

Student Name

Mathematics Stage 4

# RIGHT-ANGLED TRIANGLES

**Book 2**      Examine problems involving  
Pythagoras' theorem

Version: 260207

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# OVERVIEW

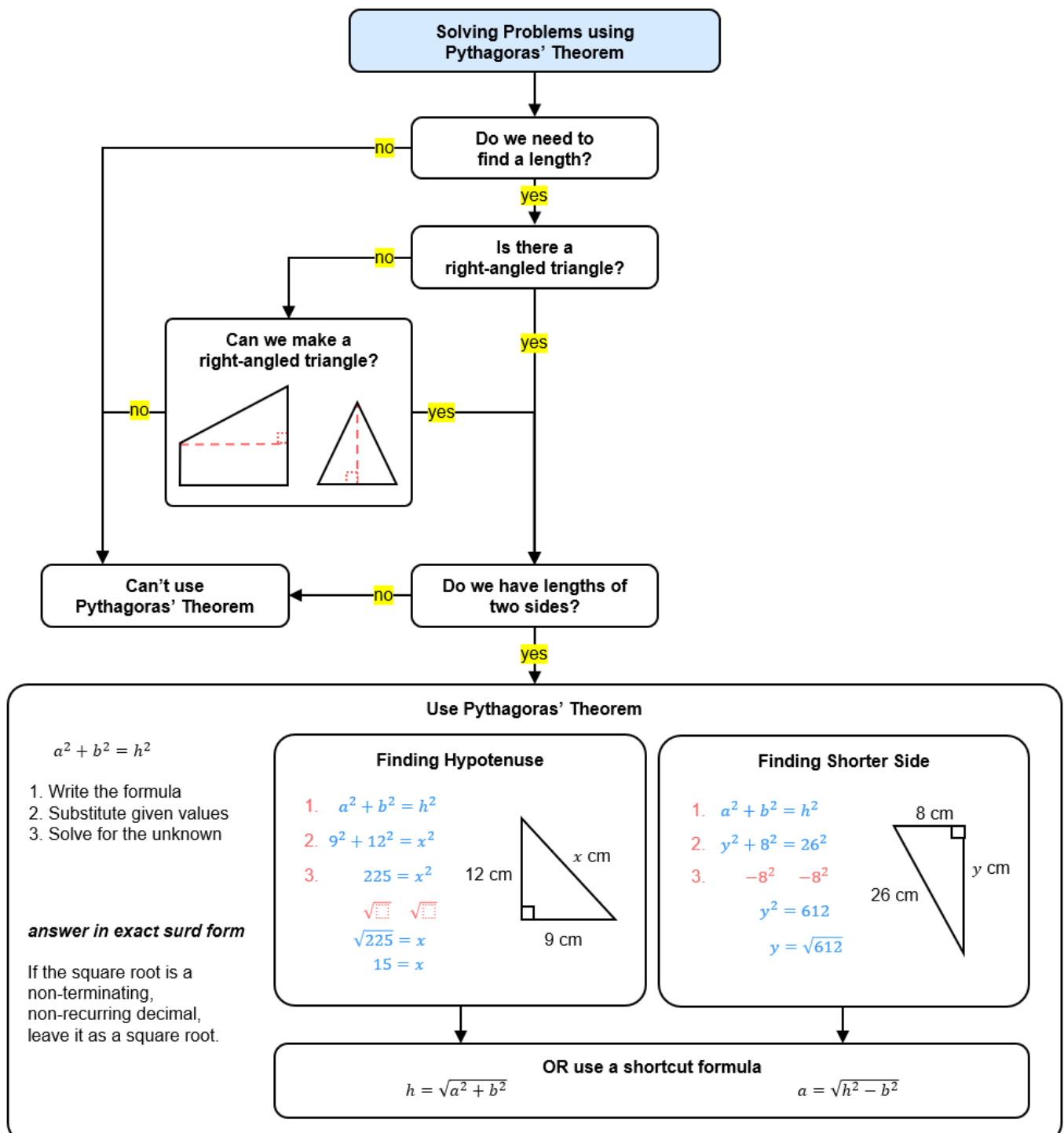
## SYLLABUS CONTENT

**MA4-PYT-C-01** applies Pythagoras' theorem to solve problems in various contexts

### Examine problems involving Pythagoras' theorem

- Apply Pythagoras' theorem to find the unknown length of a side in a right-angled triangle, giving answers in an exact form or as decimal approximations
- Apply the converse of Pythagoras' theorem to establish whether a triangle is right angled
- Solve practical problems involving Pythagoras' theorem before exploring a variety of related problems

## SUMMARY



## REVIEW OF PRIOR KNOWLEDGE

1 Square root these numbers.

a  $\sqrt{16}$

b  $\sqrt{49}$

c  $\sqrt{1600}$

d  $\sqrt{1.44}$

2 Round these numbers to 2 decimal places.

a 32.2582

b 1.2746

c 8.2811

d 272.2152

3 Round these roots to two decimal places using a calculator.

a  $\sqrt{10}$

b  $\sqrt{26}$

c  $\sqrt{65}$

d  $\sqrt{230}$

4 Evaluate these expressions. Round to 2 d.p. if appropriate.

a  $\sqrt{9}$

b  $\sqrt{11}$

c  $\sqrt{20}$

d  $\sqrt{121}$

e  $\sqrt{16 + 9}$

f  $\sqrt{10^2 + 44}$

g  $\sqrt{7^2 + 4^2}$

h  $\sqrt{20^2 - 10^2}$

5 A **surd** is a root that makes a non-terminating, non-recurring decimal.  
Which of the following are surds?

a  $\sqrt{3} = 1.732050808 \dots$

b  $\sqrt{25} = 5$

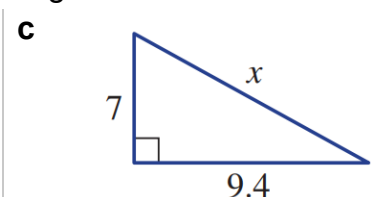
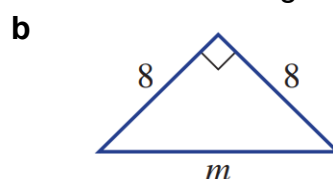
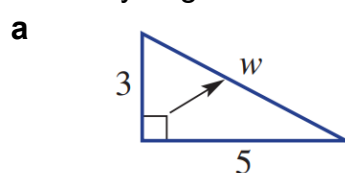
c  $\sqrt{37} = 6.08276253 \dots$

d  $\sqrt{1.44} = 1.2$

e  $\sqrt{169} = 13$

f  $\sqrt{11} = 3.31662479 \dots$

6 Write Pythagoras' theorem  $a^2 + b^2 = h^2$  for these right-angled triangles.



**7** Solve these equations.

**a**  $x^2 = 16$

**b**  $x^2 = 2$

**c**  $x^2 = 39 + 10$

**8** Solve these equations.

**a**  $x^2 + 5 = 30$

**b**  $8 + x^2 = 24$

**c**  $x^2 + 18 = 31$

**d**  $9^2 + x^2 = 128$

**e**  $x^2 + 3^2 = 5^2$

**f**  $x^2 + 8^2 = 12^2$

**9** Square roots that do not make a nice terminating decimal should be left as a root. This is called leaving in exact form or surd form.

$\sqrt{225}$  should be simplified to 15. Leave  $\sqrt{146} \approx 12.08304597 \dots$  as is.

Decide whether these roots can be written as a whole number or should be left as a root.

**a**  $\sqrt{9}$

**b**  $\sqrt{11}$

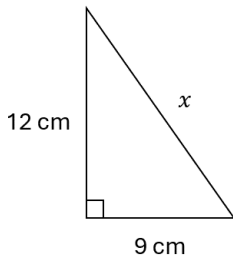
**c**  $\sqrt{121}$

**d**  $\sqrt{20}$

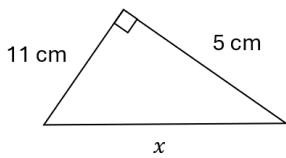
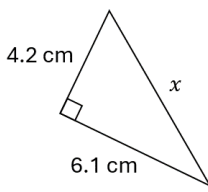
# CALCULATING THE LENGTH OF THE HYPOTENUSE

- If you have a **right-angled triangle** and the **lengths of two sides**, then you can use **Pythagoras' theorem** to calculate the length of the remaining side.
  - Write the formula  $a^2 + b^2 = h^2$
  - Substitute given values.
  - Solve for the unknown by applying inverse operations to both sides.

**Example** Calculate the length of the hypotenuse.

|                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Calculate the value of <math>x</math></p>  <ol style="list-style-type: none"> <li><math>a^2 + b^2 = h^2</math></li> <li><math>9^2 + 12^2 = x^2</math></li> <li><math>225 = x^2</math></li> </ol> $\sqrt{\quad} \quad \sqrt{\quad}$ $\sqrt{225} = x$ $15 = x$ | <p>Why can we use Pythagoras' Theorem?</p> <p><i>We can use Pythagoras' Theorem when there is a ..... - ..... triangle.</i></p> <p>Which operation did we do to go from <math>h^2 = 225</math> to <math>h = \sqrt{225}</math>?</p> <p><i>We ..... both sides.</i></p> <p>Why do we not consider the negative solution?</p> <p><i>It is not possible to have a negative .....</i></p> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Find the length of the hypotenuse. Round your answer to 2 d.p.

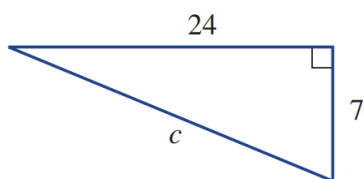
|                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>a</b></p>  <ol style="list-style-type: none"> <li><math>a^2 + b^2 = h^2</math></li> <li><math>\dots^2 + \dots^2 = x^2</math></li> <li><math>\dots + \dots = x^2</math></li> </ol> $\dots = x^2$ $\sqrt{\quad} = x$ $\dots = x$ <p style="text-align: right;"><math>x = 12.08 \text{ cm}</math></p> | <p><b>b</b></p>  <ol style="list-style-type: none"> <li><math>a^2 + b^2 = h^2</math></li> <li><math>\dots^2 + \dots^2 = x^2</math></li> <li><math>\dots + \dots = x^2</math></li> </ol> $\dots = x^2$ $\sqrt{\quad} = x$ $\dots = x$ <p style="text-align: right;"><math>x = 7.41 \text{ cm}</math></p> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## Key ideas

- If you have a right-angled triangle and the lengths of ..... sides, then you can use Pythagoras' theorem to calculate the length of the remaining side.

- 1 Find the length of the hypotenuse for these triangles.

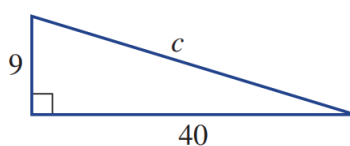
a



$$\begin{aligned} h^2 &= a^2 + b^2 \\ y^2 &= \dots^2 + \dots^2 \\ y^2 &= \dots + \dots \\ y^2 &= \dots \\ y &= \sqrt{\dots} \\ &= \dots \end{aligned}$$

25

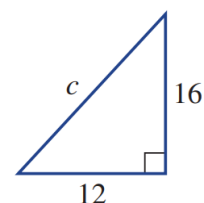
b



$$\begin{aligned} h^2 &= a^2 + b^2 \\ y^2 &= \dots^2 + \dots^2 \\ y^2 &= \dots + \dots \\ y^2 &= \dots \\ y &= \sqrt{\dots} \\ &= \dots \end{aligned}$$

41

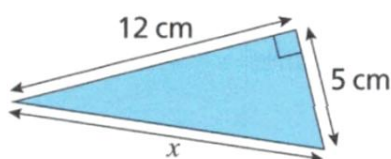
c



$$\begin{aligned} h^2 &= a^2 + b^2 \\ y^2 &= \dots^2 + \dots^2 \\ y^2 &= \dots + \dots \\ y^2 &= \dots \\ y &= \sqrt{\dots} \\ &= \dots \end{aligned}$$

20

- 2 Jayden is finding the length of the hypotenuse in this right-angled triangle. His working out is shown.



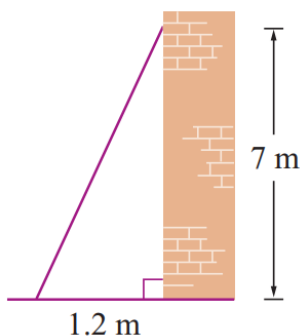
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 12^2 &= x^2 \\ 25 + 144 &= x^2 \\ 169 &= x^2 \\ \text{The length of } x &\text{ is } 169 \text{ cm} \end{aligned}$$



What mistake has he made?

He forgot to square root

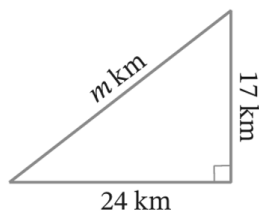
- 3 The foot of a ladder is placed 1.2 m from a vertical wall. If the ladder reaches 7 m up the wall, what is the length of the ladder? Answer in metres to 2 decimal places.



7.10 m

- 4 Find the length of the hypotenuse for these triangles. Leave your answer in exact form.

a



$$h^2 = a^2 + b^2$$

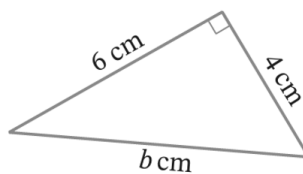
$$\dots^2 = \dots^2 + \dots^2$$

$$\dots^2 = \dots + \dots$$

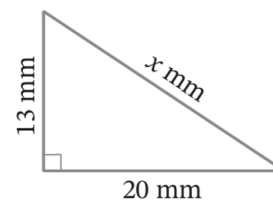
$$\dots^2 = \dots$$

$$\dots = \sqrt{\quad}$$

b



c

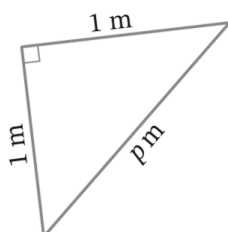


$$\sqrt{865} \text{ km}$$

$$\sqrt{52} \text{ cm}$$

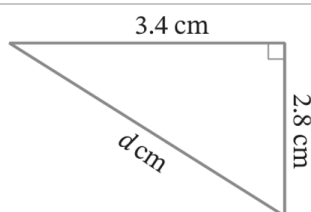
$$\sqrt{569} \text{ mm}$$

d



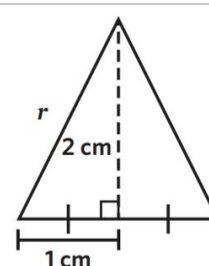
$$\sqrt{2} \text{ m}$$

e



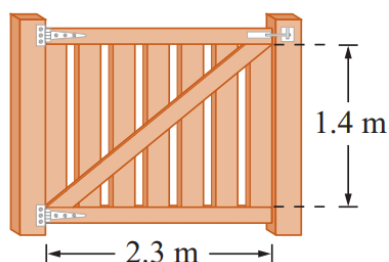
$$\sqrt{19.4} \text{ cm}$$

f



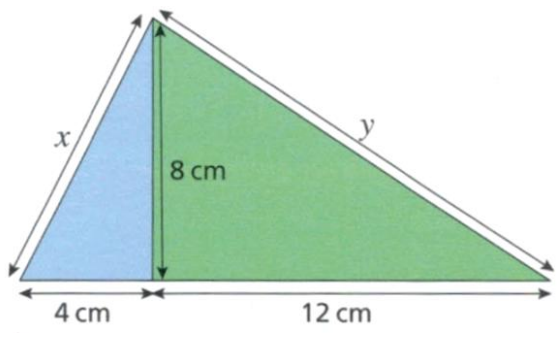
$$\sqrt{5} \text{ cm}$$

- 5 A gate is 2.3 m wide with a height of 1.4 m.  
Calculate the length of the diagonal of the gate correct to 1 decimal place.



$$2.7 \text{ m}$$

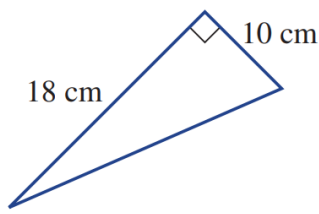
6 This shape is made up of two right-angled triangles. Find the perimeter of this shape to 2.d.p.



39.37 cm

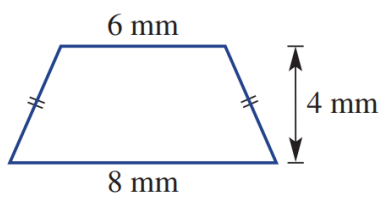
7 Find the perimeter of these shapes, correct to two decimal places.

a



48.59 cm

b

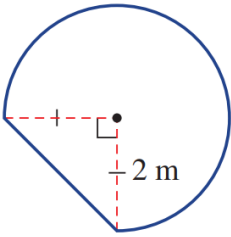


22.25 mm

c

Arc-length formula

$$l = \frac{\theta}{360} \times 2\pi r$$

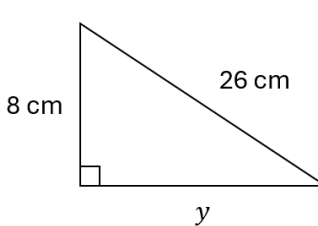


12.25 m

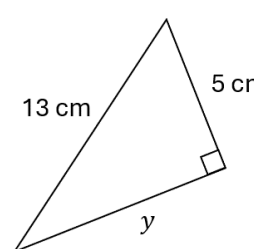
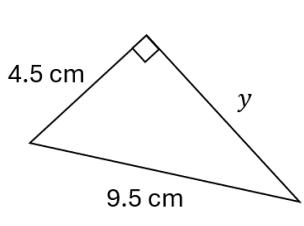
## CALCULATING THE LENGTH OF A SHORTER SIDE

- If you have a **right-angled triangle** and the **lengths of two sides**, then you can use **Pythagoras' theorem** to calculate the length of the remaining side.
  - Write the formula  $a^2 + b^2 = h^2$
  - Substitute given values.
  - Solve for the unknown by applying inverse operations to both sides.

**Example** Calculate the length of a shorter side.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                 |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Calculate the value of <math>x</math>.</p> <div style="display: flex; align-items: center;">  <div> <p>1. <math>a^2 + b^2 = h^2</math></p> <p>2. <math>y^2 + 8^2 = 26^2</math></p> <p>3. <math>-8^2 = -8^2</math></p> <p style="margin-left: 40px;"><math>y^2 = 612</math></p> <p style="margin-left: 40px;"><math>y = \sqrt{612}</math></p> </div> </div> | <p>What operation did we do to both sides to go from <math>y^2 + 8^2 = 26^2</math> to <math>y^2 = 612</math>?</p> <p>We ..... <math>8^2</math> from both sides.</p> <p>What operation did we do to both sides to go from <math>h^2 = 612</math> to <math>h = \sqrt{612}</math>?</p> <p>We ..... both sides.</p> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

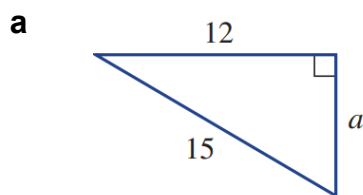
Calculate the value of  $x$ . Round your answer to 1 decimal place.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>a</b></p> <div style="display: flex; align-items: center;">  <div> <p><math>a^2 + b^2 = h^2</math></p> <p><math>y^2 + \dots^2 = \dots^2</math></p> <p><math>y^2 = \dots</math></p> <p><math>y = \sqrt{\dots}</math></p> <p><math>\approx \dots</math></p> <p style="color: blue; margin-top: 10px;"><math>y = 12 \text{ cm}</math></p> </div> </div> | <p><b>b</b></p> <div style="display: flex; align-items: center;">  <div> <p><math>a^2 + b^2 = h^2</math></p> <p><math>y^2 + \dots^2 = \dots^2</math></p> <p><math>y^2 = \dots</math></p> <p><math>y = \sqrt{\dots}</math></p> <p><math>\approx \dots</math></p> <p style="color: blue; margin-top: 10px;"><math>y = 8.4 \text{ cm}</math></p> </div> </div> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

### Key ideas

- If you have a right-angled triangle and the lengths of ..... sides, then you can use Pythagoras' theorem to calculate the length of the remaining side.
- To find a side length in a right-angled triangle:
  - W..... t..... f.....
  - ..... the given values.
  - ..... the equation by applying inverse operations to both sides.

1 Calculate the side lengths of these triangles.



$$a^2 + b^2 = h^2$$

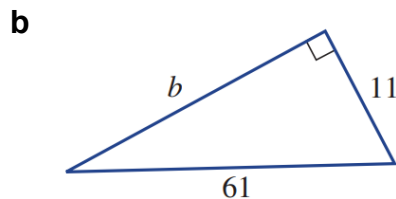
$$a^2 + \dots^2 = \dots^2$$

$$y^2 = \dots$$

$$y = \sqrt{\dots}$$

$$y = \dots$$

9



$$a^2 + b^2 = h^2$$

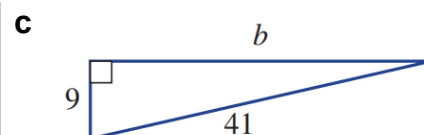
$$\dots^2 + \dots^2 = \dots^2$$

$$y^2 = \dots$$

$$y = \sqrt{\dots}$$

$$y = \dots$$

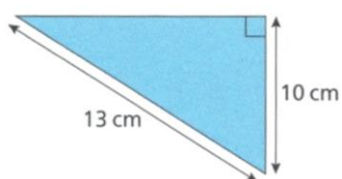
60



$$a^2 + b^2 = h^2$$

40

2 Jayden is finding the missing length in this right-angled triangle. His working out is shown.



$$a^2 + b^2 = h^2$$

$$10^2 + 12^2 = h^2$$

$$269 = h^2$$

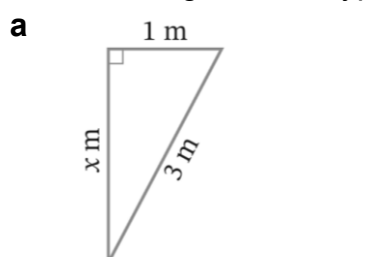
$$16.4 = h$$



What mistake has he made?

He substituted incorrectly. The hypotenuse is 13.

8 Find the length of the hypotenuse for these triangles. Leave your answer in exact form.



$$a^2 + b^2 = h^2$$

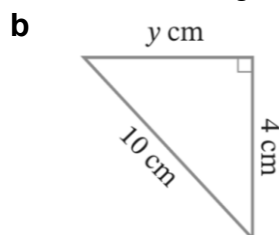
$$\dots^2 + \dots^2 = \dots^2$$

$$y^2 = \dots$$

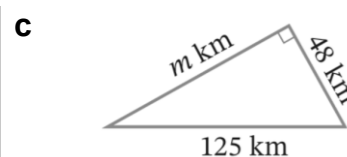
$$y = \sqrt{\dots}$$

$$y = \dots$$

$\sqrt{8}$  m

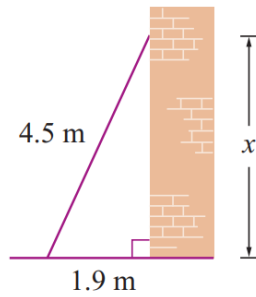


$\sqrt{84}$  cm



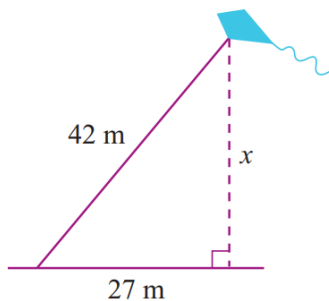
$\sqrt{13321}$  km

- 3 A ladder 4.5 m in length is placed 1.9 m from the base of a vertical wall. How far does the ladder reach up the wall to the nearest centimetre?



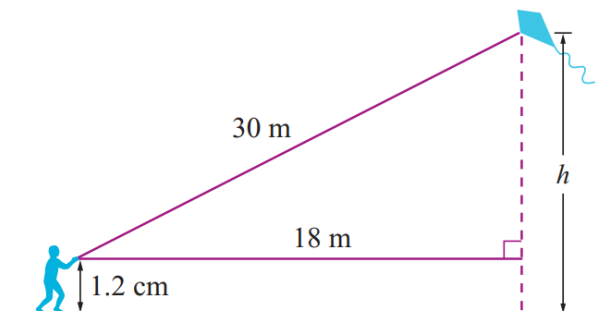
408 cm

- 4 A kite is flying with 42 m of string which is anchored to the ground. The horizontal distance from where the string is anchored is 27 m. Find the height of the kite above the ground to the nearest centimetre.



3217 cm

- 5 Oliver holds the end of the string of a kite 1.2 m above the ground. The string is 30 m long and the horizontal distance to the kite is 18 m. Find the height of the kite above the ground.

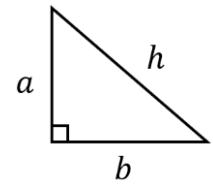


25.2 m

# CHOOSING THE APPROPRIATE STRATEGY

- These are two formulas for using Pythagoras' theorem quickly:

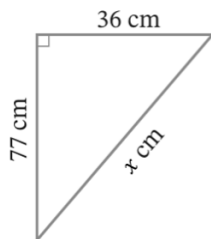
| Pythagoras' Theorem Shortcuts                     |                                                   |
|---------------------------------------------------|---------------------------------------------------|
| Finding the hypotenuse:<br>$h = \sqrt{a^2 + b^2}$ | Finding a shorter side:<br>$a = \sqrt{h^2 - b^2}$ |



## FOUNDATION

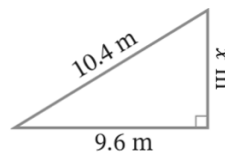
- 1 For the following questions, determine which formula you would use to find the unknown side.

a



- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

b



- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

c



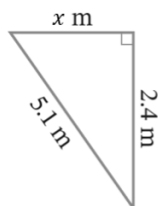
- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

85 cm

4 m

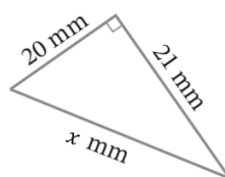
55 mm

d



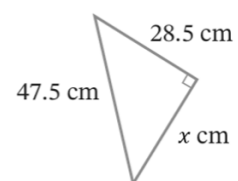
- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

e



- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

f



- ☐  $h = \sqrt{a^2 + b^2}$   
☐  $a = \sqrt{h^2 - b^2}$

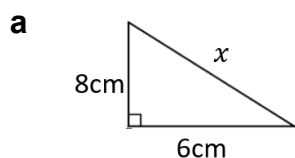
4.5 m

29 mm

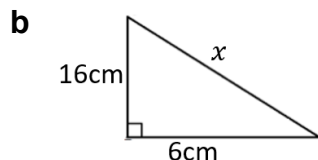
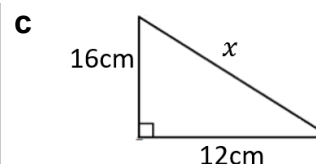
38 cm

- 2 Find  $x$  in the questions above using the chosen formula.

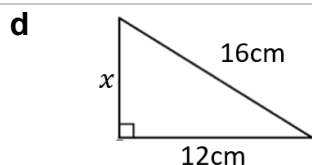
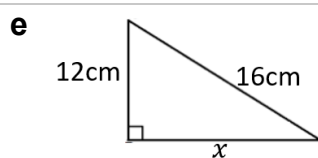
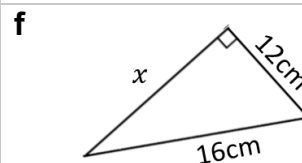
- 3 Calculate the missing side in each of these triangles. Leave your answer in exact form.



10 cm

 $\sqrt{292}$  cm

20 cm

 $\sqrt{112}$  cm $\sqrt{112}$  cm $\sqrt{112}$  cm

- 4 Consider this diagram and the unknown length  $x$ .

**a** Identify what we need to calculate before we can find  $x$ .

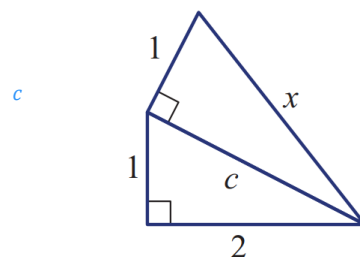
**b** Calculate the value  $c$ , then find  $x$  as an exact value.

$$c = \sqrt{5}, x = \sqrt{6}$$

**c** Was it necessary to calculate the value of  $c$  or was  $c^2$  enough?

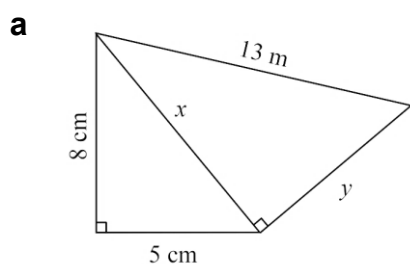
we only needed  $c^2$ 

**d** Why should we **not** round off  $c$  before finding the value of  $x$ ?



If we rounded  $c$  to 2.24, our final answer would be  $1^2 + 2.24^2 = 6.0176$ , which is inaccurate.

- 5 Use Pythagoras' Theorem twice to find  $y$ . Leave  $x$  and the answer in surd form.

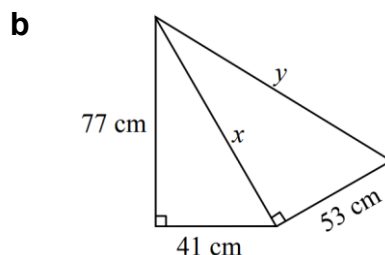


$$x = \sqrt{5^2 + 8^2} = \sqrt{89}$$

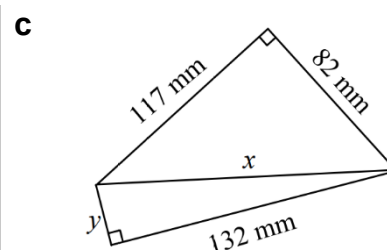
$$y = \sqrt{13^2 - (\sqrt{89})^2}$$

$$y = \dots\dots\dots$$

$$y = \sqrt{80} \text{ m}$$



$$x = \sqrt{7610}, y = \sqrt{10419} \text{ cm}$$

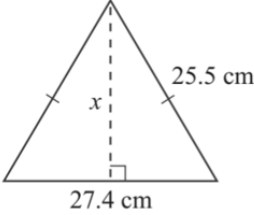
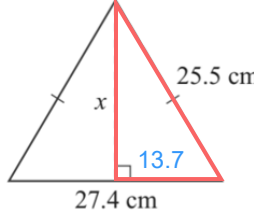
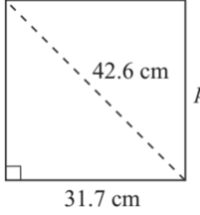
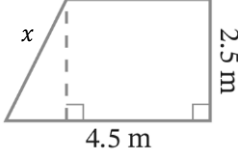
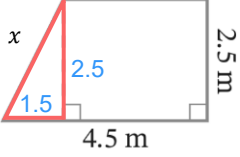
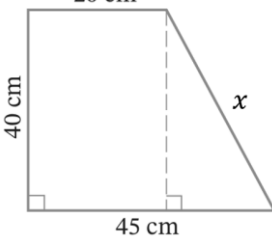


$$x = \sqrt{20413}, y = \sqrt{2989} \text{ mm}$$

# SOLVING PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES

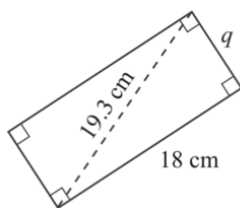
- If a question involves a right-angled triangle and the lengths of two sides is given, then we can use Pythagoras' theorem to find the third side.
  - Identify the right-angled triangle.
  - Solve for the unknown side using the appropriate formula.

**Example** Solve problems involving Pythagoras' Theorem.

| Worked Example                                                                                                                                                                                                                                                                                                 | Your Turn                                                                                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Find the value of the pronumeral, round your answer to 2 d.p.</p> <p>1.</p>   <p>2.</p> $x = \sqrt{25.5^2 - 13.7^2}$ $= 21.51 \text{ cm}$ | <p>Find the value of the pronumeral, round your answer to 2 d.p.</p>  <p><math>p = 28.46 \text{ cm}</math></p> |
| <p>1.</p>   <p>2.</p> $x = \sqrt{1.5^2 + 2.5^2}$ $= 2.92 \text{ m}$                                                                      |  <p><math>x = 47.17 \text{ cm}</math></p>                                                                    |

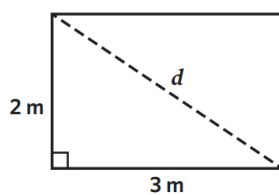
1 Calculate the missing side in these triangles. Round your answer to two decimal places.

a



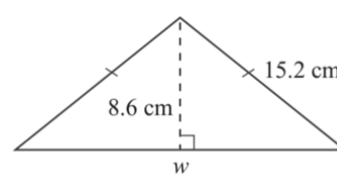
6.96 cm

b



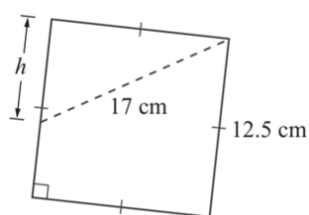
3.61 m

c



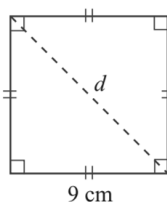
25.07 cm

d



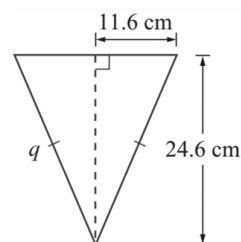
11.52 cm

e



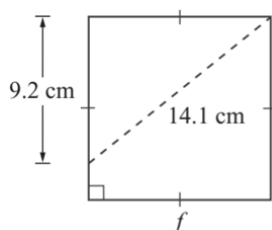
12.73 cm

f



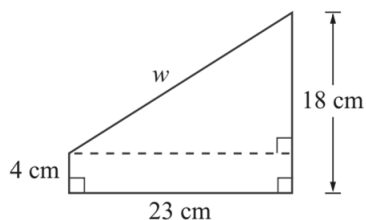
27.20 cm

g



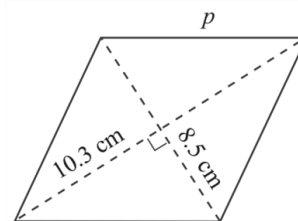
10.69 cm

h



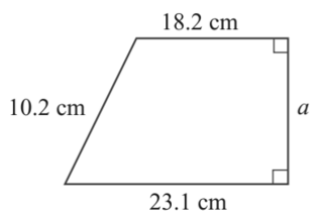
26.93 cm

i



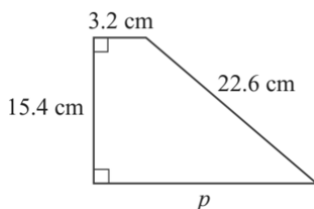
13.35 cm

j



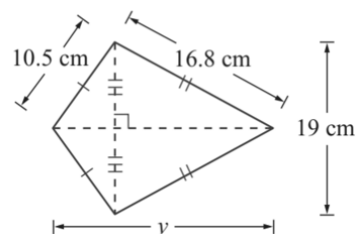
8.95 cm

k



19.74 cm

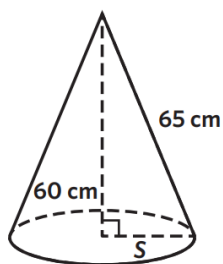
l



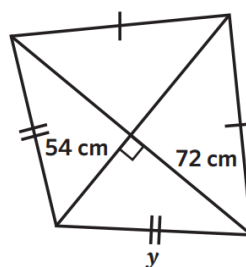
18.33 cm

2 Solve for the unknown value.

a



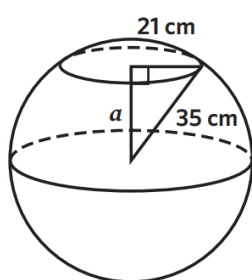
b



90 cm

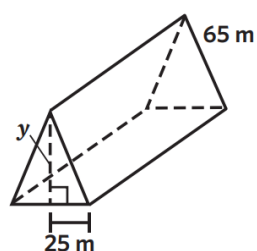
25 cm

c



28 cm

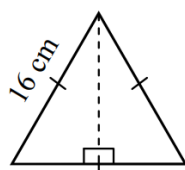
d



60 m

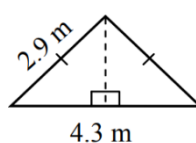
3 Find the height of these triangles. Round your answer to 1 d.p.

a



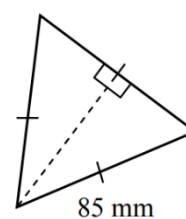
13.9 cm

b



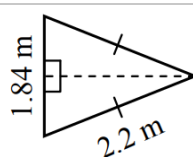
1.9 m

c



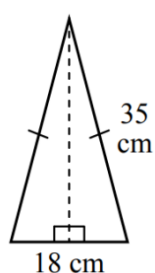
73.6 mm

d



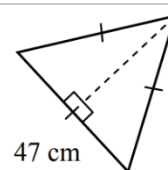
2.0 m

e



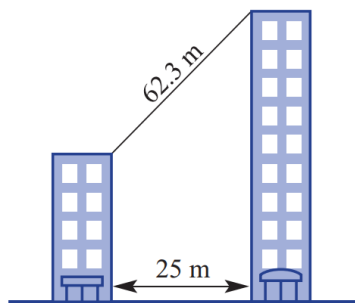
33.8 cm

f



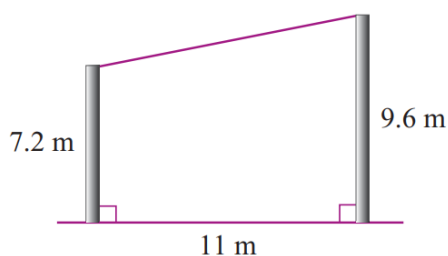
40.7 cm

- 4 Calculate the difference in height between these two buildings.  
Round your answer to 1 d.p.



57.1 m

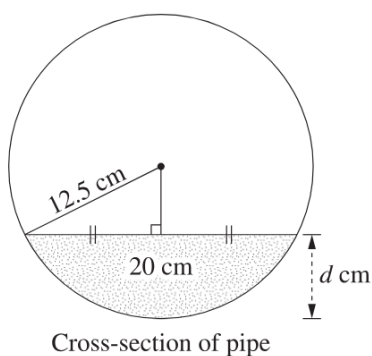
- 5 Two posts are 7.2 m tall and 9.6 m tall and 11 m apart on level ground.  
Calculate the length of string required to stretch from the top of one post to the top of the other.  
Round your answer to 1 d.p.



11.3 m

# 6 **2023 HSC Standard 2 Band 5**

A cylindrical pipe with a radius of 12.5 cm is filled with water to a depth,  $d$  cm, as shown.  
The surface of the water has a width of 20 cm.  
What is the depth of the water in the pipe?

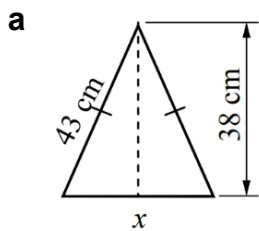


5 cm

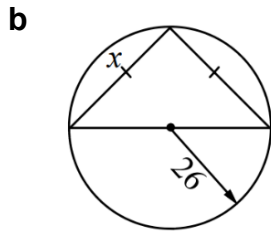
- 7 A 20, 99 and  $x$  form a Pythagorean triad. What value can  $x$  be?

101

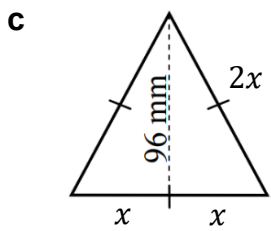
8 Find the side length of these triangles. Round your answer to 1 d.p.



40.2 cm

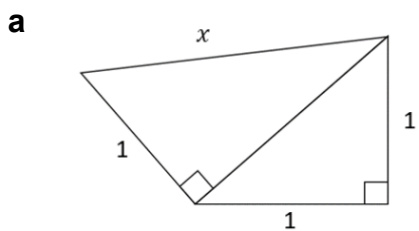


36.8

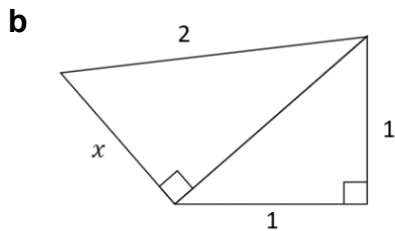


55.4 mm

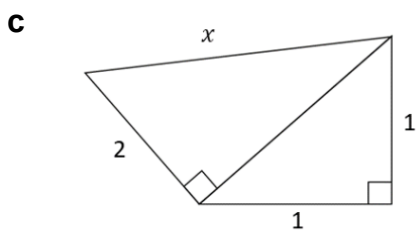
9 Find the value of  $x$ . Answer in exact form.



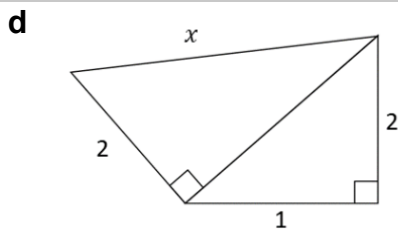
$\sqrt{3}$



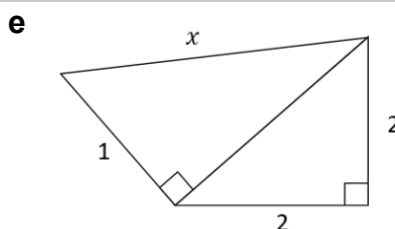
$\sqrt{2}$



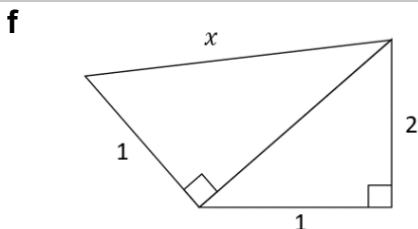
$\sqrt{6}$



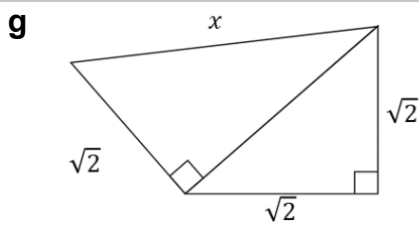
3



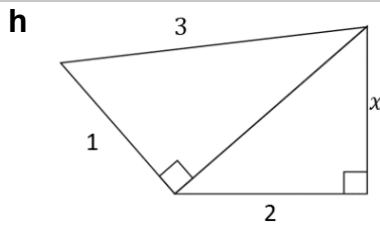
3



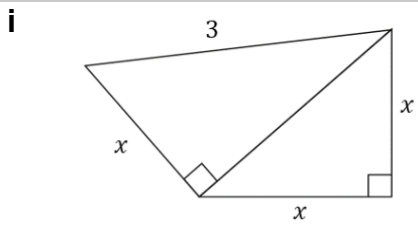
$\sqrt{6}$



$\sqrt{6}$



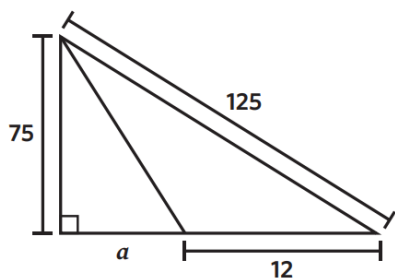
2



$\sqrt{3}$

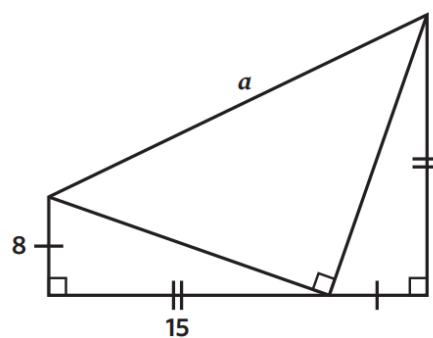
10 Determine the unknown value, give your answer in exact form.

a



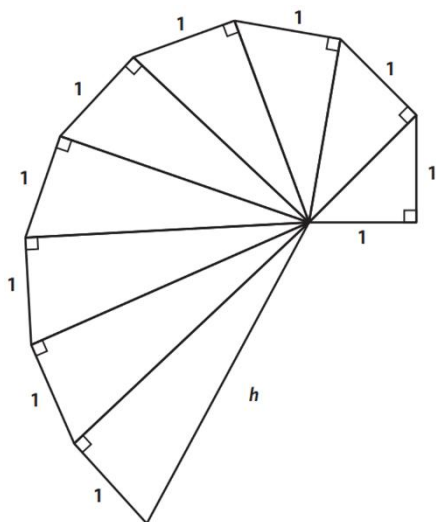
88

b



$\sqrt{578}$  cm

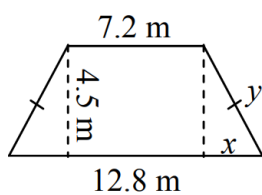
11 Determine the value of  $h$  in exact surd form.



$h = \sqrt{10}$

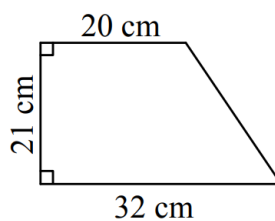
12 Find the missing sides, then find the perimeter. Round your answer to 1 d.p.

a



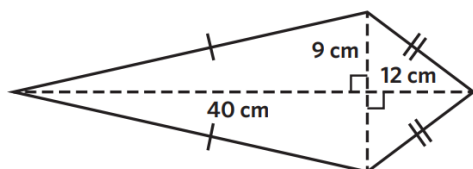
$x = 2.8, y = 4.5, P = 30.6$

b



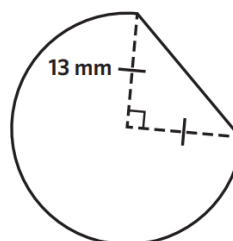
$P = 97.2$  cm

c



$P = 112$  cm

d



$P = 79.6$  cm

- 13** A right-angled triangle has one shorter side 5 m long and its longest side is exactly  $\sqrt{61}$  m long. How long is the other side?

6 cm

**14**

- a** Find the perpendicular height of an equilateral triangle, side length 10 cm. Answer in exact surd form.

$$\sqrt{10^2 - 5^2} = \sqrt{75} \text{ cm}$$

- b** Find the perpendicular height of an equilateral triangle with side length  $x$  cm.

$$\sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \sqrt{x^2 - \frac{x^2}{4}} = \sqrt{\frac{3x^2}{4}}$$

- 15** You are constructing a triangle with side lengths 18 cm and 22 cm. What length do you need to make the third side so that the triangle is right-angled? Note: there are two possible answers. Round your answers to 2 decimal places.

Possibility 1: 18 and 22 are the shorter sides.

$$\text{Third side is } \sqrt{18^2 + 22^2} = 28.43 \text{ cm}$$

Possibility 2: 22 is the hypotenuse

$$\sqrt{22^2 - 18^2} = 12.65 \text{ cm}$$

- 16** The diagonal of a rectangle is 10 cm long. The rectangle is twice as long as it is wide. What are the dimensions of the rectangle? Round your answers to 2 decimal places

Let  $x$  be width.

$$x^2 + (2x)^2 = 10^2$$

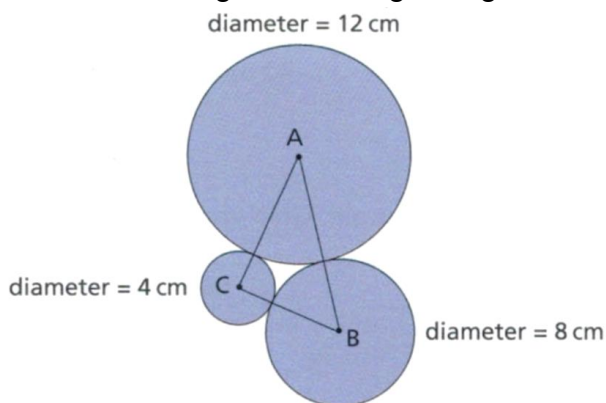
$$5x^2 = 100$$

$$x^2 = 20$$

$$x = \sqrt{20}$$

So, the rectangle is  $\sqrt{20}$  cm by  $2\sqrt{20}$  cm; 4.47 cm by 8.94 cm

- 17** The diagram shows three circles with centres A, B and C. The circles touch without overlapping. Show that triangle ABC is right-angled.



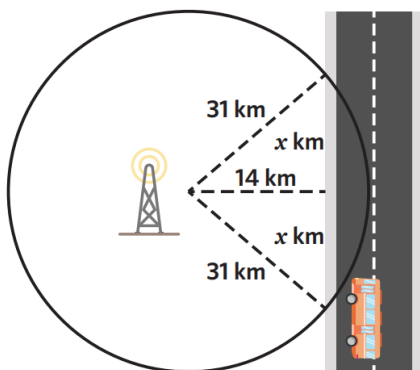
$$AB = 6 + 4 = 10, BC = 2 + 4 = 6, AC = 2 + 6 = 8$$

$$AC^2 + BC^2 = 100$$

$$AB^2 = 100$$

Since the sides satisfy  $a^2 + b^2 = c^2$ , the triangle is right angled.

- 18** A shuttle bus travels along an isolated road.  
The bus passes a cell tower 14 km away from a road.  
The cell tower provides signal in a 31 km radius.  
For how many kilometres will people on the bus have signal? Round your answer to 2 d.p.

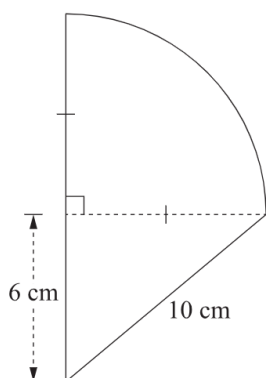


$$x = \sqrt{31^2 - 14^2} = \sqrt{765} \text{ m}$$

So, there will be signal for  $2\sqrt{765} \text{ m} = 55.32 \text{ m}$

### 19 2018 HSC Standard 2 Band 4

A shape consisting of a quadrant and a right-angled triangle is shown.  
What is the perimeter, in exact form?



$$\text{Radius of quadrant} = \sqrt{10^2 - 6^2} = 8 \text{ cm}$$

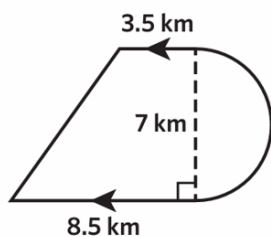
$$\text{Arc length} = \frac{3}{4} \times 2\pi(8) = 12\pi$$

$$\therefore P = 12\pi + 8 + 6 + 10$$

$$= 12\pi + 24 \text{ cm}$$

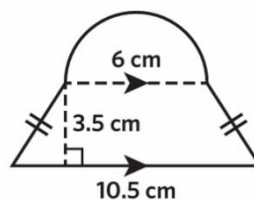
**20** Find the perimeter of these shapes. Answer to 1 d.p.

**a**



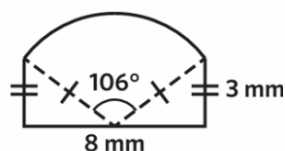
31.6 km

**b**



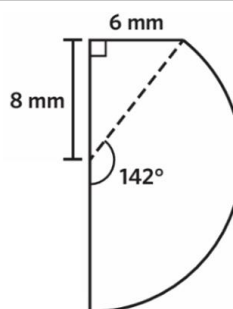
28.2 cm

**c**



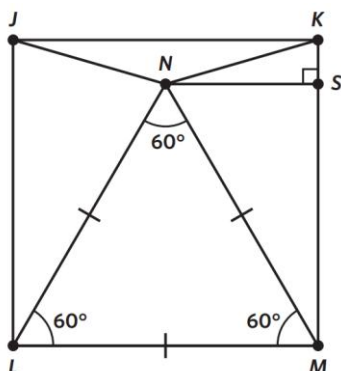
23.3 mm

**d**



48.8 mm

**21** JKML is a square. Determine the length of NK if the height of the triangle  $\triangle NML$  is 24 cm. Answer to two decimal places.



By splitting  $\triangle NML$  vertically down the middle, we create two right-angled triangles.

Let length of  $NM = x$

$$\left(\frac{x}{2}\right)^2 + 24^2 = x^2$$

$$\frac{x^2}{4} + 24^2 = x^2$$

$$24^2 = \frac{3x^2}{4}$$

$$x = \sqrt{768} \text{ cm}$$

$$KS = KM - SM$$

$$= \sqrt{768} - 24$$

$$NS = \frac{\sqrt{768}}{2}$$

$$\therefore NK = \sqrt{(\sqrt{768} - 24)^2 + \left(\frac{\sqrt{768}}{2}\right)^2} = 14.35 \text{ cm}$$

**22** A rectangle 5 m by 24 m is cut diagonally to make a right-angled triangle and a trapezium. The two shapes are then rearranged to form a large right-angled triangle. Determine the perimeter of the large right-angled triangle.

To make a cut that makes a right-angled triangle and trapezium you must start from a corner of the rectangle and go to an opposite side.

The diagonal of the trapezium and the diagonal of the small right-angled triangle must form the diagonal of the large right-angled triangle.

Since the diagonal of the trapezium and diagonal of the small right-angled triangle are the same, the only way for this to work is to make the cut connecting a vertex to a midpoint of the opposite side (two possibilities).

You can check your answer by seeing if the area of the new triangle matches the original rectangle.

60 cm or 101.26 cm

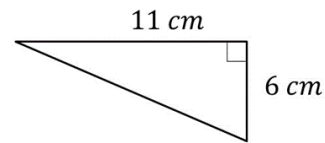
<https://www.desmos.com/geometry/oi2zmkiagn>

<https://www.desmos.com/geometry/dhogwgkq8g>

## CHECK YOUR UNDERSTANDING

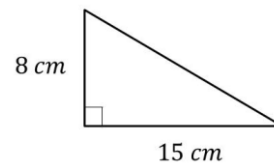
### Finding the hypotenuse

- 1 This is a right-angled triangle. Work out the length of the hypotenuse.



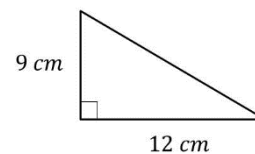
- a  $\sqrt{17}$  cm      b  $\sqrt{157}$  cm      c  $\sqrt{34}$  cm      d  $\sqrt{85}$  cm

- 2 This is a right-angled triangle. Work out the length of the hypotenuse.



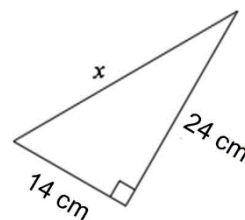
- a 17 cm      b 289 cm      c  $\sqrt{46}$  cm      d 23 cm

- 3 This is a right-angled triangle. Work out the missing length.



- a 21 cm      b 225 cm      c  $\sqrt{42}$  cm      d 15 cm

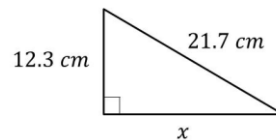
- 4 This is a right-angled triangle. Work out the missing length, correct to 1 d.p.



- a 27.8 cm      b 38 cm      c 19.5 cm      d 772 cm

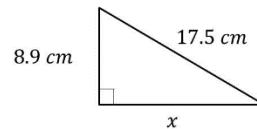
## Finding a shorter side

- 1** Work out the length of side  $x$  in this triangle, to 1 d.p.



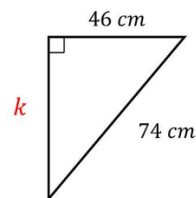
- a** 4.3 cm      **b** 24.9 cm      **c** 17.9 cm      **d** 319.6 cm

- 2** Work out the length of side  $x$  in this triangle, to 1 d.p.



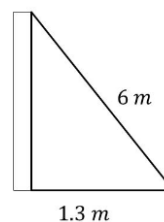
- a** 4.1 cm      **b** 15.1 cm      **c** 19.6 cm      **d** 227.0 cm

- 3** Work out the length of side  $k$  in this triangle, to 1 d.p.



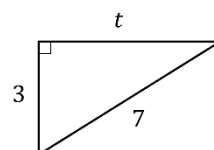
- a** 87.13 cm      **b** 3360 cm      **c** 57.97 cm      **d** 28 cm

- 4** A ladder 6 m long rests against a wall. The foot of the ladder is 1.3 m from the wall. How far up the wall is the ladder?



- a** 5.86 cm      **b** 6.14 cm      **c** 21.7 cm      **d** 34.31 cm

- 5** Which of these is the missing length?



- a**  $t = \sqrt{7^2 - 3^2}$       **b**  $t = \sqrt{7^2 + 3^2}$       **c**  $t = 7^2 - 3^2$       **d**  $t = \sqrt{3^2 - 7^2}$