Student Name

Mathematics Stage 5 Paths

LOGARITHMS

Book 1 Examine logarithms both numerically and graphically

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-LOG-P-01 establishes and applies the laws of logarithms to solve problems

Examine logarithms both numerically and graphically

- Define the term *logarithm*: the logarithm of a number to any positive base a is the index to which a is raised to give this number
- Recognise equivalence where $y = a^x$ is equivalent to $x = \log_a y$ where $a > 0$ and $a \neq 1$
- Translate statements expressing a number in index form into equivalent statements expressing the logarithm of the number
- Use graphing applications to compare and contrast graphs for the functions $y = a^x$ and $y = \log_a x$
- Generalise that $y = \log_a x$ is an increasing function when $a > 1$ and decreasing when $0 < a < 1$

REVIEW OF PRIOR KNOWLEDGE

INTRODUCTION TO LOGARITHMS

- **Roots** find the base when you know the power.

$$x^3 = 8$$
$$\sqrt[3]{} \qquad \sqrt[3]{}$$
$$x = \sqrt[3]{8}$$

- **Logarithms** find the power when you know the base.

$$2^x = 8$$
$$\log_2 \qquad \log_2$$
$$x = \log_2 8$$

Investigation Logarithms.

Solve these equations using your knowledge of exponentiation and write the logarithm.

Exponential	Logarithm
$3^x = 9$	$x = \log_3 9 =$
$3^x = 27$	$x = \log_3 27 =$
$3^x = 81$	$x = \log_3 81 =$
$2^x = 8$	$x = \log_2 8 =$
$2^x = 16$	$x = \log_2 16 =$
$2^x = 64$	
$2^x = 256$	
$4^x = 256$	
$5^x = 25$	
$5^x = \frac{1}{25}$	
$6^x = \frac{1}{36}$	

Logarithms are the inverse of exponentiation, they tell you the ………………, given a base number.

The log form of $8 = 2^3$ is ………………

The log form of $y = a^x$ is ………………

$\log_3 27$ is read as "log base three twenty-seven." It means "the power of ……… that gives 27"

Key ideas

1. For an expression $y = a^x$, we can write this as $x =$ …………………,

   where $a > 0$ and $y > 0$

Inverse Relationship between Indices and Logs

For $a > 0, y > 0$:

$$a^x = y$$

$$x = \log_a y$$

Example Convert from index form to logarithm form.

Write an equivalent statement for the following.

$6^2 = 36$ $2 = \log_6 36$	$7^0 = 1$ $0 = \log_7 1$	$10^{-2} = \frac{1}{100}$ $-2 = \log_{10} \frac{1}{100}$	$9^{\frac{1}{2}} = 3$ $\frac{1}{2} = \log_9 3$

Write an equivalent statement for the following.

a $7^2 = 49$

$\log_7 49 = 2$

b $5^0 = 1$

$\log_5 1 = 0$

c $10^{-2} = 0.01$

$\log_{10} 0.01 = -2$

d $16^{\frac{1}{2}} = 4$

$\log_{16} 4 = \frac{1}{2}$

e $10^4 = 10\,000$

$\log_{19} 10000 = 4$

f $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$\log_{\frac{3}{4}} \frac{9}{16} = 2$

Example Convert from logarithm form to index form.

Write an equivalent statement for the following.

$\log_3 9 = 2$ $9 = 3^2$	$\log_8 1 = 0$ $1 = 8^0$	$\log_8 2 = \frac{1}{3}$ $2 = 8^{\frac{1}{3}}$	$\log_6 \frac{1}{6} = -1$ $\frac{1}{6} = 6^{-1}$

Write an equivalent statement for the following.

a $\log_{10} 100 = 2$

$10^2 = 100$

b $\log_{15} 1 = 0$

$15^0 = 1$

c $\log_{10} 0.0001 = -4$

$10^{-4} = 0.0001$

d $\log_2 16 = 4$

$2^4 = 16$

e $\log_{25} 5 = \frac{1}{2}$

$25^{\frac{1}{2}} = 5$

f $\log_3 \frac{1}{9} = -2$

$3^{-2} = \frac{1}{9}$

1 Complete the table of powers.

x	0	1	2	3	4	5
2^x						
3^x						243
4^x					256	
5^x		5				
10^x			100			

2 State the value of the unknown number:

a 2 to the power of what number gives 16? ....

b 3 to the power of what number gives 81? ....

c 7 to the power of what number gives 343? ....

d 10 to the power of what number gives 10000? ....

3 Write the following using index form.

a $\log_2 16 = 4$

b $\log_{10} 100 = 2$

c $\log_3 27 = 3$

d $\log_2 \frac{1}{4} = -2$

e $\log_{10} 0.1 = -2$

f $\log_3 \frac{1}{9} = -2$

4 Write the following using logarithmic form.

a $2^3 = 8$

b $3^4 = 81$

c $2^5 = 32$

d $4^2 = 16$

e $10^{-1} = \frac{1}{10}$

f $5^{-3} = \frac{1}{125}$

5 Write an equivalent statement.

a $9^2 = 81$	**b** $2^3 = 8$	**c** $8^0 = 1$
d $3^{-2} = \frac{1}{9}$	**e** $10^5 = 100\ 000$	**f** $8^{\frac{1}{3}} = 2$
g $\log_6 36 = 2$	**h** $\log_6 216 = 3$	**i** $\log_{22} 1 = 0$
j $1 = 217^0$	**k** $125 = 5^3$	**l** $5^{\frac{1}{2}} = \sqrt{5}$
m $7^{\frac{1}{3}} = \sqrt[3]{7}$	**n** $\log_{64} 4 = \frac{1}{3}$	**o** $64^{-\frac{1}{3}} = \frac{1}{4}$

SOLVING SIMPLE EQUATIONS USING LOGARITHMS

Example Solve simple logarithmic equations.

Solve these equations.

$\log_2 x = 6$	$\log_4 x = -2$	$\log_x 144 = 2$
$x = 2^6$	$x = 4^{-2}$	$x^2 = 144$
$x = 64$	$x = \frac{1}{16}$	$x = 12$

In the third example, we do not consider -12 to be an answer because the definition of logarithms only allows for positive bases.

Solve these logarithmic equations.

a $\log_8 x = 2$

64

b $\log_4 x = 3$

64

c $\log_{13} x = -1$

$\frac{1}{13}$

d $\log_5 x = -3$

$\frac{1}{125}$

e $\log_x 49 = 2$

7

f $\log_x 11 = 1$

11

Example Evaluate logarithms.

Evaluate these logarithms

$x = \log_2 8$	$x = \log_{10} 1\,000\,000$
Let $x = \log_2 8$	Let $x = \log_{10} 1\,000\,000$
$2^x = 8$	$10^x = 1\,000\,000$
$2^x = 2^3$	$10^x = 10^6$
$\therefore x = 3$	$\therefore x = 6$

Evaluate these logarithms.

a $\log_3 9$

2

b $\log_2 32$

5

c $\log_3 \frac{1}{3}$

-1

d $\log_{10} 0.001$

-3

e $\log_2 8$

3

f $\log_8 2$

$\frac{1}{3}$

1 Evaluate these logarithms by writing them in index form.

a $x = \log_2 16$ $2^x = 16$ $2^x = 2^4$ $x =$ ……………………	**b** $x = \log_2 4$ …………………………… …………………………… ……………………………	**c** $x = \log_2 64$ …………………………… …………………………… ……………………………
d $x = \log_3 3$	**e** $x = \log_4 16$	**f** $x = \log_5 125$
g $x = \log_{10} 1000$	**h** $x = \log_7 49$	**i** $x = \log_{11} 121$
j $x = \log_2 8$	**k** $x = \log_2 \frac{1}{8}$	**l** $x = \log_2 \frac{1}{2}$
m $x = \log_{10} \frac{1}{1000}$	**n** $x = \log_7 \frac{1}{49}$	**o** $x = \log_5 \frac{1}{625}$
p $x = \log_{10} \frac{1}{10}$	**q** $x = \log_{10} 0.1$	**r** $x = \log_{10} 0.001$
s $x = \log_2 0.5$	**t** $x = \log_2 0.125$	**u** $x = \log_5 0.04$

2 Find the value of x in these equations.

a $\log_9 x = 2$ $9^2 = x$ $\dots\dots\dots = x$	**b** $\log_5 x = 3$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$	**c** $\log_2 x = 5$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$
d $\log_{10} x = 3$	**e** $\log_7 x = 1$	**f** $\log_{11} x = 0$
g $\log_7 x = -1$	**h** $\log_{10} x = -2$	**i** $\log_{12} x = -2$
j $\log_7 x = -3$	**k** $\log_2 x = -5$	**l** $\log_3 x = -4$

3 Find the value of x in these equations.

a $\log_x 8 = 3$ $x^3 = 8$ $x = \sqrt[3]{8}$ $= \dots\dots\dots\dots\dots\dots$	**b** $\log_x 27 = 3$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$	**c** $\log_x 10000 = 4$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$ $\dots\dots\dots\dots\dots\dots\dots\dots$
d $\log_x 64 = 6$	**e** $\log_x 64 = 2$	**f** $\log_x 125 = 1$
g $\log_x \frac{1}{17} = -1$	**h** $\log_x \frac{1}{6} = -1$	**i** $\log_x \frac{1}{7} = -1$
j $\log_x \frac{1}{49} = -2$	**k** $\log_x \frac{1}{8} = -3$	**l** $\log_x \frac{1}{81} = -2$

4 Find the value of x in these equations.

a $\log_3 27 = x$	**b** $\log_2 x = 4$	**c** $\log_{10} 1000 = x$
d $\log_3 x = 4$	**e** $\log_{10} x = 3$	**f** $\log_3 x = -2$
g $\log_x 27 = 3$	**h** $\log_4 0.25 = x$	**i** $\log_x 81 = 4$
j $\log_2 64 = x$	**k** $\log_4 x = -1$	**l** $\log_x 0.5 = -1$

5 Using the fact that $\sqrt{2} = 2^{\frac{1}{2}}$, so $\log_2 \sqrt{2} = \frac{1}{2}$, evaluate these logarithms

a $\log_2 \sqrt[3]{2}$	**b** $\log_2 \sqrt[4]{2}$	**c** $\log_3 \sqrt{3}$
d $\log_3 \sqrt[3]{3}$	**e** $\log_7 \sqrt{7}$	**f** $\log_{10} \sqrt[3]{10}$
g $\log_{10} \sqrt[3]{100}$	**h** $\log_2 \sqrt[3]{16}$	**i** $\log_3 \sqrt[4]{9}$
j $\log_5 \sqrt[4]{25}$	**k** $\log_2 \sqrt[5]{64}$	**l** $\log_3 \sqrt[7]{81}$

6 Solve for x

a $x = \log_a a$	**b** $\log_a x = 1$	**c** $\log_x a = 1$	**d** $x = \log_a \frac{1}{a}$
e $\log_a x = -1$	**f** $\log_x \frac{1}{a} = -1$	**g** $x = \log_a 1$	**h** $\log_a x = 0$
i $\log_x 1 = 0$	**j** $x = \log_a a^3$	**k** $x = \log_a \frac{1}{a^2}$	**l** $\log_a \frac{1}{a^5} = x$
m $x = \log_a \sqrt{a}$	**n** $x = \log_a \frac{1}{\sqrt{a}}$	**o** $\frac{1}{x} = \log_a \frac{1}{\sqrt{a}}$	**p** $\frac{1}{x} = \log_a \sqrt{a}$

7 Solve for x.

a $x = \log_7 \sqrt{7}$	**b** $x = \log_{11} \sqrt{11}$	**c** $\log_9 x = \frac{1}{2}$	**d** $\log_{144} x = \frac{1}{2}$
e $\log_x 3 = \frac{1}{2}$	**f** $\log_x 13 = \frac{1}{2}$	**g** $x = \log_6 \sqrt[3]{6}$	**h** $x = \log_9 3$
i $\log_{64} x = \frac{1}{3}$	**j** $\log_{16} x = \frac{1}{4}$	**k** $\log_x 2 = \frac{1}{3}$	**l** $\log_x 2 = \frac{1}{6}$
m $x = \log_8 2$	**n** $x = \log_{125} 5$	**o** $\log_7 x = \frac{1}{2}$	**p** $\log_7 x = -\frac{1}{2}$

LOGARITHMIC CURVES

Example Graph logarithmic relationships using a table of values.

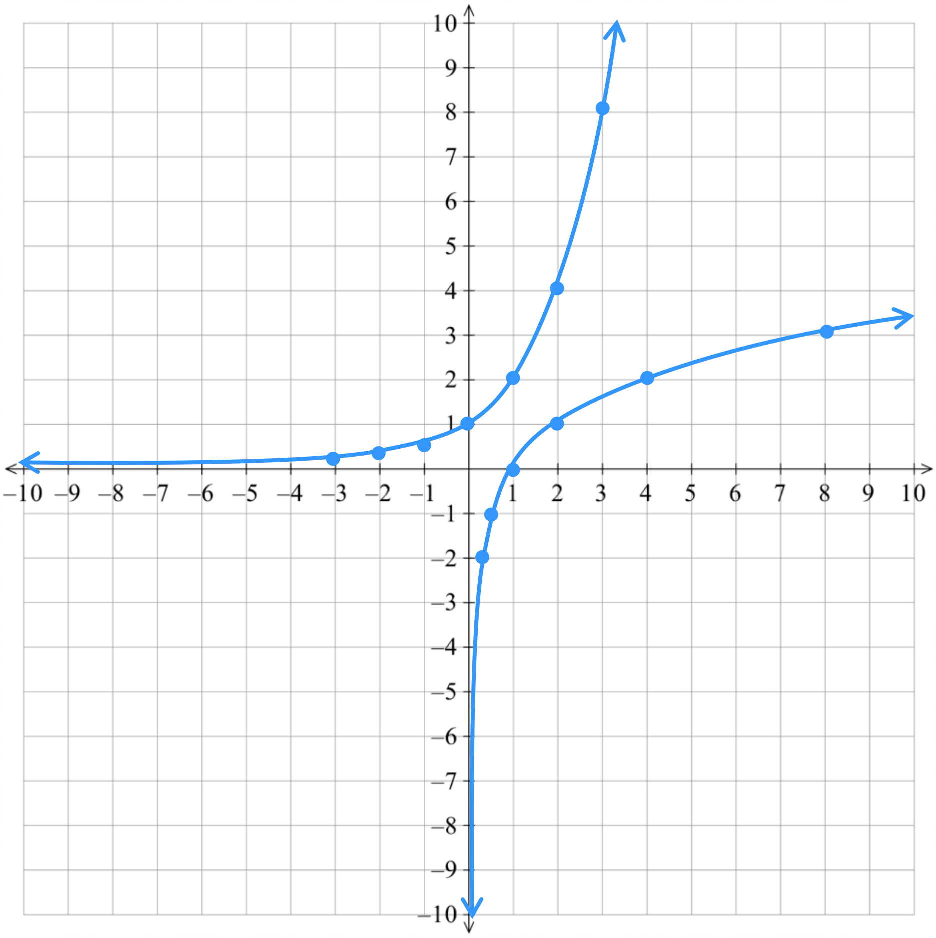
Draw the graphs of $y = 2^x$ and $y = \log_2 x$ on the same axis.

$y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$y = \log_2 x$

x	-1	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16
y	und	und	-2	-1	0	1	2	3	4



Why can't the inside of a logarithm be zero or negative?

The definition of a logarithm says that the inside of a logarithm must be ……………………

What do you notice about the graphs of $y = 2^x$ and $y = \log_2 x$?

They are …………………… of each other

Where are the asymptotes for $y = 2^x$ and $y = \log_2 x$? ………………………………………

Investigation Transformations of Logarithmic Graphs.

On Desmos https://www.desmos.com/calculator

a Sketch these graphs

$y = a^x$

$y = \log_a x$

Use the slider to change the value of a

Are the two graphs always reflections of each other?

……………………………………………………

b Sketch these graphs.

$y = \log_2 x$

$y = \log_3 x$

$y = \log_{1.1} x$

$y = \log_{0.9} x$

$y = \log_{\frac{1}{2}} x$

When $a > 1$, the graph always has a …………………… gradient.

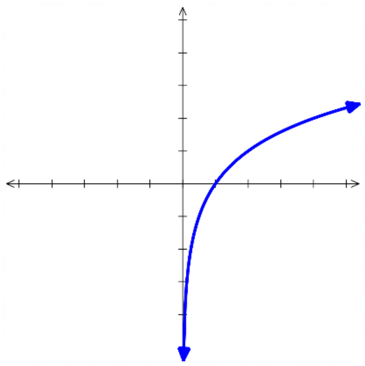
When $0 < a < 1$, the graph always has a …………………… gradient.

Log Curve Transformations

For a logarithmic curve $y = \boldsymbol{log_a(x)}$

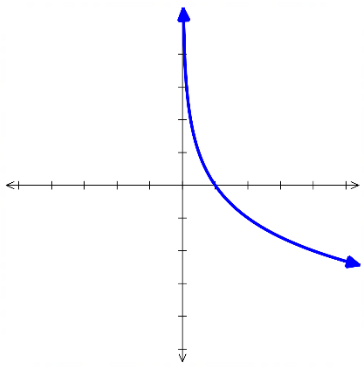
$a > 0$, the curve is increasing.

$y = \log_2 x$



$0 < a < 1$, the curve is decreasing

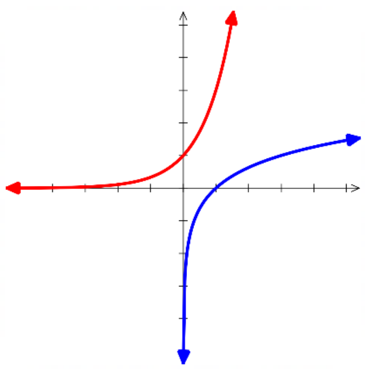
$y = \log_{0.5} x$



$y = \log_{a} x$ and $y = a^x$ are reflections of each other across the mirror line $y = x$

$y = \log_3 x$

$y = 3^x$



$y = \log_{\frac{1}{3}} x$

$y = \left(\frac{1}{3}\right)^x$