{3}$	Rational	0.33	0.333	
$\frac{7}{3}$	Rational	2.33	2.333	
-5	Rational	-5.00	-5.000	
$-\sqrt{3}$	Irrational	-1.73	-1.732	

CHECK YOUR UNDERSTANDING

Check your understanding

1 Which of these two numbers are rational?

7 and 7.27629845

a 7 **b** 7.27629845 **c** Neither **d** Both

2 Which of these two numbers are rational?

π and 3.142

a π **b** 3.142 **c** Neither **d** Both

3 Which of these two numbers are rational?

3.5 and -3.5

a 3.5 **b** -3.5 **c** Neither **d** Both

4 Which of these two numbers are rational?

$\frac{1}{3}$ and $\frac{4}{1}$

a $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** Neither **d** Both

5 Which of these two numbers are rational?

$\sqrt{36}$ and $\sqrt{12}$

a $\sqrt{36}$ **b** $\sqrt{12}$ **c** Neither **d** Both